

*IPT544000 Selected Topics in Ultrafast Optics*

# Generation and Applications of Optical Frequency Combs

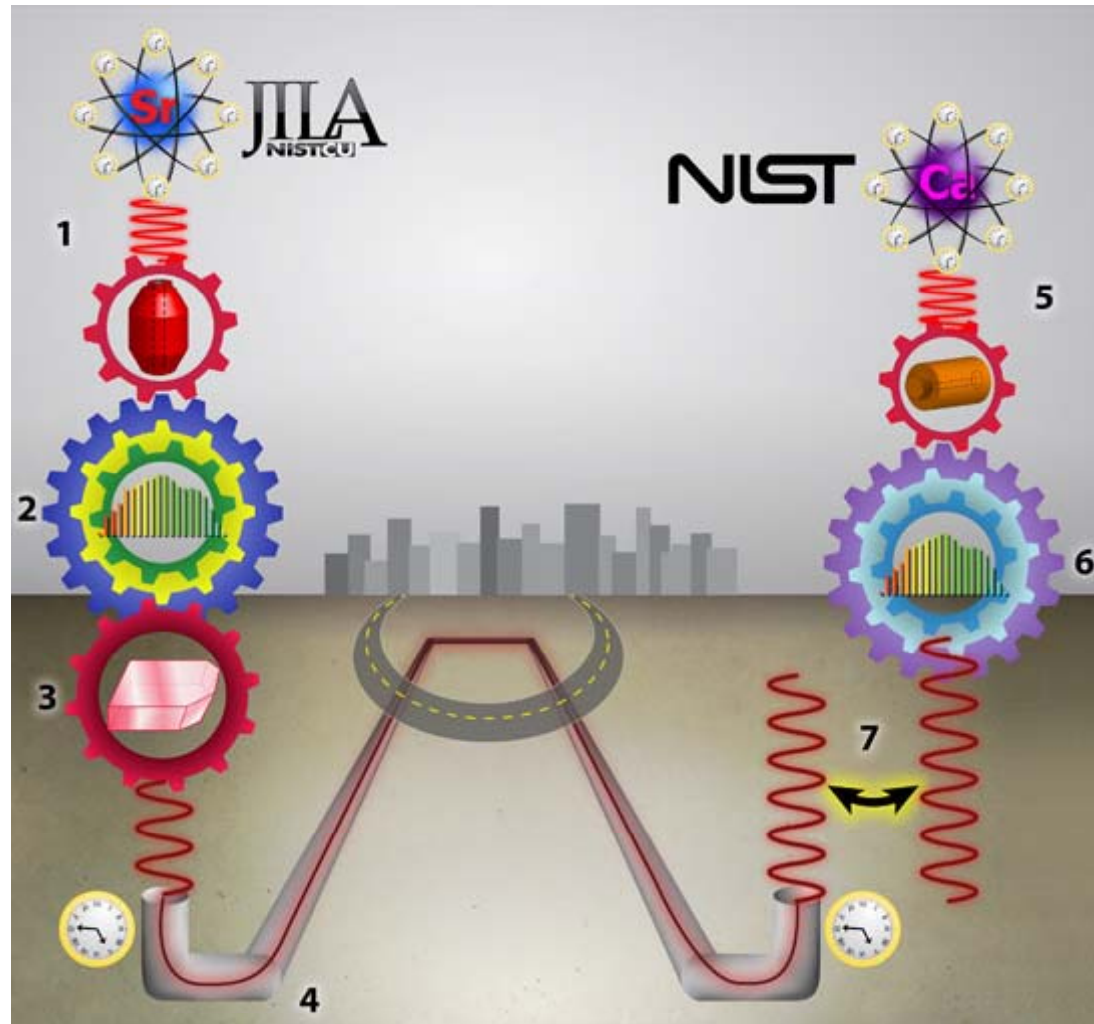
Robin (Chen-Bin) Huang

Institute of Photonics Technologies

National Tsing Hua University, Taiwan

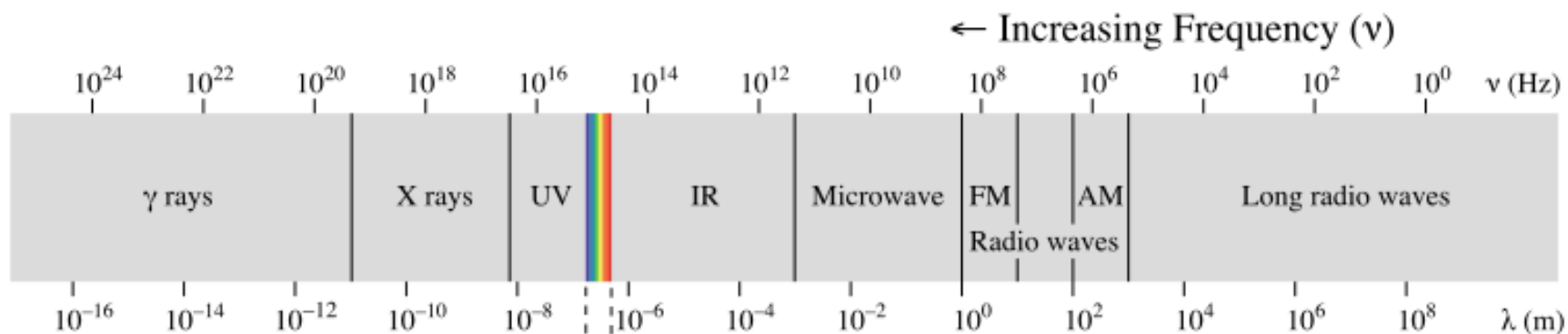


# Beautiful picture ⇔ Bright future

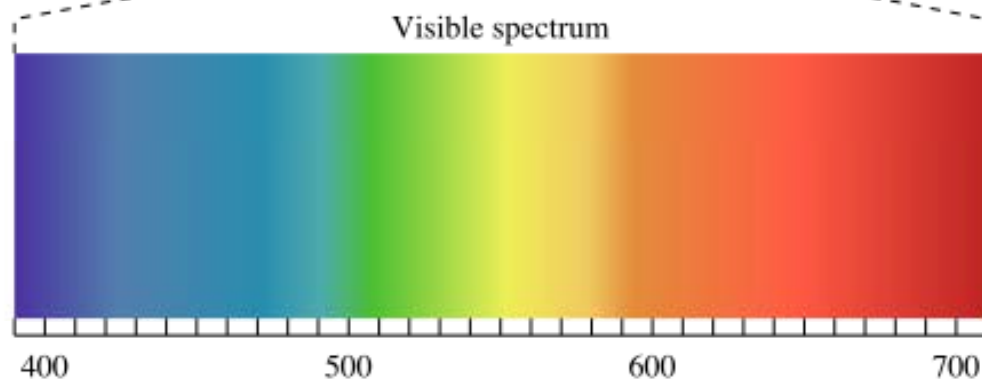


# The spectrum

- RF linking optical and vice versa?



Increasing Wavelength ( $\lambda$ ) →



# Motivations

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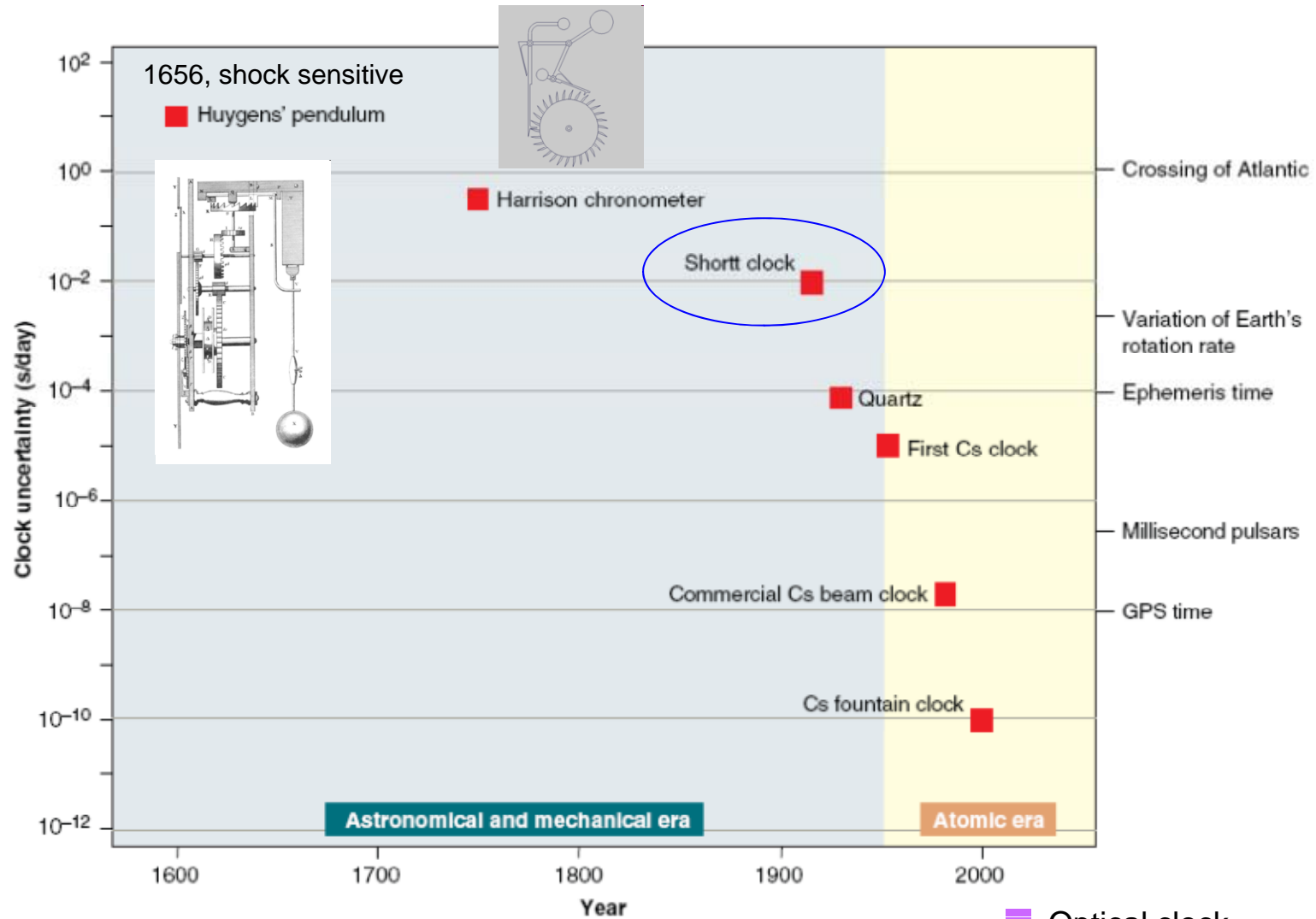
- The definition of **one second** affects our daily life
  - Internet, GPS, cell phones....
- Other units related to a second
  - SI base units: meter, ampere, candela
  - SI derived units: volt ( $\text{kg}\cdot\text{m}^2/\text{A}\cdot\text{s}^3$ ), Newton,.....
- Physical constants
  - $\alpha$  decay rate, speed of light,.....
- High quality time and frequency standards an important task throughout the world

# Definition of a second

- 2000 B.C., Egyptians: divided day and night into 12 hours
- 1000, Muslims: counting the moon,  $(1/60)^2$  of an hour
- 1670, Huygens pendulum
- 1956, the Ephemeris second:  $1/31,556,925.9747$  of Earth's one rotation around the Sun (a year in 1900) by the 11<sup>th</sup> General Conference on Weights and Measures
- 1967, the atomic time:  $1/9,192,631,770$  duration of  $^{133}\text{Cs}$  ground-state transition by 13<sup>th</sup> General Conference on Weights and Measures
- 1980's, [laser cooling](#) made possible improved frequency stability in Cs clocks (Cs fountains)

$$\sigma_y(\tau) \approx \frac{\Delta\nu}{\nu_0 SNR}$$

# Clocks-Evolution



■ Optical clock  
 Enabled by  
 frequency comb

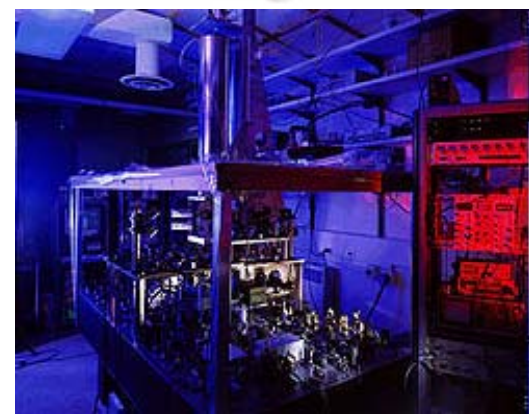
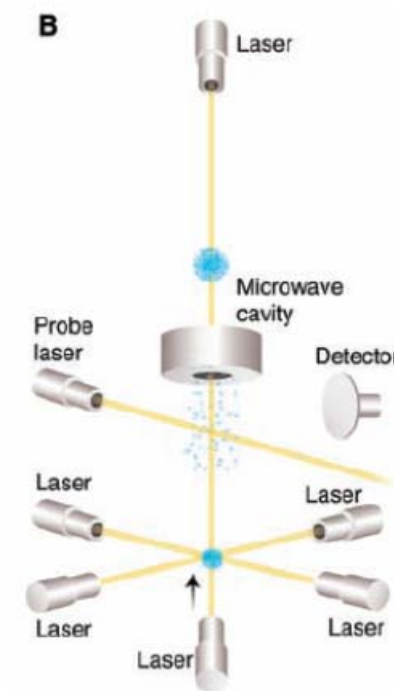
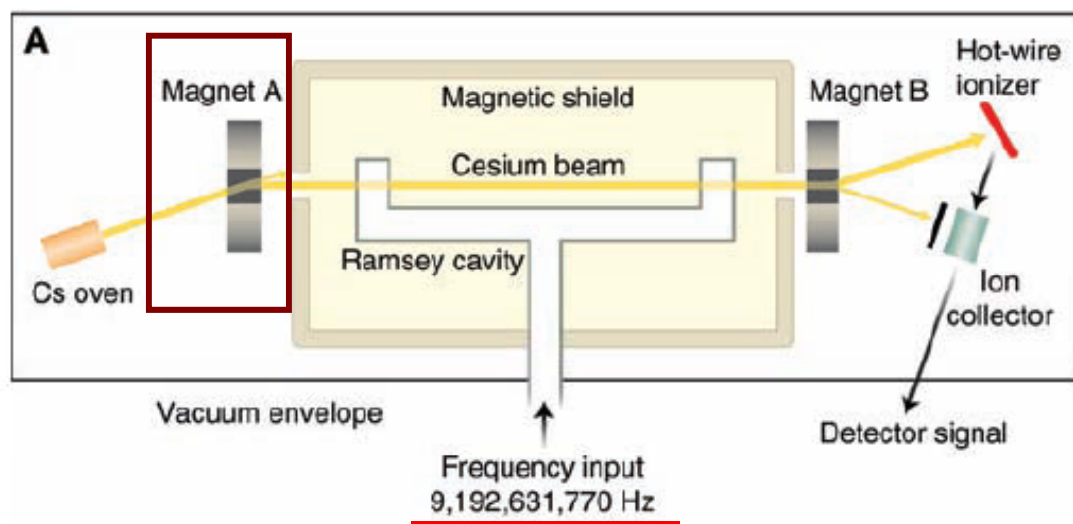
S.A. Diddams, et.al., Science **306**, 1318 (2004)

# Atomic clock: $^{133}\text{Cs}$

- Commercial: Stern-Gerlach magnets
- Cs fountain clock (envision by Zacharias in 1950s)
  - Laser cooling reduces Doppler linewidth

1997, Nobel Prize in Physics to Chu, Cohen-Tannoudji, Philips

## Doppler shifts, collisions



Think of it as polarization control in photonics

S.A. Diddams, et.al., Science **306**, 1318 (2004)

# Optical clock?

- What benefits do we gain?
- How to realize it?
- Problems?

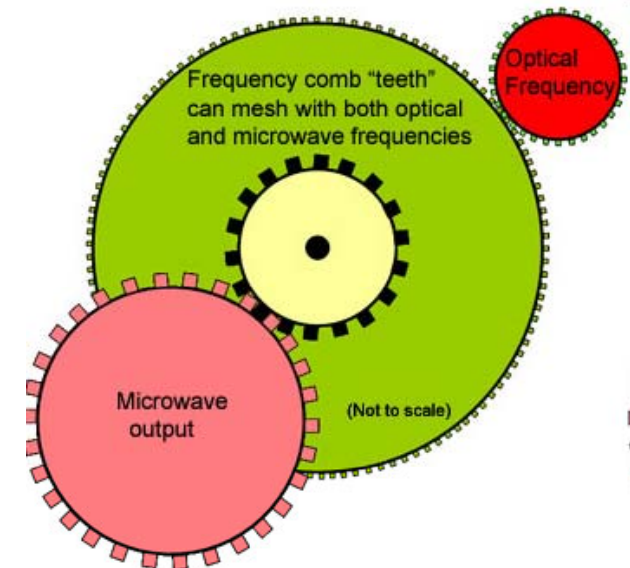
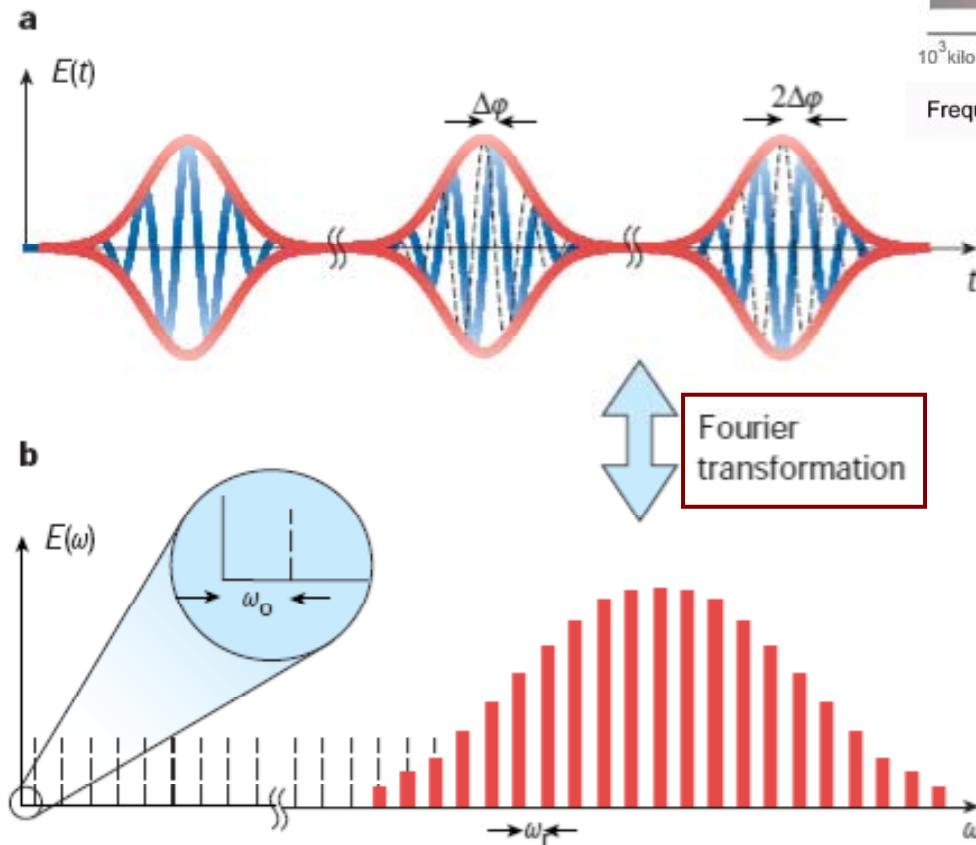
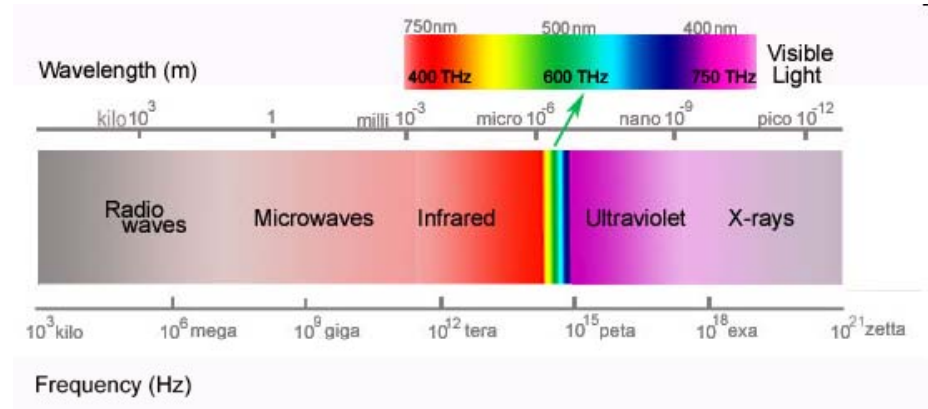
$$\sigma_y(\tau) \approx \frac{\Delta\nu}{\nu_0 SNR}$$

How to “read” optical frequencies accurately?



