Generation and Applications of Optical Frequency Combs

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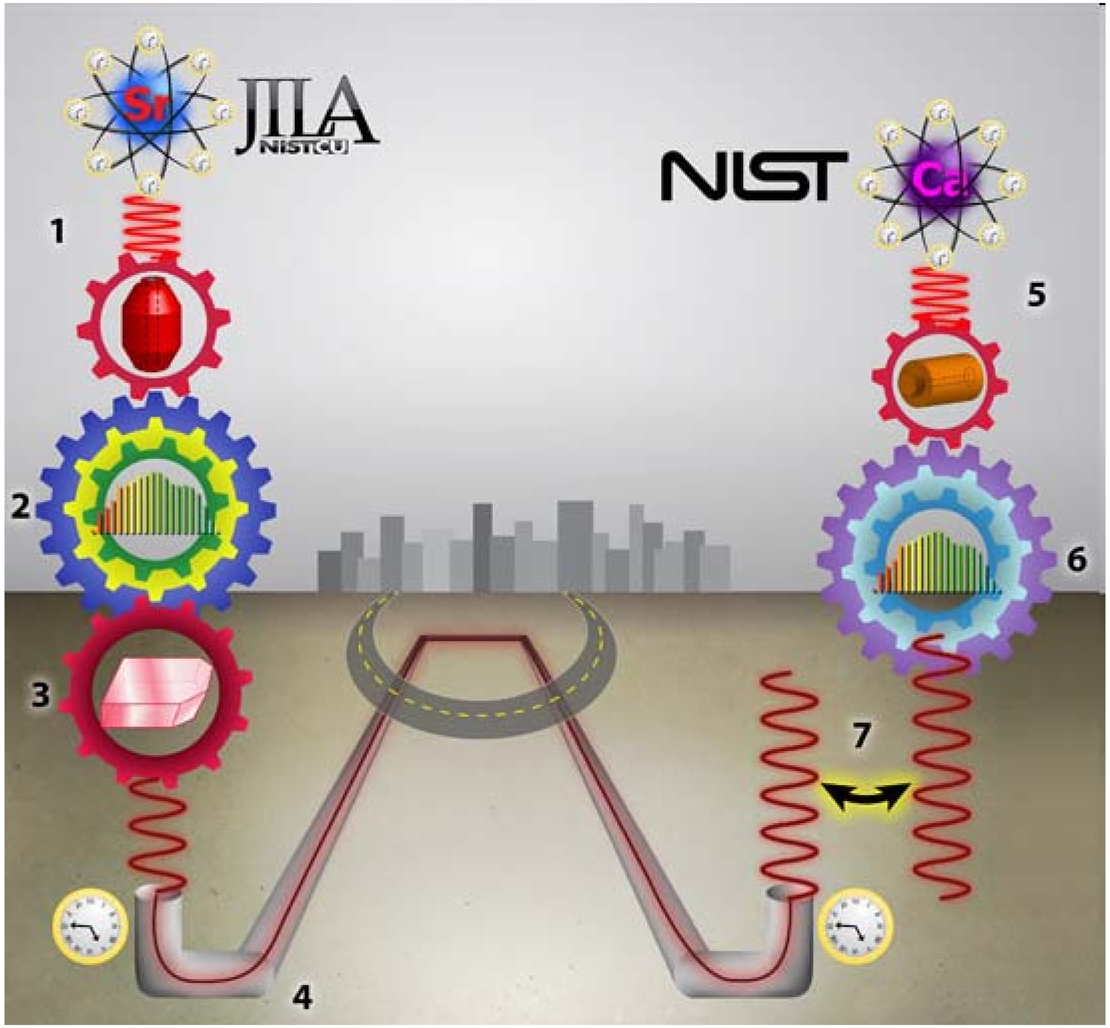
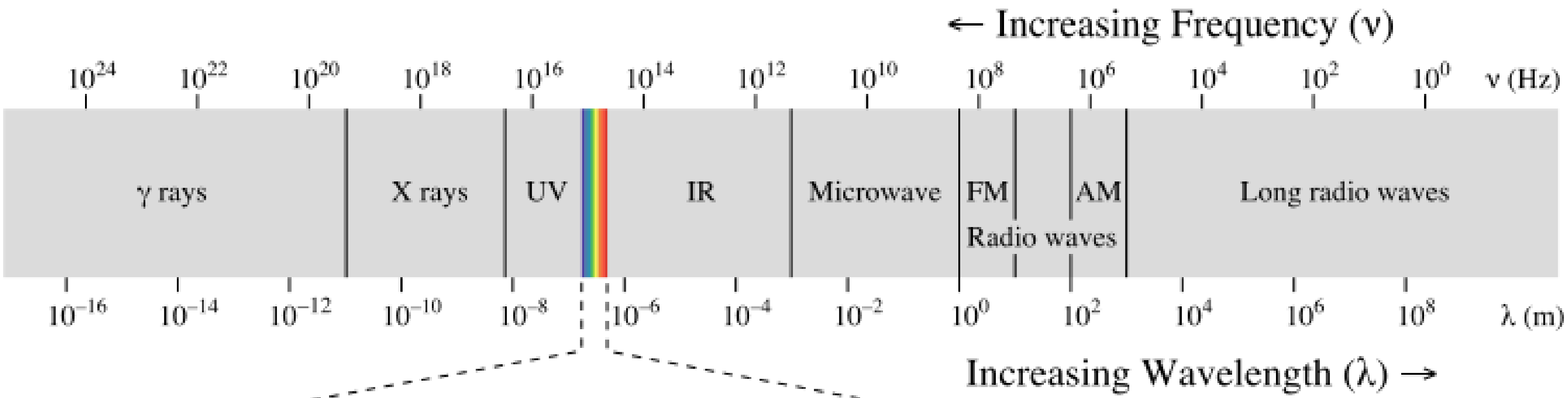
Beautiful picture ⇔ Bright future

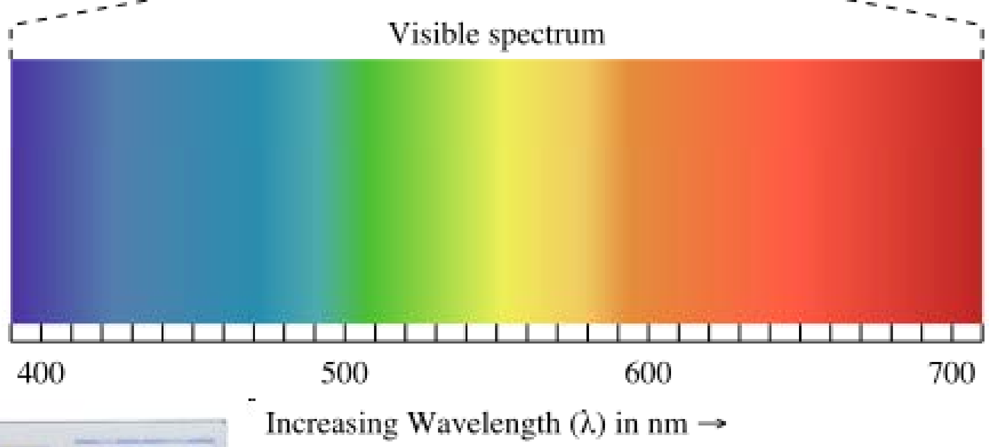


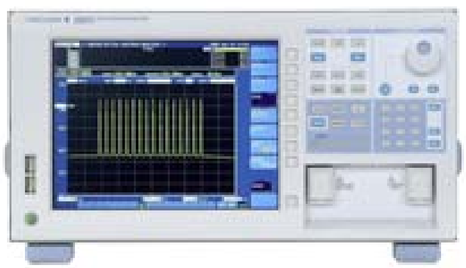
NIST

The spectrum

- RF linking optical and vise versa?







Wikepedia

Motivations

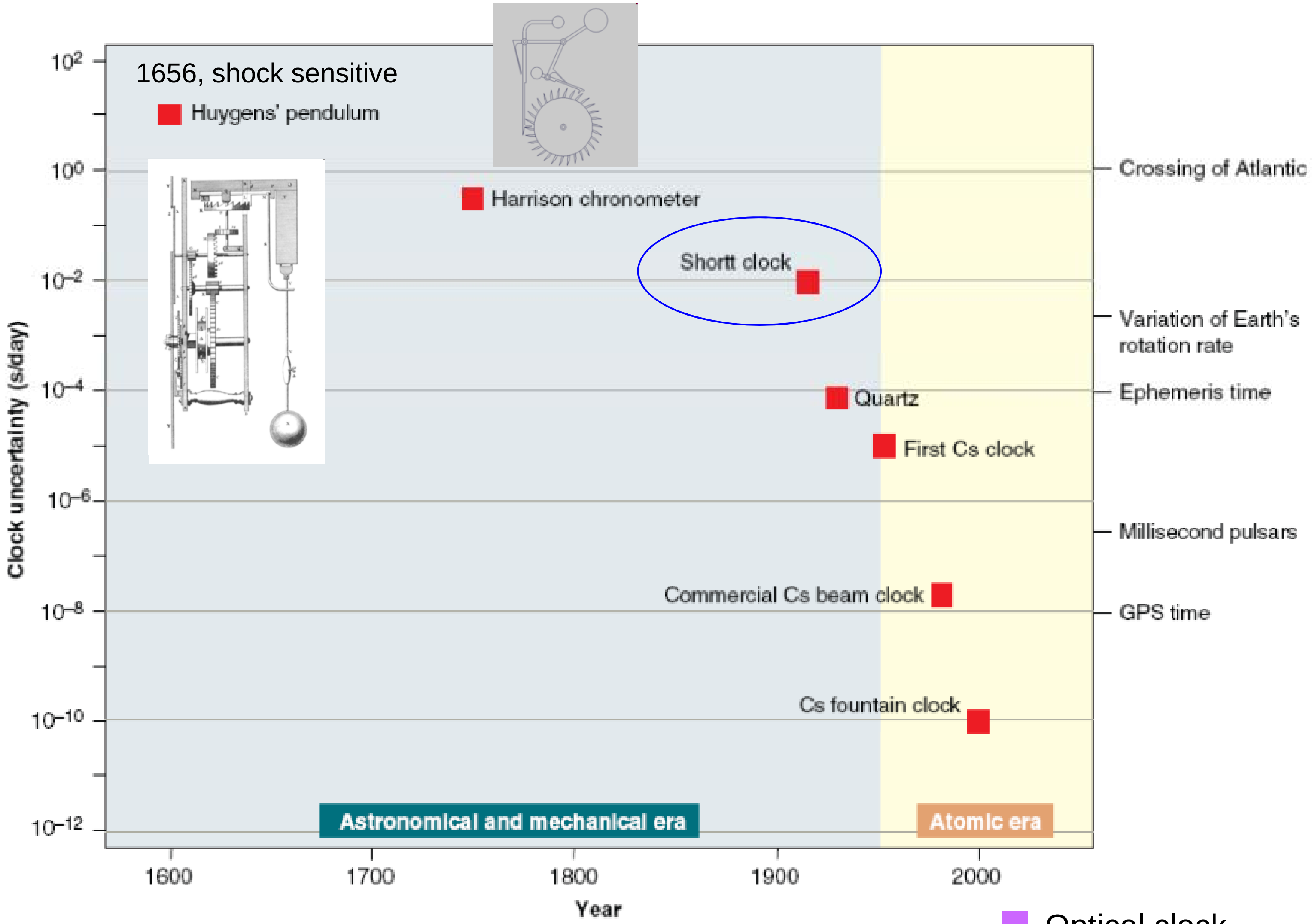
- The definition of one second affects our daily life
  - Internet, GPS, cell phones….
- Other units related to a second
  - SI base units: meter, ampere, candela
  - SI derived units: volt ($kg*m^2/A*s^3$), Newton,…..
- Physical constants
  - α decay rate, speed of light,…..
- High quality time and frequency standards an important task throughout the world

Definition of a second

- 2000 B.C., Egyptians: divided day and night into 12 hours
- 1000, Muslims: counting the moon, $(1/60)^2$ of an hour
- 1670, Huygens pendulum
- 1956, the Ephemeris second: 1/31,556,925.9747 of Earth's one rotation around the Sun (a year in 1900) by the 11^{th} General Conference on Weights and Measures
- 1967, the atomic time: 1/9,192,631,770 duration of ^{133}Cs ground-state transition by 13^{th} General Conference on Weights and Measures
- 1980's, laser cooling made possible improved frequency stability in Cs clocks (Cs fountains)

$$\sigma_y(\tau) \approx \frac{\Delta \upsilon}{\upsilon_0 SNR}$$

Clocks-Evolution



Optical clock
Enabled by
frequency comb

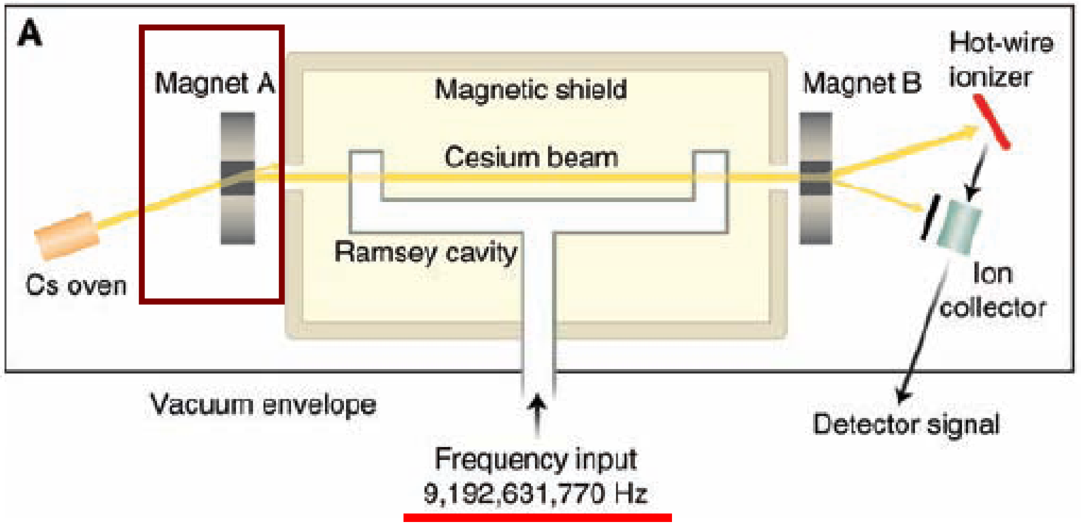
S.A. Diddams, et.al., Science **306**, 1318 (2004)

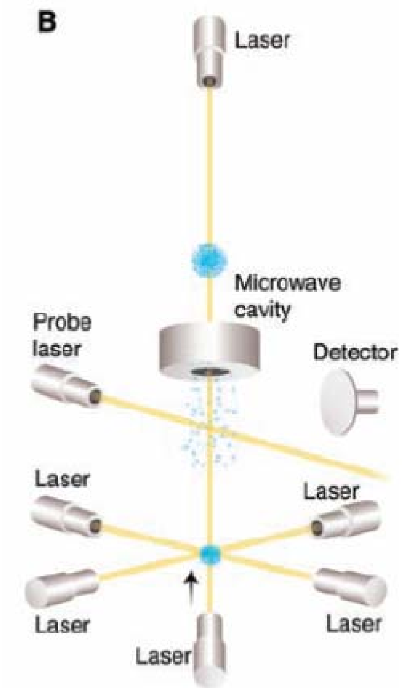
Atomic clock: ^{133}Cs

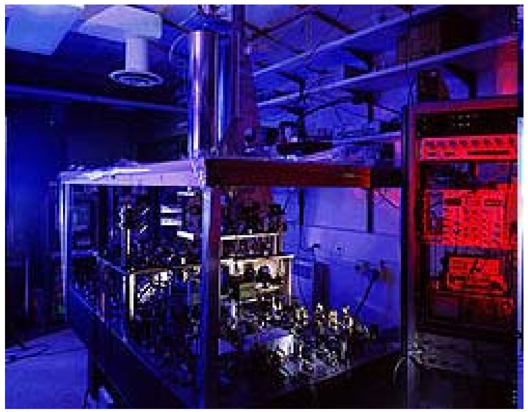
- Commercial: Stern-Gerlach magnets
- Cs fountain clock (envision by Zacharias in 1950s)
  - Laser cooling reduces Doppler linewidth

Doppler shifts, collisions

1997, Nobel Prize in Physics to Chu, Cohen-Tannoudji, Philips







Think of it as polarization control in photonics

S.A. Diddams, et.al., Science **306**, 1318 (2004)

Optical clock?

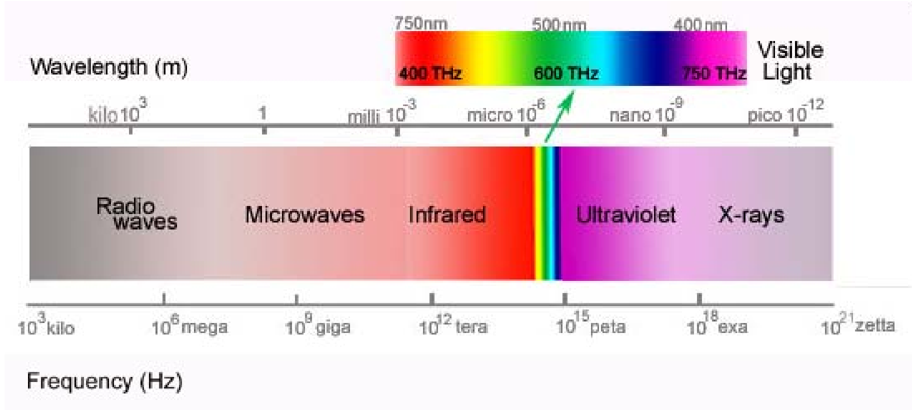
- What benefits do we gain?
- How to realize it?
- Problems?

