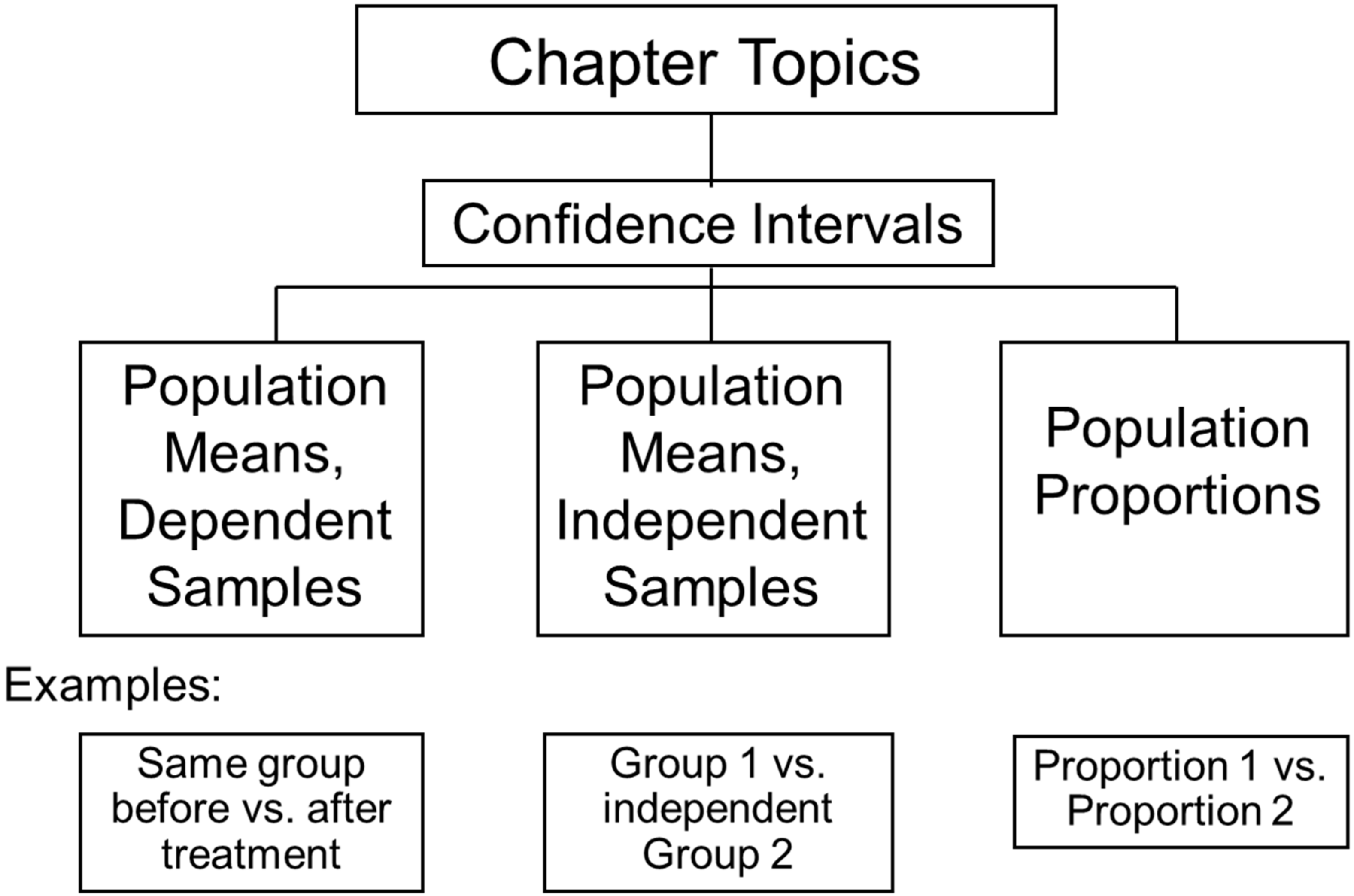
Estimation: Additional Topics



Section 8.1 Dependent Samples

Dependent samples

Confidence Interval Estimation of the Difference Between Two Normal Population Means: Dependent Samples

Tests Means of 2 Related Populations

- Paired or matched samples
- Repeated measures (before/after)
- Use difference between paired values:

$$d_i = x_i - y_i$$

- Eliminates Variation Among Subjects
- Assumptions:
  - Both Populations Are Normally Distributed

Mean Difference

Dependent samples

The i^{th} paired difference is d_i, where

$$d_i = x_i - y_i$$

The point estimate for the population mean paired difference is $\bar{d}$:

$$\bar{d} = \frac{\sum_{i=1}^{n} d_i}{n}$$

The sample standard deviation is:

$$s_d = \sqrt{\frac{\sum_{i=1}^{n} (d_i - \bar{d})^2}{n-1}}$$

n is the number of matched pairs in the sample

Confidence Interval for Mean Difference (1 of 2)

Dependent samples

The confidence interval for the difference between two population means, μ_d, is

$$\bar{d} \pm t_{n-1,\frac{\alpha}{2}} \frac{s_d}{\sqrt{n}}$$

Where

n = the sample size

(number of matched pairs in the paired sample)

Confidence Interval for Mean Difference (2 of 2)

Dependent samples

- The margin of error is

$$ME = t_{n-1,\frac{\alpha}{2}} \frac{s_d}{\sqrt{n}}$$

- $t_{n-1,\frac{\alpha}{2}}$ is the value from the Student's *t* distribution with $(n-1)$ degrees of freedom for which

$$p\left(t_{n-1} > t_{n-1,\frac{\alpha}{2}}\right) = \frac{\alpha}{2}$$

Paired Samples Example (1 of 2)

Dependent samples

- Six people sign up for a weight loss program. You collect the following data:

	Weight:		
Person	Before (x)	After (y)	Difference, d_i
1	136	125	11
2	205	195	10
3	157	150	7
4	138	140	− 2
5	175	165	10
6	166	160	6
			42

$$\bar{d} = \frac{\sum d_i}{n}$$

$$= 7.0$$

$$s_d = \sqrt{\frac{\sum (d_i - \bar{d})^2}{n-1}}$$

$$= 4.82$$

Paired Samples Example (2 of 2)

Dependent samples

- For a 95% confidence level, the appropriate *t* value is

$$t_{n-1,\frac{\alpha}{2}} = t_{5,.025} = 2.571$$

- The 95% confidence interval for the difference between means, μ_d, is

$$\bar{d} \pm t_{n-1,\frac{\alpha}{2}} \frac{S_d}{\sqrt{n}}$$

$$7 \pm (2.571) \frac{4.82}{\sqrt{6}}$$

$$-1.94 < \mu_d < 12.06$$

Since this interval contains zero, we cannot be 95% confident, given this limited data, that the weight loss program helps people lose weight