# Optical frequency comb

- A ruler that connects
  - Frequency and time
  - Microwave and optical frequencies



T. Udem, Holzwarth, Hänsch, Nature **416**, 233 (2002)

NIST

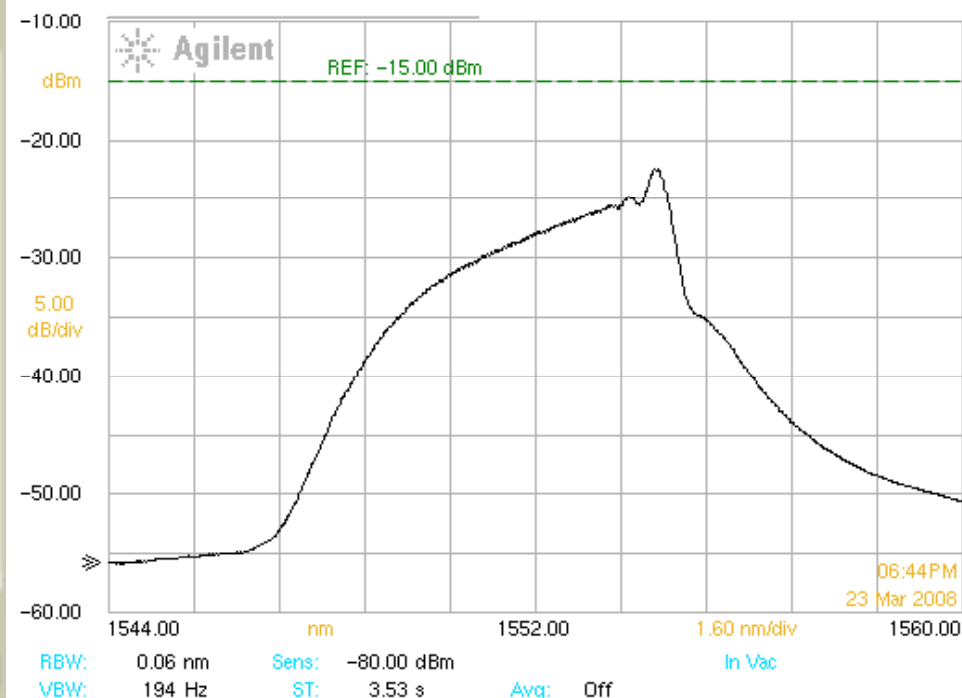
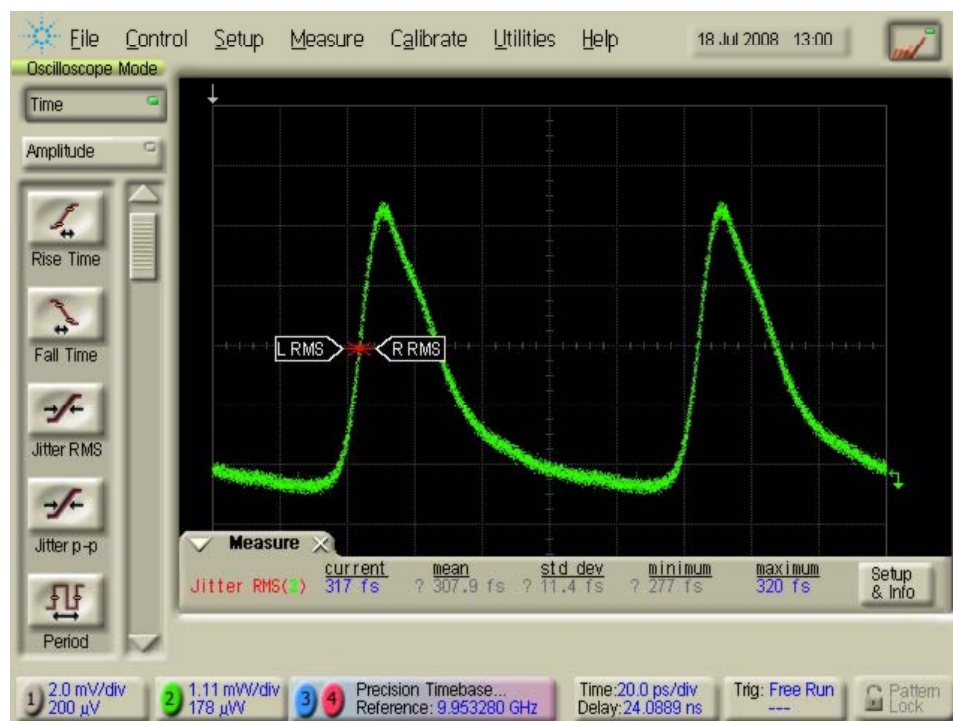
# Bigger question

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- So, if we have a pulse train, are we guaranteed a comb?
  
  
  
  
  
  
  
  
  
  
- Conversely, if we have a comb, are we guaranteed with a pulse train?

# Mode-locking $\Leftrightarrow$ frequency comb?

- Well, not quite



# Realization of a “Comb”

- 1917, Einstein laid foundation for MASER and LASER
- 1953, Townes and students invented MASER
- 1957-60's, LASER theory developed by Townes and Schawlow
- 1960, first working laser (Ruby) by Maiman
- 1962, first semiconductor (GaAs) LD by R.N. Hall
- 1978, optical frequency comb envisioned
- 1998, frequency comb realized
- Explosion on this research field and it's applications

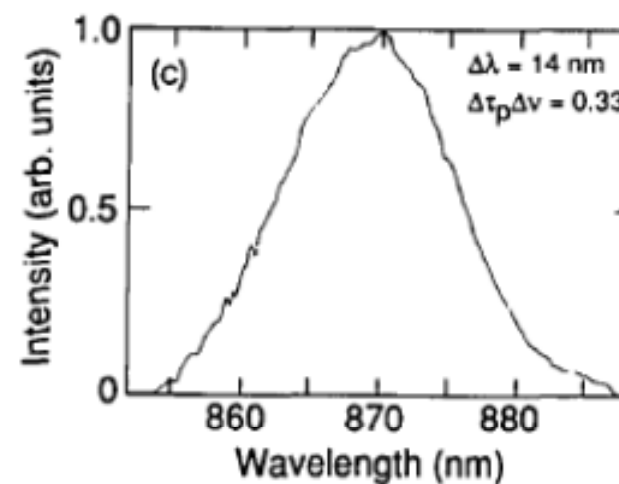
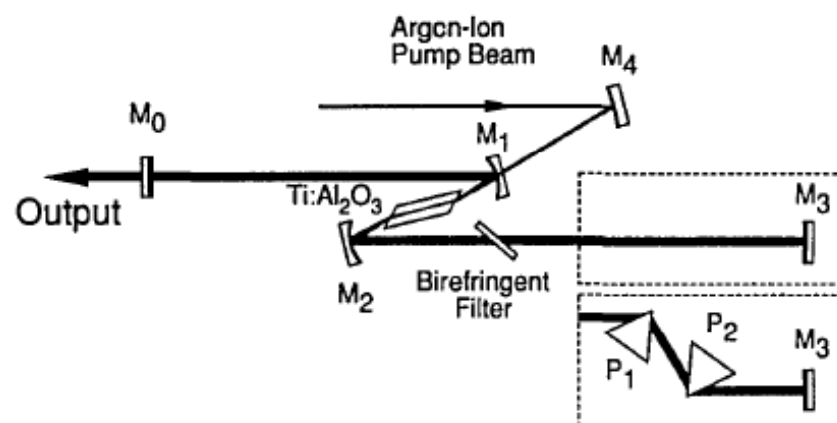
What took so long?



John L. Hall of NIST/JILA at Boulder, Colorado and  
 Theodor W. Hänsch of Max-Planck Institute, Garching, Germany.  
 (www.nobel.org)

# What took so long?

- Laser introduced
  - Spectroscopy: CW laser
  - Nonlinearity: ultrafast community
- Late 1970's: spectroscopy using sub-picosecond
  - ~800 GHz "comb" that suffered huge frequency shift
- **Kerr-lens mode-locking** introduced in 1991
  - 60 fs pulses directly from Ti:S. But no one looked in frequency domain
- 1997, white light coherence of supercontinuum observed by Hänsch
- 1999, octave spanning spectrum with photonic crystal fiber
- 2004, intrinsic octave-spanning Ti:S



D.E. Spence, et.al., Opt. Lett. **16**, 42 (1991)

# The milestone note

(CONFIDENTIAL)

## Proposal for a universal optical frequency comb synthesizer

T. W. Hänsch  
*Max-Planck-Institut für Quantenoptik*

(March 30, 1997)

### Abstract

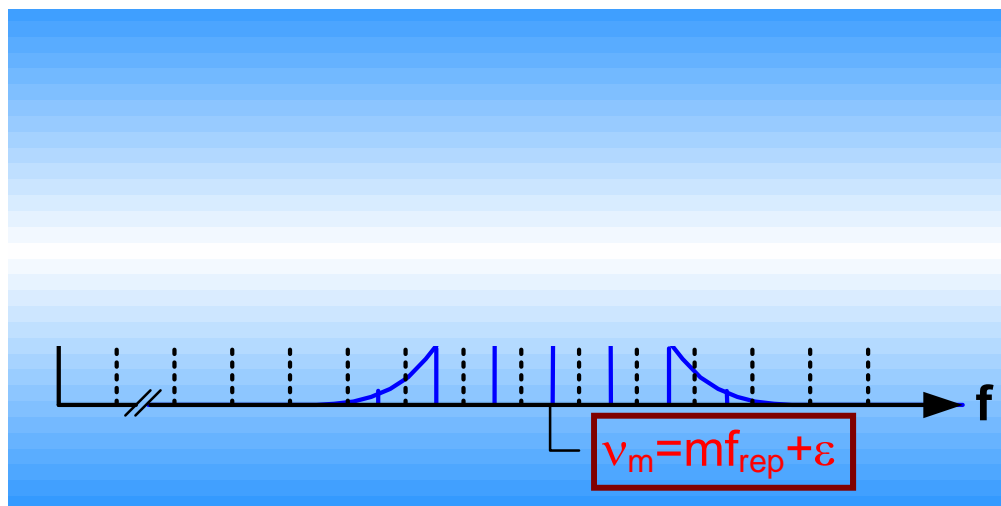
An optical frequency synthesizer is proposed which produces a wide comb of absolutely known equidistant marker frequencies throughout the infrared, visible, and ultraviolet spectral range. To this end, a white light continuum with pulse repetition rate  $f_p$  is produced by focusing the output of a mode-locked femtosecond laser into an optical fiber or bulk medium with a third order nonlinear susceptibility. The rate of phase slippage of the laser carrier relative to the pulse envelope  $f_s$  is monitored by observing a beat signal between the white light continuum and the second harmonic of the laser.

*read and understood*

*April 4, 1997 Martin Weitz*  
*April 4, 1997 T. Udem*

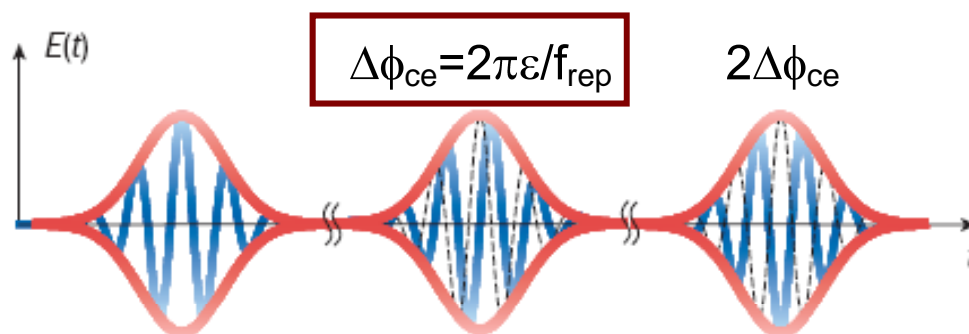
# Comb vs. Pulses

- Frequency-domain vs. time-domain
- Comb frequency offset directly linked to carrier-envelop phase!



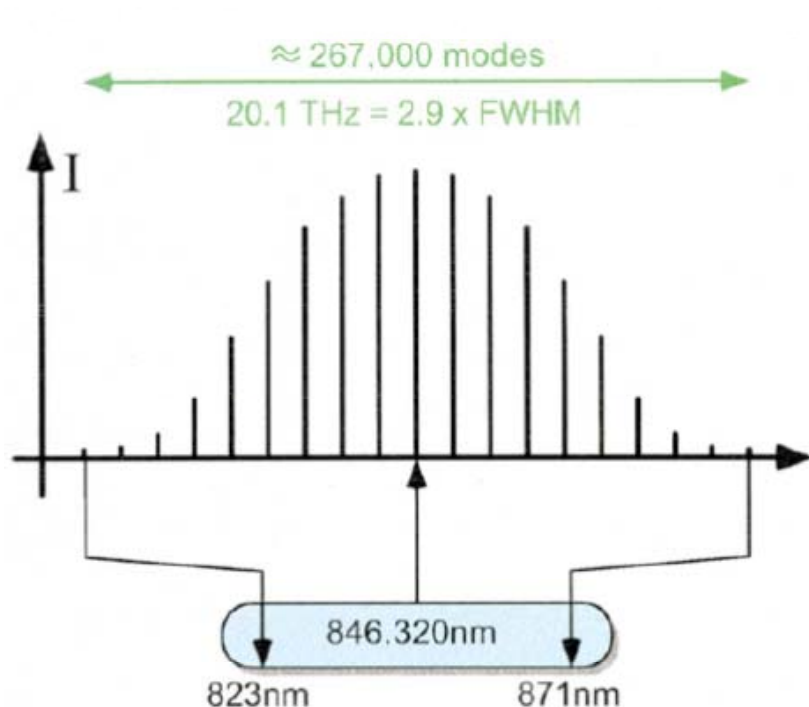
$$v_m = mf_{rep} + \epsilon$$

Let's derive the expression for carrier-envelop phase slip!

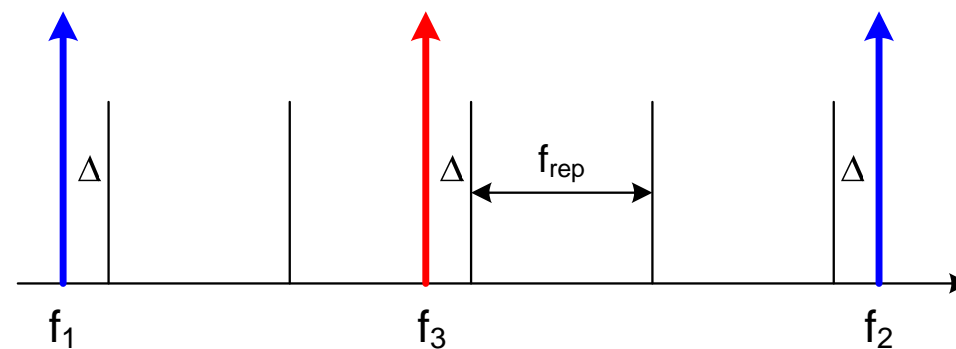


# Testing of comb spacing uniformity

- Experimental uniformity:  $3 \times 10^{-17}$ 
  - SFG of first and second diode lasers, SHG of third diode laser
  - Observing the beat of the intermediate line with third phase-locked laser



Interval divider

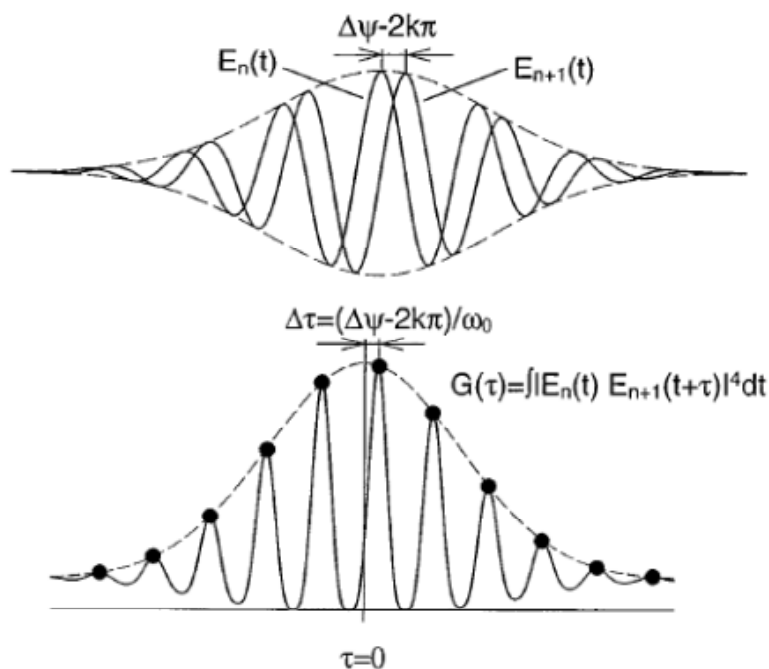


Can you think of other easy way?