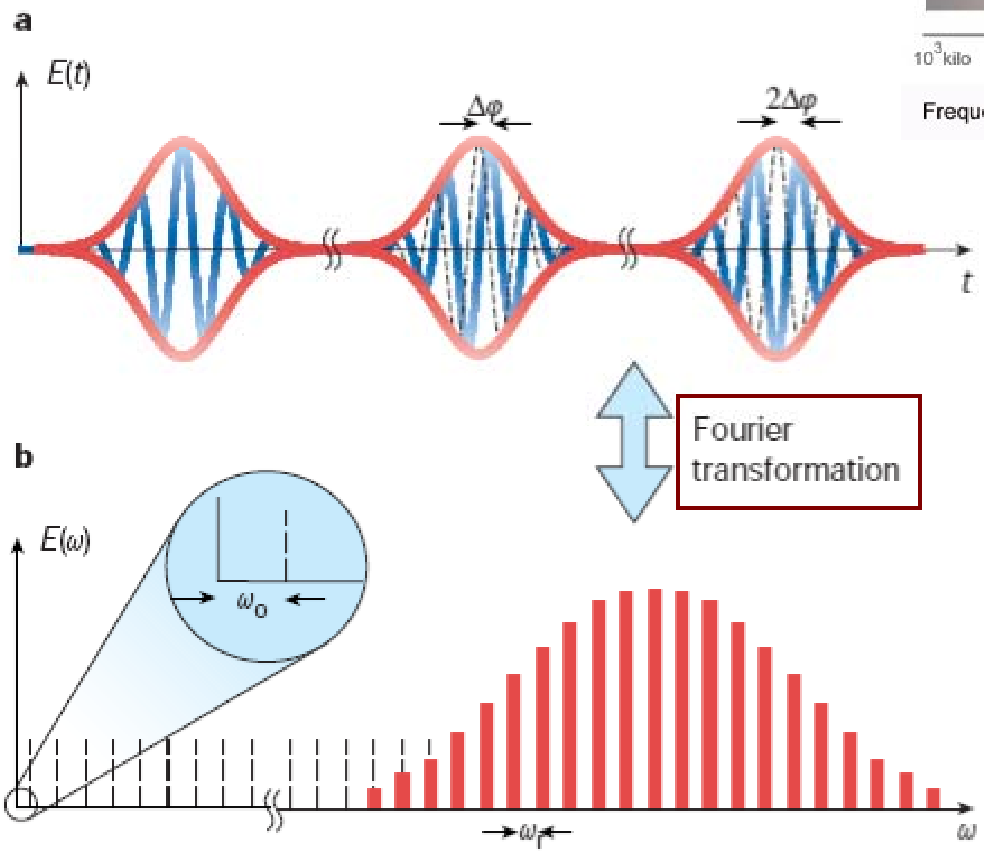
$$\sigma_y(\tau) \approx \frac{\Delta \upsilon}{\upsilon_0 SNR}$$

How to “read” optical frequencies accurately?

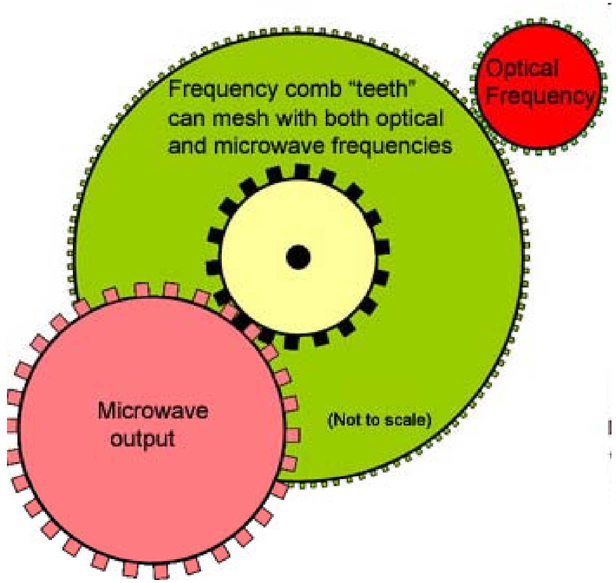
Optical frequency comb

- A ruler that connects
  - Frequency and time
  - Microwave and optical frequencies





T. Udem, Holzwarth, Hänsch, Nature **416**, 233 (2002)



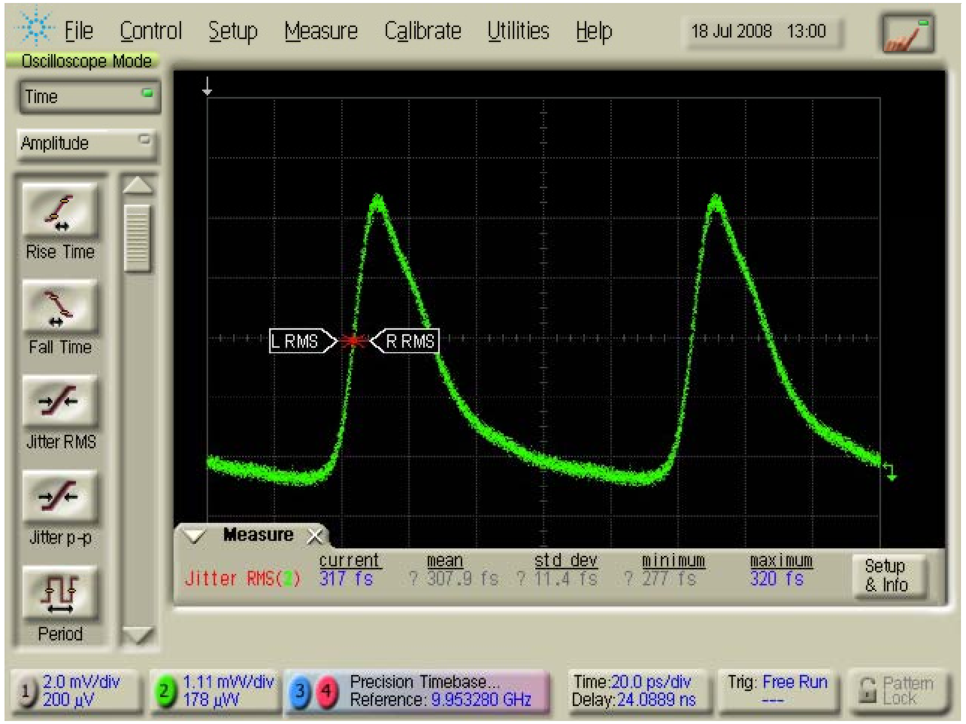
NIST

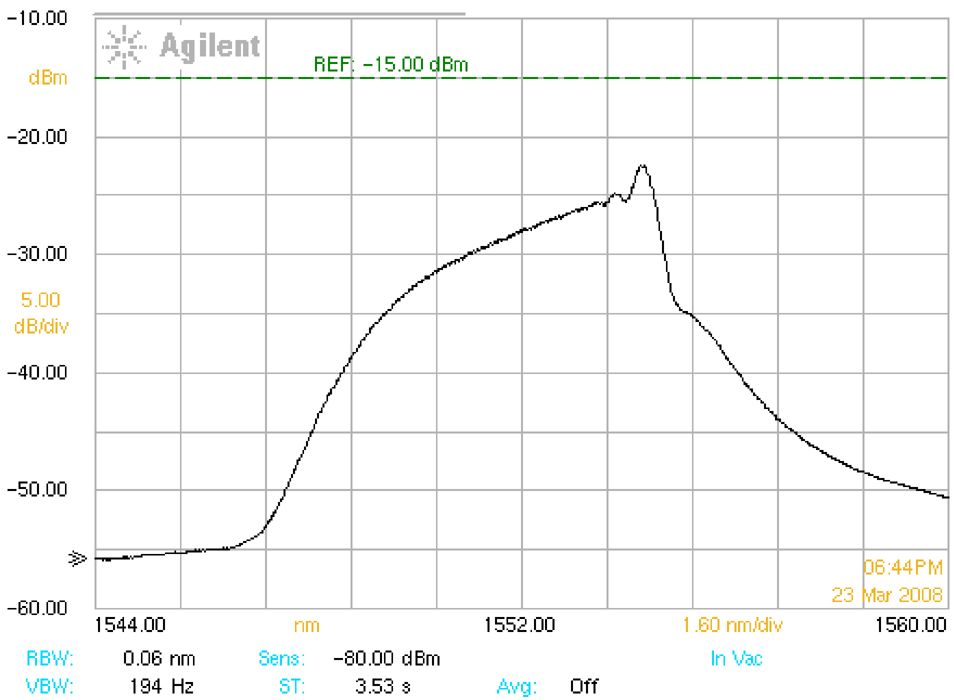
Bigger question

- So, if we have a pulse train, are we guaranteed a comb?
- Conversely, if we have a comb, are we guaranteed with a pulse train?

Mode-locking ⇔ frequency comb?

- Well, not quite





Realization of a "Comb"

- 1917, Einstein laid foundation for MASER and LASER
- 1953, Townes and students invented MASER
- 1957-60's, LASER theory developed by Townes and Schawlow
- 1960, first working laser (Ruby) by Maiman
- 1962, first semiconductor (GaAs) LD by R.N. Hall
- 1978, optical frequency comb envisioned
- 1998, frequency comb realized
- Explosion on this research field and it's applications

What took so long?

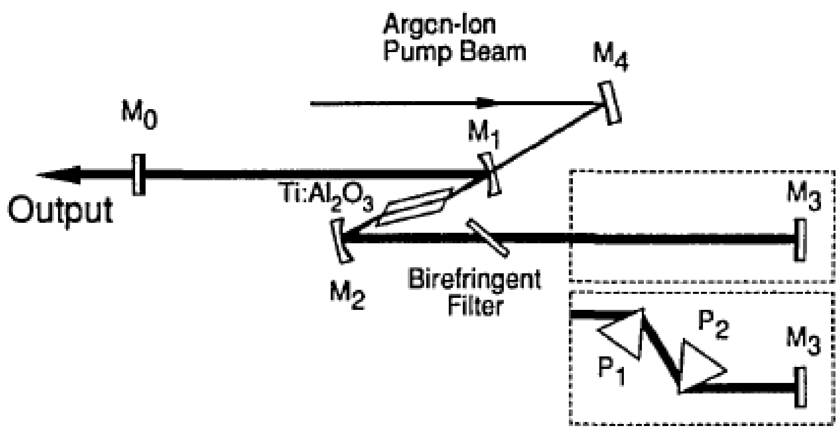
NIST National Institute of Standards and Technology

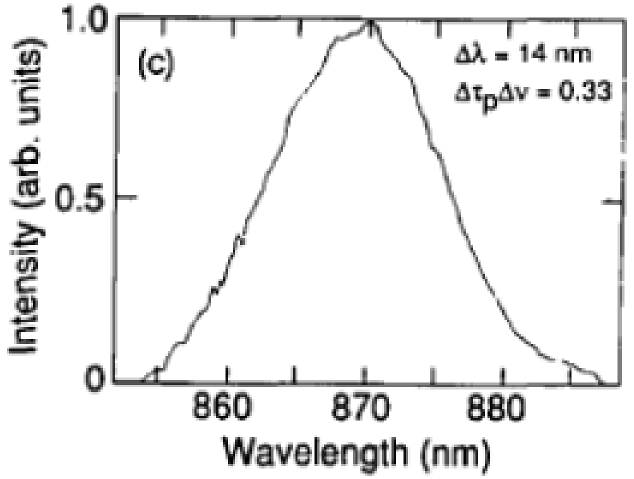
MAX-PLANCK-GESELLSCHAFT

John L. Hall of NIST/JILA at Boulder, Colorado and
Theodor W. Hänsch of Max-Plank Institute, Garching, Germany.
(www.nobel.org)

What took so long?

- Laser introduced
  - Spectroscopy: CW laser
  - Nonlinearity: ultrafast community
- Late 1970's: spectroscopy using sub-picosecond
  - ~800 GHz "comb" that suffered huge frequency shift
- Kerr-lens mode-locking introduced in 1991
  - 60 fs pulses directly from Ti:S. But no one looked in frequency domain
- 1997, white light coherence of supercontinuum observed by Hänsch
- 1999, octave spanning spectrum with photonic crystal fiber
- 2004, intrinsic octave-spanning Ti:S





D.E. Spence, et.al.,, Opt. Lett. **16**, 42 (1991)

The milestone note

(CONFIDENTIAL)

Proposal for a universal optical frequency comb synthesizer

T. W. Hänsch
Max-Planck-Institut für Quantenoptik

(March 30, 1997)

Abstract

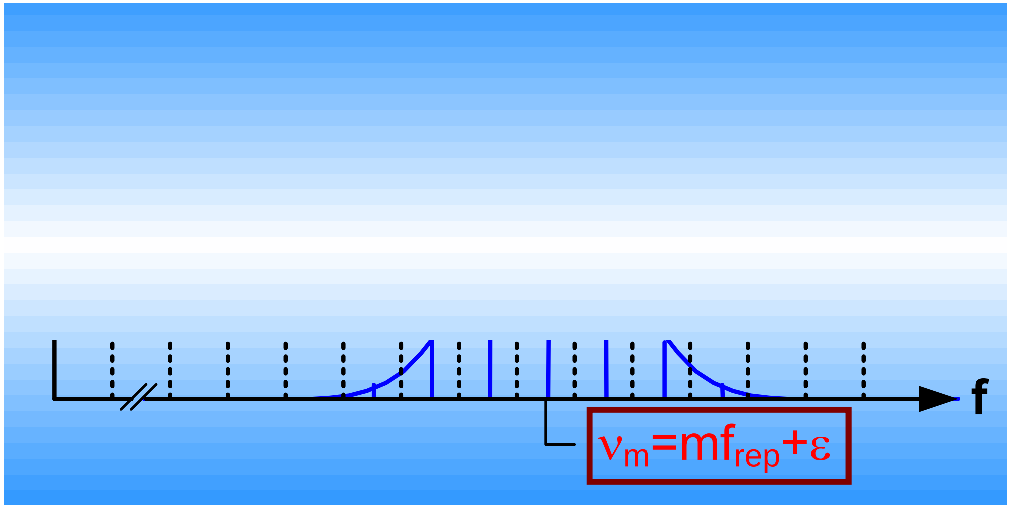
An optical frequency synthesizer is proposed which produces a wide comb of absolutely known equidistant marker frequencies throughout the infrared, visible, and ultraviolet spectral range. To this end, a white light continuum with pulse repetition rate f_p is produced by focusing the output of a mode-locked femtosecond laser into an optical fiber or bulk medium with a third order nonlinear susceptibility. The rate of phase slippage of the laser carrier relative to the pulse envelope f_s is monitored by observing a beat signal between the white light continuum and the second harmonic of the laser.

read and understood
April 4, 1997 Martin Weitz
April 4, 1997 T. Udem

http://nobelprize.org/nobel_prizes/physics/laureates/2005/hansch-lecture.html

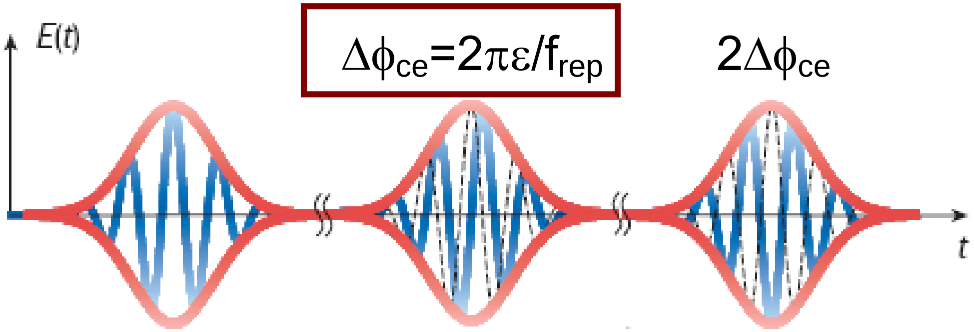
Comb vs. Pulses

- Frequency-domain vs. time-domain
- Comb frequency offset directly linked to carrier-envelop phase!



$$\upsilon_m = mf_{rep} + \varepsilon$$

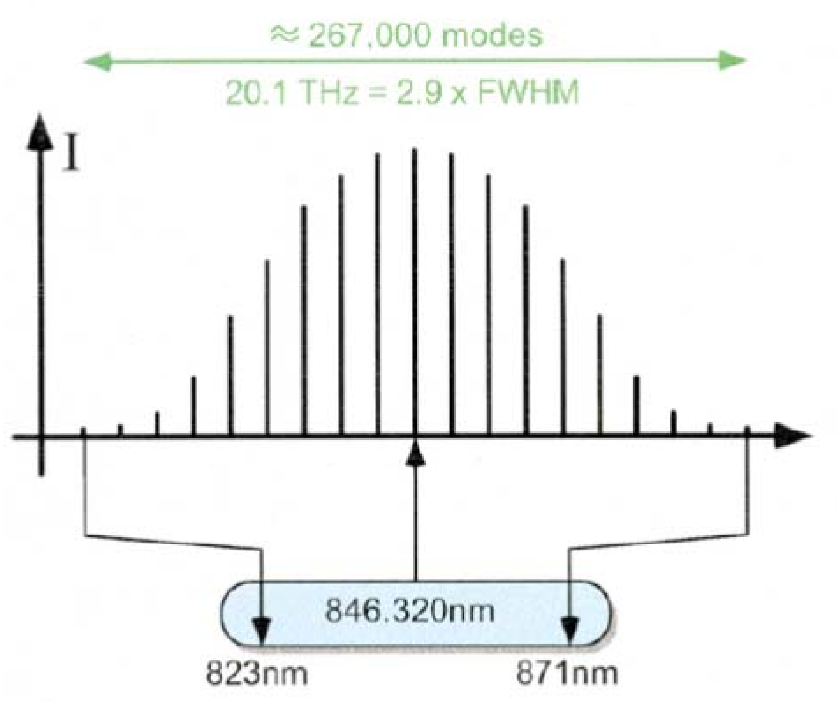
Let's derive the expression for carrier-envelop phase slip!



