Section 8.3 Two Population Proportions (1 of 2)

Population proportions

Confidence Interval Estimation of the Difference Between Two Population Proportions (Large Samples)

Goal: Form a confidence interval for the difference between two population proportions, $P_x - P_y$

Section 8.3 Two Population Proportions (2 of 2)

Population proportions

Goal: Form a confidence interval for the difference between two population proportions, $P_x - P_y$

Assumptions:

Both sample sizes are large (generally at least 40 observations in each sample)

The point estimate for the difference is $\hat{p}_x - p_y$

Two Population Proportions

Population proportions

- The random variable

$$Z = \frac{(\hat{p}_x - p_y) - (p_x - p_y)}{\sqrt{\frac{\hat{p}_x(1 - p_x)}{n_x} + \frac{\hat{p}_y(1 - p_y)}{n_y}}}$$

is approximately normally distributed

Confidence Interval for Two Population Proportions

Population proportions

The confidence limits for

$P_x - P_y$ are:

$$\left(\hat{p}_x - \hat{p}_y\right) \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_x\left(1 - \hat{p}_x\right)}{n_x} + \frac{\hat{p}_y\left(1 - \hat{p}_y\right)}{n_y}}$$

Example: Two Population Proportions (1 of 3)

Form a 90% confidence interval for the difference between the proportion of men and the proportion of women who have college degrees.

- In a random sample, 26 of 50 men and 28 of 40 women had an earned college degree

Example: Two Population Proportions (2 of 3)

Men: $\hat{p}_x = \frac{26}{50} = 0.52$

Women: $\hat{p}_y = \frac{28}{40} = 0.70$

$$\sqrt{\frac{\hat{p}_x(1-p_x)}{n_x}+\frac{\hat{p}_y(1-p_y)}{n_y}} = \sqrt{\frac{0.52(0.48)}{50}+\frac{0.70(0.30)}{40}} = 0.1012$$

For 90% confidence, $Z_{\frac{\alpha}{2}} = 1.645$

Example: Two Population Proportions (3 of 3)

The confidence limits are:

$$\left(\hat{p}_x - p_y\right) \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_x\left(1 - p_x\right)}{n_x} + \frac{\hat{p}_y\left(1 - p_y\right)}{n_y}}$$

$$= \left(.52 - .70\right) \pm 1.645\left(0.1012\right)$$

so the confidence interval is

$$-0.3465 < P_x - P_y < -0.0135$$

Since this interval does not contain zero we are 90% confident that the two proportions are not equal

Chapter Goals

After completing this chapter, you should be able to:

- Formulate null and alternative hypotheses for applications involving
  - a single population mean from a normal distribution
  - a single population proportion (large samples)
  - the variance of a normal distribution
- Formulate a decision rule for testing a hypothesis
- Know how to use the critical value and *p*-value approaches to test the null hypothesis (for both mean and proportion problems)
- Define Type I and Type II errors and assess the power of a test
- Use the chi-square distribution for tests of the variance of a normal distribution

Section 9.1 Concepts of Hypothesis Testing

- A hypothesis is a claim (assumption) about a population parameter:

  - population mean

    Example: The mean monthly cell phone bill of this city is $\mu = \$52$

  - population proportion

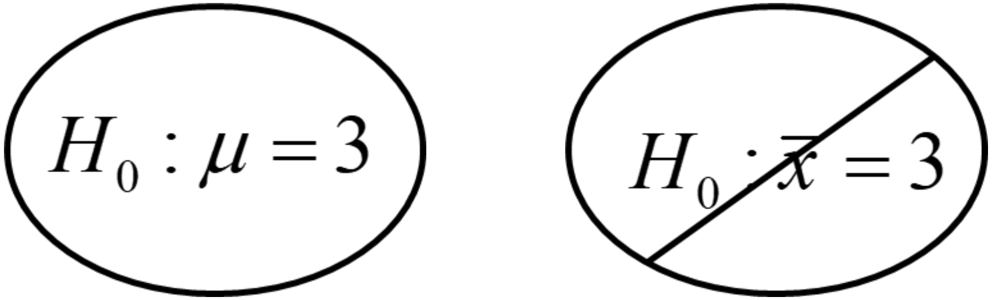
    Example: The proportion of adults in this city with cell phones is *P* = .88