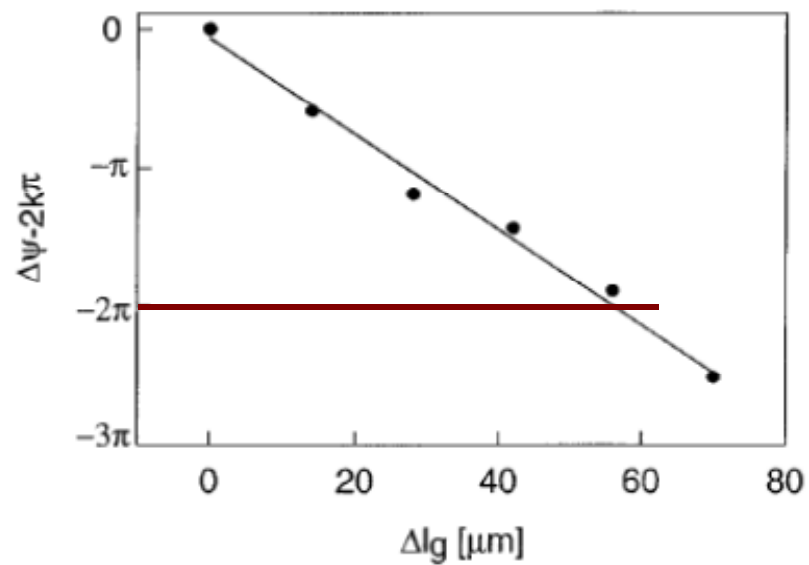
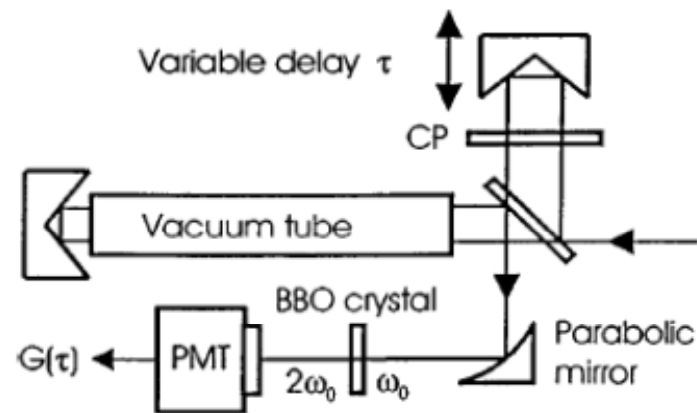


# $\Delta\Phi_{CE}$ measurement: cavity length control

- Silica wedge in the laser
- Interferometric intensity “cross”-correlation
  - Sub-10 fs Ti:S laser,  $f_{rep}=100$  MHz
  - Correlator dispersion compensated
  - Laser not stabilized



## Why vacuum tube?



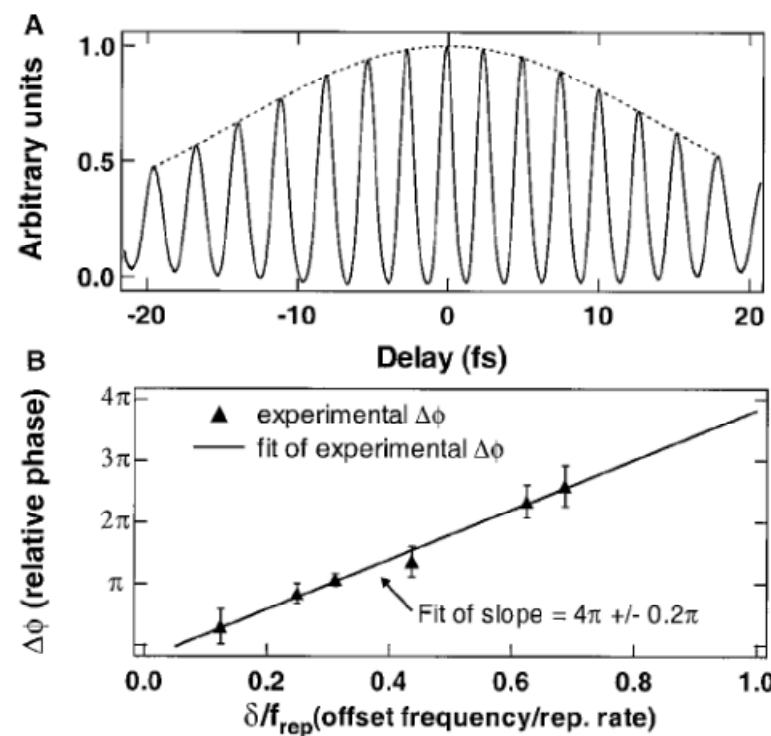
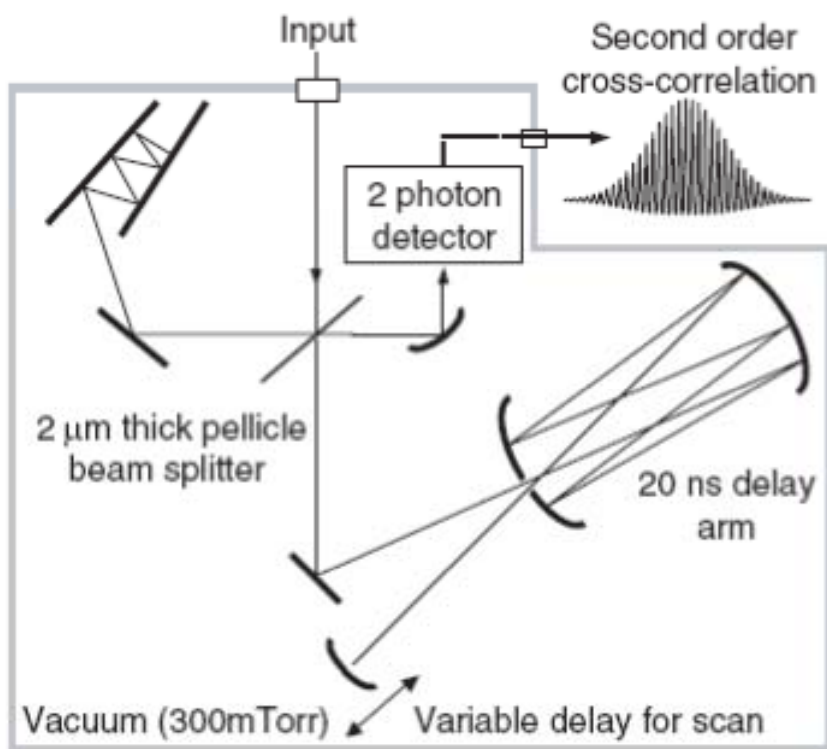
# $\Delta\Phi_{CE}$ measurement: offset frequency control

- Cross-correlation

i and i+2 pulse

Do you see problems?

$$\Delta\Phi_{CE} = 2 * 2\pi\epsilon / f_{rep}$$

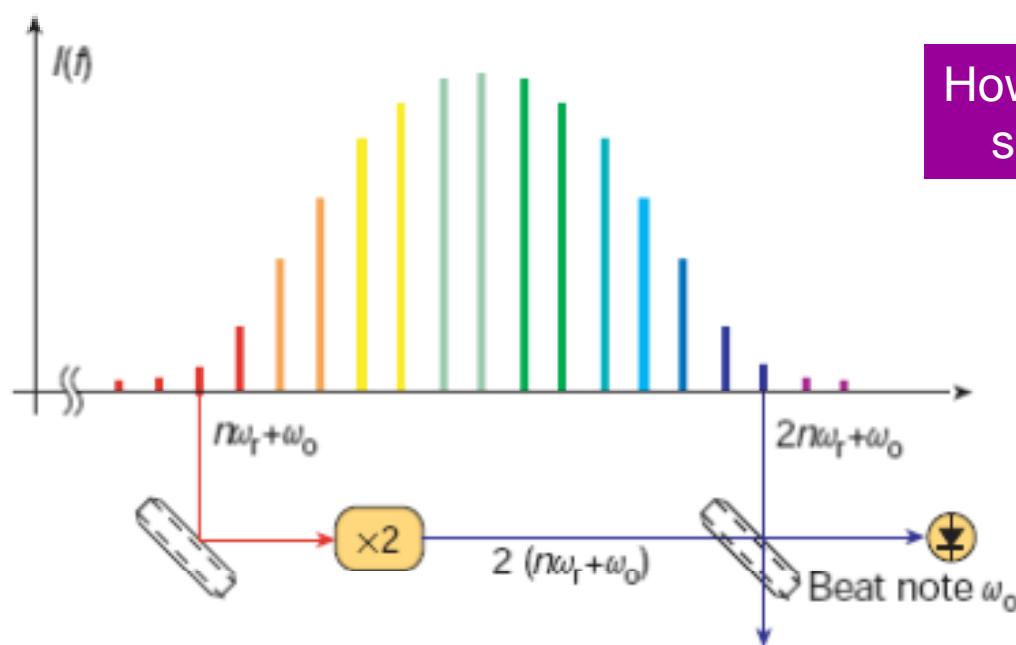


# Comb stabilization: principle

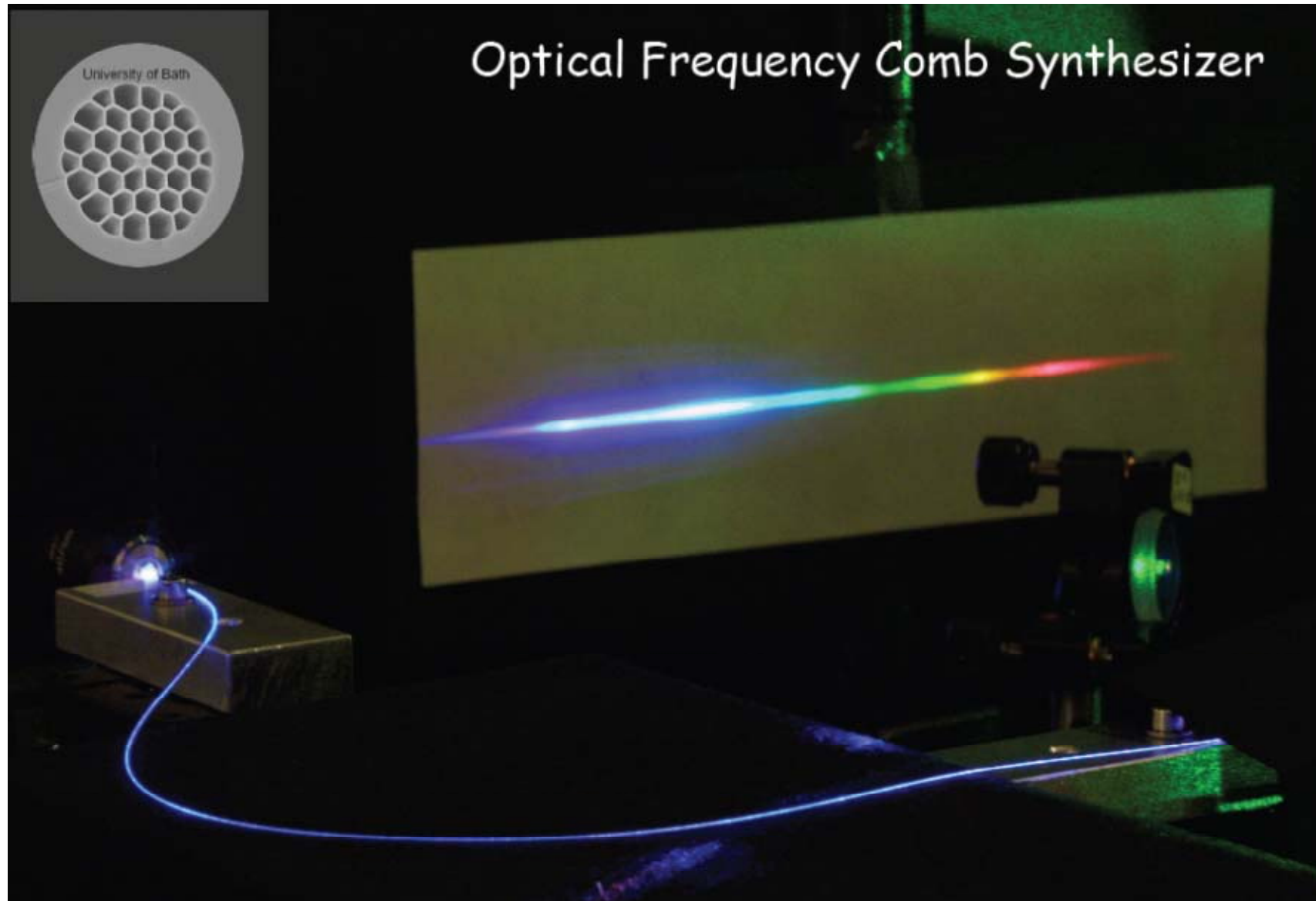
- You need to measure something before you can stabilize it!

$$\nu_m = m f_{rep} + \varepsilon$$

- Self-referencing (v-to-2v)



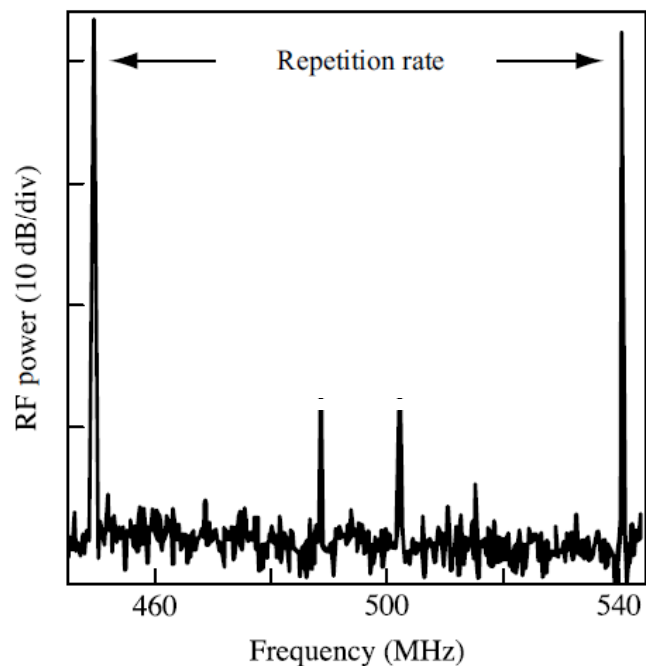
# Supercontinuum generation



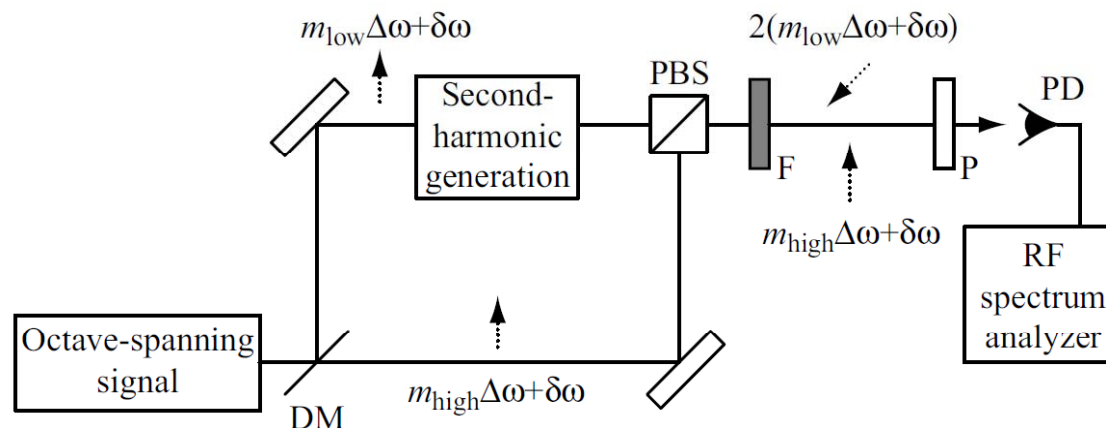
# $\epsilon$ measurement: self-referencing

- f-2f signal
- Why four signals?

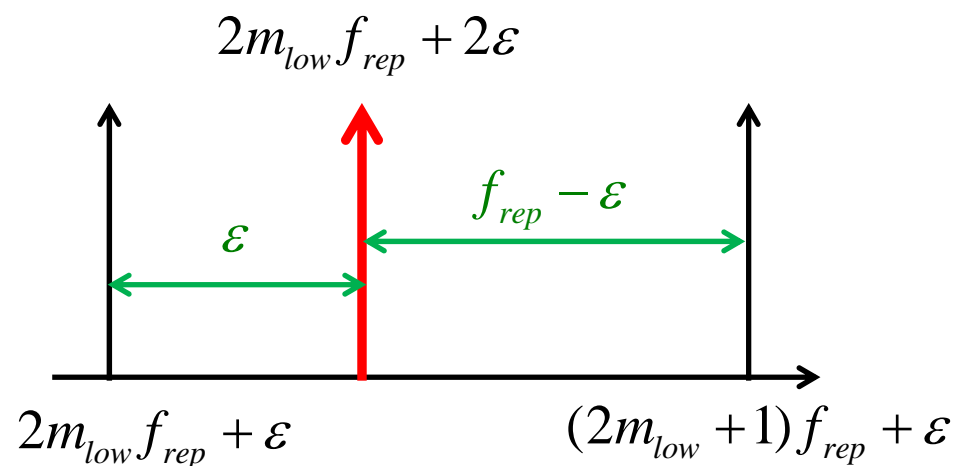
Think about how to determine the real  $\epsilon$ ?



S.T. Cundiff, J. Phys. D **35**, R43 (2002).



A. M. Weiner, *Ultrafast Optics* (Wiley, 2009)



# Comb stabilization: self-referencing

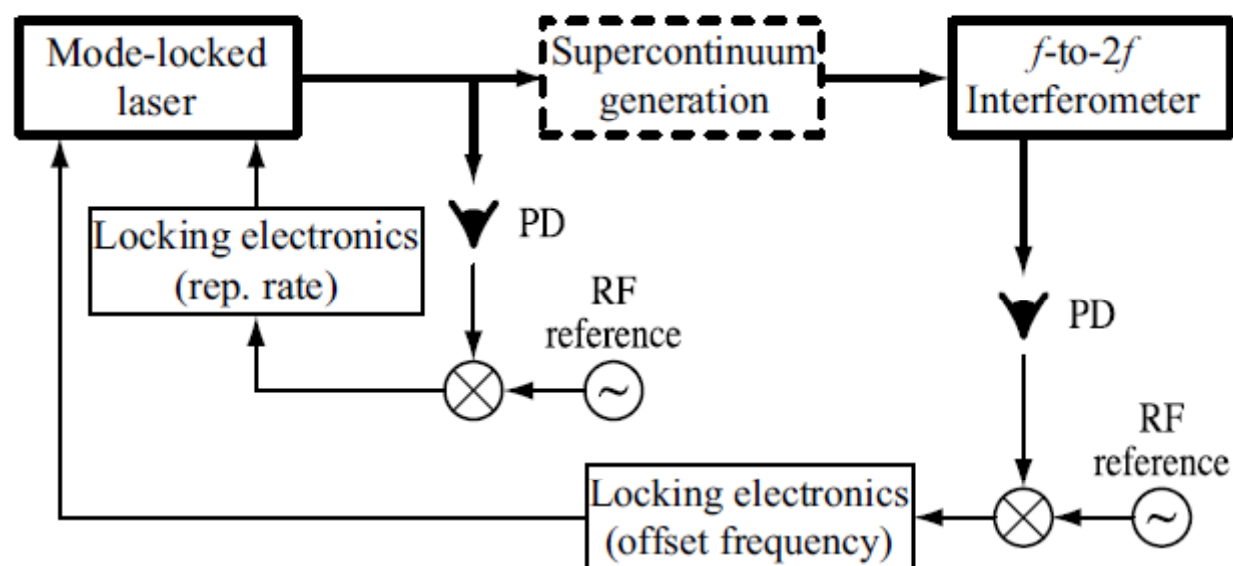
- Block diagram
- Ti:S with 2mm crystal
  - $\Delta\Phi_{CE} \sim 100\pi$
  - $\varepsilon \sim 50f_{rep}$

$$\nu_m(t) = mf_{rep}(t) + \varepsilon(t)$$

$$\nu_m(t) = mf_{rep} + \varepsilon(t)$$

$$\nu_m = mf_{rep} + \varepsilon$$

Can you think of other approaches?