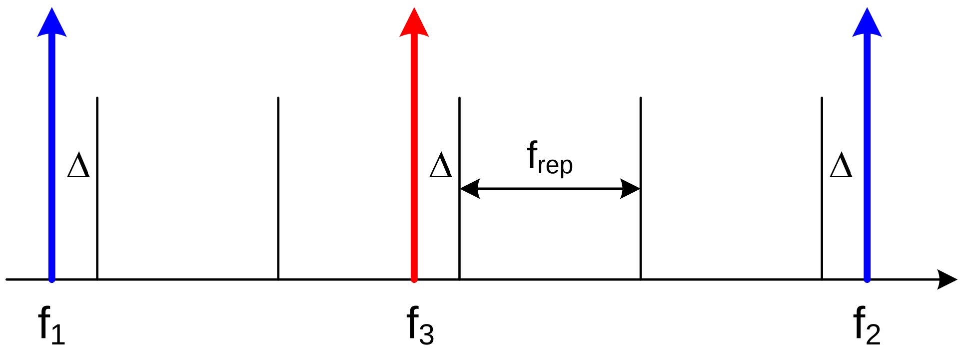
T. Udem, Holzwarth, Hänsch, Nature **416**, 233 (2002)

Testing of comb spacing uniformity

- Experimental uniformity: 3×10^{-17}
  - SFG of first and second diode lasers, SHG of third diode laser
  - Observing the beat of the intermediate line with third phase-locked laser



Interval divider



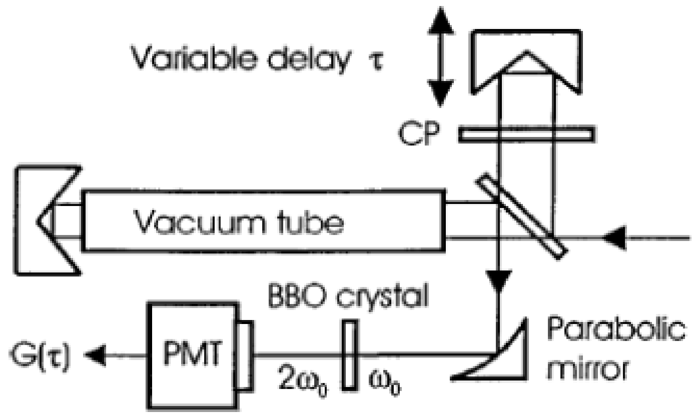
Can you think of other easy way?

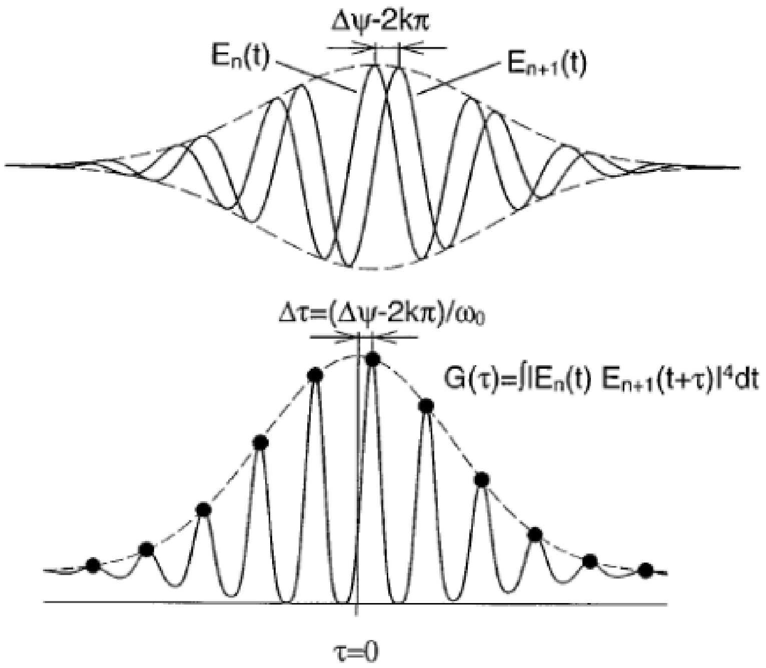
T. Udem, et.al, Opt. Lett. **24**, 881 (1999)

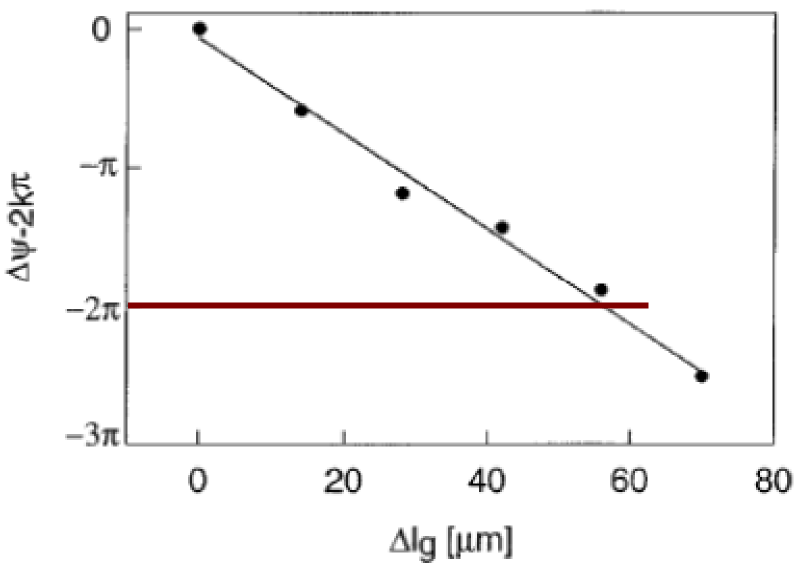
$\Delta\Phi_{CE}$ measurement: cavity length control

- Silica wedge in the laser
- Inteferometric intensity “cross”-correlation
  - Sub-10 fs Ti:S laser, f_{rep}=100 MHz
  - Correlator dispersion compensated
  - Laser not stabilized

Why vacuum tube?







Xu et.al, Opt. Lett. **21**, 2008 (1996).

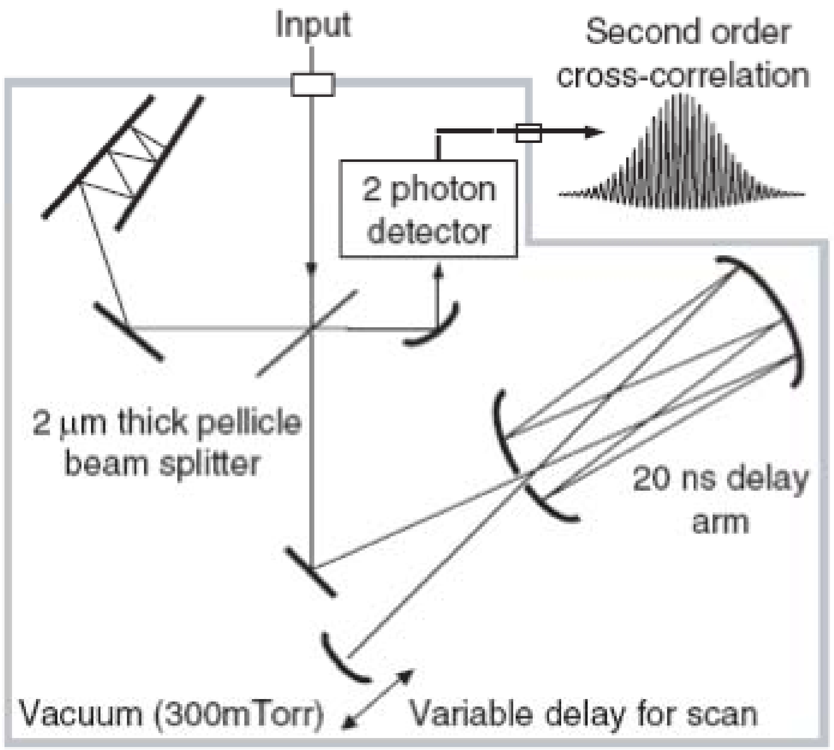
$\Delta\Phi_{CE}$ measurement: offset frequency control

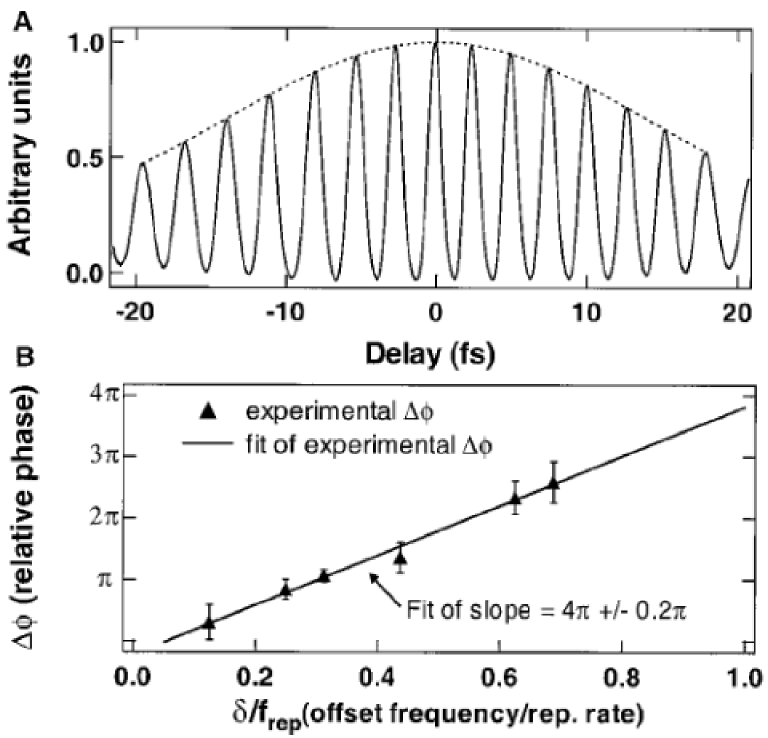
- Cross-correlation

Do you see problems?

i and i+2 pulse

$$\Delta\Phi_{CE} = 2 * 2\pi\varepsilon / f_{rep}$$





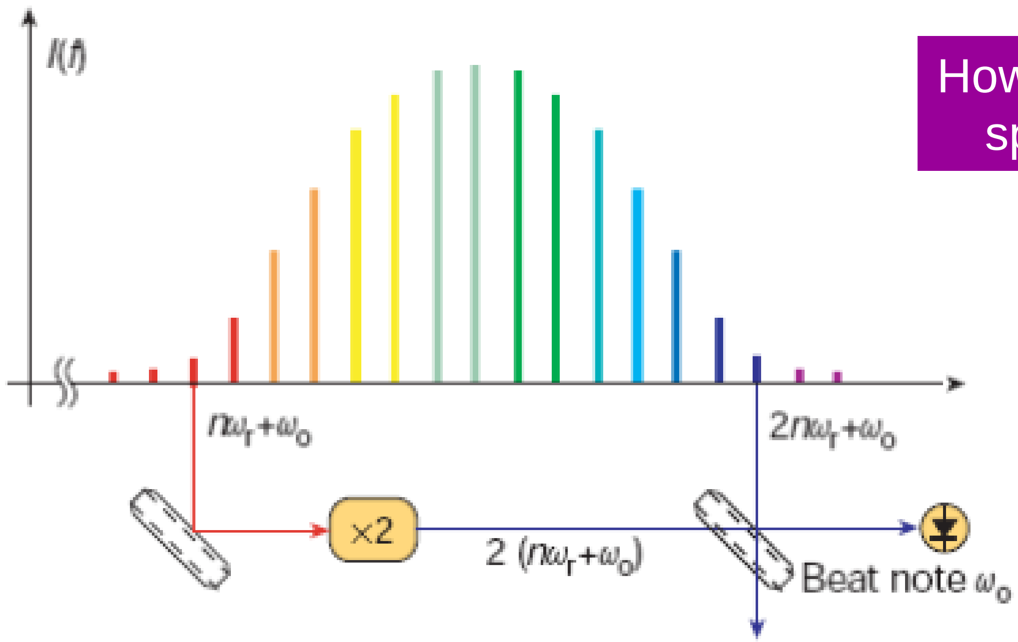
D. J. Jones, et.al., Science **288**, 635 (2000)

Comb stabilization: principle

- You need to measure something before you can stabilize it!

$$\upsilon_m = mf_{rep} + \varepsilon$$

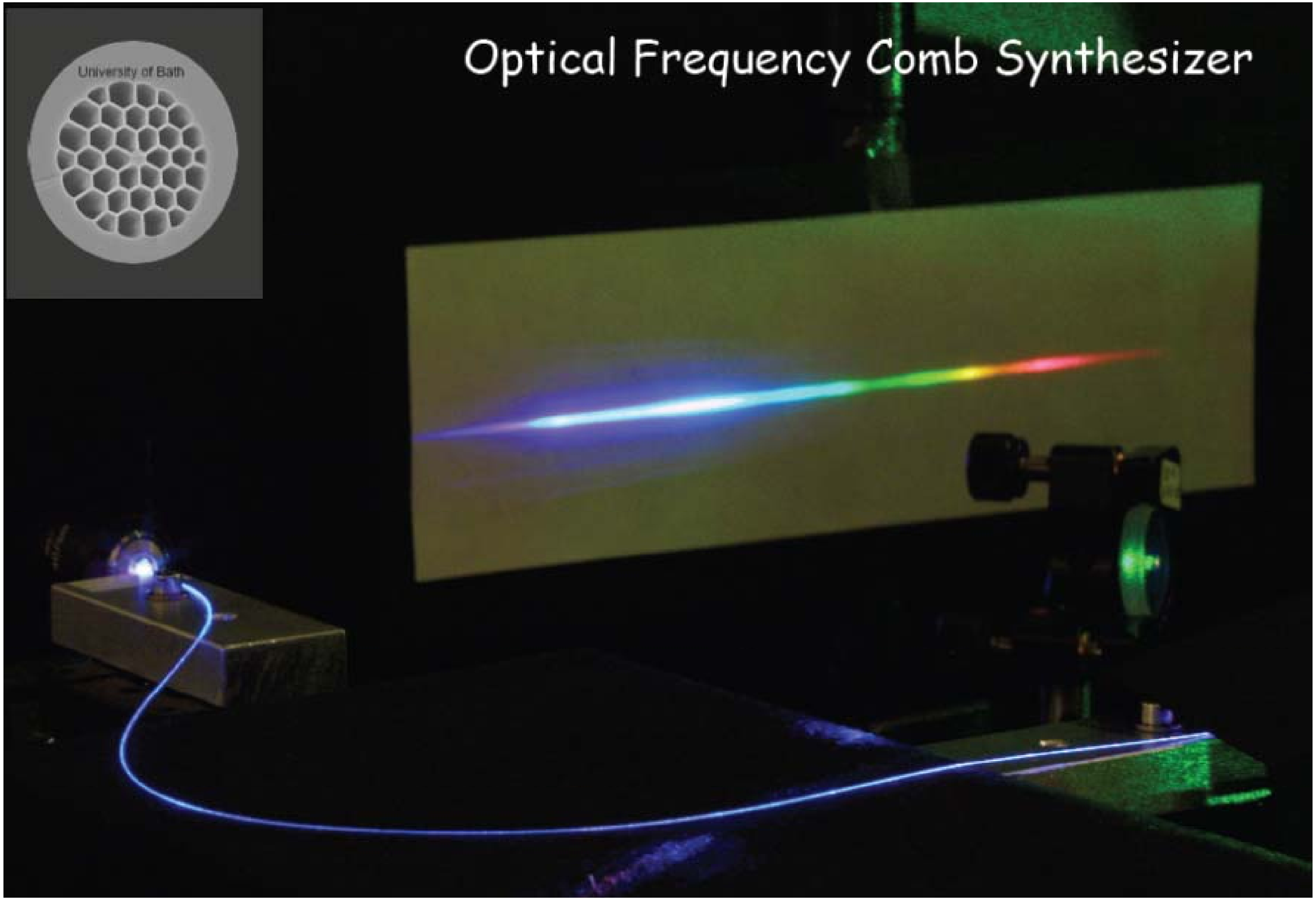
- Self-referencing (ν-to-2ν)



How to obtain octave spanning comb?

T. Udem, Holzwarth, Hänsch, Nature **416**, 233 (2002)

Supercontinuum generation

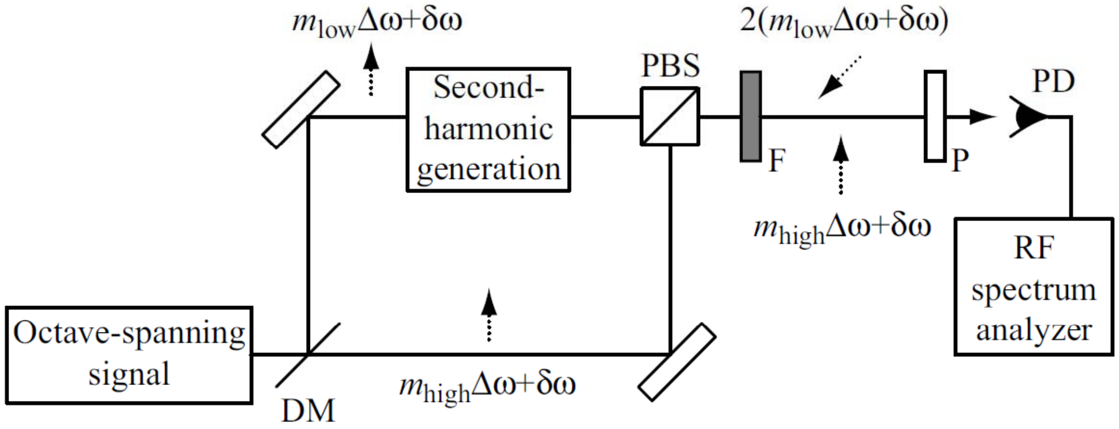


http://nobelprize.org/nobel_prizes/physics/laureates/2005/hansch-lecture.html

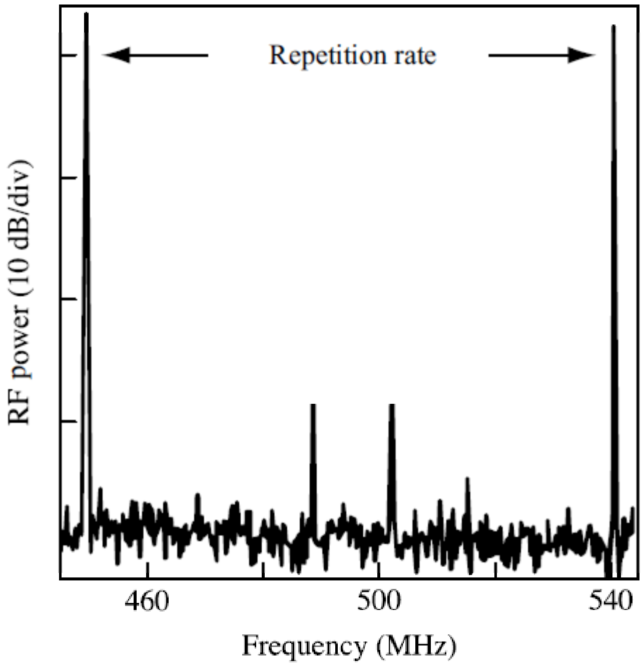
ε measurement: self-referencing

- f-2f signal
- Why four signals?