The Null Hypothesis, *H* Sub 0 (1 of 2)

- States the assumption (numerical) to be tested

  Example: The average number of TV sets in U.S. Homes is equal to three $(H_0 : \mu = 3)$

- Is always about a population parameter, not about a sample statistic



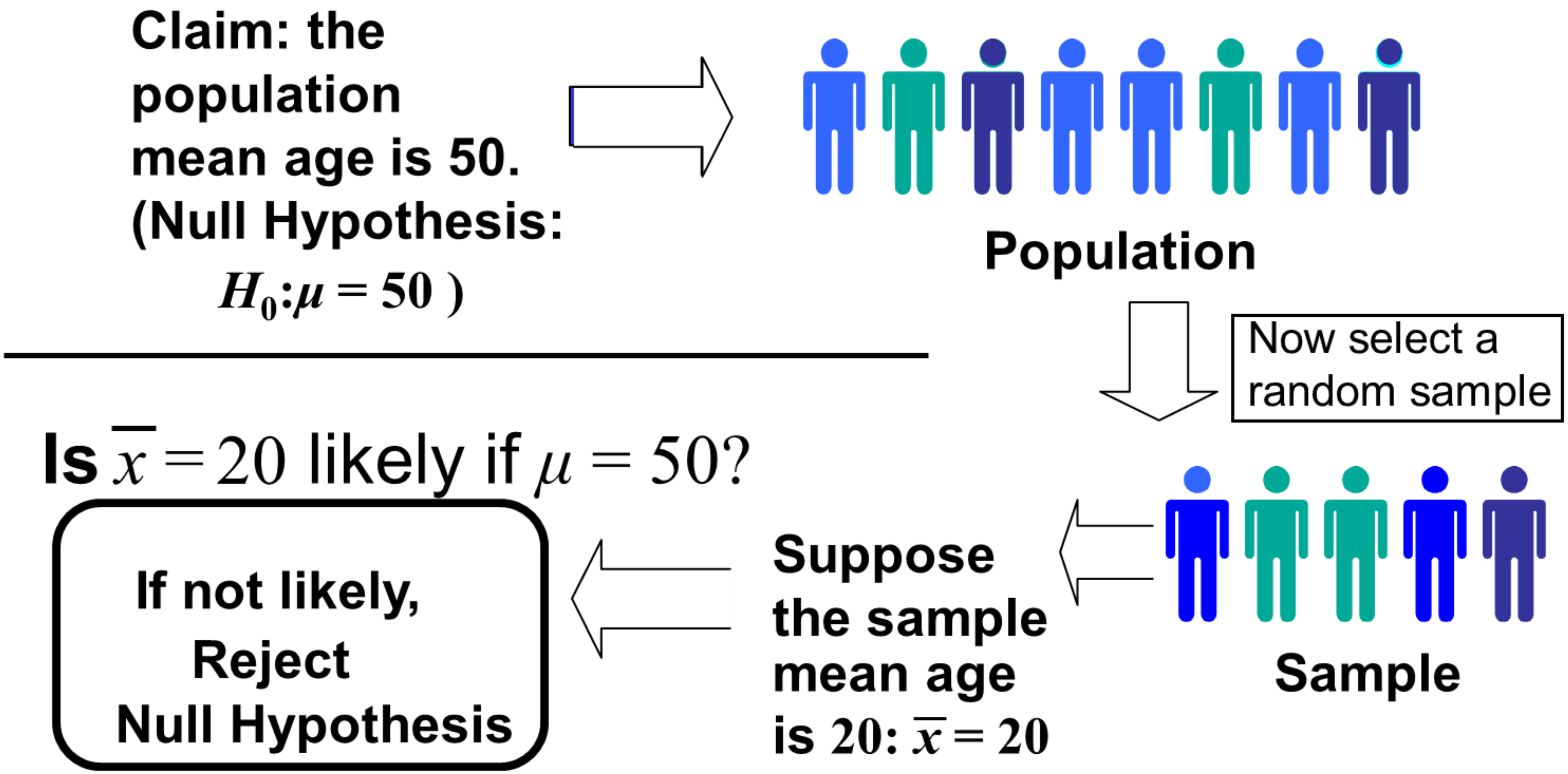
The Null Hypothesis, *H* sub 0 (2 of 2)