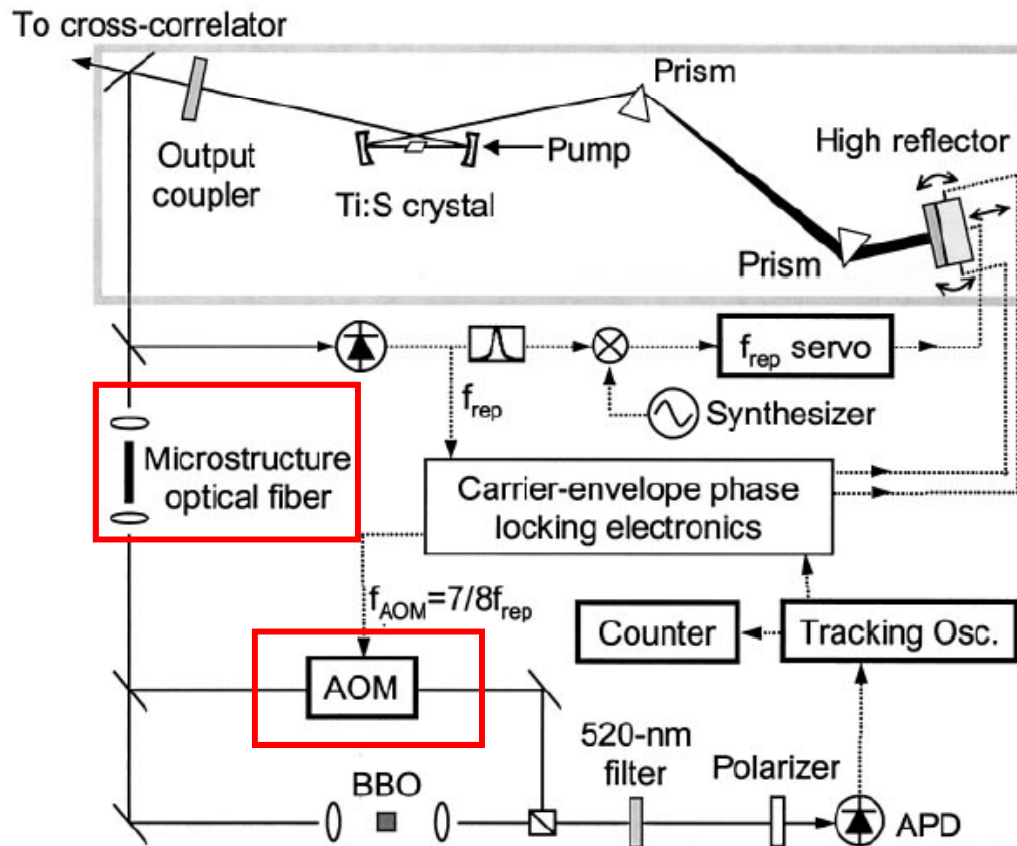
# Comb stabilization: controls

- Ti: Sapphire
  - End mirror tilt:  $\varepsilon, f_{\text{rep}}$
  - End mirror translation:  $f_{\text{rep}}$

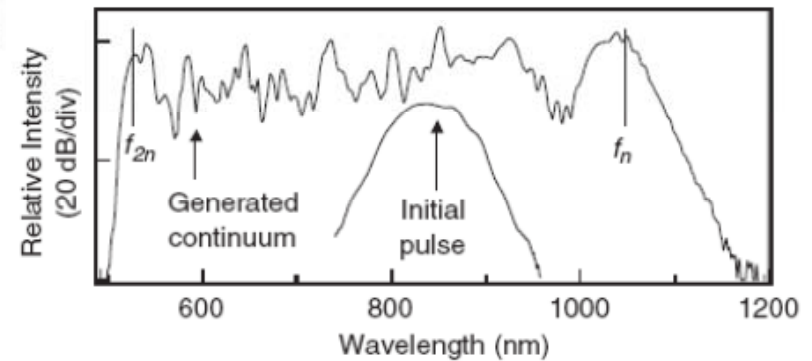
## Elastic tape model

J. Reichert, et.al., Opt. Comm. **172**, 59 (1999)

## AO modulator: stabilize and step-wise control of CEP



D.J. Jones, et.al., Science **288**, 635 (2000)



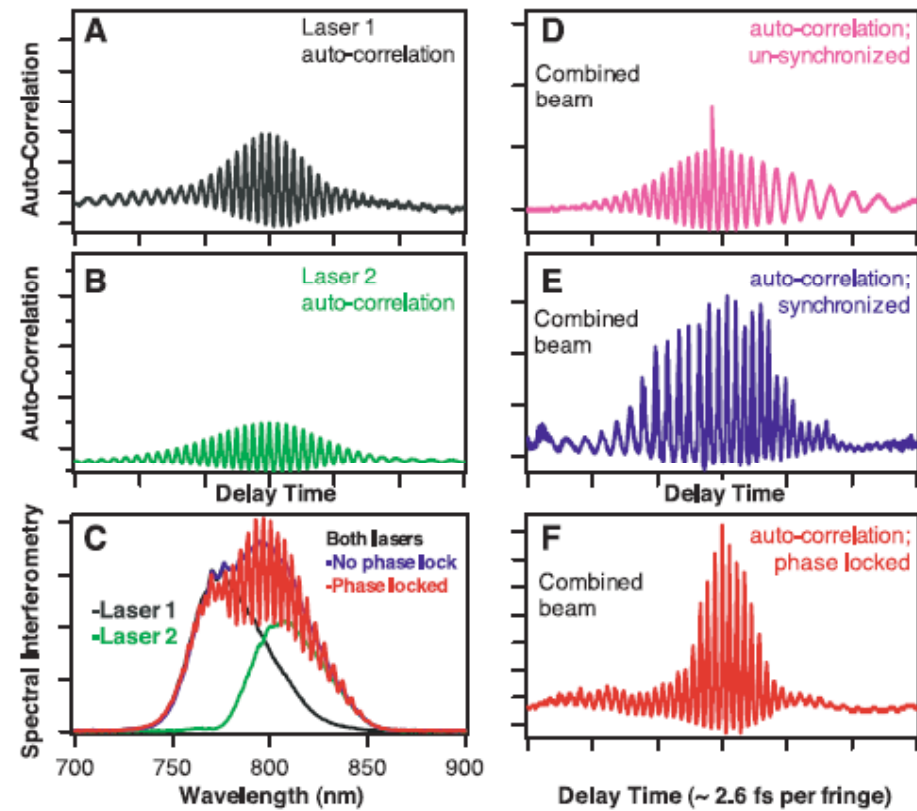
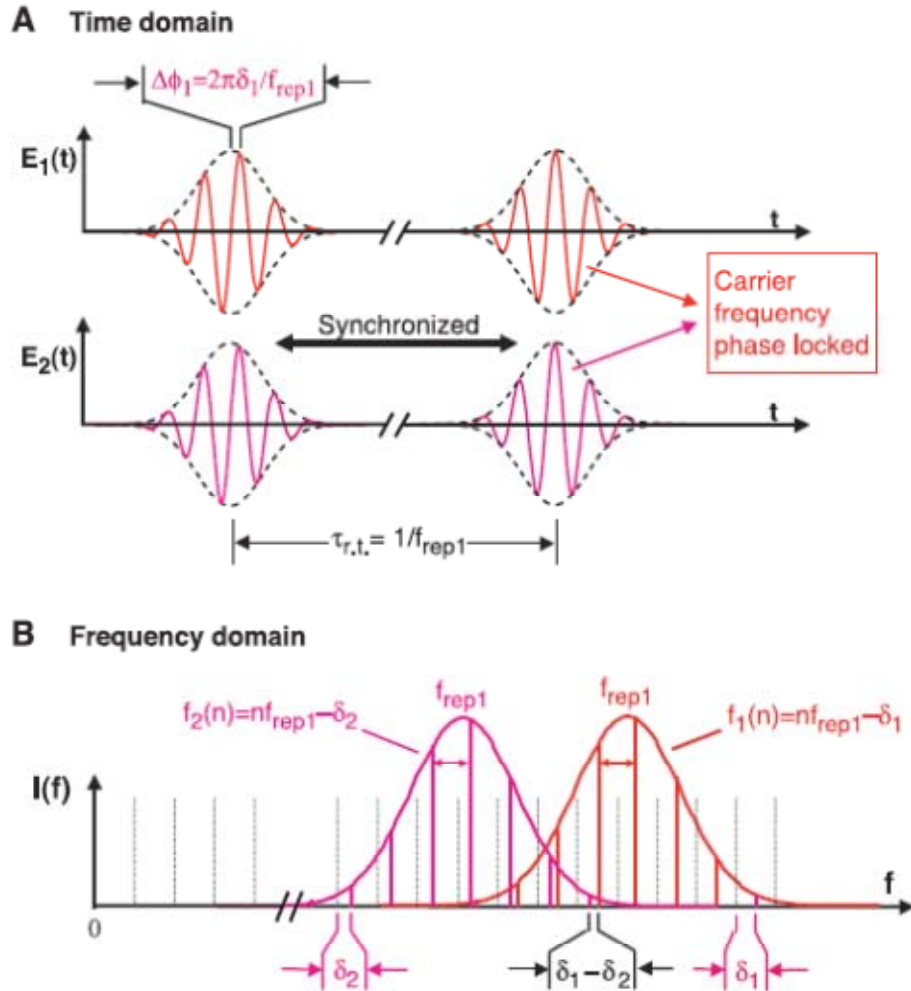
10fs, ~70nm

## Do you see a problem?

# Synchronization between two combs

- Comb stitching

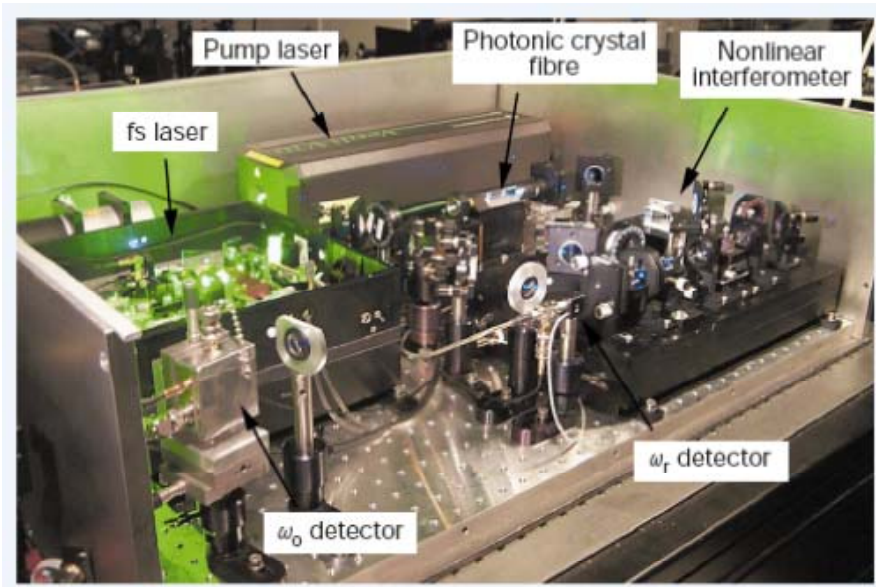
Check: A. Wirth, et.al, Science 334, 195 (2011)





# Comb and progress to octave spanning

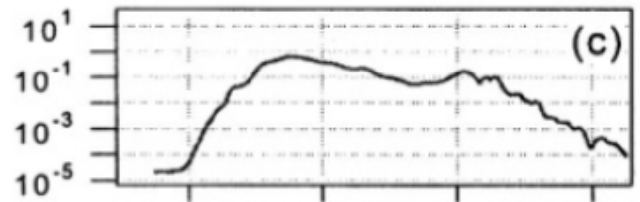
- Mode-locked Ti:Sapphire laser
  - Does not need octave 3-db bandwidth



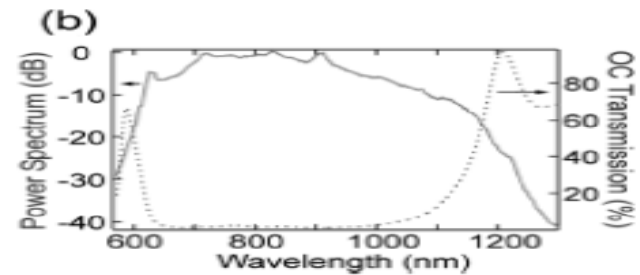
T. Udem, Holzwarth, Hänsch, Nature **416**, 233 (2002)

Extreme care in isolations

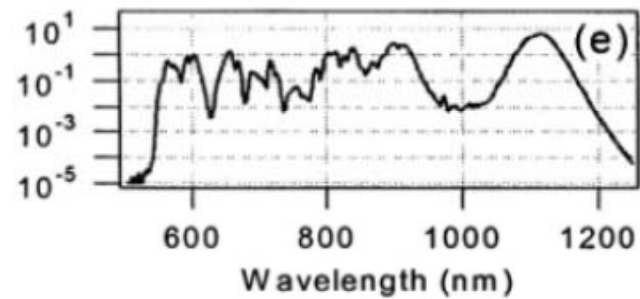
Output spectra



Chirped mirror, prism



Chirped mirror

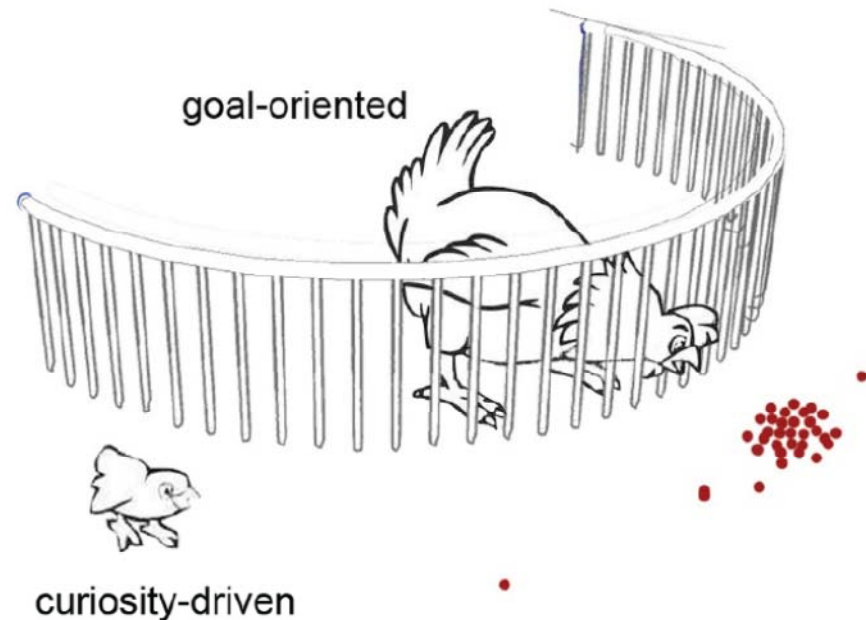


PCF

(c,e) Ye, Cundiff, *Femtosecond optical frequency comb: principle, operation and application* (Springer, 2005)  
 (b) Matos, et.al. Opt. Lett. **29**, 1683 (2004)

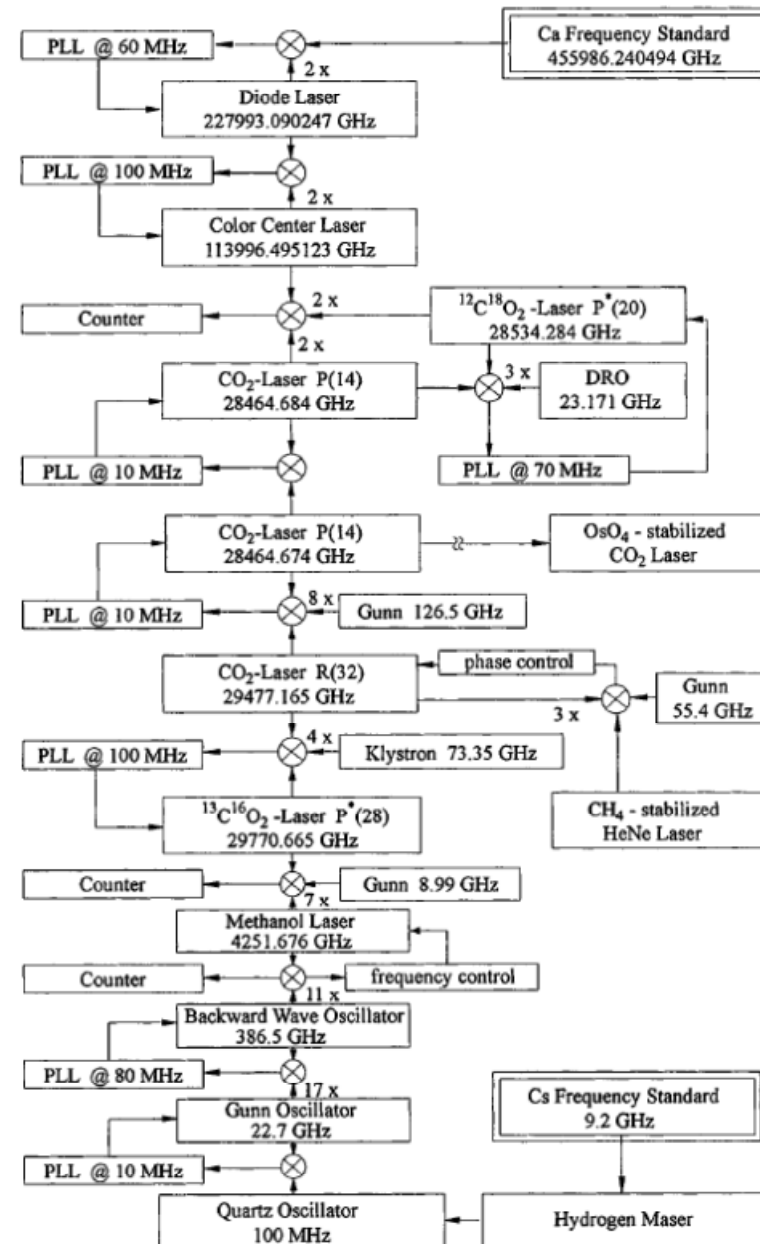
# Comb applications

- Metrology: 1978...
- Optical clock
- Synthesis of ultra-purity optical and RF frequencies
- Remote transfer of time/frequency standards
- Precision spectroscopy
- Feel free to dream!



