Appendix: Using Isoquants and Isocost Lines to Understand Production and Cost

Use isoquants and isocost lines to understand production and cost

Suppose a firm has determined it wants to produce a particular level of output. What determines the cost of that output?

1. Technology

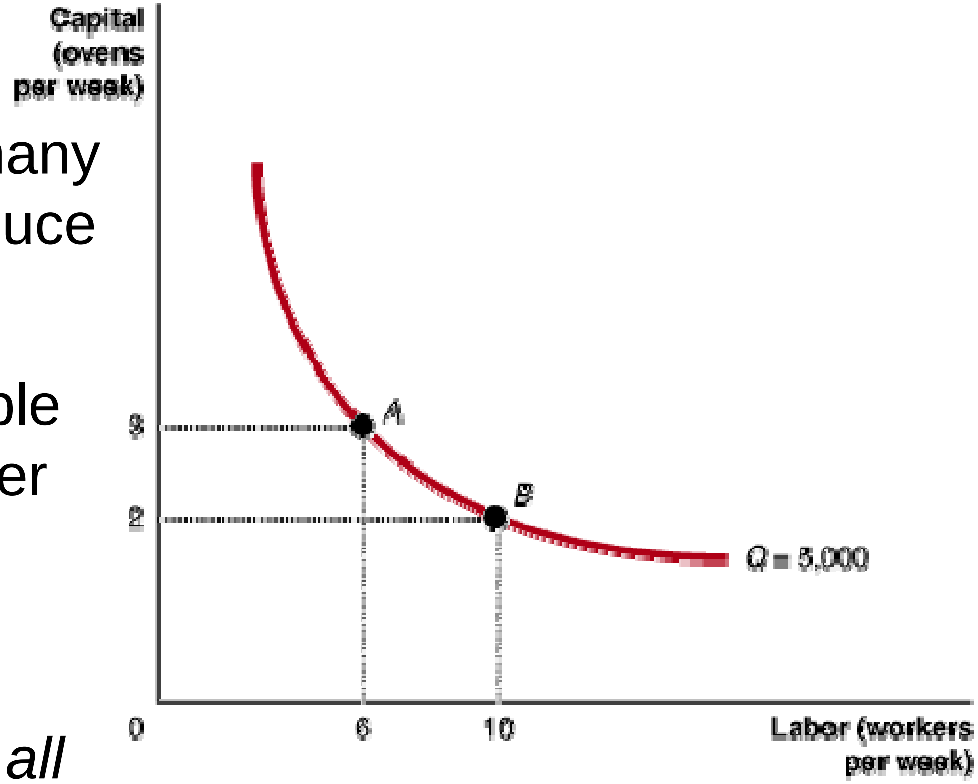
In what ways can inputs be combined to produce output?

2. Input prices

What is the cost of each input compared with the other? That is, what is the *relative price* of each input?

Figure 11A.1 Isoquants (1 of 3)

If a firm's technology allows one input to be substituted for the other in order to maintain the same level of production, then many combinations of inputs may produce the same level of output.



The pizza restaurant might be able to produce 5000 pizzas with either

- 6 workers and 3 ovens; or
- 10 workers and 2 ovens.

An ***isoquant*** is a curve showing *all* combinations of two inputs, such as capital and labor, that will produce the same level of output.

Figure 11A.1 Isoquants (2 of 3)

More inputs would allow a higher level of production; with 12 workers and 4 ovens, the restaurant could produce 10,000 pizzas.

A new isoquant describes all combinations of inputs that could produce 10,000 pizzas.

Greater production would require more inputs.

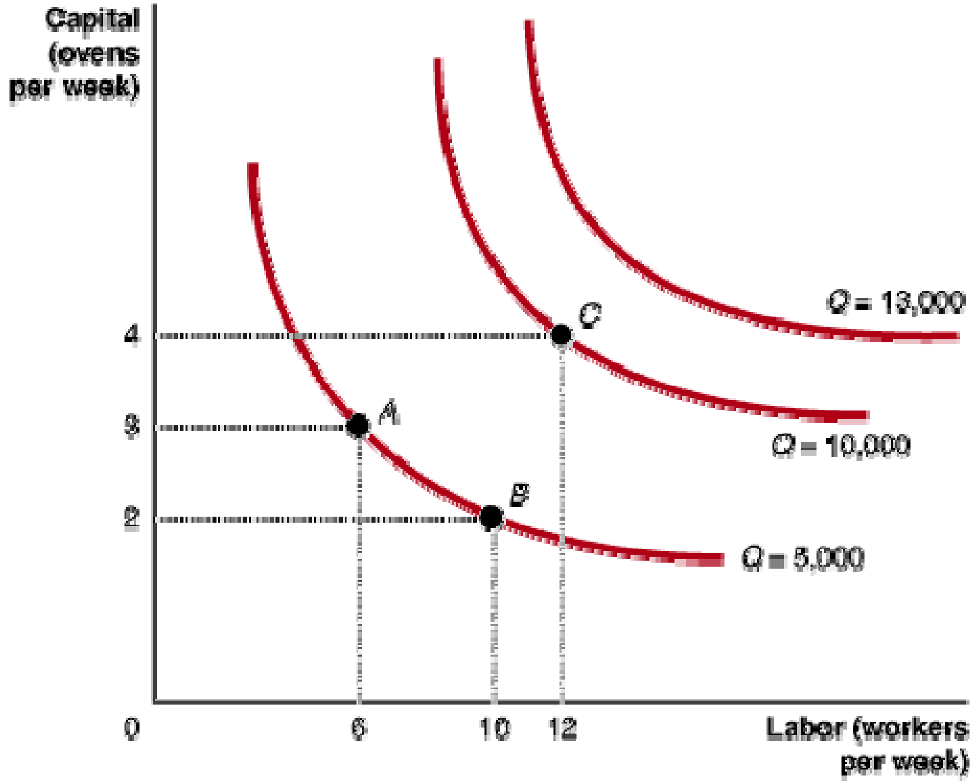


Figure 11A.1 Isoquants (2 of 3)

The slope of an isoquant describes how many units of capital are required to compensate for a unit of labor, keeping production constant.

- This is the **marginal rate of technical substitution**.

Between *A* and *B*, 1 oven can compensate for 4 workers; the MRTS=1/4.

Additional workers are poorer and poorer substitutes for capital, due to diminishing returns; so the MRTS gets smaller as we move along the isoquant, giving a convex shape.

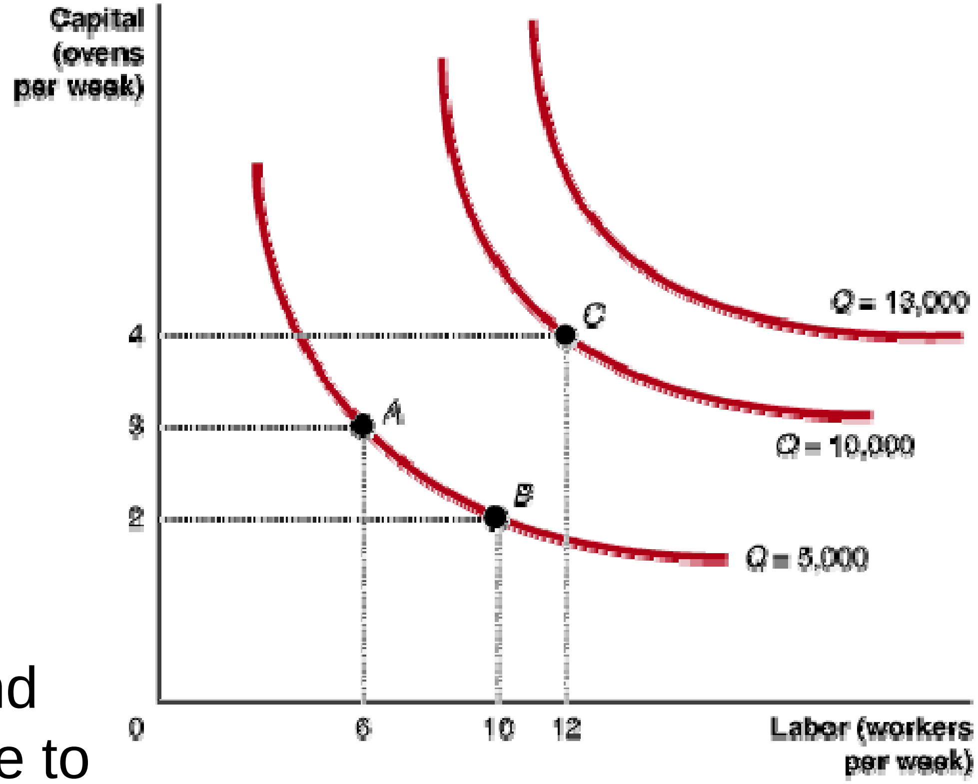


Figure 11A.2 An isocost line

For a given cost, various combinations of inputs can be purchased.