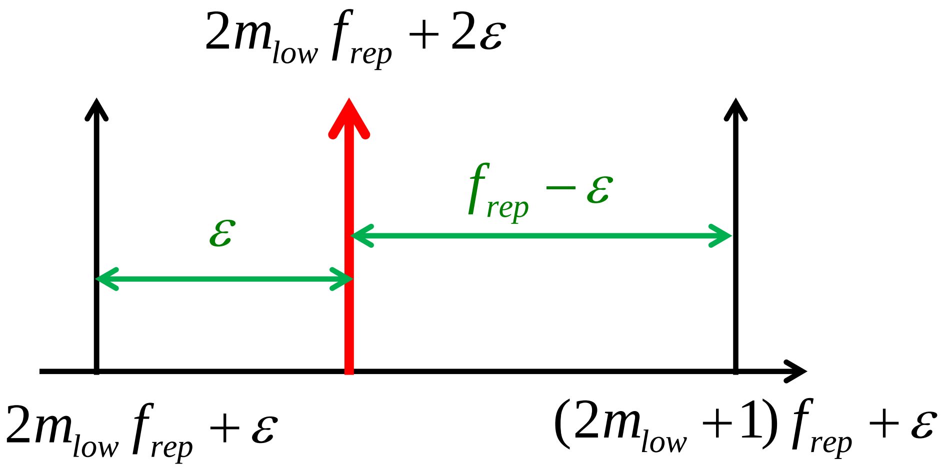
Think about how to determine the real ε?



A. M. Weiner, *Ultrafast Optics* (Wiley, 2009)



S.T. Cundiff, J. Phys. D **35**, R43 (2002).



Comb stabilization: self-referencing

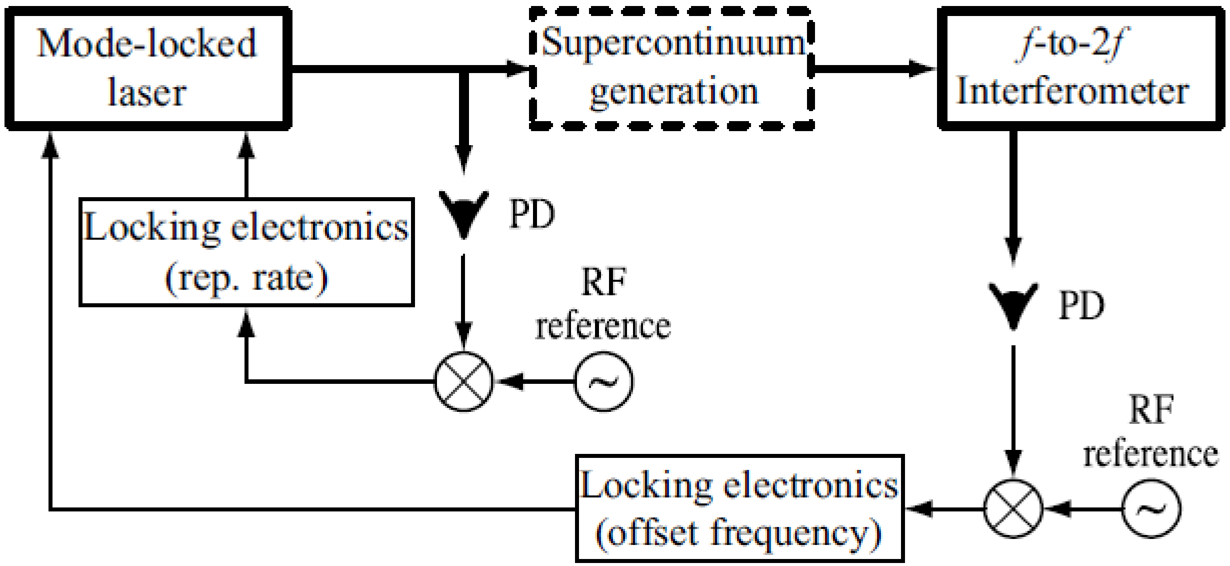
- Block diagram
- Ti:S with 2mm crystal
  - $\Delta\Phi_{CE}$~100π
  - ε~50f_{rep}

$$\upsilon_m(t) = mf_{rep}(t) + \varepsilon(t)$$

$$\upsilon_m(t) = mf_{rep} + \varepsilon(t)$$

$$\upsilon_m = mf_{rep} + \varepsilon$$

Can you think of other approaches?



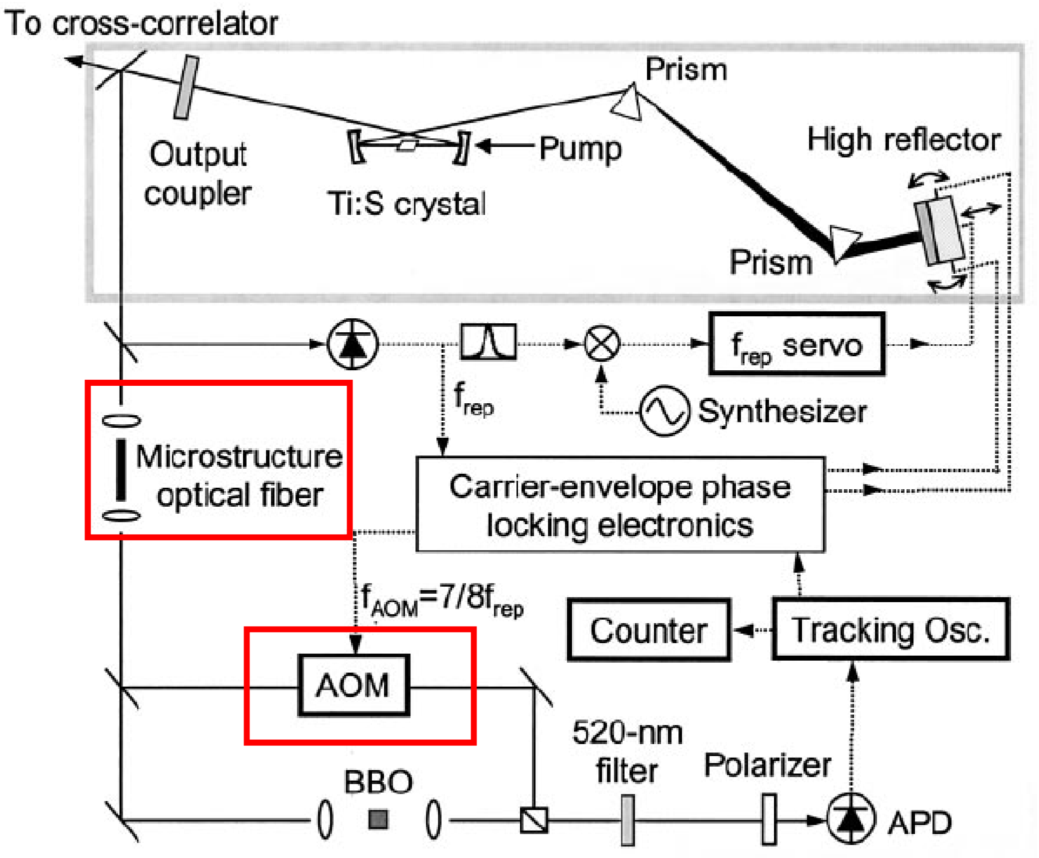
A. M. Weiner, *Ultrafast Optics* (Wiley, 2009)

Comb stabilization: controls

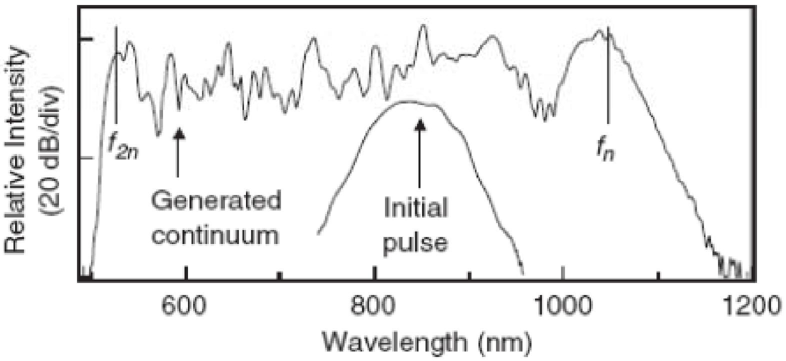
- Ti: Sapphire
  - End mirror tilt: ε, f_{rep}
  - End mirror translation: f_{rep}

Elastic tape model

J. Reichert, et.al., Opt. Comm. **172**, 59 (1999)



AO modulator: stabilize and step-wise control of CEP



10fs, ~70nm

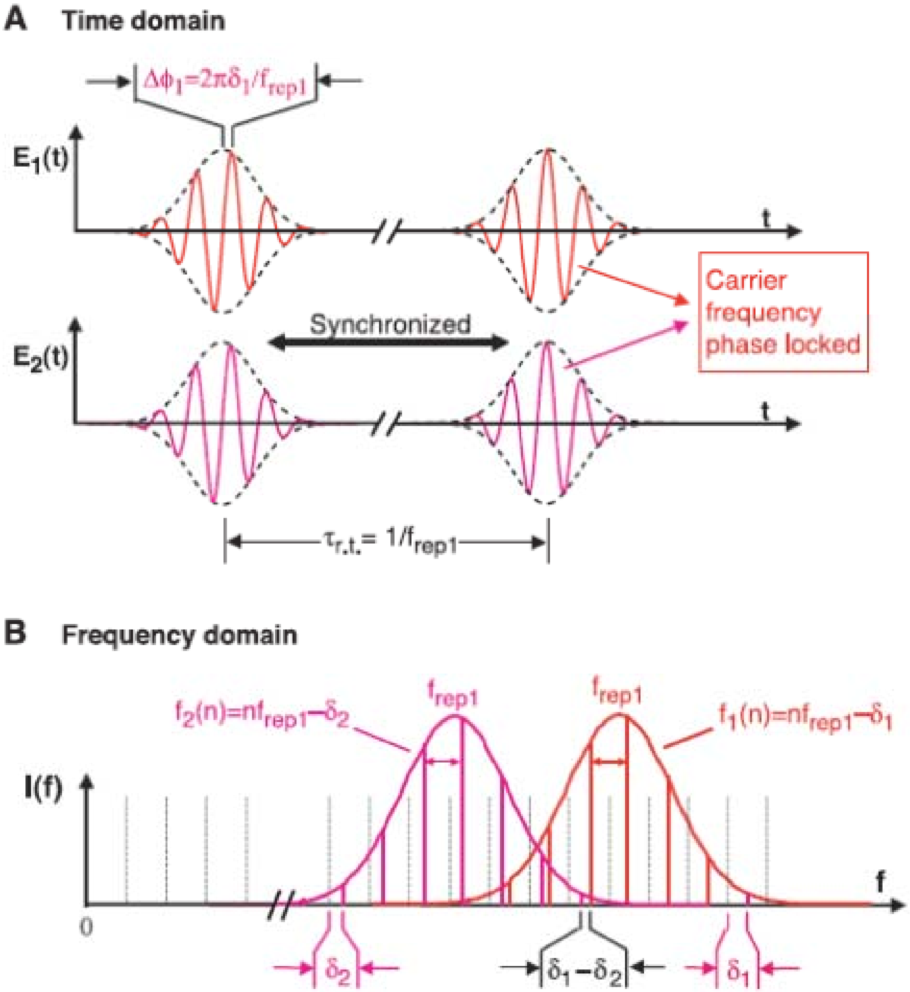
Do you see a problem?

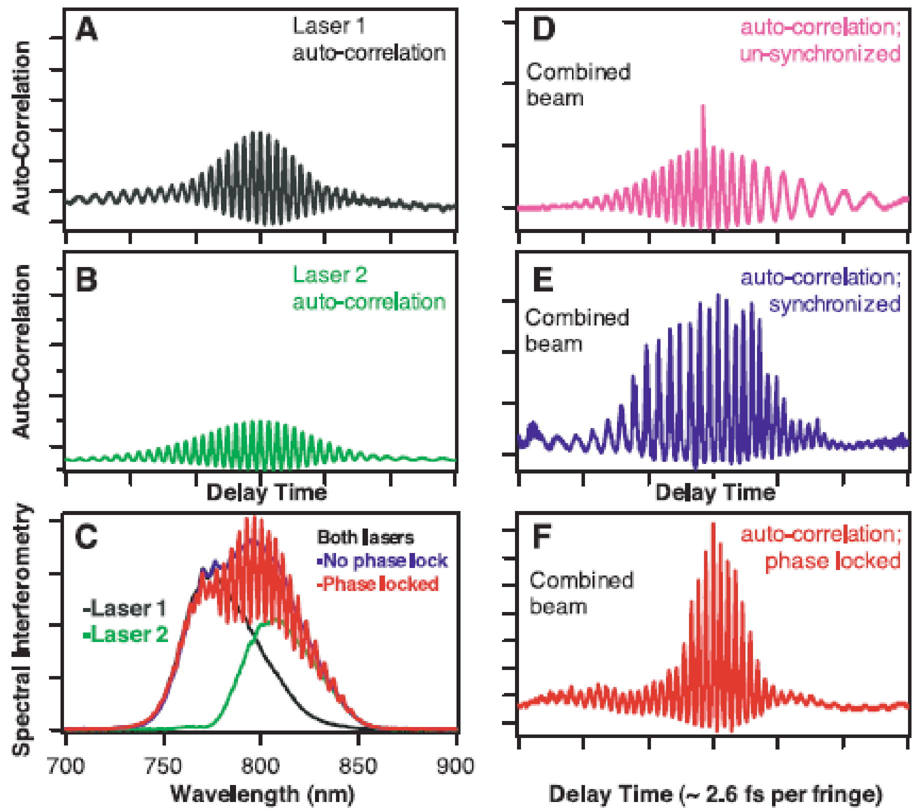
D.J. Jones, et.al., Science **288**, 635 (2000)

Synchronization between two combs

- Comb stitching

Check: A. Wirth, et.al, Science **334**, 195 (2011)



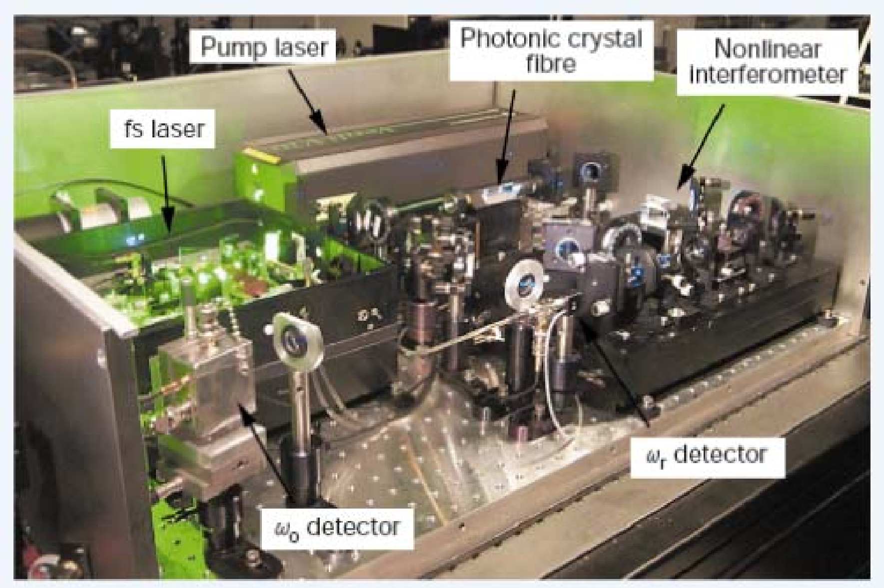


Shelton et.al,, Science **293**, 1286 (2001)

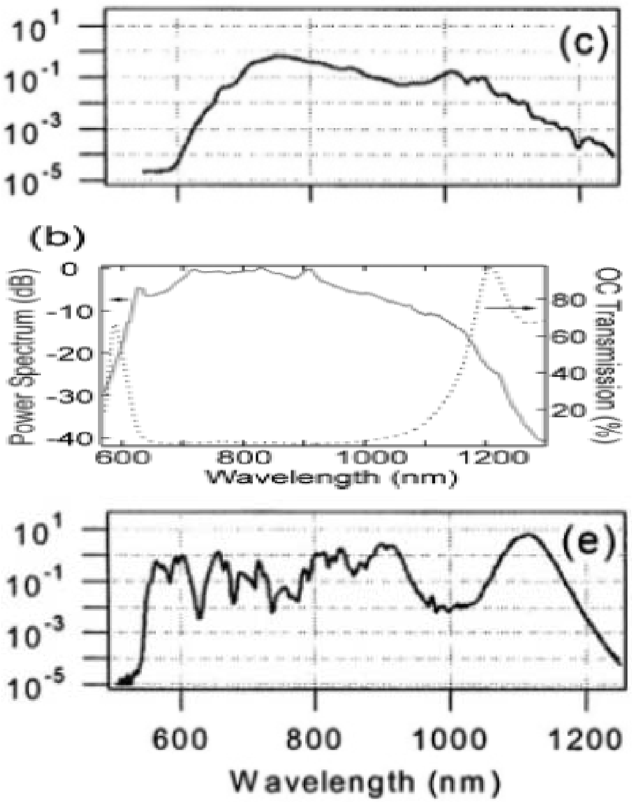
Comb and progress to octave spanning

- Mode-locked Ti:Sapphire laser
  - Does not need octave 3-db bandwidth

Output spectra



T. Udem, Holzwarth, Hänsch, Nature **416**, 233 (2002)

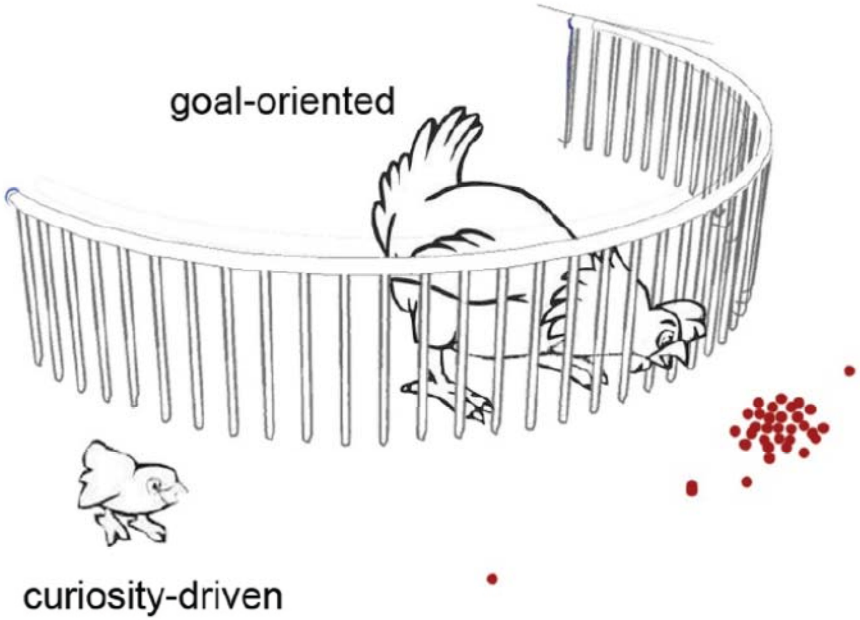


Extreme care in isolations

(c,e) Ye, Cundiff, *Femtosecond optical frequency comb: principle, operation and application* (Springer, 2005)
(b) Matos, et.al. Opt. Lett. **29**, 1683 (2004)