# Frequency metrology: frequency chain

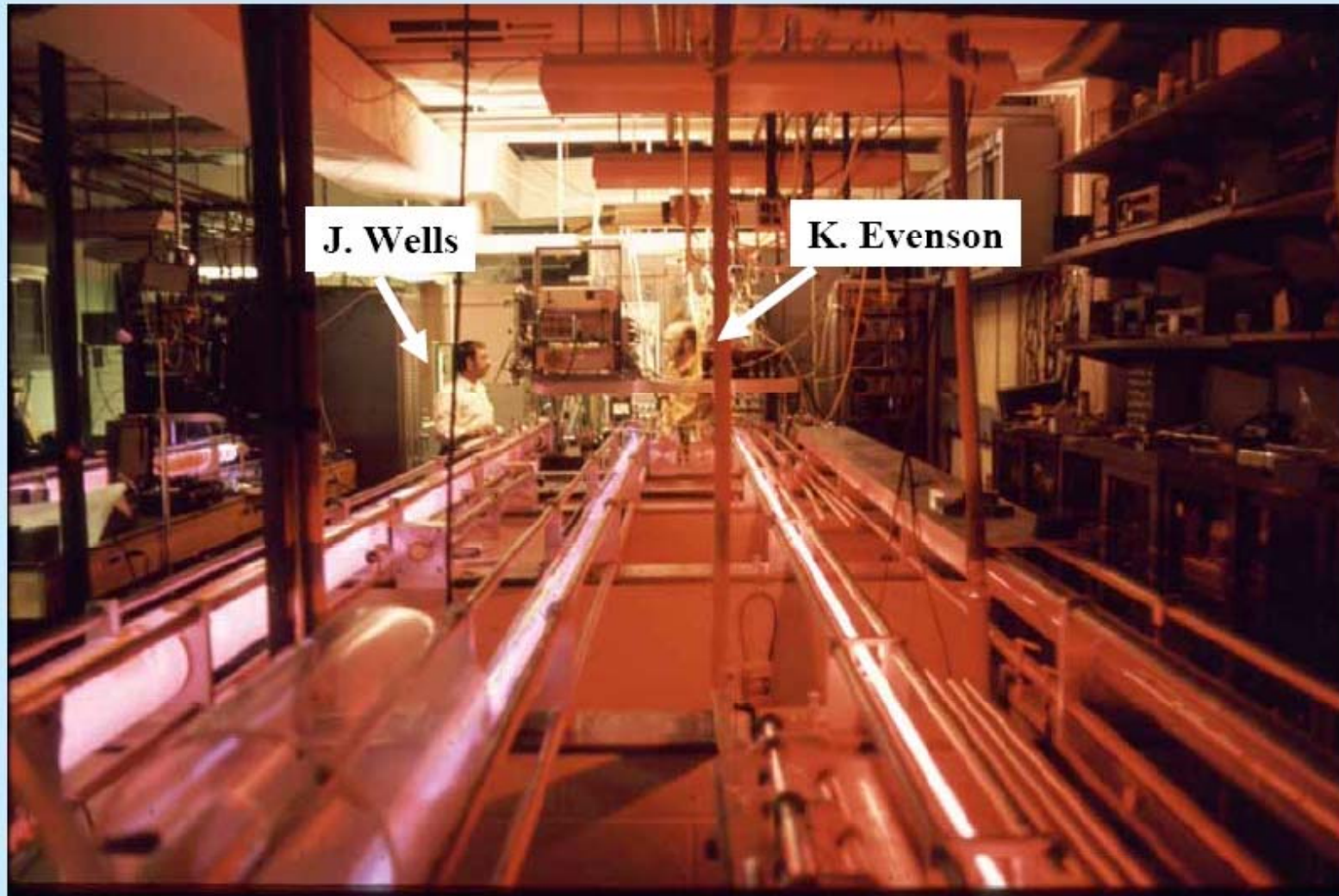
- Multiplicative up-conversion
  - Complicated
  - Expensive
  - Not affordable unless \$\$\$
  - Three labs, two buildings



H. Schnatz, et.al., Phys. Rev. Lett. **76**, 18 (1996)

# Optical frequency chain

NBS (NIST): measurement of speed of light, 1972

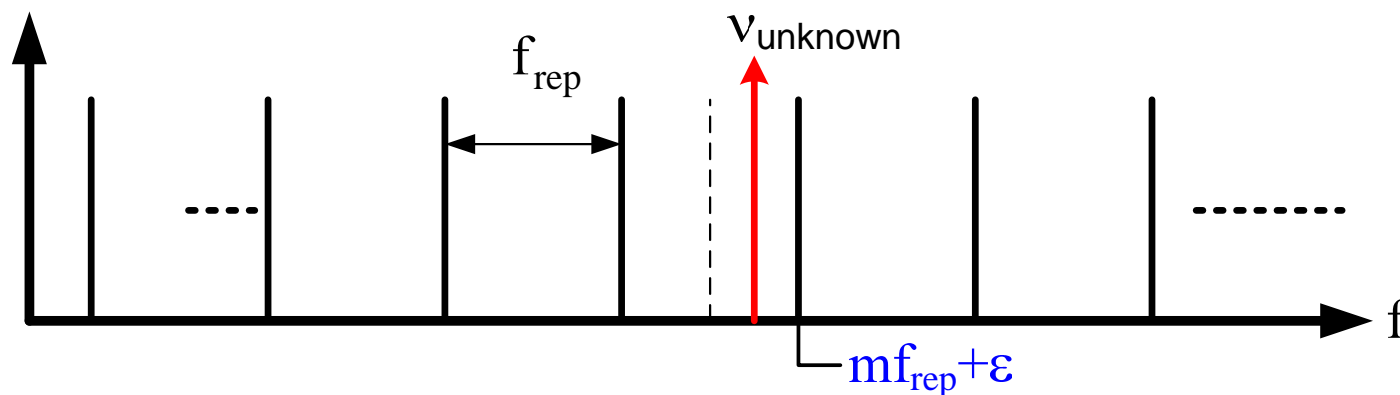


J. L. Hall & J. Ye, "NIST 100th birthday", *Optics & Photonics News* 12, 44, Feb. 2001

# Optical metrology: using combs

- How frequency “ruler” helps
  - Optical frequencies linked through 2 microwave frequencies
- Larger comb spacing better

$$\nu_m = m f_{rep} + \epsilon$$



$$\nu_{unknown} = m f_{rep} + \epsilon \pm f_{beat}$$

How to remove sign ambiguity?  
How to get the right  $m$ ?

# fs comb measured frequencies

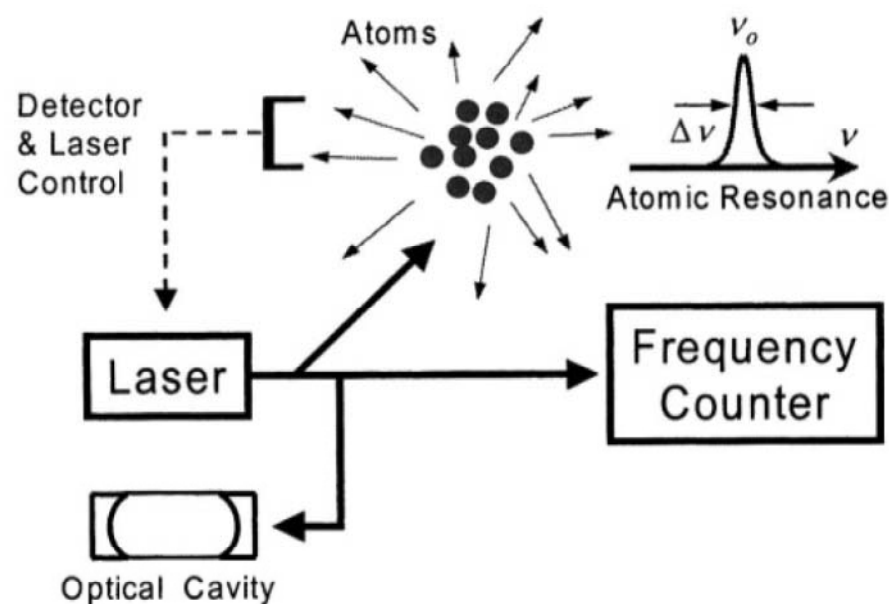
|                                 |        |                   |                       |
|---------------------------------|--------|-------------------|-----------------------|
| • Ca                            | 657 nm | Schnatz – PTB     | PRL 1 Jan '96         |
| • C <sub>2</sub> H <sub>2</sub> | 1.5 μm | Nakagawa - NRLM   | JOSA-B Dec '96        |
| • Sr <sup>+</sup>               | 674 nm | Bernard – NRC     | PRL 19 Apr '99        |
| • In <sup>+</sup>               | 236 nm | v. Zanthier - MPQ | Opt.Comm. Aug'99      |
| • H                             | 243 nm | Reichert - MPQ    | PRL 10 Apr '00        |
| • Rb                            | 778 nm | D. Jones - JILA   | Science 28 Apr 00     |
| • I <sub>2</sub>                | 532 nm | Diddams - JILA    | PRL 29 May '00        |
| • H                             | 243 nm | Niering - MPQ     | PRL 12 June '00       |
| • Yb <sup>+</sup>               | 467 nm | Roberts - NPL     | PRA 7 July '00        |
| • In <sup>+</sup>               | 236 nm | v. Zanthier – MPQ | Opt. Lett. 1 Dec.'00  |
| • Ca                            | 657 nm | Stenger – PTB     | PRA 17 Jan '01        |
| • Hg <sup>+</sup>               | 282 nm | Udem – NIST       | PRL 28 May '01        |
| • Ca                            | 657 nm | Udem – NIST       | PRL 28 May '01        |
| • Yb <sup>+</sup>               | 435 nm | Stenger – PTB     | Opt. Lett. 15 Oct '01 |

# Optical clock

- 1950's:  $^{133}\text{Cs}$  at 9,192,631,770 Hz
- 2000: optical clock
  - Narrow linewidth oscillator
    - Narrow quantum transition
    - Probe laser
  - Frequency counter
- Much better instability:  $10^5$  better ideally
- Shorten measurement time
- Choices
  - Atom (Ca, Sr, Mg, H...)
  - Ion ( $\text{Hg}^+$ ,  $\text{Yb}^+$ ,  $\text{In}^+$ ,  $\text{Sr}^+$ ...)
  - Molecule ( $\text{I}_2$ ,  $\text{CH}_4$ ,  $\text{CO}_2$ ...)

## Allan deviation

$$\sigma_y(\tau) = \frac{\Delta\nu}{\pi\nu_0} \sqrt{\frac{T}{2N\tau}}$$



# Periodic table



## PERIODIC TABLE OF THE ELEMENTS

<http://www.ktf-split.hr/periodni/en/>

| PERIOD | GROUP I IA                         | GROUP IIA                           | GROUP IIIA                          | GROUP IVA                               | GROUP VA                            | GROUP VIA                            | GROUP VIIA                          | GROUP VIIIA                         |                                      |                                     |                                      |                                     |  |                                    |                                    |                                     |                                   |                                  |
|--------|------------------------------------|-------------------------------------|-------------------------------------|---|-------------------------------------|--------------------------------------|-------------------------------------|-------------------------------------|--------------------------------------|-------------------------------------|--------------------------------------|-------------------------------------|--|------------------------------------|------------------------------------|-------------------------------------|-----------------------------------|----------------------------------|
| 1      | 1 1.0079<br><b>H</b><br>HYDROGEN   |                                     |                                     |   |                                     |                                      |                                     | 2 4.0026<br><b>He</b><br>HELIUM     |                                      |                                     |                                      |                                     |  |                                    |                                    |                                     |                                   |                                  |
| 2      | 3 6.941<br><b>Li</b><br>LITHIUM    | 4 9.0122<br><b>Be</b><br>BERYLLIUM  | 5 10.811<br><b>B</b><br>BORON       | 6 12.011<br><b>C</b><br>CARBON          | 7 14.007<br><b>N</b><br>NITROGEN    | 8 15.999<br><b>O</b><br>OXYGEN       | 9 18.998<br><b>F</b><br>FLUORINE    | 10 20.180<br><b>Ne</b><br>NEON      |                                      |                                     |                                      |                                     |  |                                    |                                    |                                     |                                   |                                  |
| 3      | 11 22.990<br><b>Na</b><br>SODIUM   | 12 24.305<br><b>Mg</b><br>MAGNESIUM | 13 26.982<br><b>Al</b><br>ALUMINIUM | 14 28.086<br><b>Si</b><br>SILICON       | 15 30.974<br><b>P</b><br>PHOSPHORUS | 16 32.065<br><b>S</b><br>SULPHUR     | 17 35.453<br><b>Cl</b><br>CHLORINE  | 18 39.948<br><b>Ar</b><br>ARGON     |                                      |                                     |                                      |                                     |  |                                    |                                    |                                     |                                   |                                  |
| 4      | 19 39.098<br><b>K</b><br>POTASSIUM | 20 40.078<br><b>Ca</b><br>CALCIUM   | 21 44.956<br><b>Sc</b><br>SCANDIUM  | 22 47.867<br><b>Ti</b><br>TITANIUM      | 23 50.942<br><b>V</b><br>VANADIUM   | 24 51.996<br><b>Cr</b><br>CHROMIUM   | 25 54.938<br><b>Mn</b><br>MANGANESE | 26 55.845<br><b>Fe</b><br>IRON      | 27 58.933<br><b>Co</b><br>COBALT     | 28 58.693<br><b>Ni</b><br>NICKEL    | 29 63.546<br><b>Cu</b><br>COPPER     | 30 65.39<br><b>Zn</b><br>ZINC       | 31 69.723<br><b>Ga</b><br>GALLIUM      | 32 72.64<br><b>Ge</b><br>GERMANIUM | 33 74.922<br><b>As</b><br>ARSENIC  | 34 78.96<br><b>Se</b><br>SELENIUM   | 35 79.904<br><b>Br</b><br>BROMINE | 36 83.80<br><b>Kr</b><br>KRYPTON |
| 5      | 37 85.468<br><b>Rb</b><br>RUBIDIUM | 38 87.62<br><b>Sr</b><br>STRONTIUM  | 39 88.906<br><b>Y</b><br>YTRIUM     | 40 91.224<br><b>Zr</b><br>ZIRCONIUM     | 41 92.906<br><b>Nb</b><br>NIOBIUM   | 42 95.94<br><b>Mo</b><br>MOLYBDENUM  | 43 (98)<br><b>Tc</b><br>TECHNETIUM  | 44 101.07<br><b>Ru</b><br>RUTHENIUM | 45 102.91<br><b>Rh</b><br>RHODIUM    | 46 106.42<br><b>Pd</b><br>PALLADIUM | 47 107.87<br><b>Ag</b><br>SILVER     | 48 112.41<br><b>Cd</b><br>CADMIUM   | 49 114.82<br><b>In</b><br>INDIUM       | 50 118.71<br><b>Sn</b><br>TIN      | 51 121.76<br><b>Sb</b><br>ANTIMONY | 52 127.60<br><b>Te</b><br>TELLURIUM | 53 126.90<br><b>I</b><br>IODINE   | 54 131.29<br><b>Xe</b><br>XENON  |
| 6      | 55 132.91<br><b>Cs</b><br>CAESIUM  | 56 137.33<br><b>Ba</b><br>BARIUM    | 57-71<br><b>La-Lu</b><br>Lanthanide | 72 178.49<br><b>Hf</b><br>HAFNIUM       | 73 180.95<br><b>Ta</b><br>TANTALUM  | 74 183.84<br><b>W</b><br>TUNGSTEN    | 75 186.21<br><b>Re</b><br>RHENIUM   | 76 190.23<br><b>Os</b><br>OSMIUM    | 77 192.22<br><b>Ir</b><br>IRIDIUM    | 78 195.08<br><b>Pt</b><br>PLATINUM  | 79 196.97<br><b>Au</b><br>GOLD       | 80 200.59<br><b>Hg</b><br>MERCURY   | 81 204.38<br><b>Tl</b><br>THALLIUM     | 82 207.2<br><b>Pb</b><br>LEAD      | 83 208.98<br><b>Bi</b><br>BISMUTH  | 84 (209)<br><b>Po</b><br>POLONIUM   | 85 (210)<br><b>At</b><br>ASTATINE | 86 (222)<br><b>Rn</b><br>RADON   |
| 7      | 87 (223)<br><b>Fr</b><br>FRANCIUM  | 88 (226)<br><b>Ra</b><br>RADIUM     | 89-103<br><b>Ac-Lr</b><br>Actinide  | 104 (261)<br><b>Rf</b><br>RUTHERFORDIUM | 105 (262)<br><b>Db</b><br>DUBNIUM   | 106 (266)<br><b>Sg</b><br>SEABORGIUM | 107 (264)<br><b>Bh</b><br>BOHRIUM   | 108 (277)<br><b>Hs</b><br>HASSIUM   | 109 (268)<br><b>Mt</b><br>MEITNERIUM | 110 (281)<br><b>Uun</b><br>UNUNNIUM | 111 (272)<br><b>Uuu</b><br>UNUNUNIUM | 112 (285)<br><b>Uub</b><br>UNUNBIUM | 114 (289)<br><b>Uuq</b><br>UNUNQUADIUM |                                    |                                    |                                     |                                   |                                  |

Legend for element classification:

- Metal
- Semimetal
- Nonmetal

Classification by group:

- 1 Alkali metal
- 2 Alkaline earth metal
- 13-10 Transition metals
- 16 Chalcogens element
- 17 Halogens element
- 18 Noble gas

Other categories:

- Lanthanide
- Actinide

STANDARD STATE (25 °C; 101 kPa)

- Ne** - gas
- Ga** - liquid
- Fe** - solid
- Tc** - synthetic

### LANTHANIDE

|                                     |                                  |  |                                     |                                     |                                    |                                    |                                      |                                   |                                      |                                   |                                  |                                   |                                    |                                    |
|-------------------------------------|----------------------------------|--|-------------------------------------|-------------------------------------|------------------------------------|------------------------------------|--------------------------------------|-----------------------------------|--------------------------------------|-----------------------------------|----------------------------------|-----------------------------------|------------------------------------|------------------------------------|
| 57 138.91<br><b>La</b><br>LANTHANUM | 58 140.12<br><b>Ce</b><br>CERIUM | 59 140.91<br><b>Pr</b><br>PRASEODYMIUM | 60 144.24<br><b>Nd</b><br>NEODYMIUM | 61 (145)<br><b>Pm</b><br>PROMETHIUM | 62 150.36<br><b>Sm</b><br>SAMARIUM | 63 151.96<br><b>Eu</b><br>EUROPIUM | 64 157.25<br><b>Gd</b><br>GADOLINIUM | 65 158.93<br><b>Tb</b><br>TERBIUM | 66 162.50<br><b>Dy</b><br>DYSPROSIUM | 67 164.93<br><b>Ho</b><br>HOLMIUM | 68 167.26<br><b>Er</b><br>ERBIUM | 69 168.93<br><b>Tm</b><br>THULIUM | 70 173.04<br><b>Yb</b><br>YTTERIUM | 71 174.97<br><b>Lu</b><br>LUTETIUM |
|-------------------------------------|----------------------------------|--|-------------------------------------|-------------------------------------|------------------------------------|------------------------------------|--------------------------------------|-----------------------------------|--------------------------------------|-----------------------------------|----------------------------------|-----------------------------------|------------------------------------|------------------------------------|

### ACTINIDE

|                                   |                                   |  |                                  |                                    |                                    |                                    |                                 |                                    |                                      |                                      |                                   |                                       |                                    |                                      |
|-----------------------------------|-----------------------------------|--|----------------------------------|------------------------------------|------------------------------------|------------------------------------|---------------------------------|------------------------------------|--------------------------------------|--------------------------------------|-----------------------------------|---------------------------------------|------------------------------------|--------------------------------------|
| 89 (227)<br><b>Ac</b><br>ACTINIUM | 90 232.04<br><b>Th</b><br>THORIUM | 91 231.04<br><b>Pa</b><br>PROTACTINIUM | 92 238.03<br><b>U</b><br>URANIUM | 93 (237)<br><b>Np</b><br>NEPTUNIUM | 94 (244)<br><b>Pu</b><br>PLUTONIUM | 95 (243)<br><b>Am</b><br>AMERICIUM | 96 (247)<br><b>Cm</b><br>CURIUM | 97 (247)<br><b>Bk</b><br>BERKELIUM | 98 (251)<br><b>Cf</b><br>CALIFORNIUM | 99 (252)<br><b>Es</b><br>EINSTEINIUM | 100 (257)<br><b>Fm</b><br>FERMIUM | 101 (258)<br><b>Md</b><br>MENDELEVIUM | 102 (259)<br><b>No</b><br>NOBELIUM | 103 (262)<br><b>Lr</b><br>LAWRENCIUM |
|-----------------------------------|-----------------------------------|--|----------------------------------|------------------------------------|------------------------------------|------------------------------------|---------------------------------|------------------------------------|--------------------------------------|--------------------------------------|-----------------------------------|---------------------------------------|------------------------------------|--------------------------------------|

(1) Pure Appl. Chem., 73, No. 4, 667-683 (2001)  
Relative atomic mass is shown with five significant figures. For elements having no stable nuclides, the value enclosed in brackets indicates the mass number of the longest-lived isotope of the element.  
However three such elements (Th, Pa, and U) do have a characteristic terrestrial isotopic composition, and for these an atomic weight is tabulated.