The table shows combinations of ovens and workers that could be produced with $6000, if ovens cost $1000 and workers cost $500.

<u>Isocost line</u>: All the combinations of two inputs, such as capital and labor, the have the same total cost.

Combinations of Workers and Ovens with a Total Cost of $6,000

Point	Ovens	Workers	Total Cost
A	6	0	(6 x $1,000) + (0 x $500) = $6,000
B	5	2	(5 x $1,000) + (2 x $500) = 6,000
C	4	4	(4 x $1,000) + (4 x $500) = 6,000
D	3	6	(3 x $1,000) + (6 x $500) = 6,000
E	2	8	(2 x $1,000) + (8 x $500) = 6,000
F	1	10	(1 x $1,000) + (10 x $500) = 6,000
G	0	12	(0 x $1,000) + (12 x $500) = 6,000

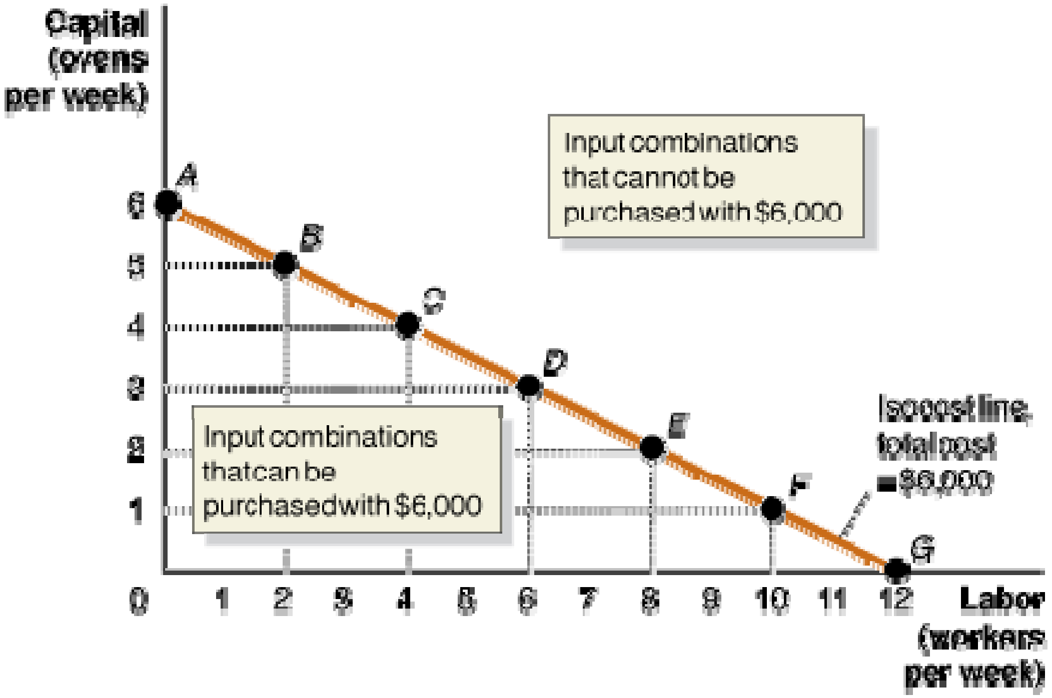


Figure 11A.3 The position of the isocost line

With more money, more inputs can be purchased.

The slope of the isocost line remains constant, because it is always equal to the *price of the input on the horizontal axis divided by the price of the input on the vertical axis, multiplied by -1.*

The slope indicates the rate at which prices allow one input to be traded for the other: here, 1 oven costs the same as 2 workers:

slope = -1/2.

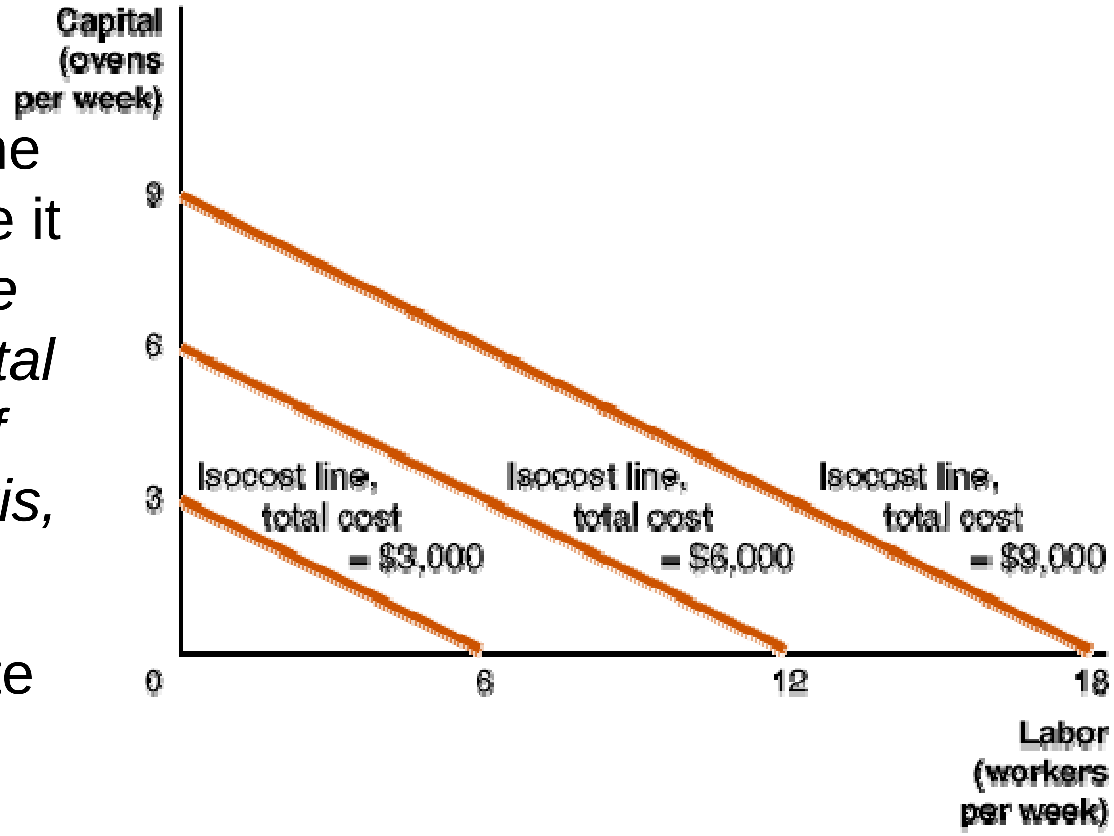
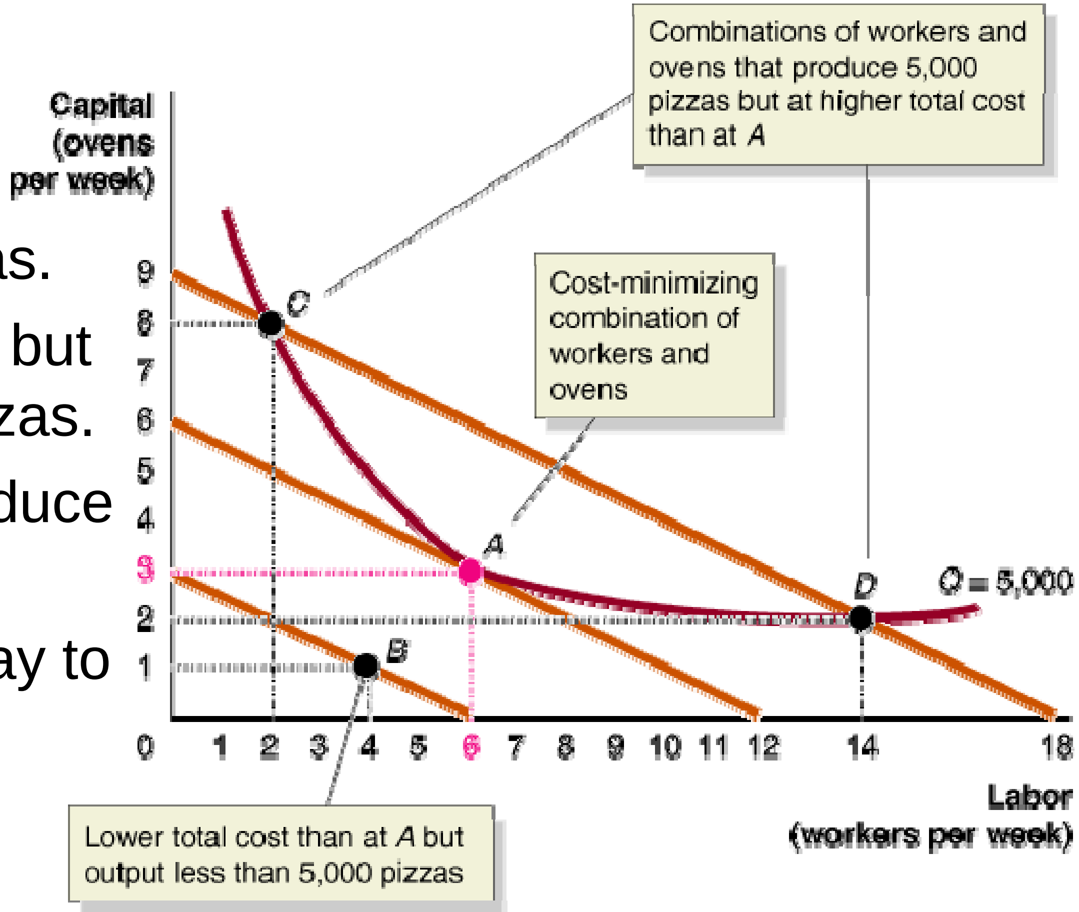


Figure 11A.4 Choosing capital and labor to minimize total cost



Suppose the restaurant wants to produce 5000 pizzas.