- Begin with the assumption that the null hypothesis is true
  - Similar to the notion of innocent until proven guilty
- Refers to the status quo
- Always contains ?= , ?≤ or ?≥ sign
- May or may not be rejected

The Alternative Hypothesis, *H* Sub 1

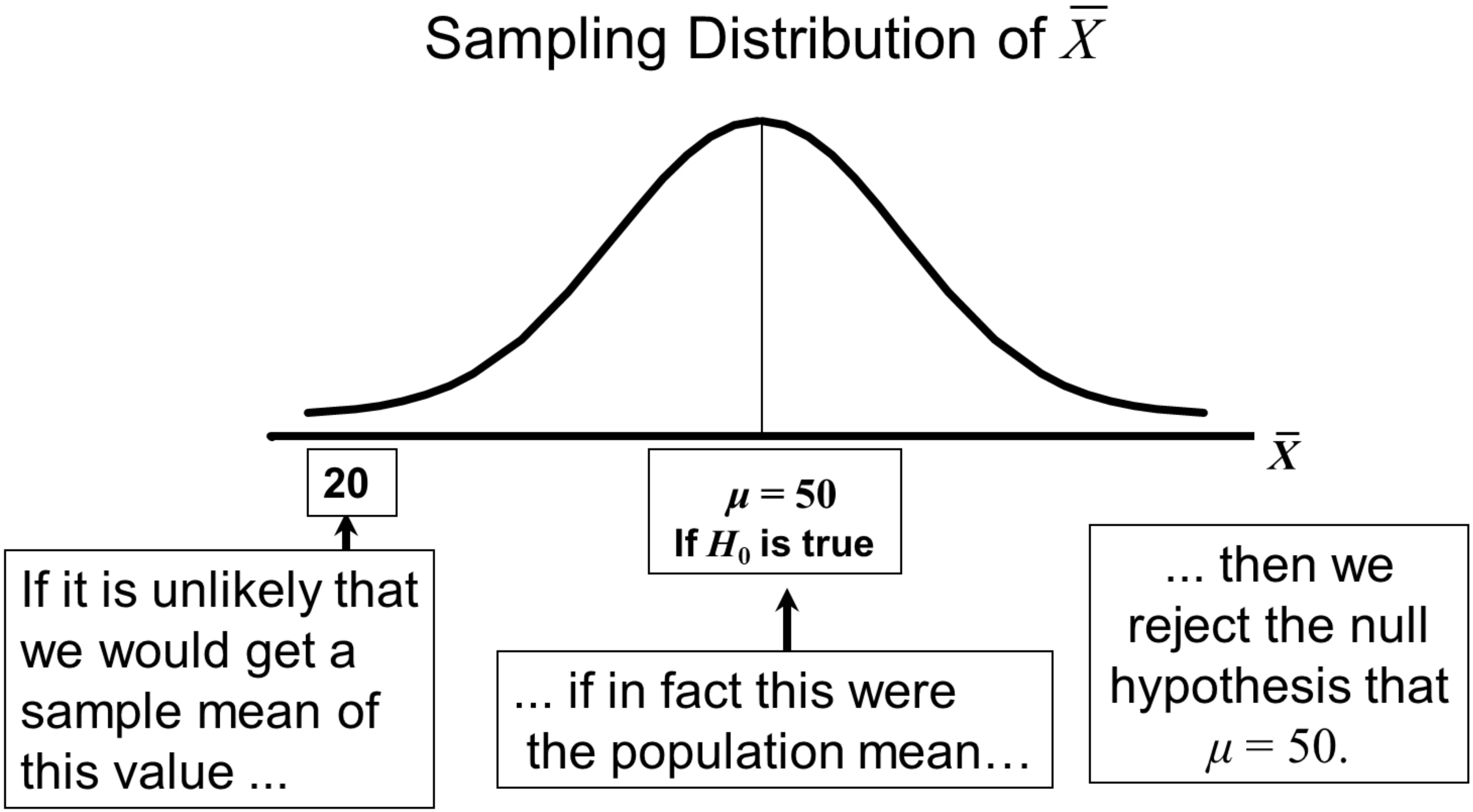
- Is the opposite of the null hypothesis
  - e.g., The average number of TV sets in U.S. homes is not equal to 3 $(H_1 : \mu \neq 3)$
- Challenges the status quo
- Never contains the ?$=$, ?$\leq$ or ?$\geq$ sign
- May or may not be supported
- Is generally the hypothesis that the researcher is trying to support