Comb applications

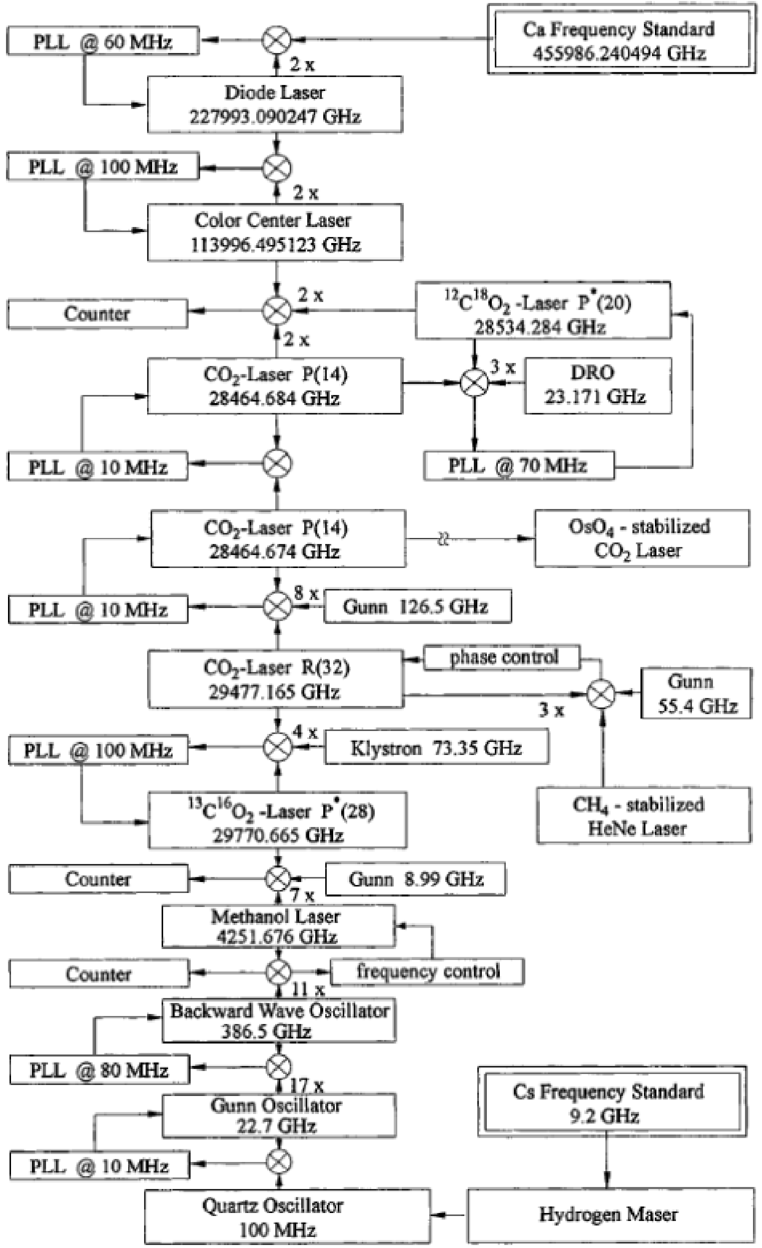
- Metrology: 1978…
- Optical clock
- Synthesis of ultra-purity optical and RF frequencies
- Remote transfer of time/frequency standards
- Precision spectroscopy
- Feel free to dream!



http://nobelprize.org/nobel_prizes/physics/laureates/2005/hansch-lecture.html

Frequency metrology: frequency chain

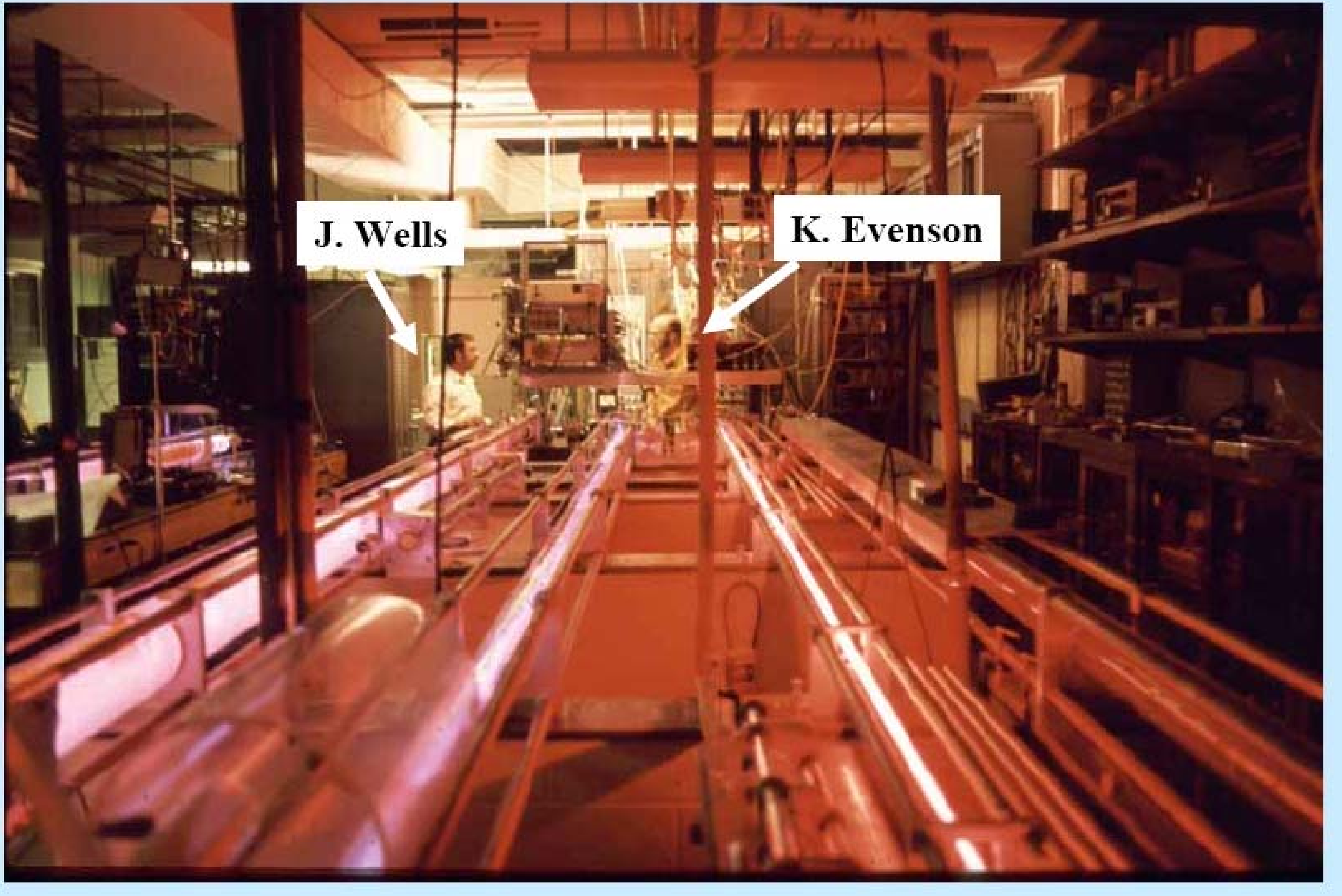
- Multiplicative up-conversion
  - Complicated
  - Expensive
  - Not affordable unless $$$
  - Three labs, two buildings



H. Schnatz, et.al., Phys. Rev. Lett. **76**, 18 (1996)

Optical frequency chain

NBS (NIST): measurement of speed of light, 1972

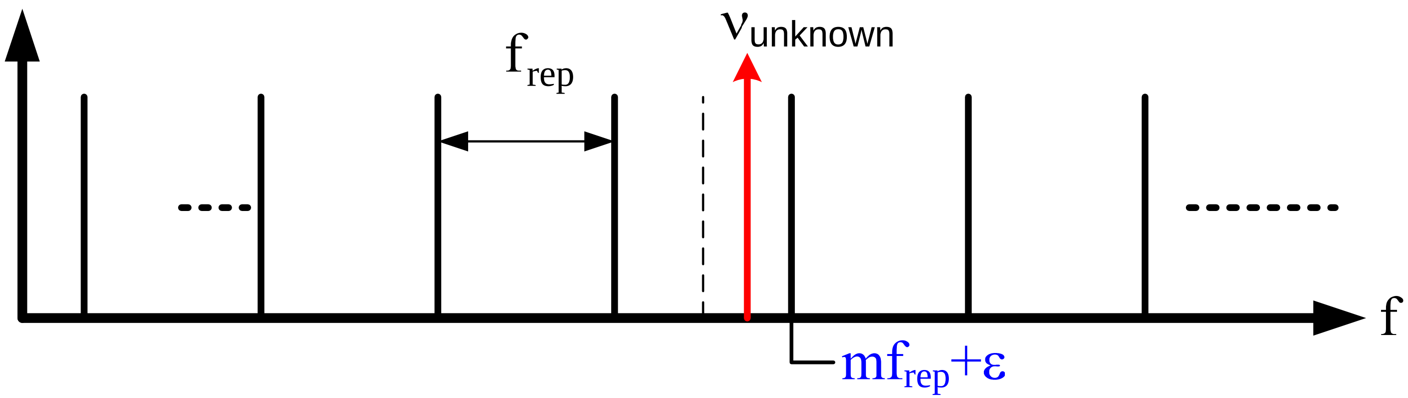


J. L. Hall & J. Ye, "NIST 100th birthday", Optics & Photonics News 12, 44, Feb. 2001

Optical metrology: using combs

- How frequency "ruler" helps
  - Optical frequencies linked through 2 microwave frequencies
- Larger comb spacing better

$$\upsilon_m = m f_{rep} + \varepsilon$$



$$\nu_{unknown} = m f_{rep} + \varepsilon \pm f_{beat}$$

How to remove sign ambiguity?
How to get the right *m*?

fs comb measured frequencies

Ca	657 nm	Schnatz – PTB	PRL 1 Jan '96
C_2H_2	1.5 μm	Nakagawa - NRLM	JOSA-B Dec '96
Sr^+	674 nm	Bernard – NRC	PRL 19 Apr '99
In^+	236 nm	v. Zanthier - MPQ	Opt.Comm. Aug'99
H	243 nm	Reichert - MPQ	PRL 10 Apr '00
Rb	778 nm	D. Jones - JILA	Science 28 Apr 00
I_2	532 nm	Diddams - JILA	PRL 29 May '00
H	243 nm	Niering - MPQ	PRL 12 June '00
Yb^+	467 nm	Roberts - NPL	PRA 7 July '00
In^+	236 nm	v. Zanthier – MPQ	Opt. Lett. 1 Dec.'00
Ca	657 nm	Stenger – PTB	PRA 17 Jan '01
Hg^+	282 nm	Udem – NIST	PRL 28 May '01
Ca	657 nm	Udem – NIST	PRL 28 May '01
Yb^+	435 nm	Stenger – PTB	Opt. Lett.15 Oct '01

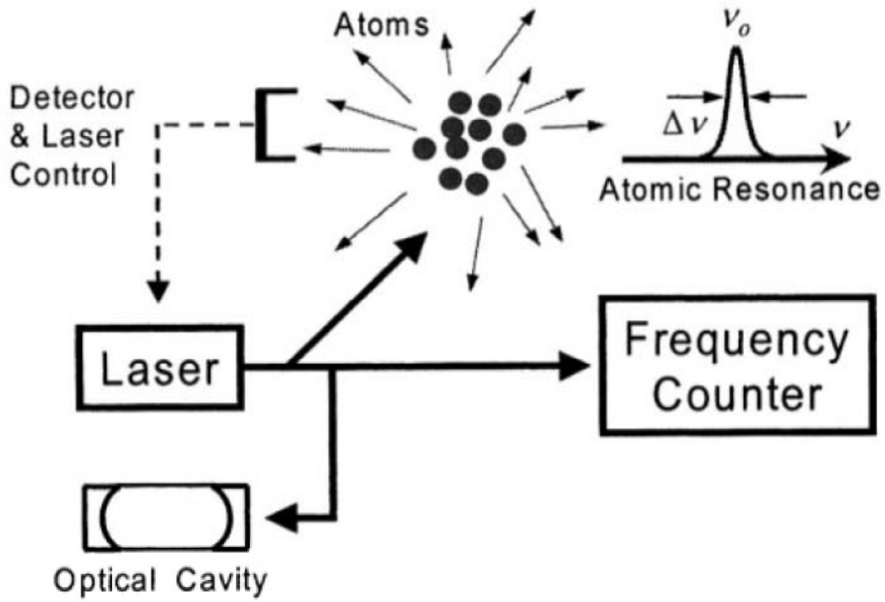
http://nobelprize.org/nobel_prizes/physics/laureates/2005/hall-lecture.html

Optical clock

- 1950's: ^{133}Cs at 9,192,631,770 Hz
- 2000: optical clock
  - Narrow linewidth oscillator
    - Narrow quantum transition
    - Probe laser
  - Frequency counter
- Much better instability: 10^5 better ideally
- Shorten measurement time
- Choices
  - Atom (Ca, Sr, Mg, H…)
  - Ion (Hg^+, Yb^+, In^+, Sr^+…)
  - Molecule (I_2, CH_4, CO_2...)

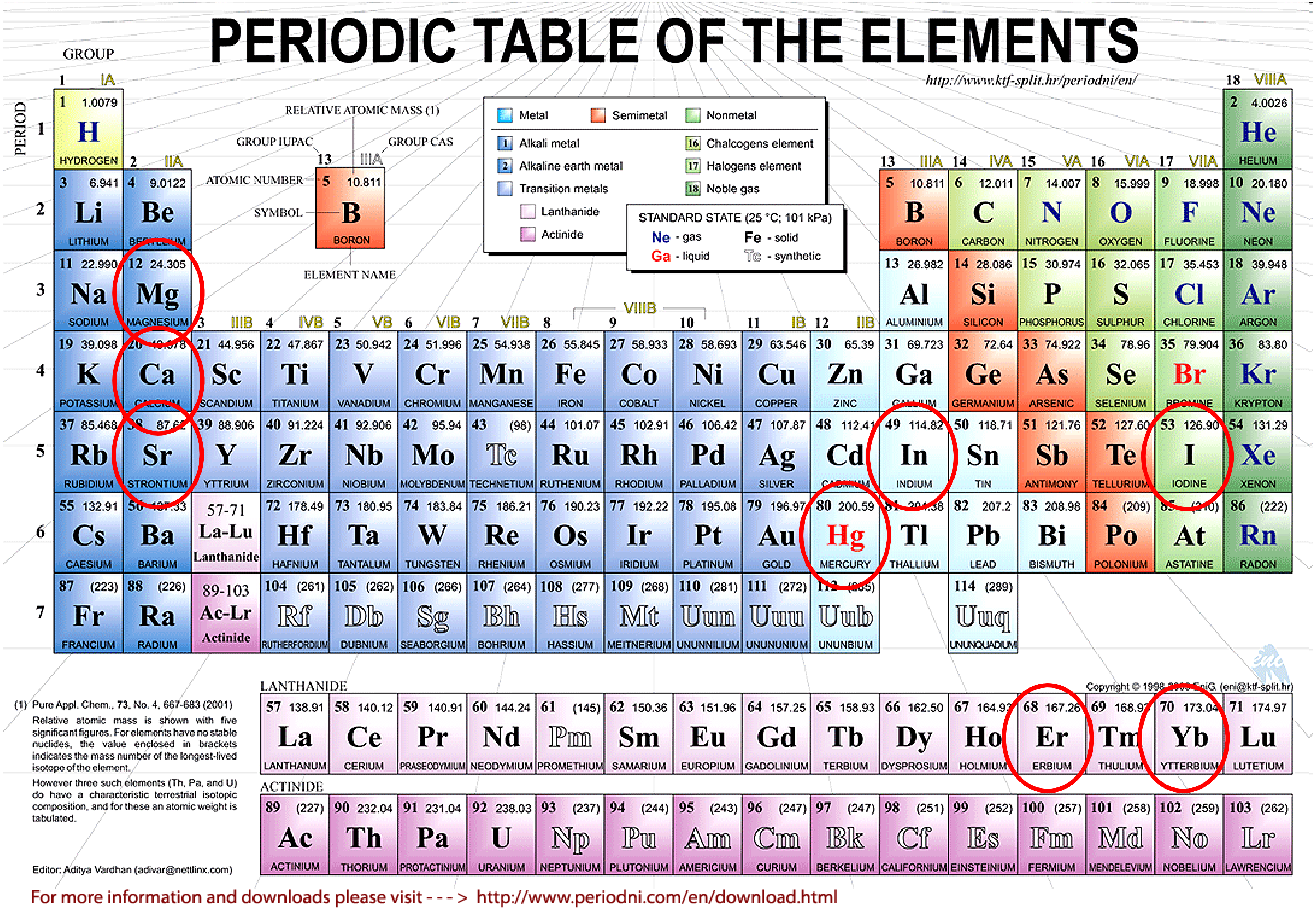
Allan deviation

$$\sigma_y(\tau) = \frac{\Delta\nu}{\pi\nu_0}\sqrt{\frac{T}{2N\tau}}$$



Ye, Cundiff, *Femtosecond optical frequency comb: principle, operation and application* (Springer, 2005)

Periodic table



Molecular frequency standards ~1997

• HeNe Laser w CH_4 Absorber	3.39 μm
• HeNe vis Laser w I_2 Absorber	~5 vis λ's
• CO_2 Laser w CO_2 Absorber	10.6 μm
• CO_2 Laser w OsO_4 Absorber	10.6 μm
• Ar^+ Laser w I_2 Absorber	514 nm
• Nd:YAG Laser w I_2 Absorber	1064 nm
• Nd:YAG Laser w C_2HD Abs.	1064 nm
• Yb:YAG Laser w C_2H_2 Abs.	1030 nm
• Diode Lasers w C_2H_2 Abs.	1550 nm