- Point B costs only $3000, but doesn't produce 5000 pizzas.
- Points A, C, and D all produce 5000 pizzas.
- Point A is the cheapest way to produce 5000 pizzas; the isocost line going through it is the lowest.

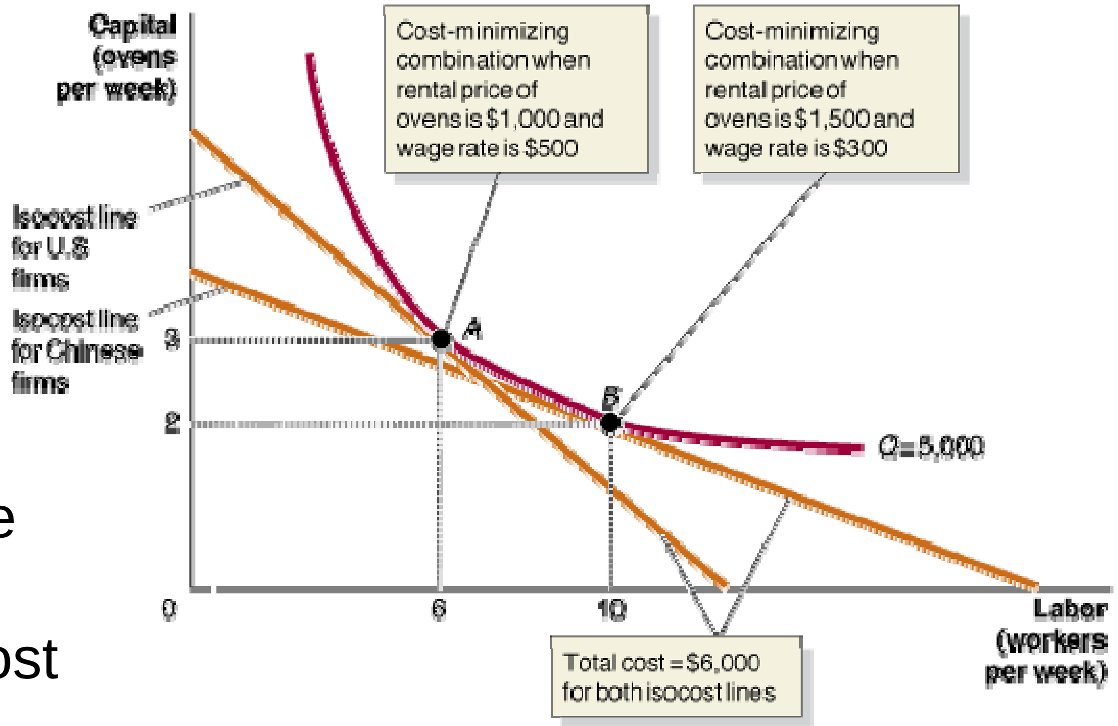
Observe that at this point, the slope of the isoquant and isocost line are equal.

Figure 11A.5 Changing input prices affects the cost-minimizing input choice

If prices change, so does the cost-minimizing combination of capital and labor.

Suppose we open a pizza franchise in China, where ovens are more expensive ($1500) and workers are cheaper ($300). The isocost lines are now flatter.

To obtain the same level of production, we would *substitute* toward the input that is now relatively cheaper: workers.



Another look at cost minimization

We observed that at the minimum cost level of production, the slopes of the isocost line and the isoquant were equal.

Generally, writing labor as *L* and capital as *K*, we have:

$$\text{Slope of isoquant} = -MRTS = -\frac{P_L}{P_K} = \text{Slope of isocost line}$$

So at the cost-minimizing level of production, $MRTS = P_L/P_K$.

- The MRTS tells us the rate at which a firm is able to substitute labor for capital, *given existing technology*.
- The slope of the isocost line tells us the rate at which a firm is able to substitute labor for capital, *given current input prices*.
- These are equal at the cost-minimizing level of production, but there is no reason they should be equal elsewhere.

Moving along an isoquant (1 of 2)

Suppose we move between two points on an isoquant, increasing labor and decreasing the capital used.

When we increase labor, we increase production by the number of workers we add, times their marginal production:

$$\text{Change in quantity of workers} \times MP_L$$

We can interpret the reduction in output from reducing capital in the same way: it is equal to the amount of capital we remove, times the marginal production of that capital:

$$-\text{Change in quantity of capital} \times MP_K$$

Moving along an isoquant (2 of 2)

Since we are moving along an isoquant, production stays constant

- So the decrease in output from using less capital must exactly equal the increase in output from increasing labor:

$$-\text{Change in quantity of capital} \times MP_K = \text{Change in quantity of workers} \times MP_L$$

Rearranging this equation gives:

$$\frac{-\text{Change in the quantity of capital}}{\text{Change in the quantity of workers}} = \frac{MP_L}{MP_K}$$

The left hand side is the slope of the isoquant: the *MRTS*. So:

$$MRTS = \frac{MP_L}{MP_K}$$

Optimality condition for a firm

The slope of the isocost line is the wage rate (*w*) divided by the rental price of capital (*r*). Earlier in the appendix, we showed that at the cost-minimizing point,

$$MRTS = \frac{w}{r}$$

So now we know that at the cost-minimizing point:

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

Rearranging gives us:

$$\frac{MP_L}{w} = \frac{MP_K}{r}$$

Interpreting the firm's optimality condition

$$\frac{MP_L}{w} = \frac{MP_K}{r}$$

At the cost-minimizing input combination, the marginal output of the last dollar spent on labor should be equal to the marginal output of the last dollar spent on capital.

We could use this idea to determine whether a firm was producing efficiently or not: if an extra dollar spent on capital produced more (less) output than an extra dollar spent on labor, then the firm is *not* minimizing costs; it could:

- increase (decrease) capital, and
- decrease (increase) labor,

maintaining the same level of output and lowering cost.

Making the Connection: Do NFL teams behave efficiently?

NFL teams face a salary cap, and try to maximize wins subject to this total cost.

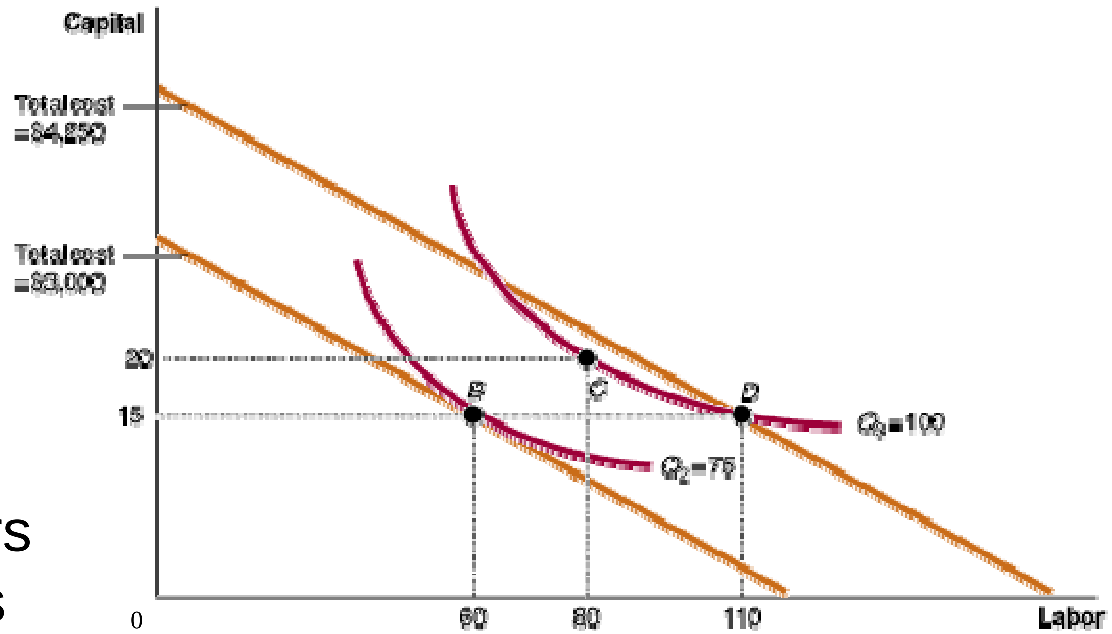
Teams face an important choice between trying to win with veterans or rookies. If the teams are optimizing, the marginal productivity per dollar spent on each type of player should be equal.