Editor: Aditya Vardhan (adivard@netlinx.com)

For more information and downloads please visit ---> <http://www.periodni.com/en/download.html>



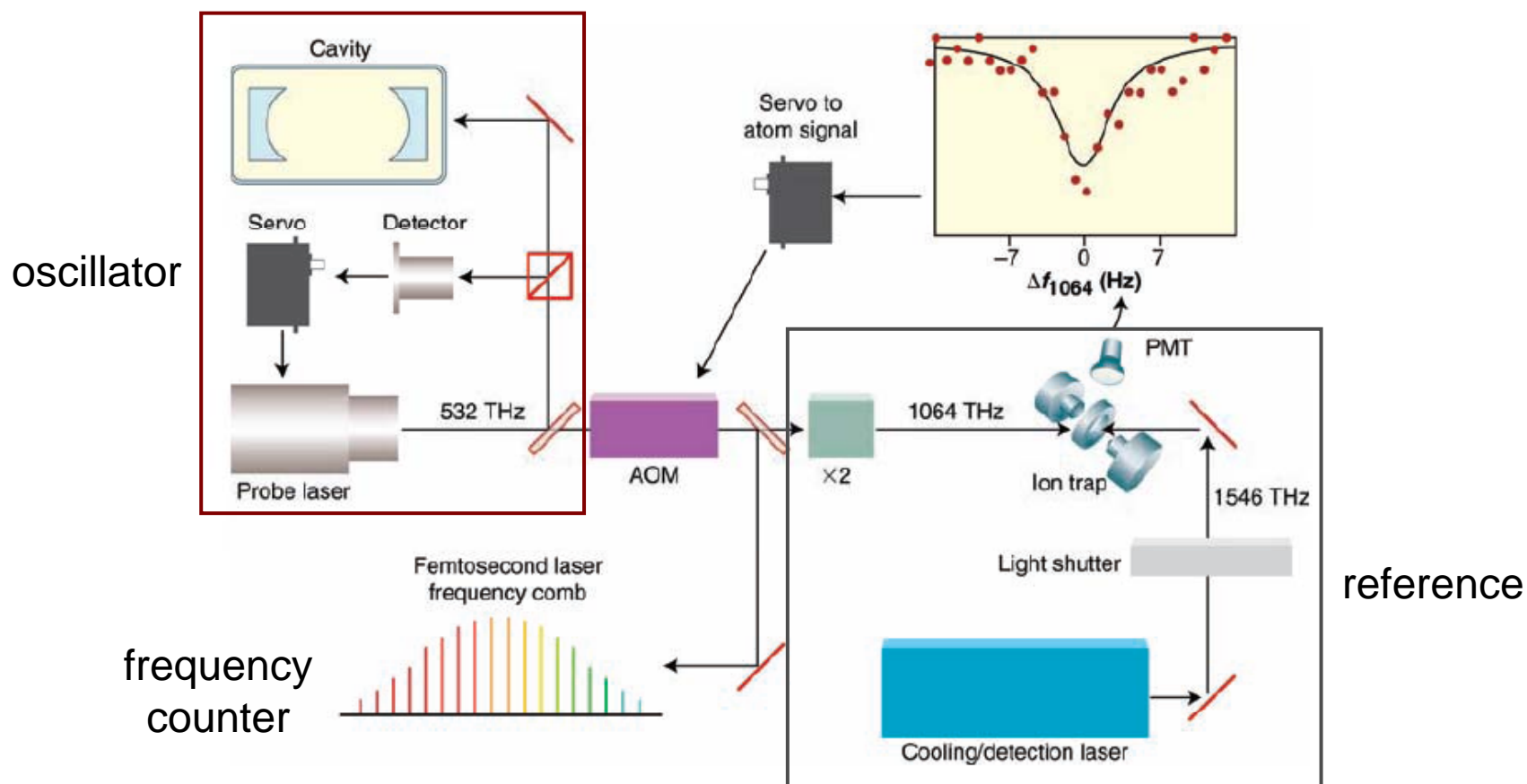
## Molecular frequency standards ~1997

- HeNe Laser w  $\text{CH}_4$  Absorber 3.39  $\mu\text{m}$
- HeNe vis Laser w  $\text{I}_2$  Absorber ~5 vis  $\lambda$ 's
- $\text{CO}_2$  Laser w  $\text{CO}_2$  Absorber 10.6  $\mu\text{m}$
- $\text{CO}_2$  Laser w  $\text{OsO}_4$  Absorber 10.6  $\mu\text{m}$
- $\text{Ar}^+$  Laser w  $\text{I}_2$  Absorber 514 nm
- Nd:YAG Laser w  $\text{I}_2$  Absorber 1064 nm
- Nd:YAG Laser w  $\text{C}_2\text{HD}$  Abs. 1064 nm
- Yb:YAG Laser w  $\text{C}_2\text{H}_2$  Abs. 1030 nm
- Diode Lasers w  $\text{C}_2\text{H}_2$  Abs. 1550 nm

# Hg<sup>+</sup> optical clock

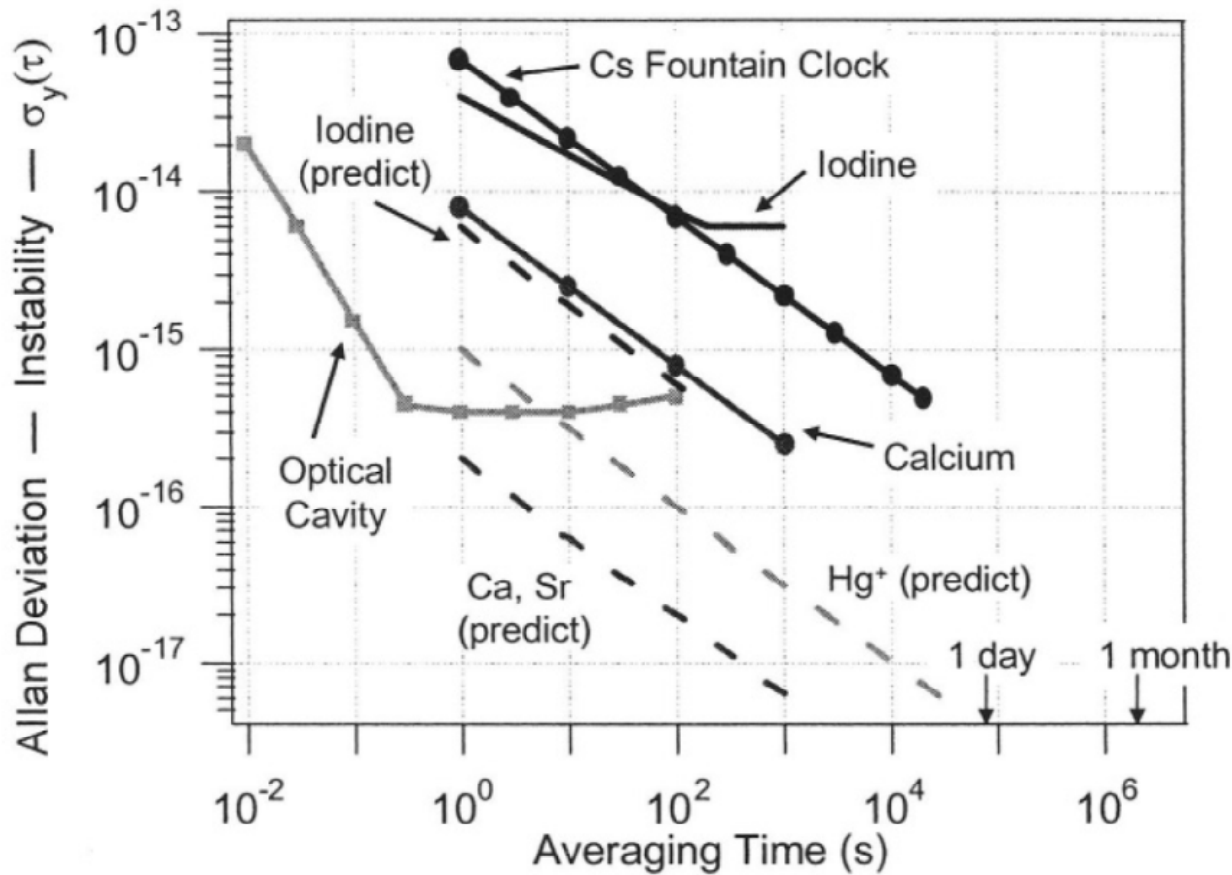
- Cooled ion that provides atomic transition (282 nm)
- Stabilized probe laser that provides the “clock tick”
- Optical frequency comb that provides an “optical” frequency divider

$$Q = \nu_0 / \Delta\nu \sim 10^{14}$$



# Optical clocks: some results

- Striking improvements



$$f_{beat} = mf_{rep} + \varepsilon - \nu_{opt}$$

locked

$$f_{rep} = \frac{\nu_{opt} + f_{beat} - \varepsilon}{m}$$

Why can't we just use the optical cavity?

# Low-noise frequency synthesis



- RF → optical
- RF → RF
- Optical → optical
- **Optical → RF**

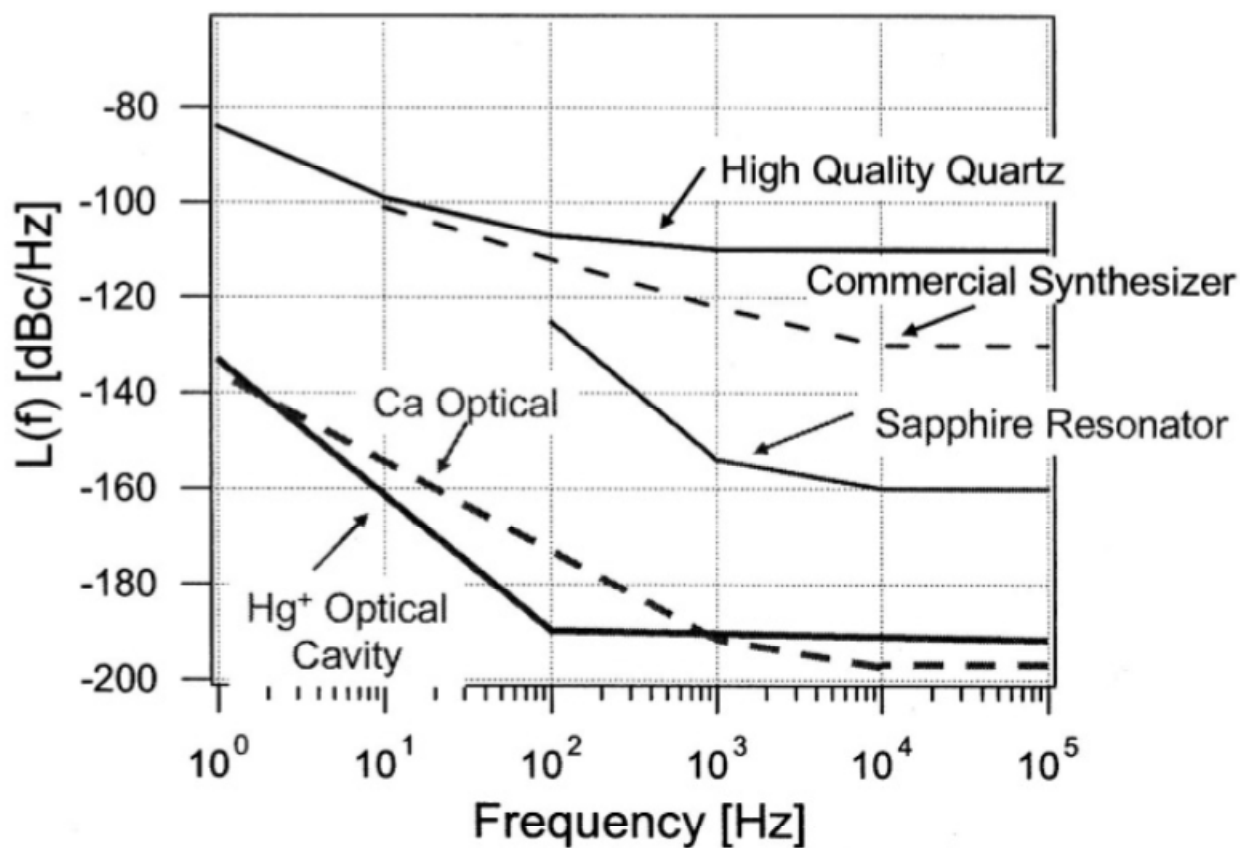
$$f_{rep} = \frac{\nu_{opt} + f_{beat} - \epsilon}{m}$$

Extremely low phase noise RF signals

# Phase noise issue

- Transfer of RF reference onto optical: disastrous
- Optical reference a better choice

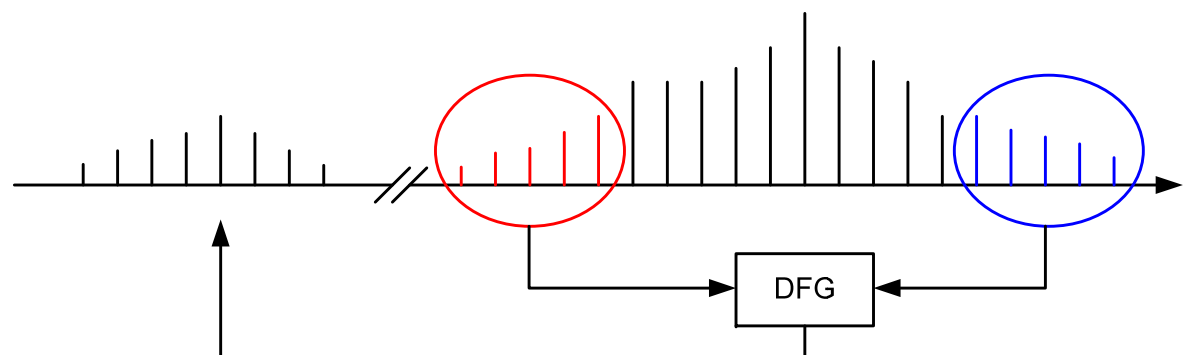
$$L(f) \propto m^2$$



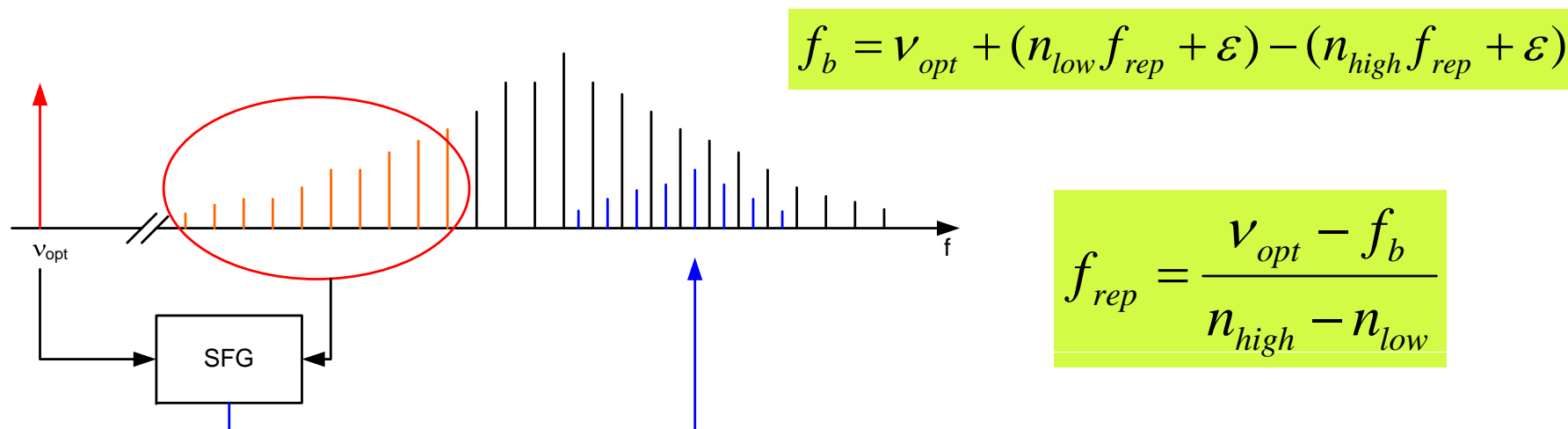
Ye, Cundiff, *Femtosecond optical frequency comb: principle, operation and application* (Springer, 2005)

# Removing $\varepsilon$ dependence

- Difference frequency generation



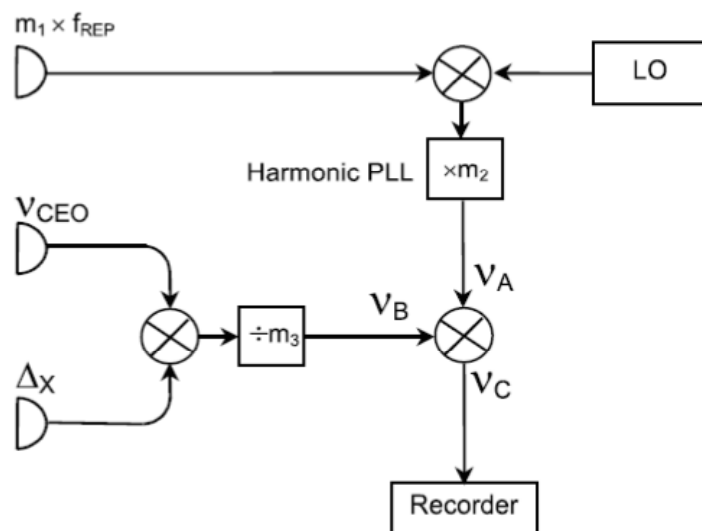
- Sum frequency generation with an optical standard



# Comb as transfer oscillator

- Comb can be un-stabilized!
- Ratio of widely separated optical frequencies or microwave

## Linking optical and microwave frequencies

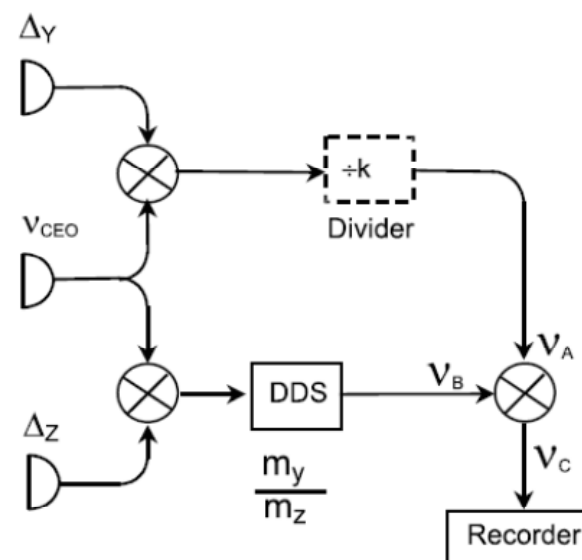


reference

$$f_{lo} = \frac{f_c}{m_2} + \frac{v_{opt}}{m_2 m_3}$$

Detected signals independent of laser noise properties!