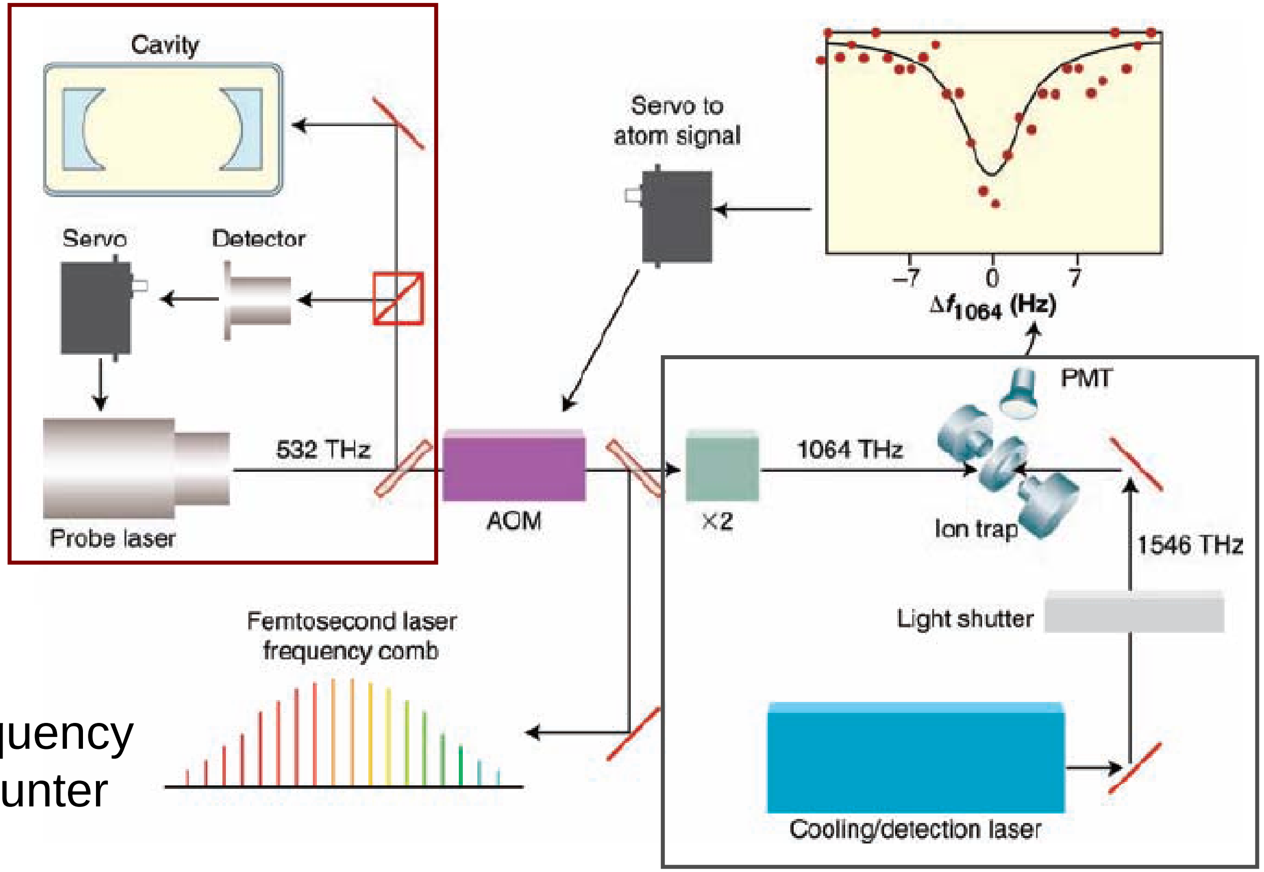
http://nobelprize.org/nobel_prizes/physics/laureates/2005/hall-lecture.html

Hg⁺ optical clock

- Cooled ion that provides atomic transition (282 nm)
- Stabilized probe laser that provides the “clock tick”
- Optical frequency comb that provides an “optical” frequency divider

$Q=\nu_0/\Delta\nu \sim 10^{14}$

oscillator



frequency
counter

reference

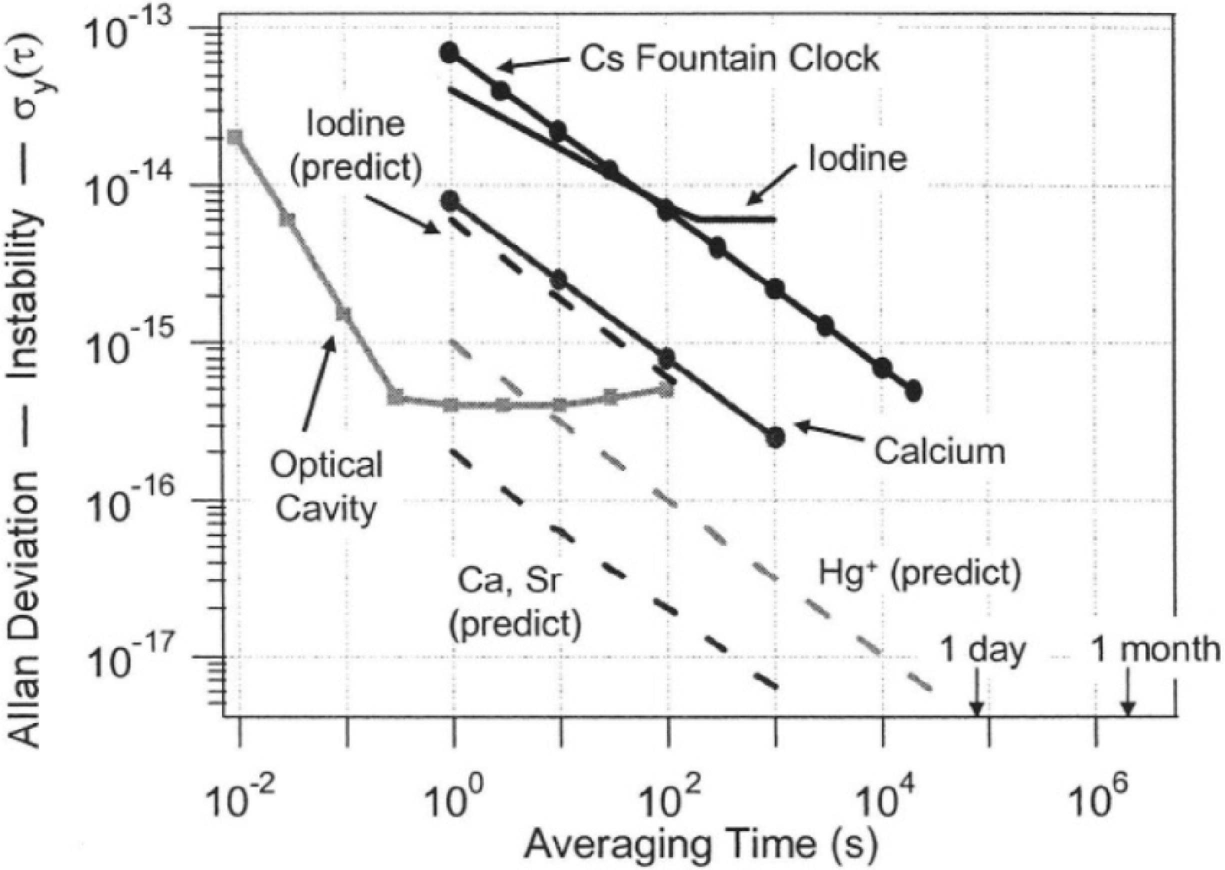
S.A. Diddams, et.al., Science **306**, 1318 (2004).

Optical clocks: some results

- Striking improvements

$$f_{beat} = mf_{rep} + \varepsilon - \nu_{opt}$$

locked



$$f_{rep} = \frac{\upsilon_{opt} + f_{beat} - \varepsilon}{m}$$

Why can't we just use the optical cavity?

Ye, Cundiff, *Femtosecond optical frequency comb: principle, operation and application* (Springer, 2005)

Low-noise frequency synthesis

- RF ➔ optical
- RF ➔ RF
- Optical ➔ optical
- Optical ➔ RF

$$f_{rep} = \frac{\upsilon_{opt} + f_{beat} - \varepsilon}{m}$$

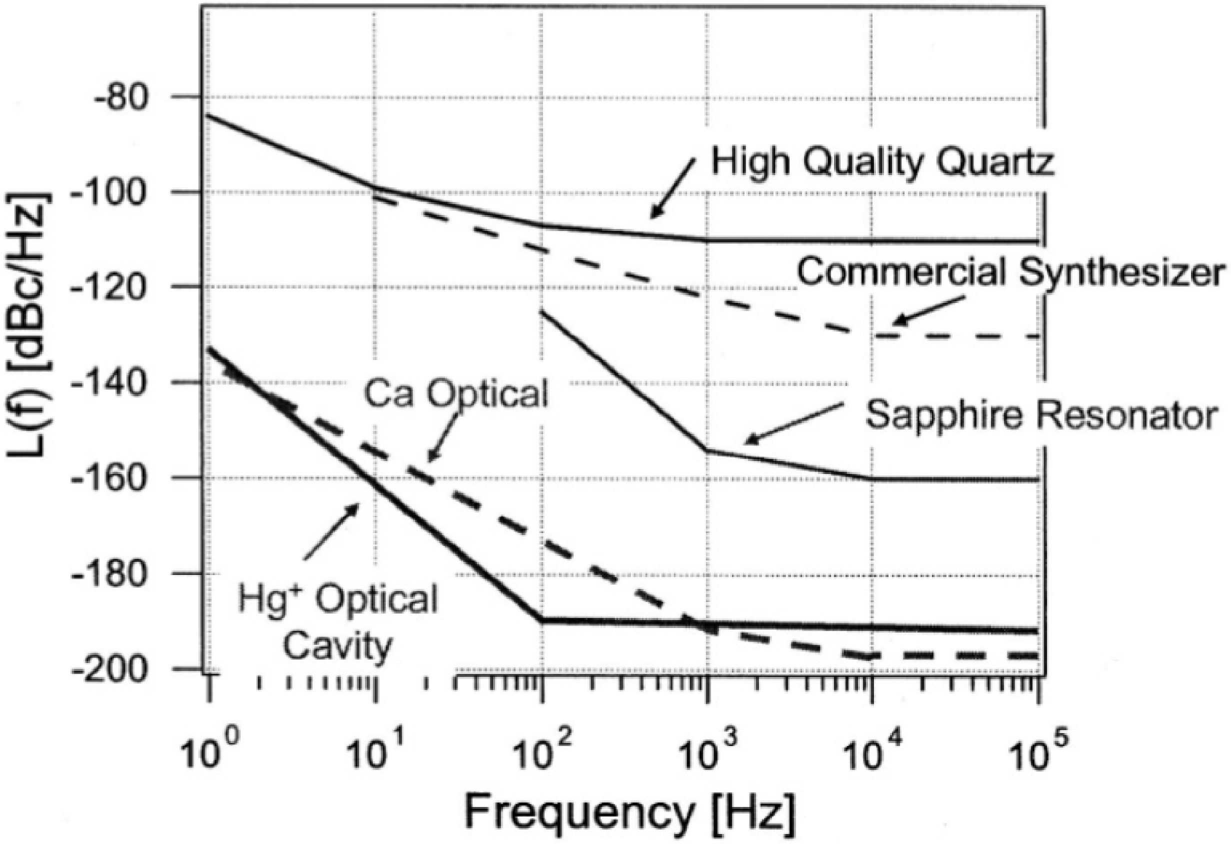
Extremely low phase noise RF signals

J. Ye et.al., PRL **87**, 270801 (2001).

Phase noise issue

- Transfer of RF reference onto optical: disastrous
- Optical reference a better choice

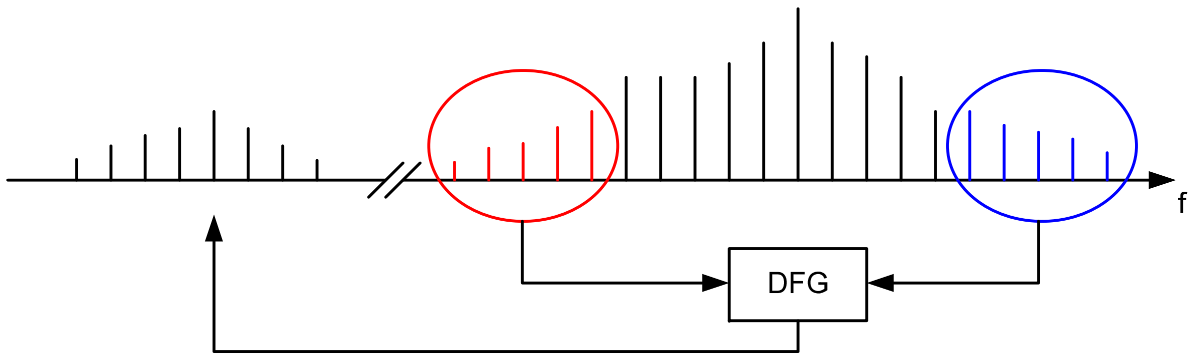
$$L(f) \propto m^2$$



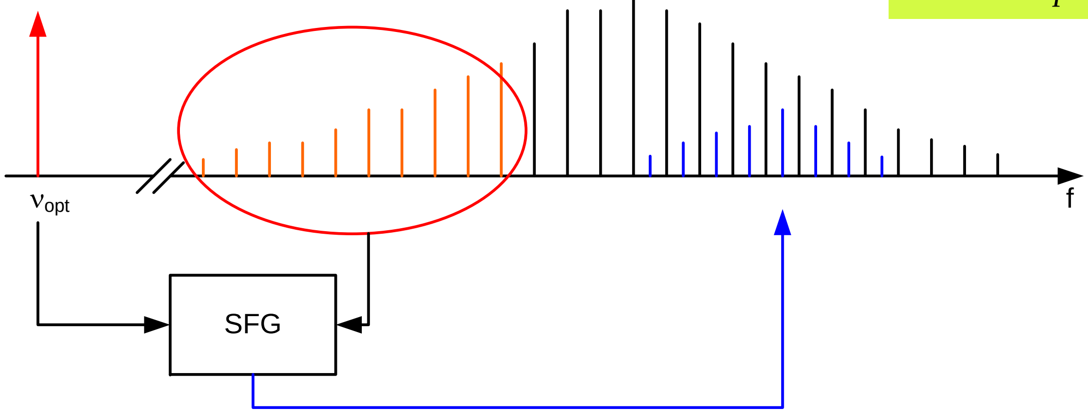
Ye, Cundiff, *Femtosecond optical frequency comb: principle, operation and application* (Springer, 2005)

Removing ε dependence

- Difference frequency generation



- Sum frequency generation with an optical standard



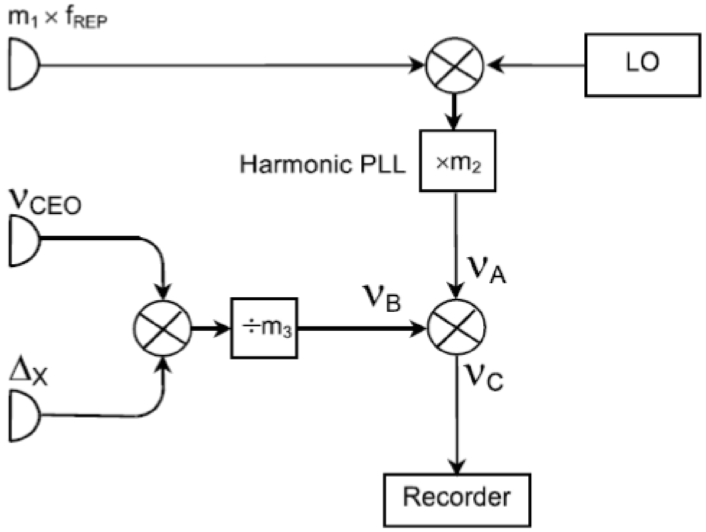
$$f_b = \nu_{opt} + (n_{low} f_{rep} + \varepsilon) - (n_{high} f_{rep} + \varepsilon)$$

$$f_{rep} = \frac{\nu_{opt} - f_b}{n_{high} - n_{low}}$$

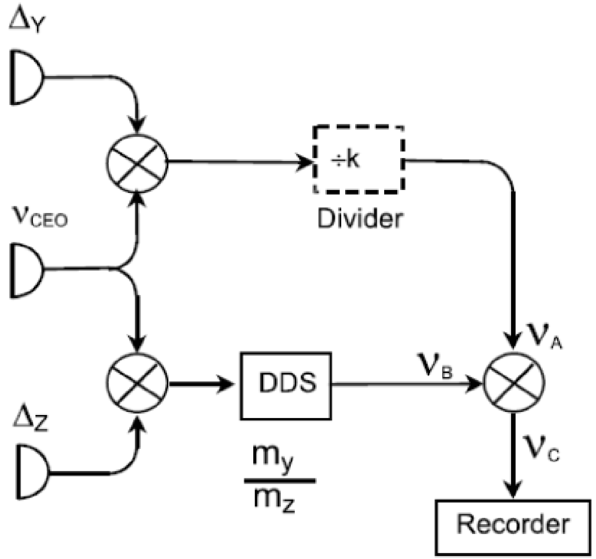
Comb as transfer oscillator

- Comb can be un-stabilized!
- Ratio of widely separated optical frequencies or microwave

Linking optical and microwave frequencies



Linking two optical frequencies



reference

$$f_{lo} = \frac{f_c}{m_2} + \frac{\nu_{opt}}{m_2 m_3}$$

Detected signals independent of laser noise properties!