Economists Cade Massey and Richard Thaler found that teams were over-spending on high draft picks; they attributed this to NFL general managers being *overconfident* in their ability to spot NFL-level talent among college players.

Figure 11A.6 The expansion path (1 of 2)

A bookcase manufacturing firm produces 75 bookcases a day, using 60 workers and 15 machines.

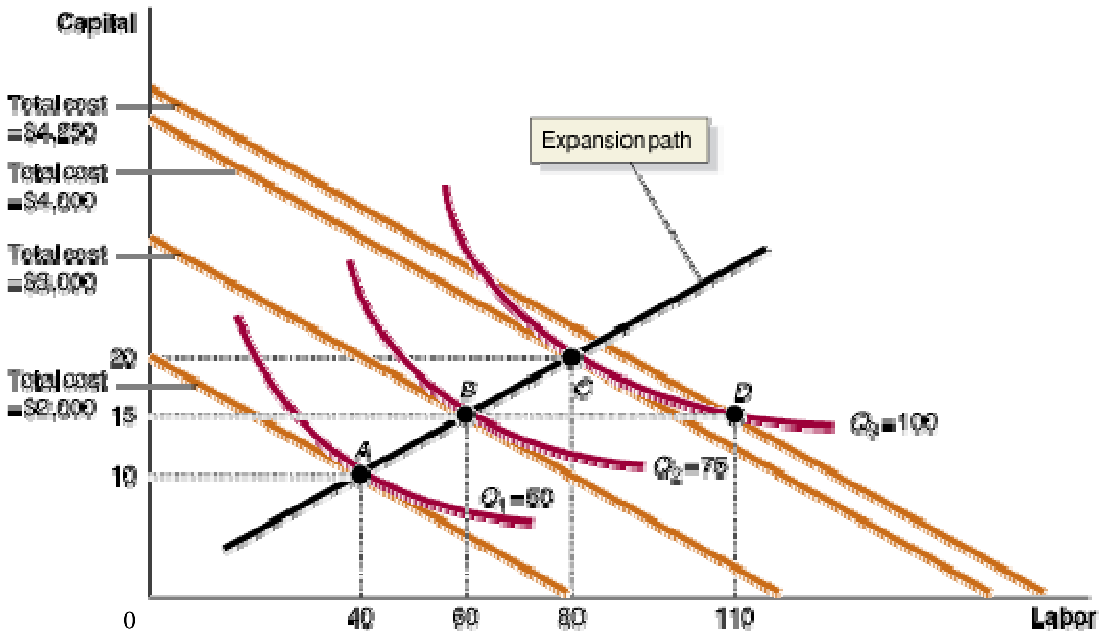
In the short run, if the firm wants to expand production to 100 bookcases, it must do so by employing more workers only; the number of machines is fixed.



Notice that there is are lower-cost combination of inputs (like point C) that would produce 100 bookcases; in the long run, the firm will switch to one of those.

Figure 11A.6 The expansion path (2 of 2)

Point C is a combination on the long-run **expansion path** for the firm: a curve that shows the firm's cost-minimizing combination of inputs for every level of output.



We can tell because the isocost line and isoquant are tangent at point C.

Point A minimizes costs for a lower quantity (50).

The expansion path is the set of all cost-minimizing bundle, given a particular set of input prices.

Appendix

Using Isoquants and Isocost Lines to Understand Production and Cost

LEARNING OBJECTIVE: Use isoquants and isocost lines to understand production and cost.

Isoquants

In this chapter, we studied the important relationship between a firm's level of production and its costs. In this appendix, we look more closely at how firms choose the combination of inputs to produce a given level of output. Firms usually have a choice about how they will produce their output. For example, Jill Johnson is able to produce 5,000 pizzas per week by using 10 workers and 2 ovens or by using 6 workers and 3 ovens. We will see that firms search for the *cost-minimizing* combination of inputs that will allow them to produce a given level of output. The cost-minimizing combination of inputs depends on two factors: technology—which determines how much output a firm receives from employing a given quantity of inputs—and input prices—which determine the total cost of each combination of inputs.

An Isoquant Graph

We begin by graphing the levels of output that Jill can produce using different combinations of two inputs: labor—the quantity of workers she hires per week—and capital—the quantity of ovens she uses per week. In reality, of course, Jill uses more than just these two inputs to produce pizzas, but nothing important would change if we expanded the discussion to include many inputs instead of just two. Figure 11A.1 measures the quantity of capital along the vertical axis and the quantity of labor along the horizontal axis. The curves in the graph are **isoquants**, which show all the combinations of two inputs—in this case capital and labor—that will produce the same level of output.

Isoquant A curve that shows all the combinations of two inputs, such as capital and labor, that will produce the same level of output.

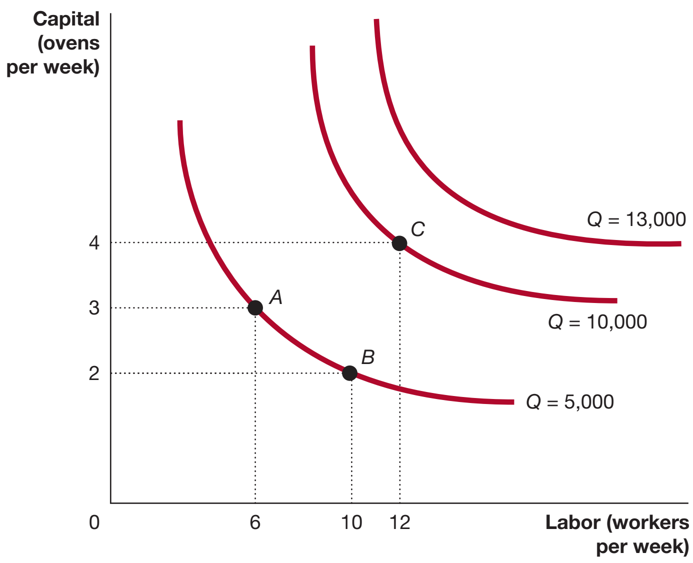


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Figure 11A.1

Isoquants

Isoquants show all the combinations of two inputs—in this case capital and labor—that will produce the same level of output. For example, the isoquant labeled $Q = 5{,}000$ shows all the combinations of ovens and workers that enable Jill to produce that quantity of pizzas per week. At point *A*, she produces 5,000 pizzas using 3 ovens and 6 workers, and at point *B*, she produces the same output using 2 ovens and 10 workers. With more ovens and workers, she can move to a higher isoquant. For example, with 4 ovens and 12 workers, she can produce at point *C* on the isoquant $Q = 10{,}000$. With even more ovens and workers, she could move to the isoquant $Q = 13{,}000$.

The isoquant labeled $Q = 5{,}000$ shows all the combinations of workers and ovens that enable Jill to produce that quantity of pizzas per week. For example, at point *A*, she produces 5,000 pizzas using 6 workers and 3 ovens, and at point *B*, she produces the same output using 10 workers and 2 ovens. With more workers and ovens, she can move to a higher isoquant. For example, with 12 workers and 4 ovens, she can produce at point *C* on the isoquant $Q = 10{,}000$. With even more workers and ovens, she could move to the isoquant $Q = 13{,}000$. The higher the isoquant—that is, the further to the upper right on the graph—the more output the firm produces. Although we have shown only three isoquants in this graph, there is, in fact, an isoquant for every level of output. MyEconLab Concept Check

The Slope of an Isoquant

Marginal rate of technical substitution (*MRTS*) The rate at which a firm is able to substitute one input for another while keeping the level of output constant.

Remember that the slope of a curve is the ratio of the change in the variable on the vertical axis to the change in the variable on the horizontal axis. Along an isoquant, the slope tells us the rate at which a firm is able to substitute one input for another while keeping the level of output constant. This rate is called the **marginal rate of technical substitution (*MRTS*)**.

We expect that the *MRTS* will change as we move down an isoquant. In Figure 11A.1, at a point like *A* on isoquant $Q = 5{,}000$, the isoquant is relatively steep. As we move down the curve, it becomes less steep at a point like *B*. This shape is the usual one for isoquants: They are bowed in, or convex. The reason isoquants have this shape is that as we move down the curve, we continue to substitute labor for capital. As the firm produces the same quantity of output using less capital, the additional labor it needs increases because of diminishing returns. Remember from the chapter that, as a consequence of diminishing returns, for a given decline in capital, increasing amounts of labor are necessary to produce the same level of output. Because the *MRTS* is equal to the change in capital divided by the change in labor, it will become smaller (in absolute value) as we move down an isoquant. MyEconLab Concept Check

Isocost Lines

Isocost line All the combinations of two inputs, such as capital and labor, that have the same total cost.