## Linking two optical frequencies

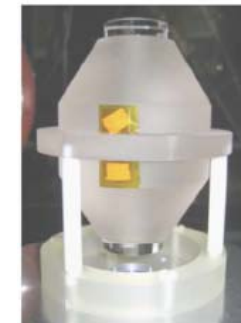
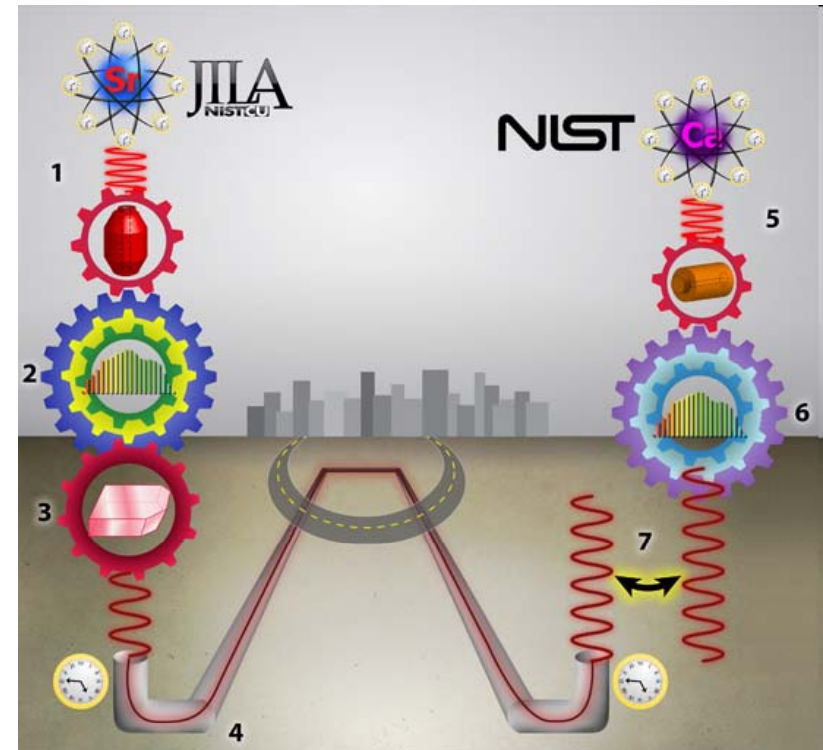
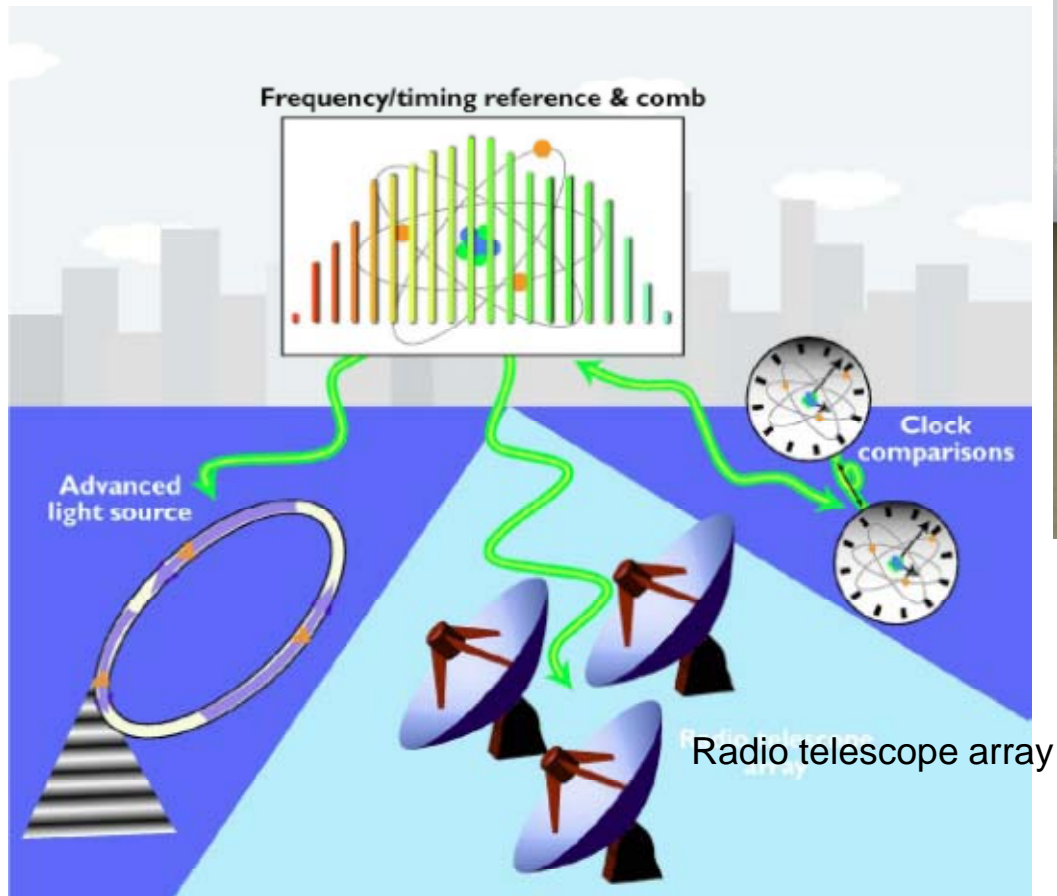


reference

$$v_y = f_c + \frac{m_y}{m_z} v_z$$

# Remote transfer

- Fiber as coherence transfer media



S.M. Foreman, et. al., Rev. Sci. Instr. **78**, 021101 (2007).



# Comb alternatives

---

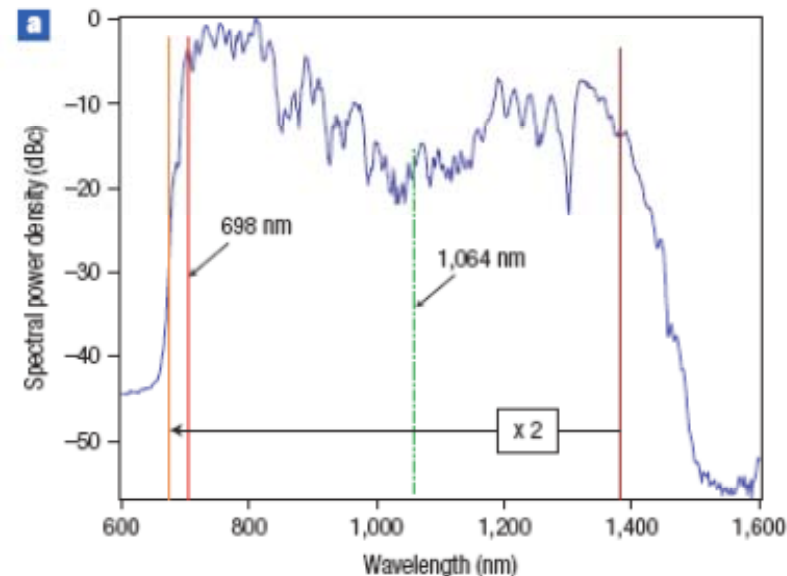
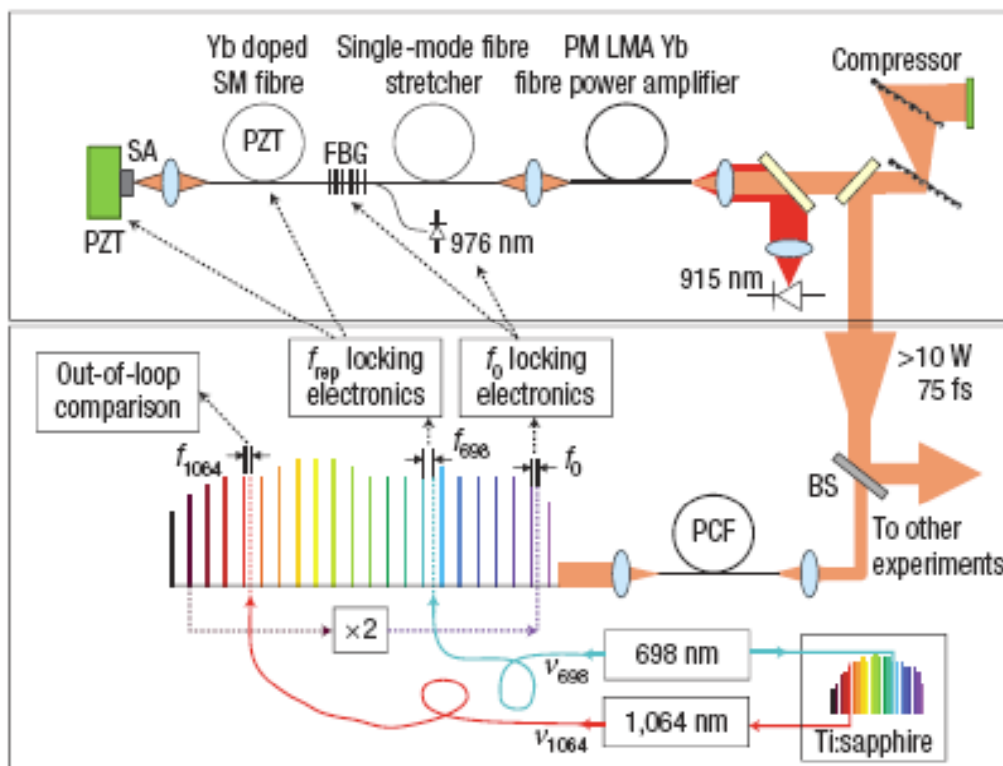
- Fiber laser combs
- Externally-modulated CW comb
  - Simple phase modulator
  - Dual-electrode intensity modulator
- Optical frequency comb generator
  - Actually dominated large-span optical frequency difference measurements during 1993-1998
- Compact comb generator using micro-toroid

# Fiber laser combs

- Compact, high power (>10 W)
- Different wavelength windows
  - Er (1.5 $\mu\text{m}$ ), Yb (1.05 $\mu\text{m}$ )

Cladding pumped CPA linear amplification avoids nonlinearity induced phase and amplitude noise

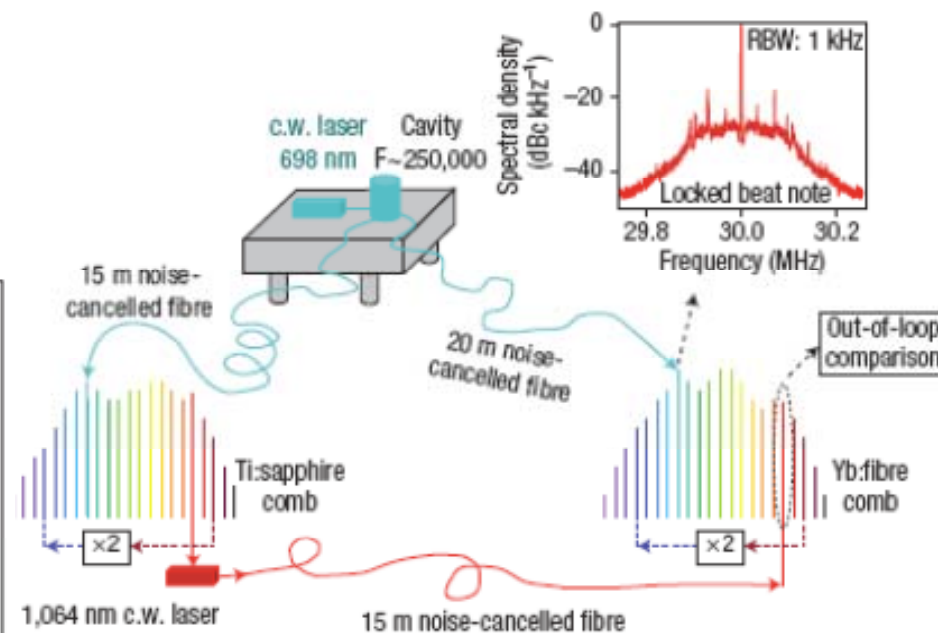
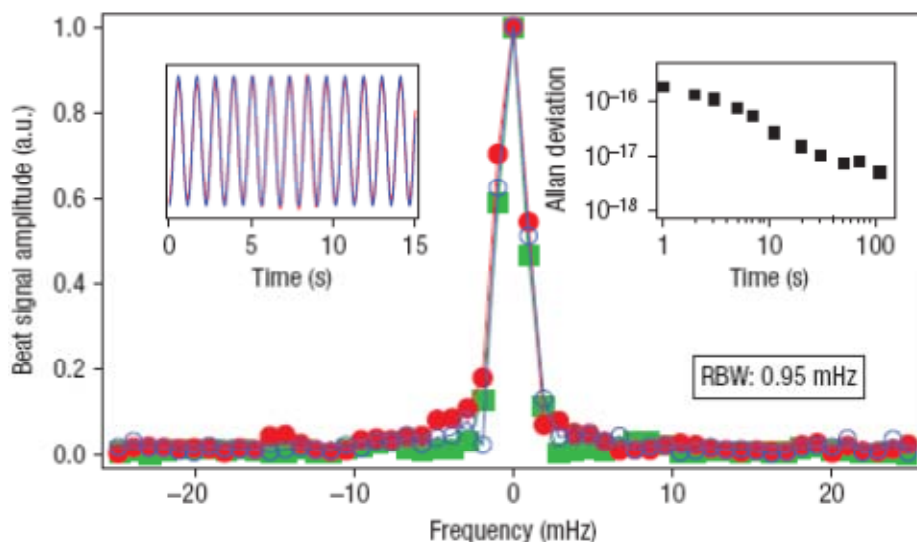
Best phase stability ever reported



698 nm: LD locked to Sr optical lattice clock

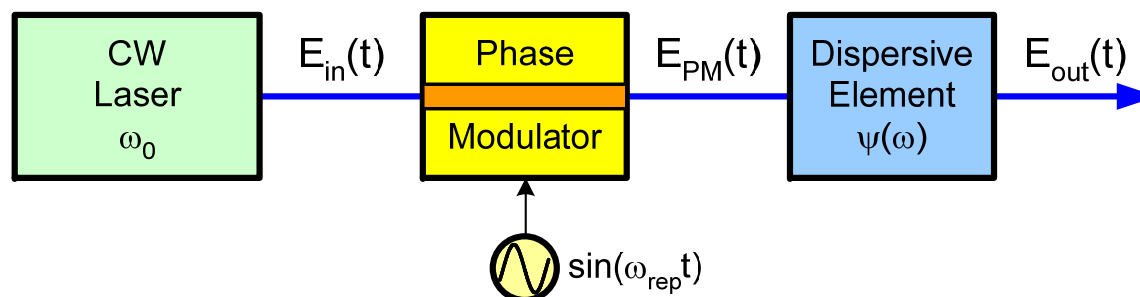
# Fiber laser combs (cont.)

- Unlocked
  - Fast  $f_0$  linewidth < 10 kHz, record for fiber systems (~50 kHz)
  - Long term (~min)  $f_0$  linewidth < 100 kHz (Ti:S ~MHz)
- Locked  $f_0$  < 1 mHz
- Frequency comparison with Ti:S
  - < 1 mHz (1.05 ks time)