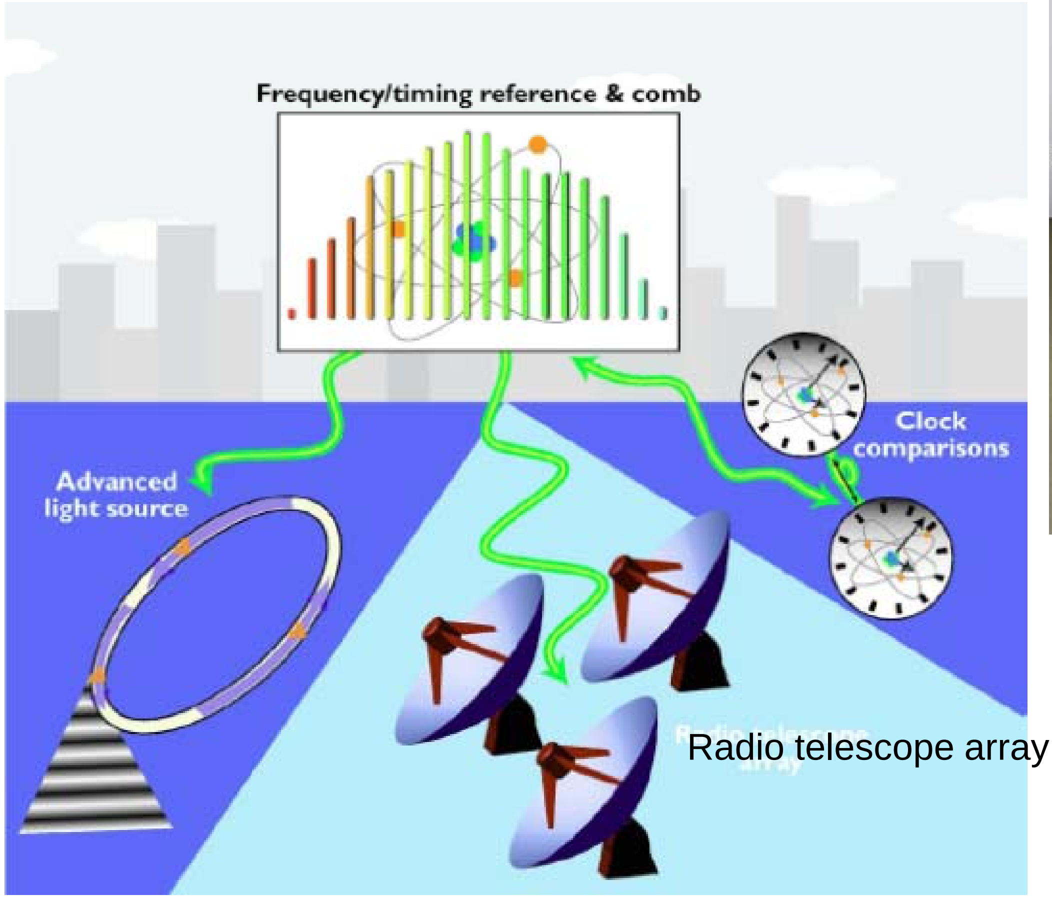
reference

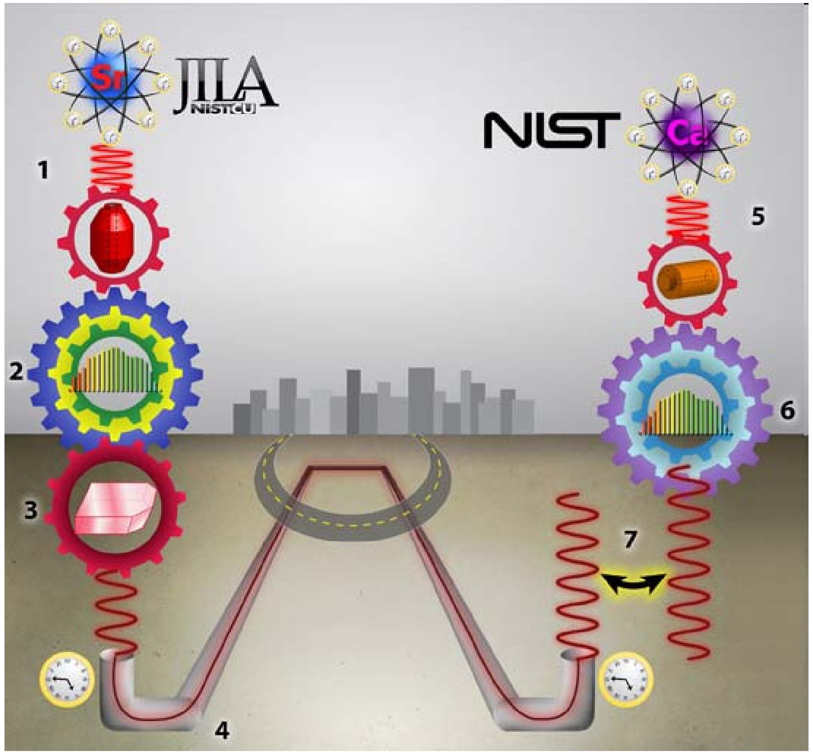
$$\nu_y = f_c + \frac{m_y}{m_z}\nu_z$$

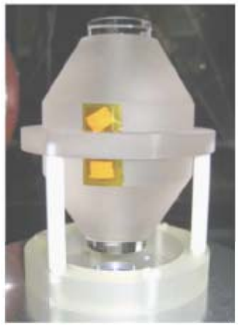
H.R. Telle, et.al., Appl. Phys. B **74**, 1 (2002).

Remote transfer

- Fiber as coherence transfer media







S.M. Foreman, et. al., Rev. Sci. Instr. **78**, 021101 (2007).

Comb alternatives

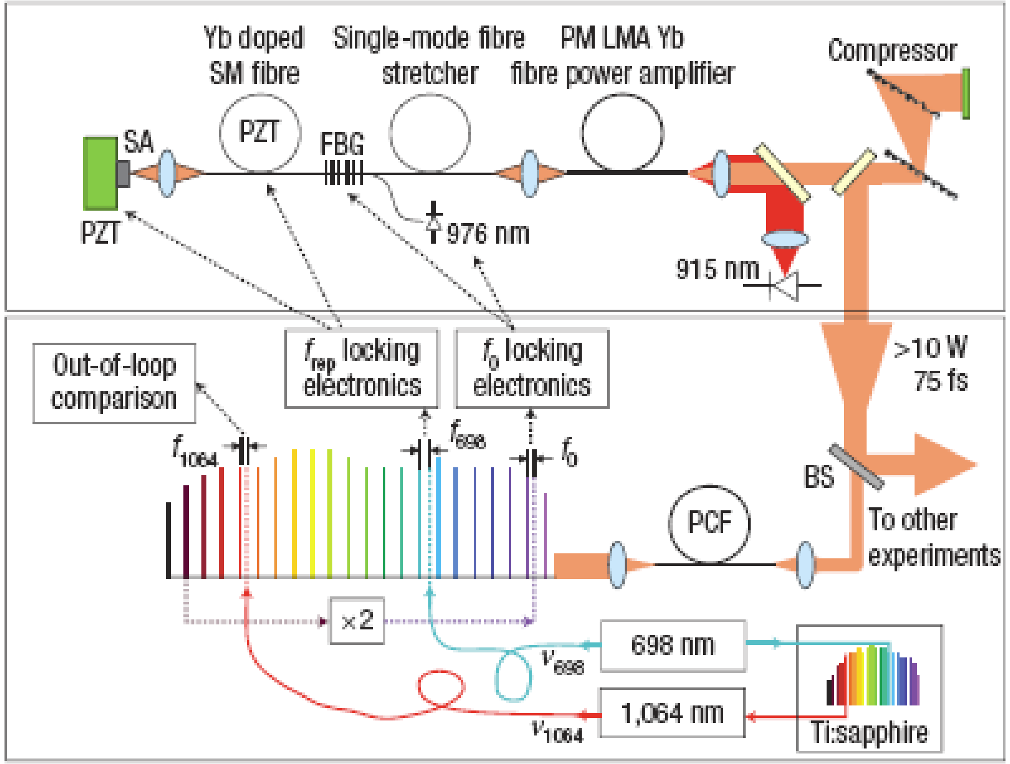
- Fiber laser combs
- Externally-modulated CW comb
  - Simple phase modulator
  - Dual-electrode intensity modulator
- Optical frequency comb generator
  - Actually dominated large-span optical frequency difference measurements during 1993-1998
- Compact comb generator using micro-toroid

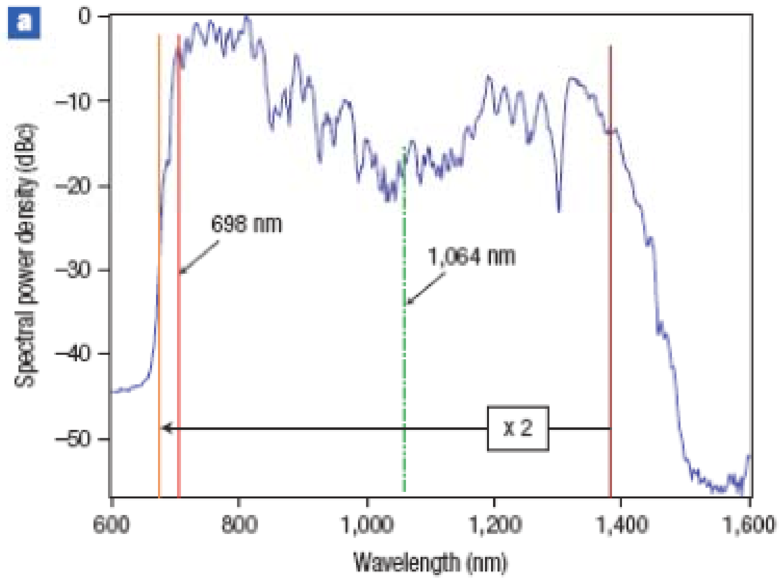
Fiber laser combs

- Compact, high power (>10 W)
- Different wavelength windows
  - Er (1.5μm), Yb (1.05μm)

Cladding pumped CPA linear amplification avoids nonlinearity induced phase and amplitude noise

Best phase stability ever reported



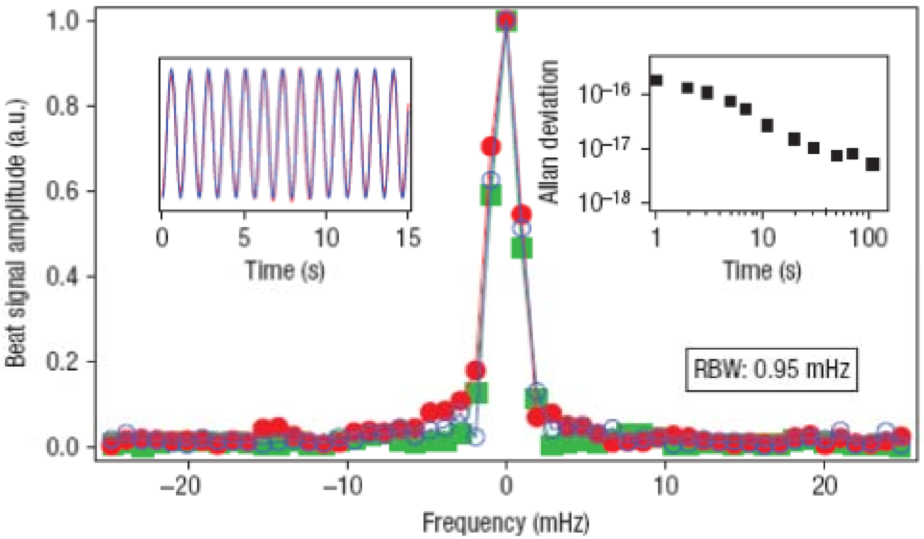


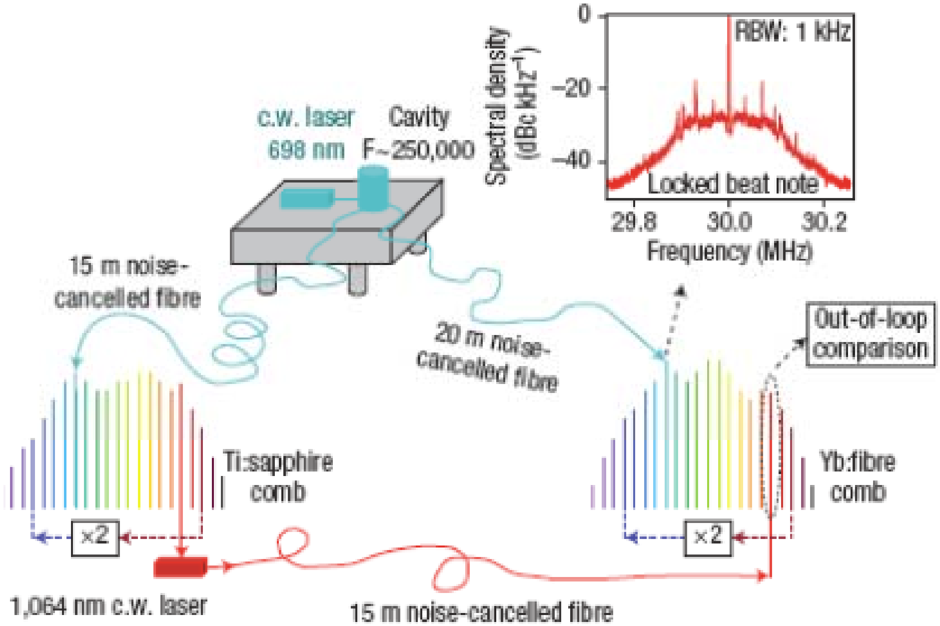
698 nm: LD locked to Sr optical lattice clock

T.R. Schibli et.al., Nat. Photonics **2**, 355 (2008)

Fiber laser combs (cont.)

- Unlocked
  - Fast f_o linewidth < 10 kHz, record for fiber systems (~50 kHz)
  - Long term (~min) f_0 linewidth < 100 kHz (Ti:S ~MHz)
- Locked f_0 < 1 mHz
- Frequency comparison with Ti:S
  - < 1 mHz (1.05 ks time)

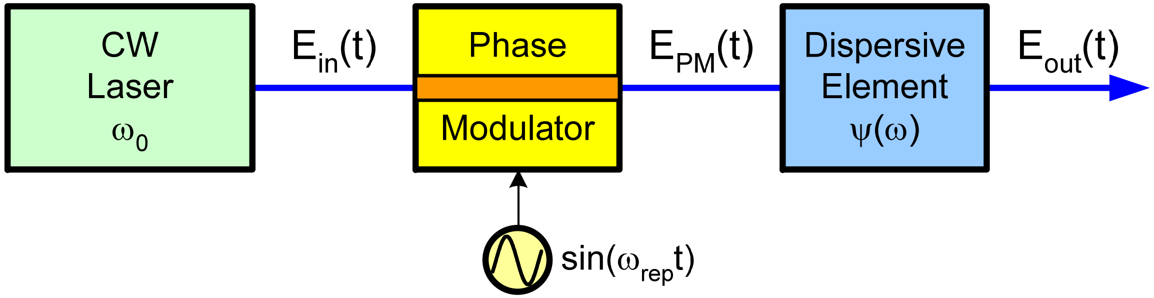




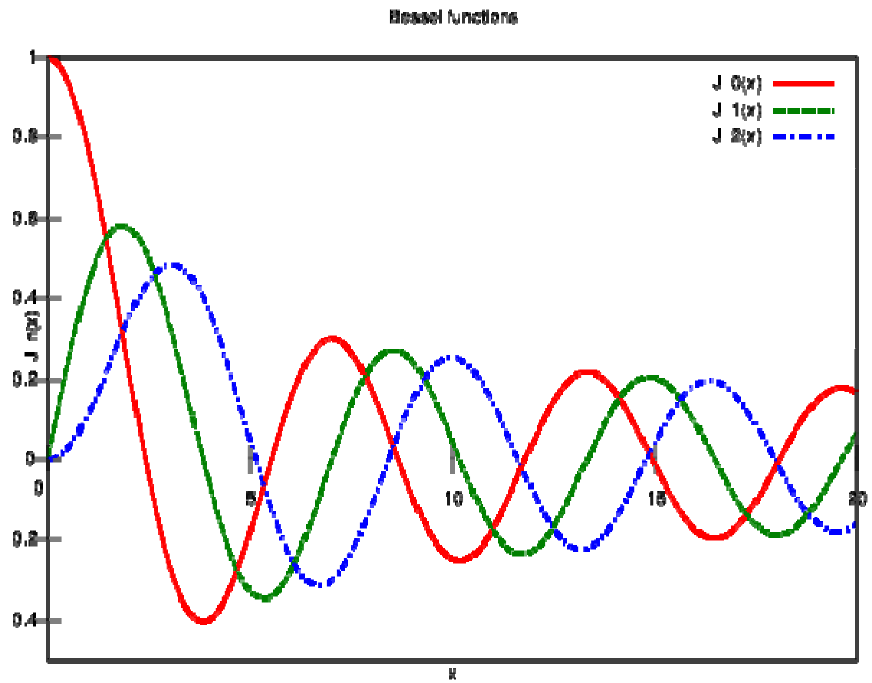
T.R. Schibli et.al., Nat. Photonics **2**, 355 (2008)

Externally modulated laser combs

- Phase or dual-electrode intensity modulation



- Bessel's series



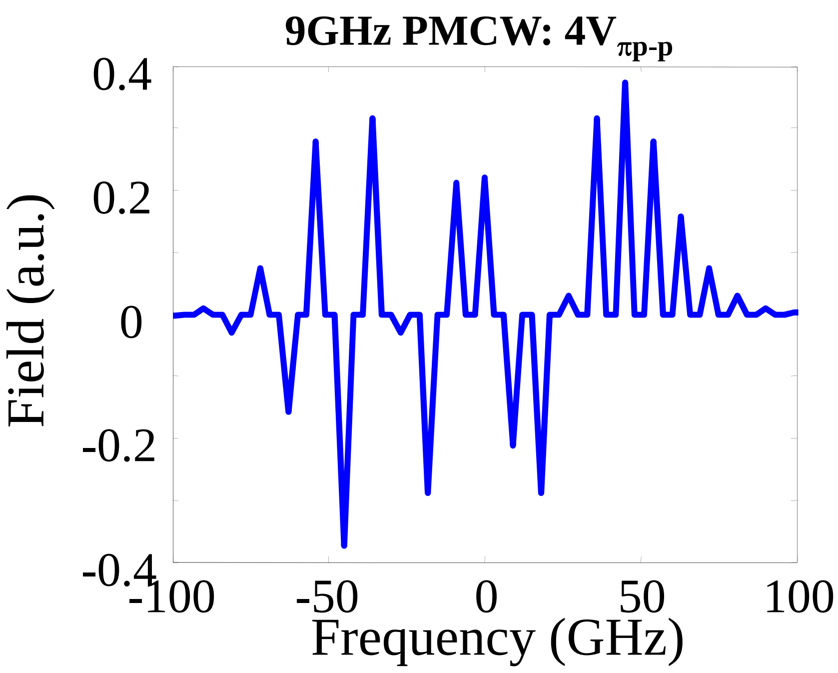
$$\upsilon_m = m f_{rep} + \varepsilon$$

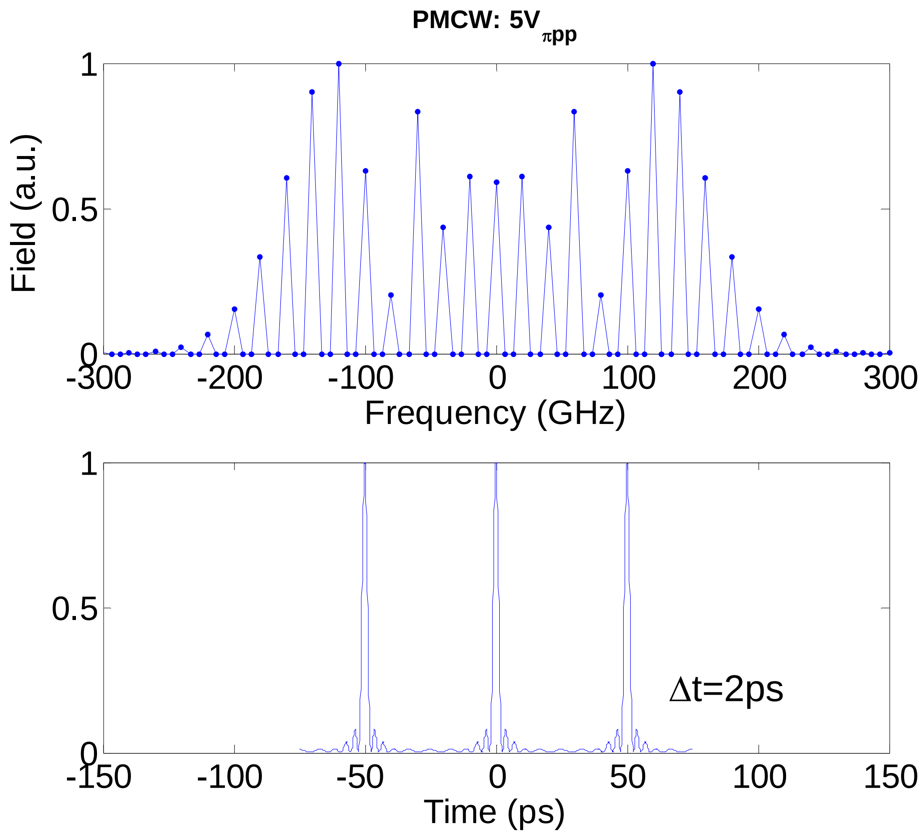
De-coupled!