A firm wants to produce a given quantity of output at the lowest possible cost. We can show the relationship between the quantity of inputs used and the firm's total cost by using an *isocost* line. An **isocost line** shows all the combinations of two inputs, such as capital and labor, that have the same total cost.

Graphing the Isocost Line

Suppose that Jill has \$6,000 per week to spend on capital and labor. Suppose, to simplify the analysis, that Jill can rent pizza ovens by the week. The table in Figure 11A.2 shows the combinations of capital and labor available to her if the rental price of ovens is \$1,000 per week and the wage rate is \$500 per week. The graph uses the data in the table to construct an isocost line. The isocost line intersects the vertical axis at the maximum number of ovens Jill can rent per week, which is shown by point *A*. The line intersects the horizontal axis at the maximum number of workers Jill can hire per week, which is point *G*. As Jill moves down the isocost line from point *A*, she gives up renting 1 oven for every 2 workers she hires. Any combination of inputs along the line or inside the line can be purchased with \$6,000. Any combination that lies outside the line cannot be purchased because it would have a total cost to Jill of more than \$6,000. MyEconLab Concept Check

The Slope and Position of the Isocost Line

The slope of the isocost line is constant and equals the change in the quantity of ovens divided by the change in the quantity of workers. In this case, in moving from any point on the isocost line to any other point, the change in the quantity of ovens equals -1, and the change in the quantity of workers equals 2, so the slope equals $-1/2$. Notice that with a rental price of ovens of \$1,000 per week and a wage rate for labor of

Combinations of Workers and Ovens with a Total Cost of $6,000			
Point	Ovens	Workers	Total Cost
A	6	0	(6 x $1,000) + (0 x $500) = $6,000
B	5	2	(5 x $1,000) + (2 x $500) = $6,000
C	4	4	(4 x $1,000) + (4 x $500) = $6,000
D	3	6	(3 x $1,000) + (6 x $500) = $6,000
E	2	8	(2 x $1,000) + (8 x $500) = $6,000
F	1	10	(1 x $1,000) + (10 x $500) = $6,000
G	0	12	(0 x $1,000) + (12 x $500) = $6,000

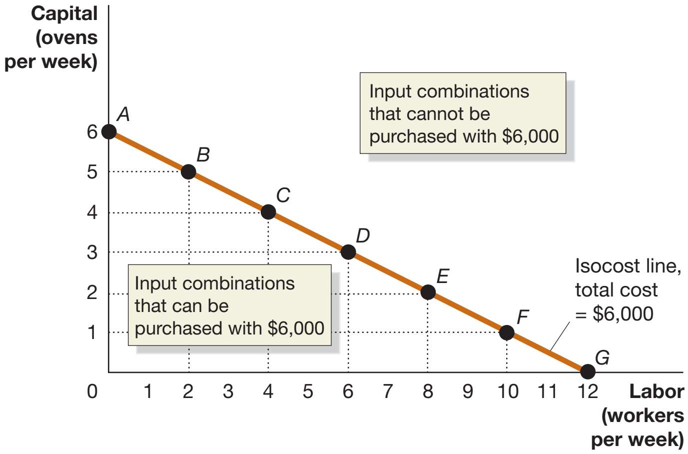

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Figure 11A.2

An Isocost Line

The isocost line shows the combinations of inputs with a total cost of $6,000. The rental price of ovens is $1,000 per week, so if Jill spends the whole $6,000 on ovens, she can rent 6 ovens (point *A*). The wage rate is $500 per week, so if Jill spends the whole $6,000 on workers, she can hire 12 workers (point *G*). As she moves down the isocost line, she gives up renting 1 oven for every 2 workers she hires. Any combination of inputs along the line or inside the line can be purchased with $6,000. Any combination that lies outside the line cannot be purchased with $6,000.

$500 per week, the slope of the isocost line is equal to the ratio of the wage rate divided by the rental price of capital, multiplied by −1, or −$500/$1,000 = −1/2. In fact, this result will always hold, whatever inputs are involved and whatever their prices may be: *The slope of the isocost line is equal to the ratio of the price of the input on the horizontal axis divided by the price of the input on the vertical axis multiplied by −1.*

The position of the isocost line depends on the level of total cost. Higher levels of total cost shift the isocost line outward, and lower levels of total cost shift the isocost line inward. This can be seen in Figure 11A.3, which shows isocost lines for total costs of $3,000, $6,000, and $9,000. We have shown only three isocost lines in the graph, but there is, in fact, a different isocost line for each level of total cost. MyEconLab Concept Check

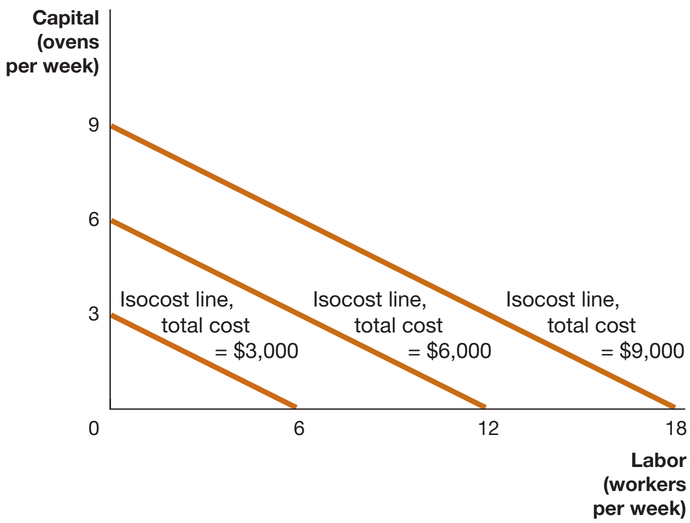

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Figure 11A.3

The Position of the Isocost Line

The position of the isocost line depends on the level of total cost. As total cost increases from $3,000 to $6,000 to $9,000 per week, the isocost line shifts outward. For each isocost line shown, the rental price of ovens is $1,000 per week, and the wage rate is $500 per week.

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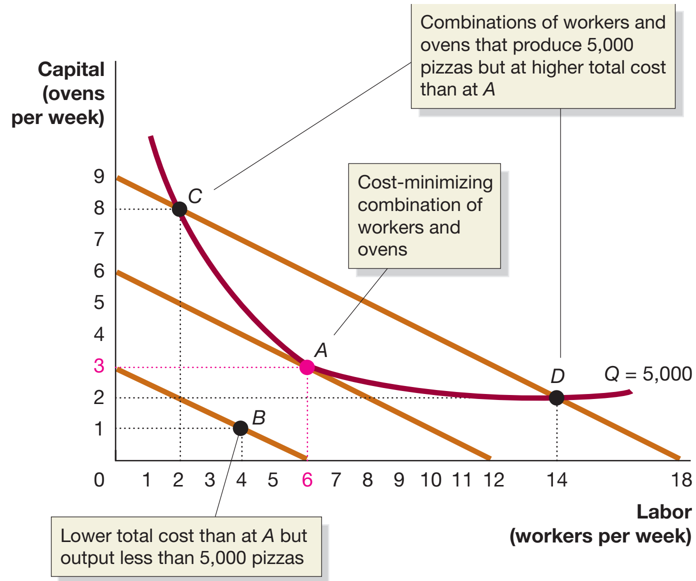
Figure 11A.4

Choosing Capital and Labor to Minimize Total Cost

Jill wants to produce 5,000 pizzas per week at the lowest total cost. Point *B* is the lowest-cost combination of inputs shown in the graph, but this combination of 1 oven and 4 workers will produce fewer than the 5,000 pizzas needed. Points *C* and *D* are combinations of ovens and workers that will produce 5,000 pizzas, but their total cost is $9,000. The combination of 3 ovens and 6 workers at point *A* produces 5,000 pizzas at the lowest total cost of $6,000.



Choosing the Cost-Minimizing Combination of Capital and Labor

Suppose Jill wants to produce 5,000 pizzas per week. Figure 11A.1 shows that there are many combinations of ovens and workers that will allow Jill to produce this level of output. There is only one combination of ovens and workers, however, that will allow her to produce 5,000 pizzas *at the lowest total cost*. Figure 11A.4 shows the isoquant Q = 5,000 along with three isocost lines. Point *B* is the lowest-cost combination of inputs shown in the graph, but this combination of 1 oven and 4 workers will produce fewer than the 5,000 pizzas needed. Points *C* and *D* are combinations of ovens and workers that will produce 5,000 pizzas, but their total cost is $9,000. The combination of 3 ovens and 6 workers at point *A* produces 5,000 pizzas at the lowest total cost of $6,000.