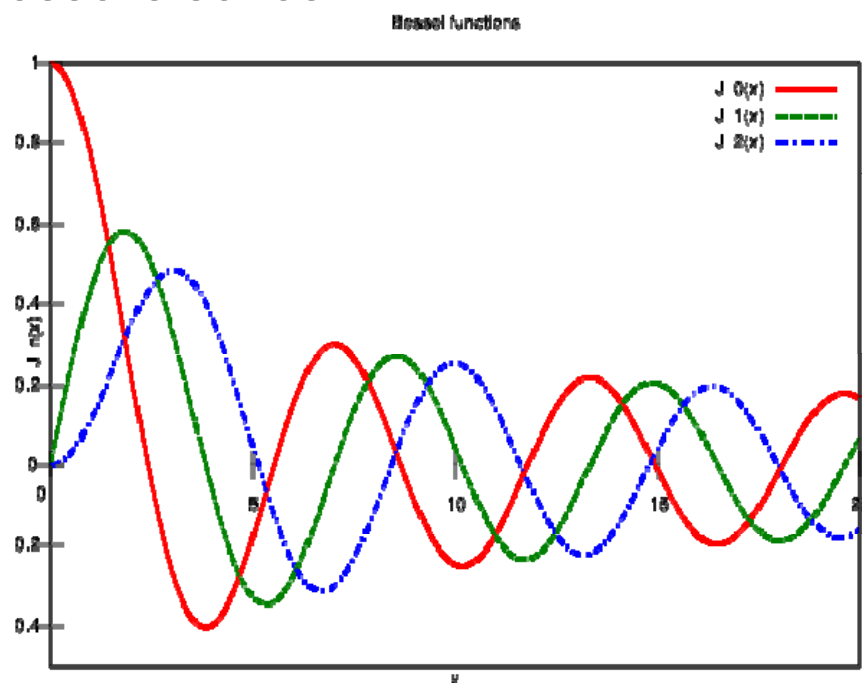


# Externally modulated laser combs

- Phase or dual-electrode intensity modulation



- Bessel's series



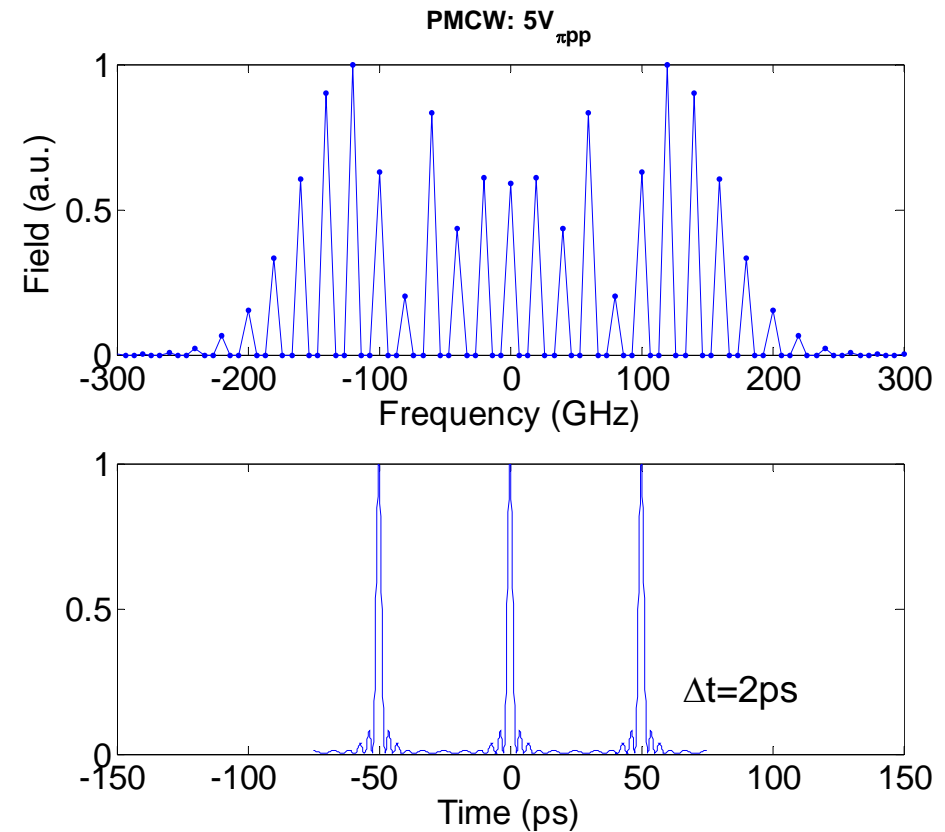
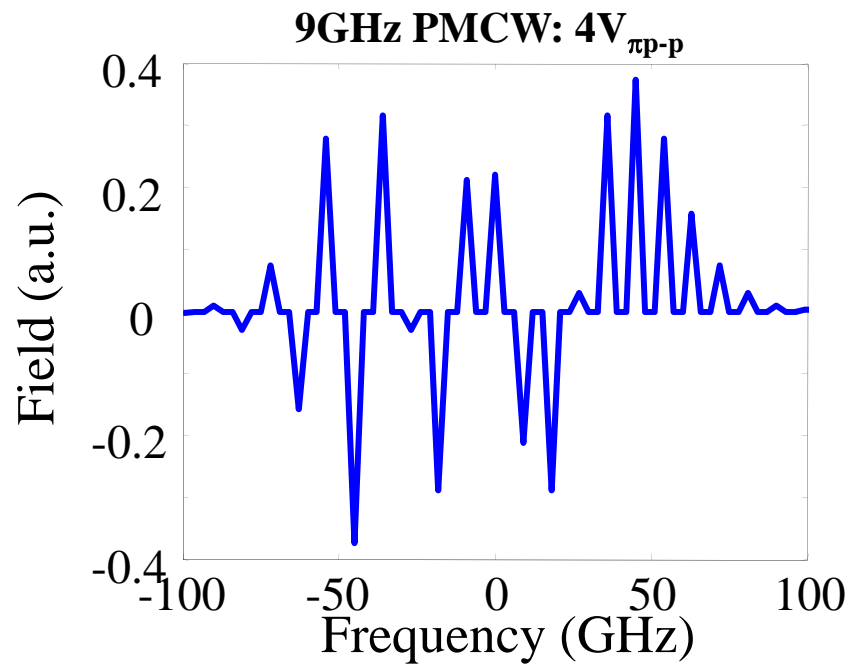
$$\nu_m = m f_{rep} + \epsilon$$

De-coupled!

Let's derive it's output spectrum!

# Comb spectral phases

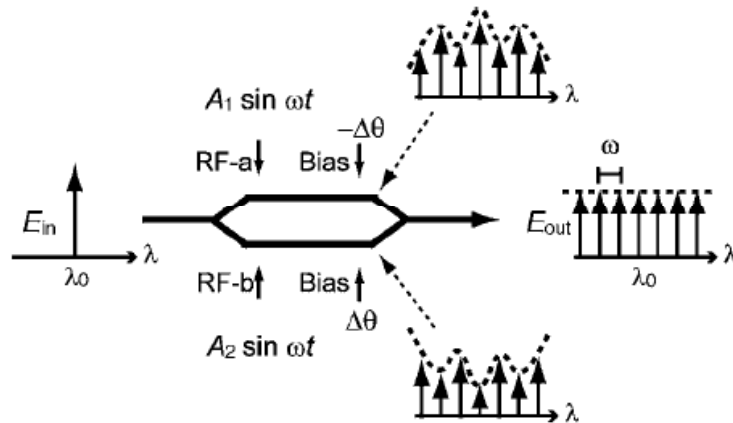
- Not self-pulsing



# Flat externally modulated CW laser comb

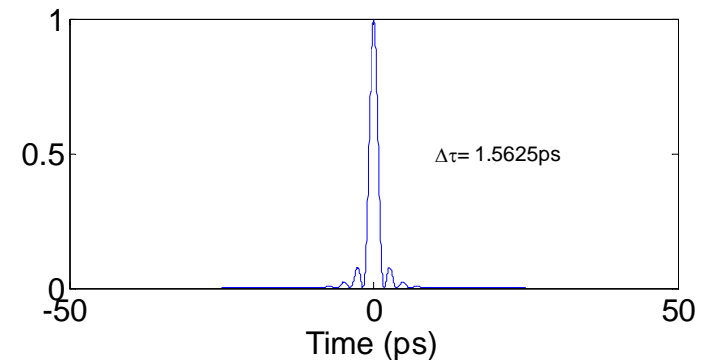
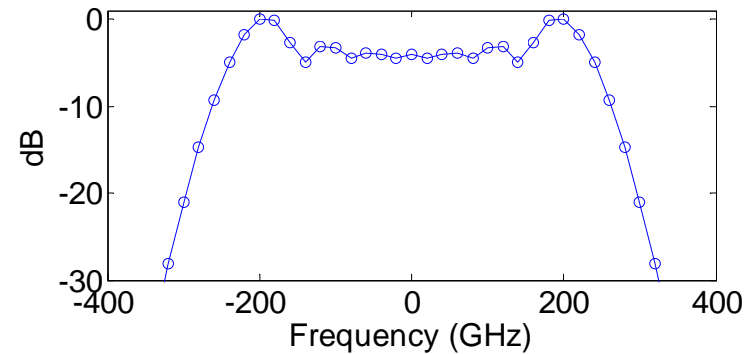
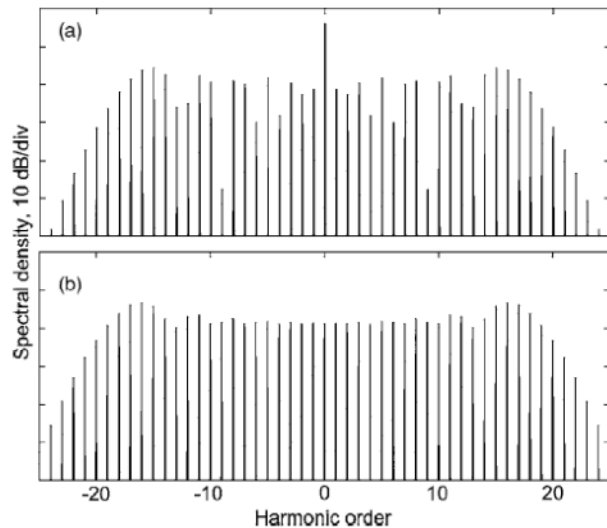


- Dual electrode intensity modulator



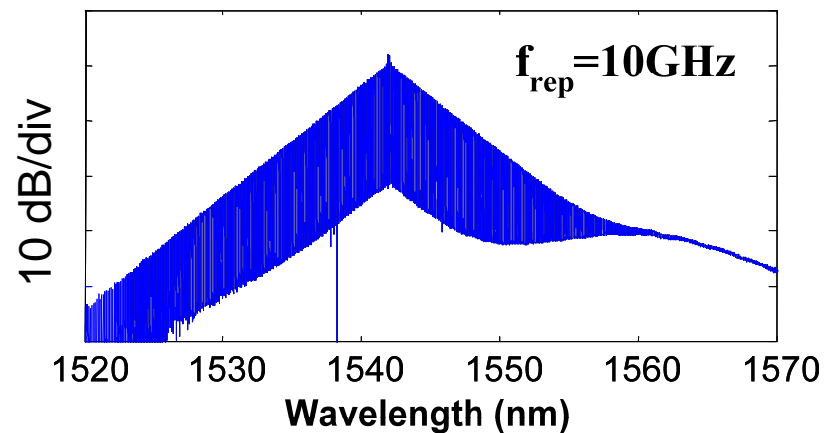
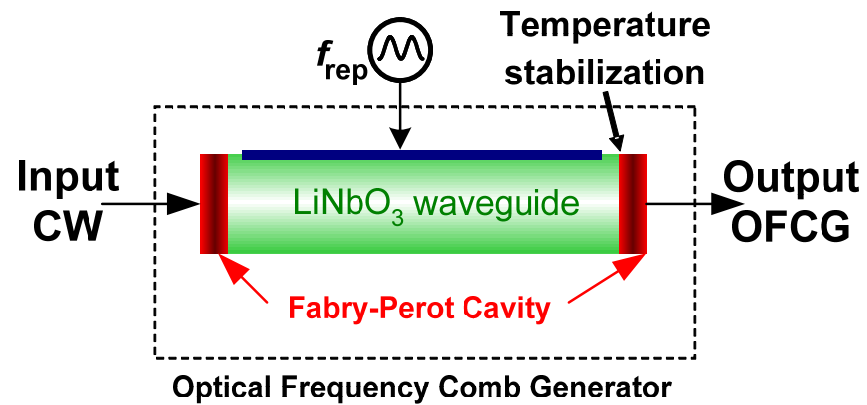
$$\frac{P_k}{P_{in}} = \frac{1}{2\pi A} \left\{ 1 + \cos(2\Delta\theta) \cos(2\Delta A) + [\cos(2\Delta\theta) + \cos(2\Delta A)] \cos\left[2\bar{A} - \frac{(2k+1)\pi}{2}\right] \right\}$$

$$\bar{A} \equiv (A_1 + A_2)/2 \quad \Delta A \equiv (A_1 - A_2)/2 \quad \Delta\theta \equiv (\theta_1 - \theta_2)/2$$



T. Sakamoto et.al, Opt. Lett. **32**, 1515 (2007).

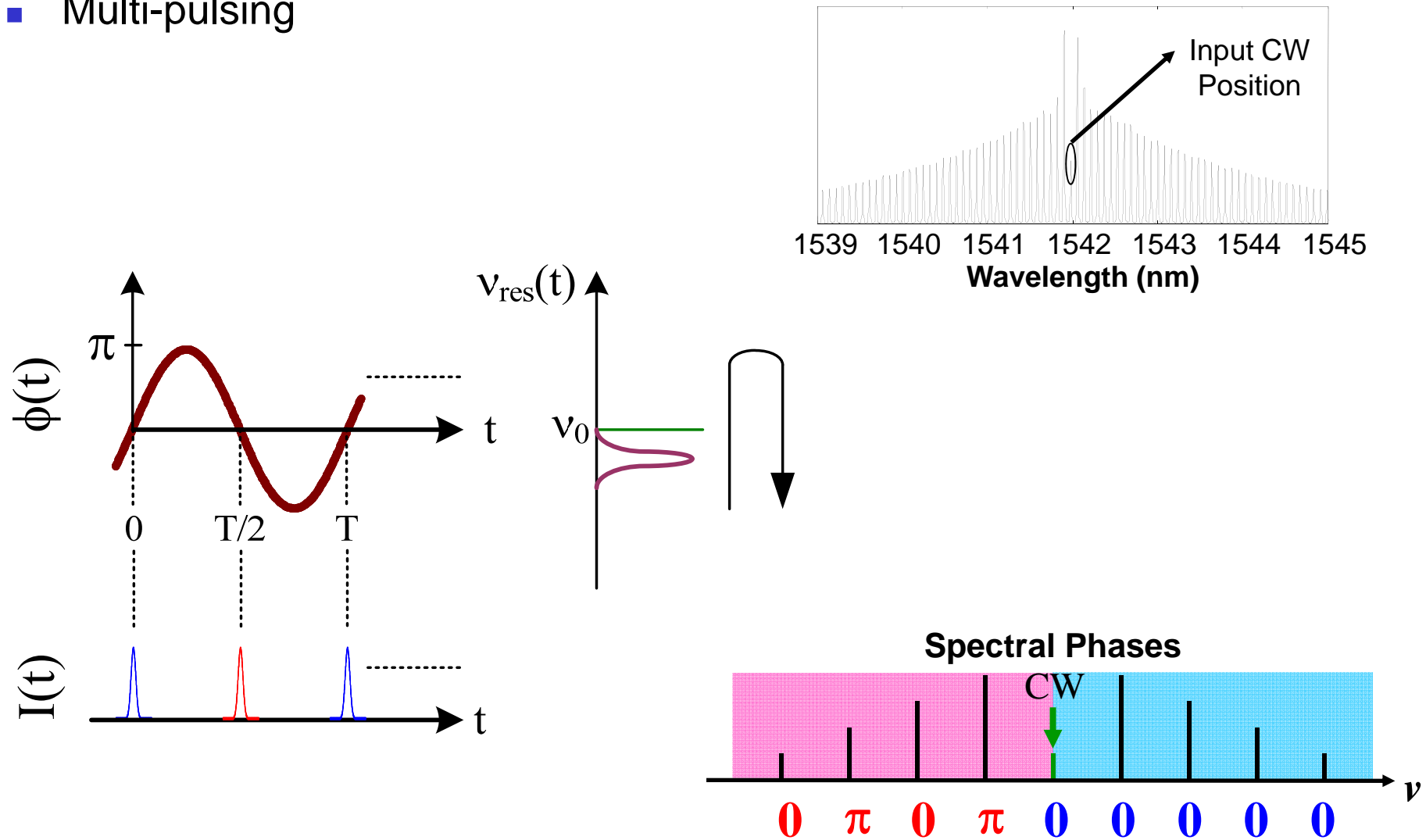
- Phase modulation inside cavity
  - Smooth spectrum: exponential
  - Wide bandwidth



# OFCG: pulsing property



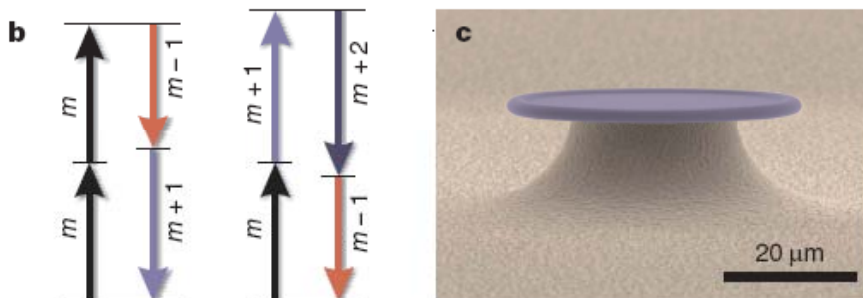
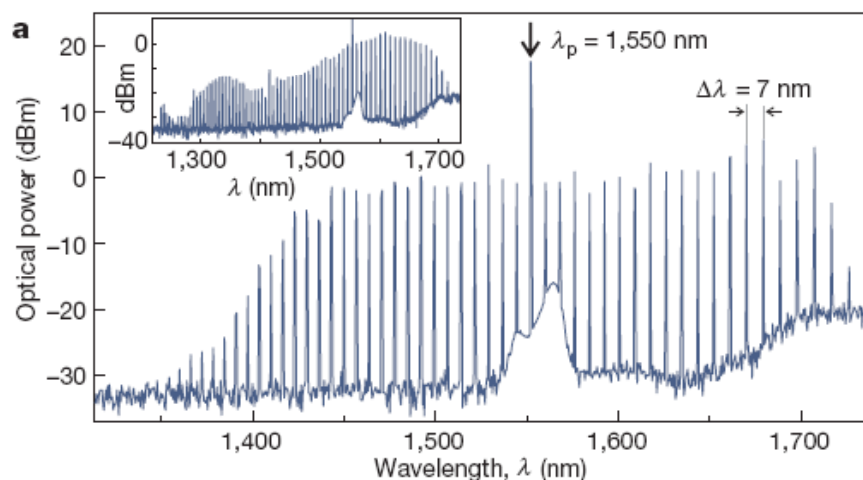
- Multi-pulsing



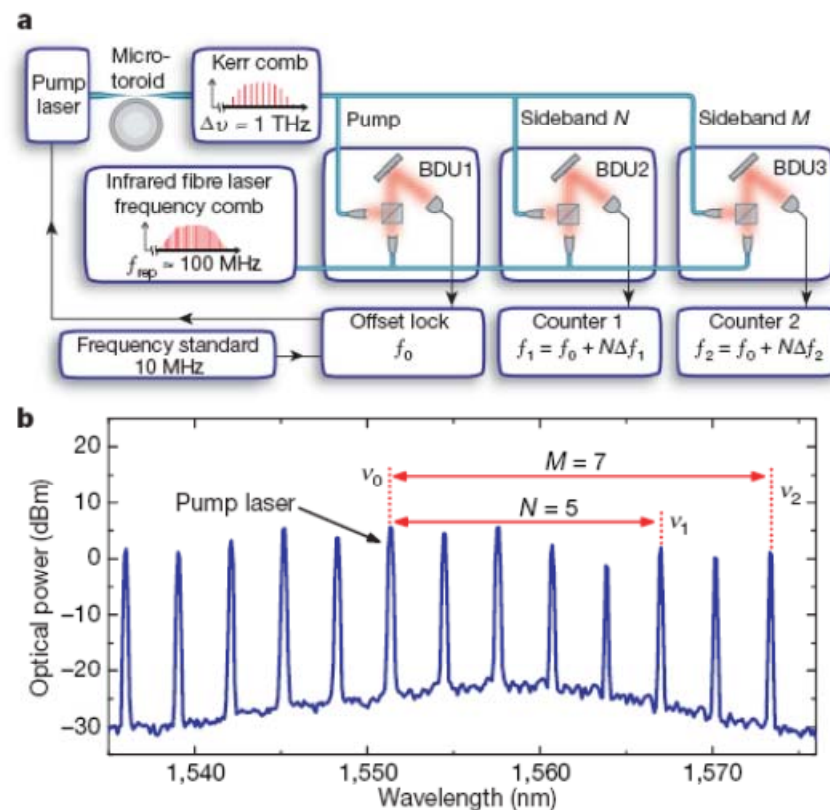


# Micro-toroid comb generator

- Extremely high-Q cavity
- $\chi^3$  nonlinearity: four-wave mixing
- Large comb spacing!



Comb spacing uniformity:  $7.3 \times 10^{-18}$



P. Del'Haye, et.al., Nature **450**, 1214 (2007)  
 P. Del'Haye, et.al., PRL **101**, 053903 (2008)  
 P. Del'Haye, et.al., Nat. Photonics **3**, 529 (2009)  
 Ferdous, et.al., Nat. Photonics **6**, xxx (2011)  
 Kippenberg, et.al., Science **332**, 555 (2011)

# Summary

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- History of a second and optical frequency comb explained
- Derivation and method for measuring the carrier-envelope phase slippage discussed
- Stabilization of OFC
  - Self-referencing
- Applications of OFC
  - Optical frequency metrology, optical clock, .....
- Alternative OFC generation methods