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HIGH-ORDER MOMENT-CLOSURE APPROXIMATIONS OF THE BOLTZMANN EQUATION

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The Boltzmann equation (BE) is the keystone of the kinetic theory of fluids, describing flow in the transitional molecular/continuum regime [1]. BE provides an evolution equation for the so-called one-particle marginal, viz., the probability density of particles in the position/velocity space. Accordingly, BE is high-dimensional (2D+1 for D spatial dimensions, where +1 accounts for time dependence). The BE has several fundamental structural properties, notably, certain invariance properties, symmetries and decay of an entropy functional (the celebrated H theorem). These structural properties underly the connection between BE and conventional continuum models, and it can be shown that (BE) encapsulates all conventional continuum models, such as the Navier-Stokes-Fourier system, as limit solutions [2]. Accordingly, (BE) inherently corresponds to a multiscale model [3].

The vast majority of numerical techniques for kinetic equations is based on stochastic particle methods. From a rigorous approximation perspective, such particle methods have several shortcomings. A relatively sparsely investigated approach, is provided by moment-closure systems [4-5]. Moment-closure approximation inherently exploit the structural properties of (BE) to arrive at efficient approximations. One of the fundamental properties of moment-closure systems, is that they yield a natural hierarchy of hyperbolic systems, which makes them ideally suited for (goal-)adaptive approximations.

In this presentation, I will present our recent progress in the development of high-order moment-closure methods for the Boltzmann equation.

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