Figure 11A.4 shows that moving to an isocost line with a total cost of less than $6,000 would mean producing fewer than 5,000 pizzas. Being at any point along the isoquant Q = 5,000 other than point *A* would increase total cost above $6,000. In fact, the combination of inputs at point *A* is the only one on isoquant Q = 5,000 that has a total cost of $6,000. All other input combinations on this isoquant have higher total costs. Notice also that at point *A*, the isoquant and the isocost lines are tangent, so the slope of the isoquant is equal to the slope of the isocost line at that point.

Different Input Price Ratios Lead to Different Input Choices

Jill's cost-minimizing choice of 3 ovens and 6 workers is determined jointly by these two factors:

1. The technology available to her, as represented by her firm's isoquants
2. Input prices, as represented by her firm's isocost lines

If the technology of making pizzas changes, perhaps because new ovens are developed, her isoquants will be affected, and her choice of inputs may change. If her isoquants remain unchanged but input prices change, then her choice of inputs may also change. This fact can explain why firms in different countries that face different input prices may produce the same good using different combinations of capital and labor, even though they have the same technology available.

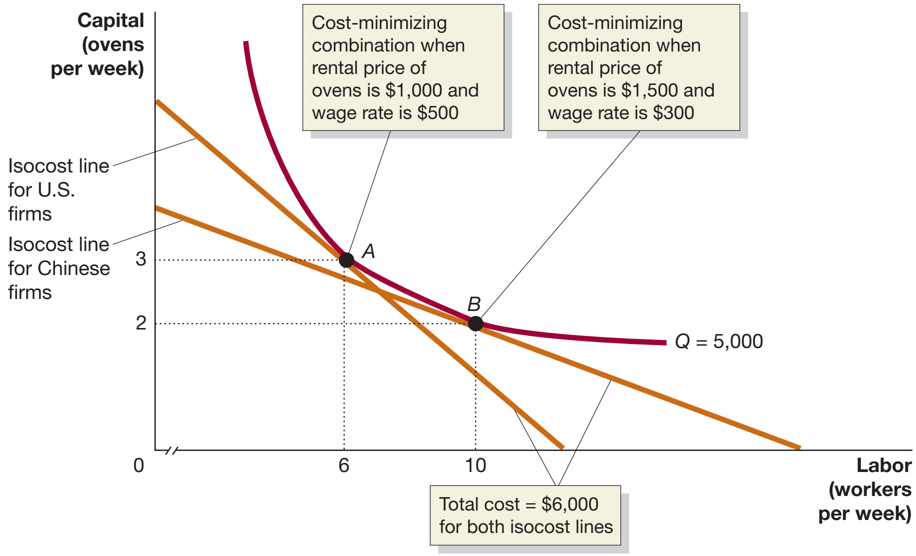


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Figure 11A.5

Changing Input Prices Affects the Cost-Minimizing Input Choice

As the graph shows, the input combination at point *A*, which was optimal for Jill, is not optimal for a businessperson in China. Using the input combination at point *A* would cost businesspeople in China more than $6,000. Instead, the Chinese isocost line is tangent to the isoquant at point *B*, where the input combination is 2 ovens and 10 workers. Because ovens cost more in China but workers cost less, a Chinese firm will use fewer ovens and more workers than a U.S. firm, even if it has the same technology as the U.S. firm.

For example, suppose that in China, pizza ovens are higher priced and labor is lower priced than in the United States. In our example, Jill Johnson pays $1,000 per week to rent pizza ovens and $500 per week to hire workers. Suppose a businessperson in China must pay a price of $1,500 per week to rent the identical pizza ovens but can hire Chinese workers who are as productive as U.S. workers at a wage of $300 per week. Figure 11A.5 shows how the cost-minimizing input combination for the businessperson in China differs from Jill's.

Remember that the slope of the isocost line equals the wage rate divided by the rental price of capital multiplied by −1. The slope of the isocost line that Jill and other U.S. firms face is −$500/$1,000, or −1/2. Firms in China, however, face an isocost line with a slope of −$300/$1,500, or −1/5. As Figure 11A.5 shows, the input combination at point *A*, which was optimal for Jill, is not optimal for a firm in China. Using the input combination at point *A* would cost a firm in China more than $6,000. Instead, the Chinese isocost line is tangent to the isoquant at point *B*, where the input combination is 2 ovens and 10 workers. This result makes sense: Because ovens cost more in China but workers cost less, a Chinese firm will use fewer ovens and more workers than a U.S. firm, even if it has the same technology as the U.S. firm. MyEconLab Concept Check

Solved Problem 11A.1

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Firms Responding to Differences in Input Price Ratios

David Autor, an economist at MIT, has written that, "When Nissan Motor Company builds cars in Japan, it makes extensive use of industrial robots to reduce labor costs. When it assembles cars in India, it uses robots far more sparingly." Explain why Nissan uses this strategy. Illustrate your answer with an isoquant–isocost line graph.

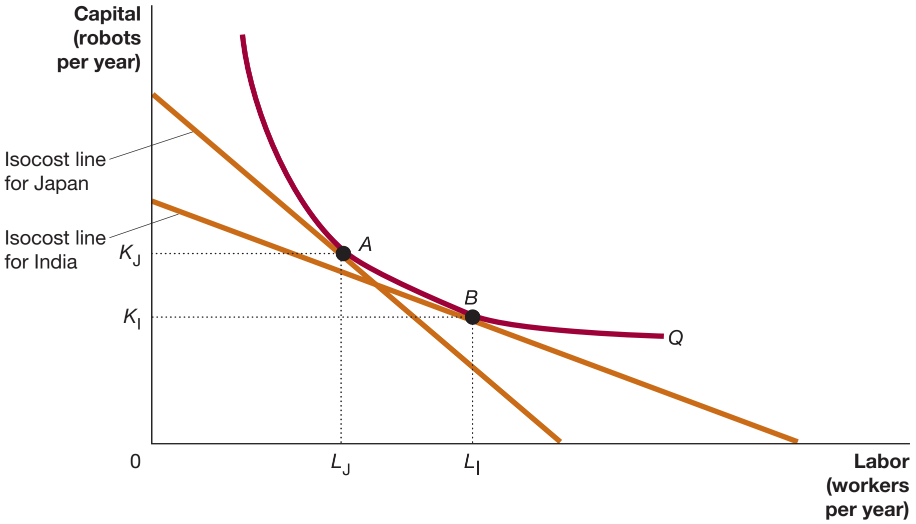
Source: David Autor, "The 'Task' Approach to Labor Markets: An Overview," *Journal of Labor Market Research*, Vol. 46, No. 3 (February 2013), pp. 185–199.

Solving the Problem

Step 1: Review the chapter material. This problem is about determining the optimal choice of inputs when input price ratios differ, so you may want to review the section "Different Input Price Ratios Lead to Different Input Choices," which begins on page 454.

Step 2: Answer the question by explaining Nissan's strategy. Nissan must be pursuing this strategy because labor costs are higher in Japan than in India. The higher cost of labor relative to capital (industrial robots) in Japan means that Nissan faces an isocost line that is steeper in Japan than in India. Assuming that Nissan's technology is the same in the two countries, the steeper isocost line in Japan will be tangent to the isoquant at a point that represents more capital and less labor than in India. This conclusion is illustrated by the graph in step 3.

Step 3: Finish your answer by drawing a graph to illustrate your explanation in step 2. Nissan will produce in Japan at point *A*, using L_J units of labor and K_J units of capital. It will produce in India at point *B*, using L_I units of labor and K_I units of capital.



MyEconLab Study Plan

Your Turn: For more practice, do related problem 11A.6 on page 461 at the end of this appendix.

Another Look at Cost Minimization

We know that consumers maximize utility when they consume each good up to the point where the marginal utility per dollar spent is the same for every good. We can derive a very similar cost-minimization rule for firms. Remember that at the point of cost minimization, the isoquant and the isocost line are tangent, so they have the same slope. Therefore, *at the point of cost minimization, the marginal rate of technical substitution (MRTS) is equal to the wage rate divided by the rental price of capital.*

The slope of the isoquant tells us the rate at which a firm is able to substitute labor for capital, *keeping the level of output constant*. The slope of the isocost line tells us the rate at which a firm is able to substitute labor for capital, *given current input prices*. Only at the point of cost minimization are these two rates the same.