Let's derive it's output spectrum!

Comb spectral phases

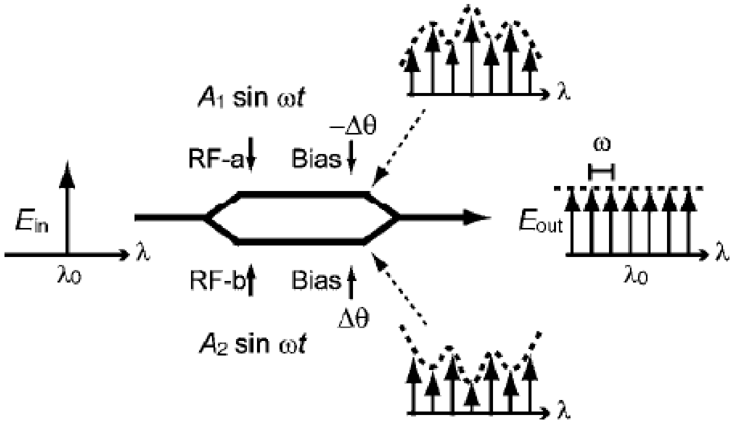
- Not self-pulsing

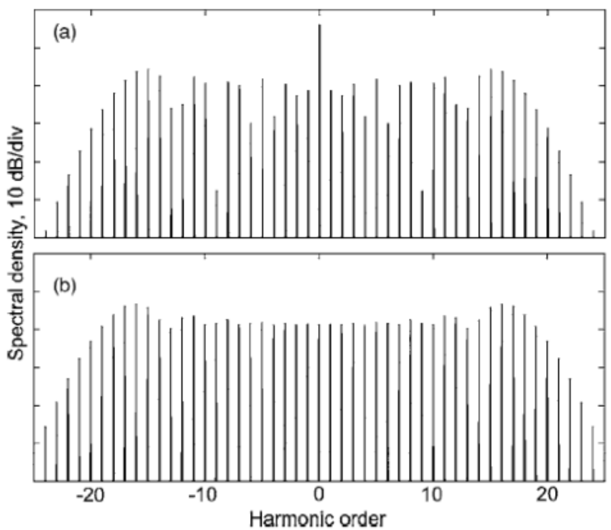




Flat externally modulated CW laser comb

- Dual electrode intensity modulator



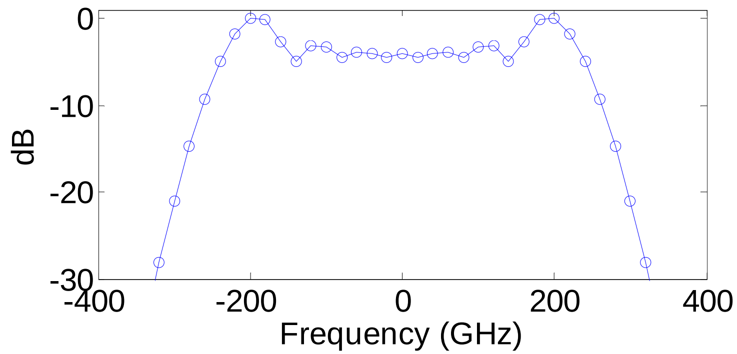


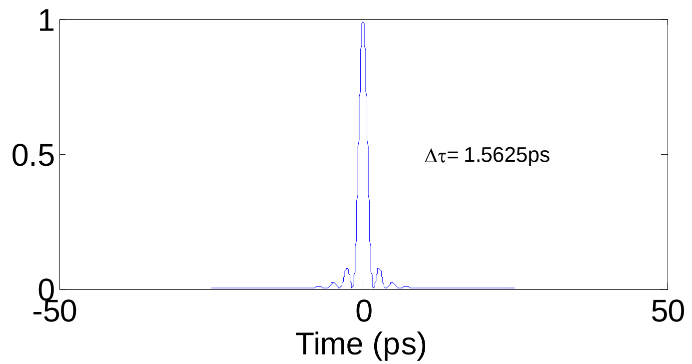
T. Sakamoto et.al, Opt. Lett. **32**, 1515 (2007).

$$\frac{P_k}{P_{in}} = \frac{1}{2\pi\overline{A}}\{1+\cos(2\Delta\theta)\cos(2\Delta A)+$$

$$[\cos(2\Delta\theta)+\cos(2\Delta A)]\cos[2\overline{A}-\frac{(2k+1)\pi}{2}]\}$$

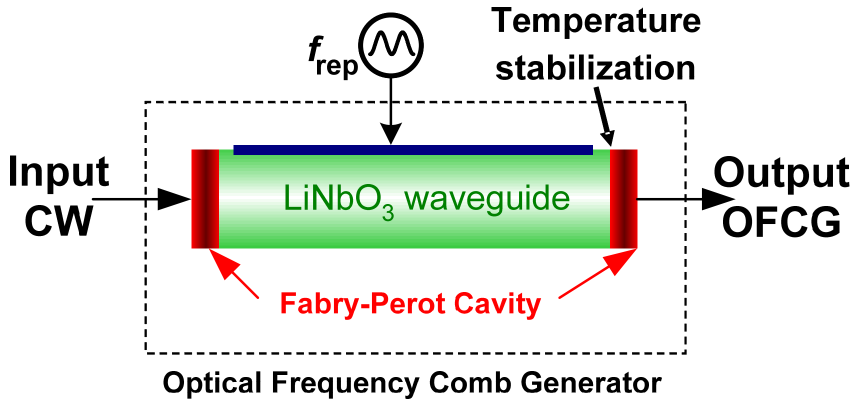
$\overline{A} \equiv (A_1 + A_2)/2$ $\Delta A \equiv (A_1 - A_2)/2$ $\Delta\theta \equiv (\theta_1 - \theta_2)/2$

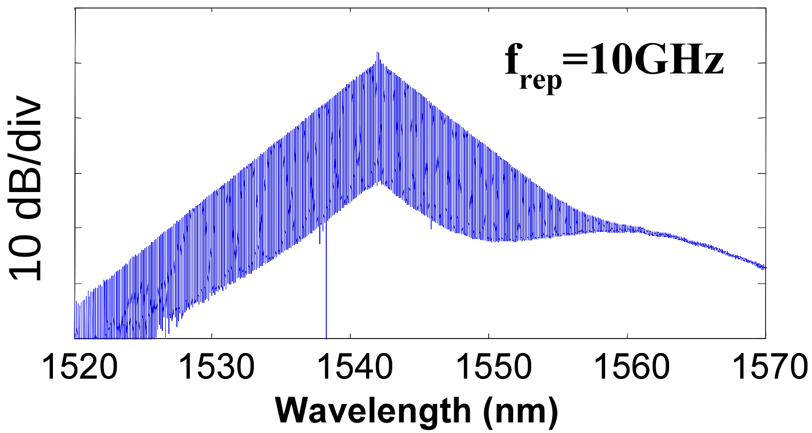




OFCG

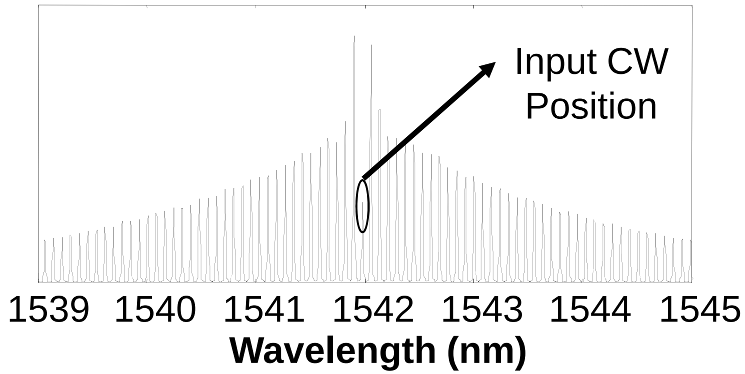
- Phase modulation inside cavity
  - Smooth spectrum: exponential
  - Wide bandwidth

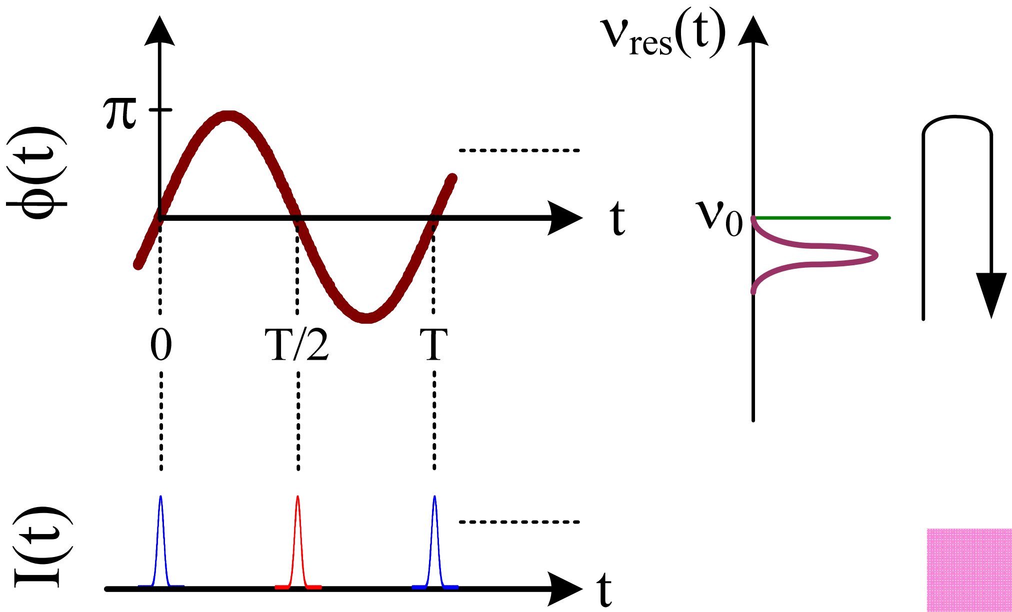


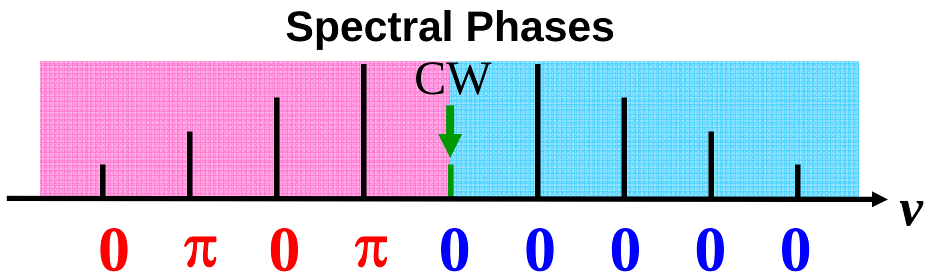


OFCG: pulsing property

- Multi-pulsing

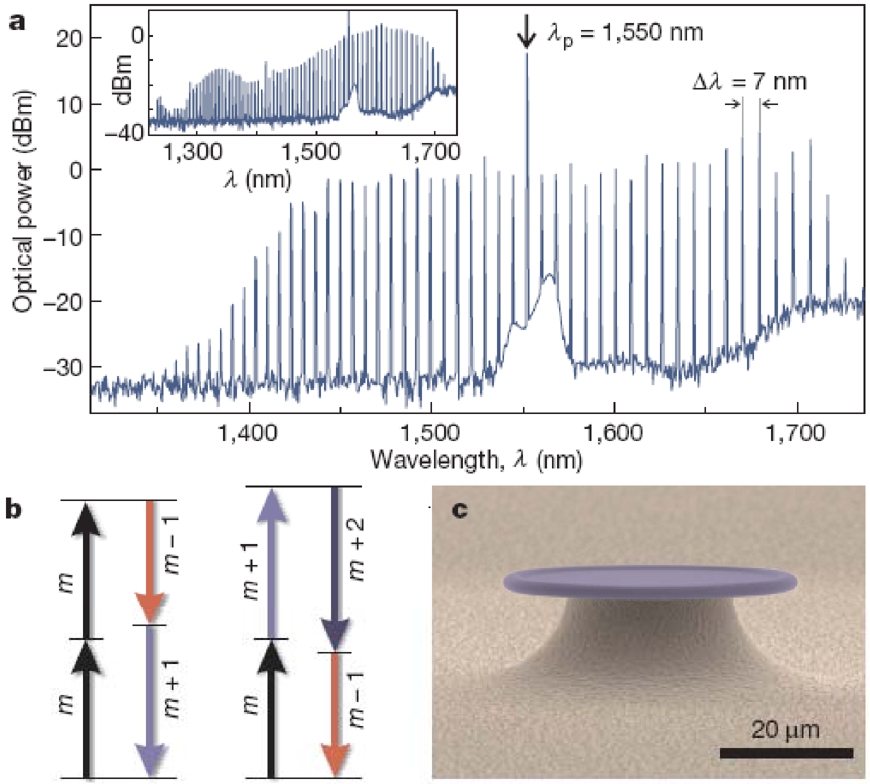




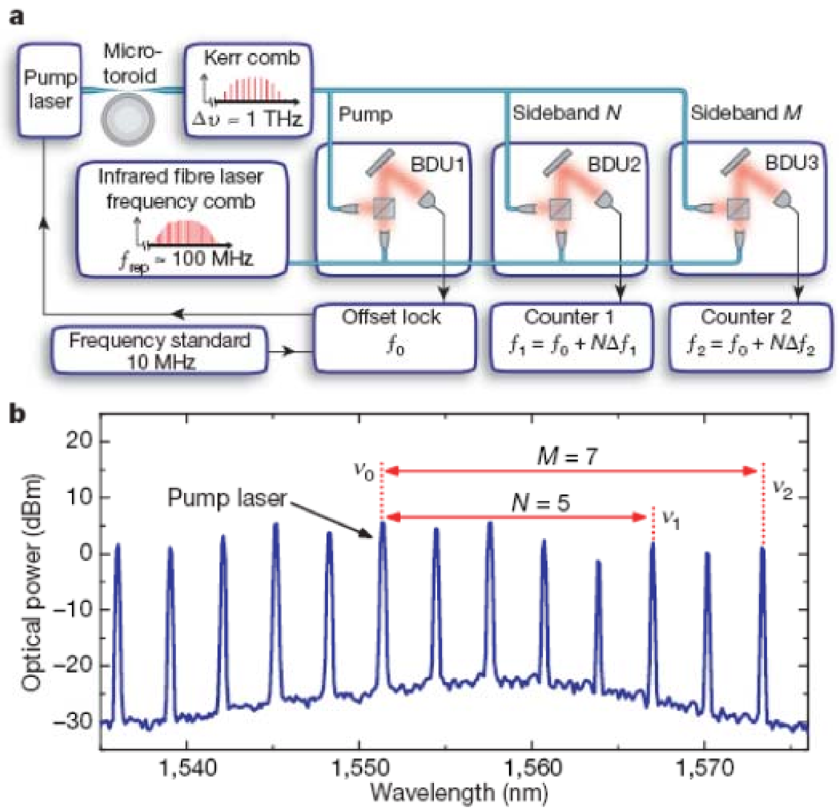


Micro-toroid comb generator

- Extremely high-Q cavity
- χ^3 nonlinearity: four-wave mixing
- Large comb spacing!



Comb spacing uniformity: 7.3×10^{-18}



P. Del'Haye, et.al., Nature **450**, 1214 (2007)
P. Del'Haye, et.al., PRL **101,** 053903 (2008)
P. Del'Haye, et.al., Nat. Photonics **3**, 529 (2009)
Ferdous, et.al.,, Nat. Photonics **6,** xxx (2011)
Kippenberg, el.al., Science **332**, 555 (2011)

Summary

- History of a second and optical frequency comb explained
- Derivation and method for measuring the carrier-envelop phase slippage discussed
- Stabilization of OFC
  - Self-referencing
- Applications of OFC
  - Optical frequency metrology, optical clock, …..
- Alternative OFC generation methods