When we move from one point on an isoquant to another, we end up using more of one input and less of the other input, but the level of output remains the same. For example, as Jill moves down an isoquant, she uses fewer ovens and more workers but produces the same quantity of pizzas. In this chapter, we defined the *marginal product of labor* (MP_L) as the additional output produced by a firm as a result of hiring one more worker. Similarly, we can define the *marginal product of capital* (MP_K) as the additional output produced by a firm as a result of using one more machine. So, when Jill uses fewer ovens by moving down an isoquant, she loses output equal to:

$$-\text{Change in the quantity of ovens} \times MP_K.$$

But she uses more workers, so she gains output equal to:

$$\text{Change in the quantity of workers} \times MP_L.$$

We know that the gain in output from the additional workers is equal to the loss from the smaller quantity of ovens because total output remains the same along an isoquant. Therefore, we can write:

$$-\text{Change in the quantity of ovens} \times MP_K = \text{Change in the quantity of workers} \times MP_L.$$

Loss in output from using fewer ovens

Gain in output from using more workers

If we rearrange terms, we have the following:

$$\frac{-\text{Change in the quantity of ovens}}{\text{Change in the quantity of workers}} = \frac{MP_L}{MP_K}.$$

Because:

$$\frac{-\text{Change in the quantity of ovens}}{\text{Change in the quantity of workers}}$$

is the slope of the isoquant, it is equal to the marginal rate of technical substitution (multiplied by negative 1). So, we can write:

$$\frac{-\text{Change in the quantity of ovens}}{\text{Change in the quantity of workers}} = MRTS = \frac{MP_L}{MP_K}.$$

The slope of the isocost line equals the wage rate (w) divided by the rental price of capital (r). We saw earlier in this appendix that at the point of cost minimization, the *MRTS* equals the ratio of the prices of the two inputs. Therefore:

$$\frac{MP_L}{MP_K} = \frac{w}{r}.$$

We can rewrite this to show that at the point of cost minimization:

$$\frac{MP_L}{w} = \frac{MP_K}{r}.$$

This last expression tells us that to minimize cost for a given level of output, a firm should hire inputs up to the point where the last dollar spent on each input results in the same increase in output. If this equality did not hold, a firm could lower its costs by using more of one input and less of the other. For example, if the left side of the equation were greater than the right side, a firm could rent fewer ovens, hire more workers, and produce the same output at lower cost. MyEconLab Concept Check

Solved Problem 11A.2

MyEconLab Interactive Animation

Determining the Optimal Combination of Inputs

Consider the information in the following table for Jill Johnson's restaurant:

Marginal product of capital	3,000 pizzas per oven
Marginal product of labor	1,200 pizzas per worker
Wage rate	$300 per week
Rental price of ovens	$600 per week

Briefly explain whether Jill is minimizing costs. If she is not minimizing costs, explain whether she should rent more ovens and hire fewer workers or rent fewer ovens and hire more workers.

Solving the Problem

Step 1: Review the chapter material. This problem is about determining the optimal choice of inputs by comparing the ratios of the marginal products of inputs to their prices, so you may want to review the section "Another Look at Cost Minimization," which begins on page 456.

Step 2: Compute the ratios of marginal product to input price to determine whether Jill is minimizing costs. If Jill is minimizing costs, the following relationship should hold:

$$\frac{MP_L}{w} = \frac{MP_K}{r}.$$

In this case, we have:

$$MP_L = 1{,}200$$
$$MP_K = 3{,}000$$
$$w = \$300$$
$$r = \$600.$$

So:

$$\frac{MP_L}{w} = \frac{1{,}200}{\$300} = 4 \text{ pizzas per dollar, and } \frac{MP_K}{r} = \frac{3{,}000}{\$600} = 5 \text{ pizzas per dollar.}$$

Because the two ratios are not equal, Jill is not minimizing cost.

Step 3: Determine how Jill should change the mix of inputs she uses. Jill produces more pizzas per dollar from the last oven than from the last worker. This indicates that she has too many workers and too few ovens. Therefore, to minimize cost, Jill should use more ovens and hire fewer workers.

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Your Turn: For more practice, do related problems 11A.7 and 11A.8 on page 461 at the end of this appendix.

Making the Connection

MyEconLab Video

Do National Football League Teams Behave Efficiently?

Did a rule change keep Tampa Bay from paying Jameis Winston too much?

In the National Football League (NFL), the "salary cap" is the maximum amount each team can spend in a year on salaries for football players. Each year's salary cap results from negotiations between the league and the union representing the players. To achieve efficiency, an NFL team should distribute salaries among players so as to maximize the level of output—in this case, winning football games—given the constant level of cost represented by the salary cap. (Notice that maximizing the level of output for a given level of cost is equivalent to minimizing cost for a given level of output. To see why, think about the situation in which an isocost line is tangent to an isoquant. At the point of tangency, the firm has simultaneously minimized the cost of producing the level of output represented by the isoquant and maximized the output produced at the level of cost represented by the isocost line.)

In distributing the fixed amount of salary payments available, teams should equalize the ratios of the marginal productivity of players, as represented by their contribution to winning games, to the salaries players receive. Just as a firm may not use a machine that has a very high marginal product if its rental price is very high, a football team may not want to hire a superstar player if the salary the team would need to pay is too high.

Economists Cade Massey, of the University of Pennsylvania, and Richard Thaler, of the University of Chicago, have analyzed whether NFL teams distribute their salaries efficiently. NFL teams obtain their players either by signing free agents—who are players whose contracts with other teams have expired—or by signing players chosen in the annual draft of eligible college players. The college draft consists of seven rounds, with the teams with the worst records the previous year choosing first. Massey and Thaler find that, in fact, NFL teams do not allocate salaries efficiently. In particular, the players chosen with the first few picks of the first round of the draft tend to be paid salaries that are much higher relative to their marginal products than are players taken later in the first round. A typical team with a high draft pick would increase its ability to win football games at the constant cost represented by the salary cap if it traded for lower draft picks, providing it could find another team willing to make the trade. Why do NFL teams apparently make the error of not efficiently distributing salaries? Massey and Thaler argue that general managers of NFL teams tend to be overconfident in their ability to forecast how well a college player is likely to perform in the NFL.

General managers of NFL teams are not alone in suffering from overconfidence. Studies have shown that, in general, people tend to overestimate their ability to forecast an uncertain outcome. Because NFL teams tend to overestimate the future marginal productivity of high draft picks, they pay them salaries that are inefficiently high compared to salaries other draft picks receive. NFL teams were aware that they were probably overpaying high draft picks. In 2011, they negotiated a new contract with the NFL Players Union that limited the salaries that drafted players could receive.

This example shows that the concepts developed in this chapter provide powerful tools for analyzing whether firms are operating efficiently.

Source: Cade Massey and Richard Thaler, "The Loser's Curse: Decision Making and Market Efficiency in the National Football League Draft," *Management Science*, Vol. 59, No. 7, (July 2013), pp. 1479–1495.

Your Turn: Test your understanding by doing related problem 11A.14 on page 462 at the end of this appendix.

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The Expansion Path

We can use isoquants and isocost lines to examine what happens as a firm expands its level of output. Figure 11A.6 shows three isoquants for a firm that produces bookcases. The isocost lines are drawn under the assumption that the machines used in producing bookcases can be rented for $100 per day and the wage rate is $25 per day. The point where each isoquant is tangent to an isocost line determines the cost-minimizing combination of capital and labor for producing that level of output. For example, 10 machines and 40 workers is the cost-minimizing combination of inputs for producing 50 bookcases per day. The cost-minimizing points *A*, *B*, and *C* lie along the firm's **expansion path**, which is a curve that shows the cost-minimizing combination of inputs for every level of output.

Expansion path A curve that shows a firm's cost-minimizing combination of inputs for every level of output.

An important point to note is that the expansion path represents the least-cost combination of inputs to produce a given level of output *in the long run*, when the firm is able to vary the levels of all of its inputs. We know, though, that in the short run, at least one input is fixed. We can use Figure 11A.6 to show that as the firm expands in the short run, its costs will be higher than in the long run. Suppose that the firm is currently at point *B*, using 15 machines and 60 workers to produce 75 bookcases per day. The firm wants to expand its output to 100 bookcases per day, but in the short run, it is unable to increase the quantity of machines it uses. Therefore, to expand output, it must hire more workers. The figure shows that in the short run, to produce 100 bookcases per day using 15 machines, the lowest costs it can attain are at point *D*, where it employs 110 workers. With a rental price of machines of $100 per day and a wage rate of $25 per day, in the short run, the firm will have total costs

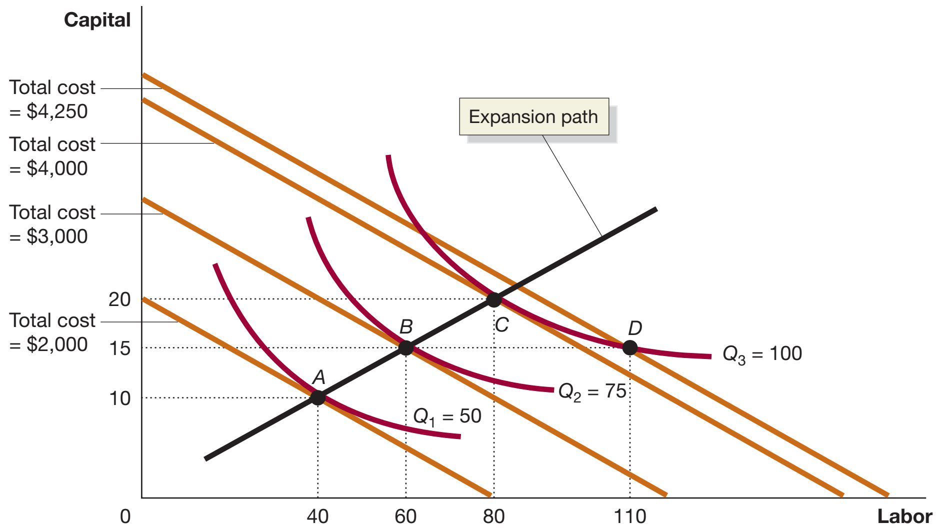

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Figure 11A.6 The Expansion Path

The tangency points *A*, *B*, and *C* lie along the firm's expansion path, which is a curve that shows the cost-minimizing combination of inputs for every level of output. In the short run, when the quantity of machines is fixed, the firm can expand output from 75 bookcases per day to 100 bookcases per day at the lowest cost only by moving from point *B* to point *D* and increasing the number of workers from 60 to 110. In the long run, when it can increase the quantity of machines it uses, the firm can move from point *D* to point *C*, thereby reducing its total costs of producing 100 bookcases per day from $4,250 to $4,000.

of $4,250 to produce 100 bookcases per day. In the long run, though, the firm can increase the number of machines it uses from 15 to 20 and reduce the number of workers from 110 to 80. This change allows it to move from point *D* to point *C* on its expansion path and to lower its total costs of producing 100 bookcases per day from $4,250 to $4,000. The firm's minimum total costs of production are lower in the long run than in the short run.

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Key Terms

Expansion path, p. 459

Isocost line, p. 452

Isoquant, p. 451

Marginal rate of technical substitution (*MRTS*), p. 452

11A Using Isoquants and Isocost Lines to Understand Production and Cost, pages 451–460

LEARNING OBJECTIVE: Use isoquants and isocost lines to understand production and cost.

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Review Questions

11A.1 What is an isoquant? What is the slope of an isoquant?

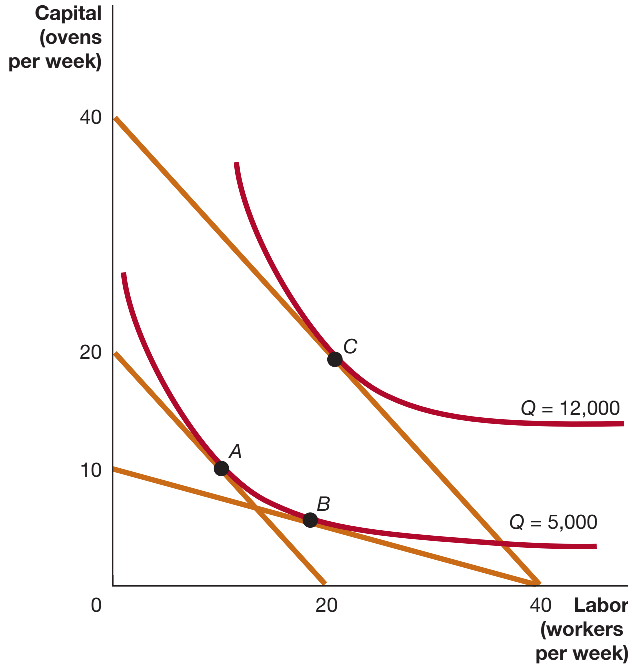
11A.2 What is an isocost line? What is the slope of an isocost line?

11A.3 How do firms choose the optimal combination of inputs?

Problems and Applications

11A.4 A company called Avis can rent cars for $500 per week and hire private drivers for $250 per week. It is currently using 20 cars and 10 private drivers to offer its services and has a total cost of $5,000. Draw an isoquant-isocost line to show the cost-minimizing input combination and the maximum quantity of labor and capital Avis can use with total costs of $5,000.

11A.5 Use the following graph to answer the questions.



a. If the wage rate and the rental price of ovens are both $100 and total cost is $2,000, is the cost-minimizing point *A*, *B*, or *C*? Briefly explain.

b. If the wage rate is $25, the rental price of ovens is $100, and total cost is $1,000, is the cost-minimizing point *A*, *B*, or *C*? Briefly explain.

c. If the wage rate and the rental price of ovens are both $100 and total cost is $4,000, is the cost-minimizing point *A*, *B*, or *C*? Briefly explain.

11A.6 **(Related to Solved Problem 11A.1 on page 455)** India has less arable land per farmer than the United States. As a result, the price of labor in India is less relative to the price of land than it is in the United States. Assume that United Stated and India had access to the same technology for producing grains. Use an isoquant-isocost graph to illustrate why the combination of land and labor for producing grains in India would have been different from the combination used to produce grains in the United States.

11A.7 **(Related to Solved Problem 11A.2 on page 457)** Consider the information in the following table for Jill Johnson's restaurant:

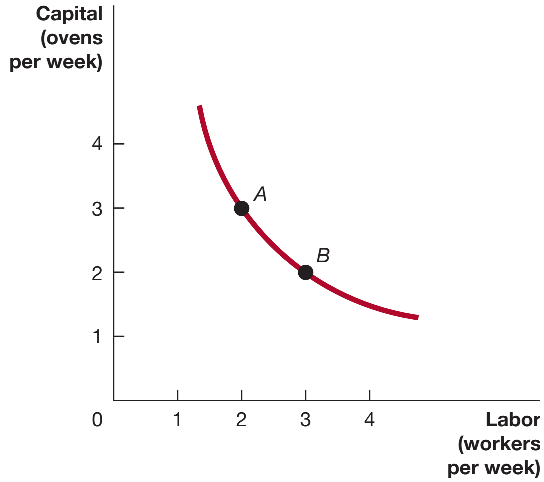
Marginal product of capital	4,000
Marginal product of labor	100
Wage rate	$10
Rental price of pizza ovens	$500

Briefly explain whether Jill is minimizing costs. If she is not minimizing costs, explain whether she should rent more ovens and hire fewer workers or rent fewer ovens and hire more workers.

11A.8 **(Related to Solved Problem 11A.2 on page 457)** Draw an isoquant–isocost line graph to illustrate the following situation: Jill Johnson can rent pizza ovens for $200 per week and hire workers for $100 per week. Currently, she is using 5 ovens and 10 workers to produce 20,000 pizzas per week and has total costs of $2,000. Jill's marginal rate of technical substitution (*MRTS*) equals −1. Explain why this means that she's not minimizing costs and what she could do to minimize costs.

11A.9 Draw an isoquant–isocost line graph to illustrate the following situation and the change that occurs: Jill Johnson can rent pizza ovens for $2,000 per week and hire workers for $1,000 per week. Currently, she is using 5 ovens and 10 workers to produce 20,000 pizzas per week and has total costs of $20,000. Then Jill reorganizes the way things are done in her business and achieves positive technological change.

11A.10 Use the following graph to answer the following questions about Jill Johnson's isoquant curve.



a. Which combination of inputs yields more output: combination *A* (3 ovens and 2 workers) or combination *B* (2 ovens and 3 workers)?

b. What will determine whether Jill selects *A*, *B*, or some other point along this isoquant curve?

c. Is the marginal rate of technical substitution (*MRTS*) greater at point *A* or point *B*?

11A.11 A company sells eggs for $1,000 per week and hire workers at $500 per week. The company can minimize the cost of producing 100 eggs by breeding 20 chickens and using 5 workers, at a total cost of $10,000. The company can minimize the cost by breeding 100 chickens and selling 300 eggs per week using 15 workers, at a total cost of $20,000. The company can minimize the cost by breeding 200 chickens and selling 600 eggs per week using 30 workers, at a total cost of $40,000. Draw an isoquant-isocost curve and discuss its economies and diseconomies of scale.

11A.12 In Brazil, a grove of oranges is picked using 20 workers, ladders, and baskets. In Florida, a grove of oranges is picked using 1 worker and a machine that shakes the oranges off the trees and scoops up the fallen oranges. Using an isoquant–isocost line graph, illustrate why these two different methods are used to pick the same number of oranges per day in these two locations.

11A.13 Jill Johnson is minimizing the costs of producing pizzas. The rental price of one of her ovens is $2,000 per week, and the wage rate is $600 per week. The marginal product of capital in her business is 12,000 pizzas. What must be the marginal product of her workers?

11A.14 **(Related to the Making the Connection on page 458)** If Cade Massey and Richard Thaler are correct, should the team that has the first pick in the draft keep the pick or trade it to another team for a lower pick? Briefly explain. Does the 2011 agreement that limits the salaries of drafted players affect your answer?

11A.15 New Balance produces sneakers. It needs 50 workers and $300 to produce 100 sneakers per week. Suppose that both wage rate and the maintenance costs of machineries triples.

a. Draw a new isocost line to reflect this change in the wage rate and maintenance cost of machineries. Label this combination A1.

b. Draw a new isoquant to show the combination of capital and labor that minimizes total cost given the increase in input prices. Label this combination A2.

c. Comparing point A1 and point A2, how can we be sure that at point A2 New Balance will be using more or less labor and more or less capital? Briefly explain.