Hypothesis Testing Process



Pearson

Reason for Rejecting *H* Sub 0



Level of Significance, α

- **Defines the unlikely values of the sample statistic if the null hypothesis is true**
  – Defines rejection region of the sampling distribution
- Is designated by $\boldsymbol{\alpha}$, (level of significance)
  – Typical values are 0.01, 0.05, or 0.10
- Is selected by the researcher at the beginning
- Provides the critical value(s) of the test

Level of Significance and the Rejection Region

Level of significance = α

$H_0: \mu = 3$

$H_1: \mu \neq 3$

Two-tail test

$H_0: \mu \leq 3$

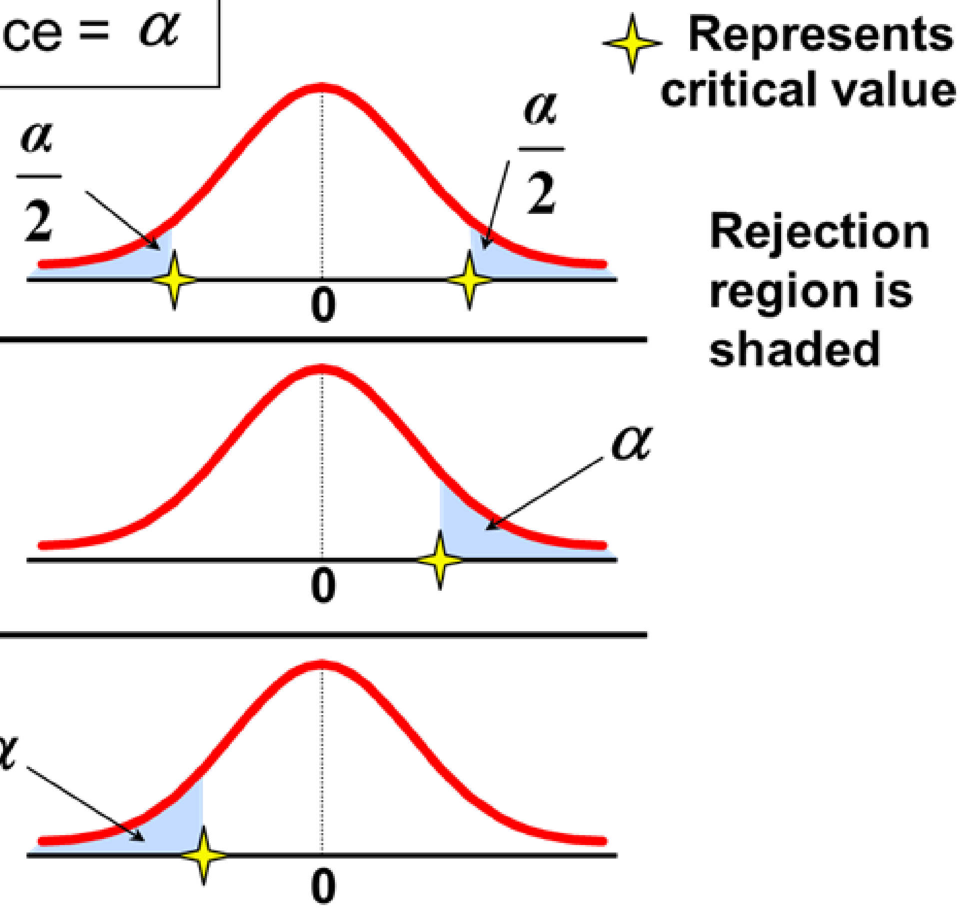
$H_1: \mu > 3$

Upper-tail test

$H_0: \mu \geq 3$

$H_1: \mu < 3$

Lower-tail test



Errors in Making Decisions (1 of 2)

- **Type I Error**
  - Reject a true null hypothesis
  - Considered a serious type of error

The probability of Type I Error is $\boldsymbol{\alpha}$

- Called level of significance of the test
- Set by researcher in advance

Errors in Making Decisions (2 of 2)

- **Type II Error**
  - Fail to reject a false null hypothesis

The probability of Type II Error is β

Outcomes and Probabilities

Possible Hypothesis Test Outcomes

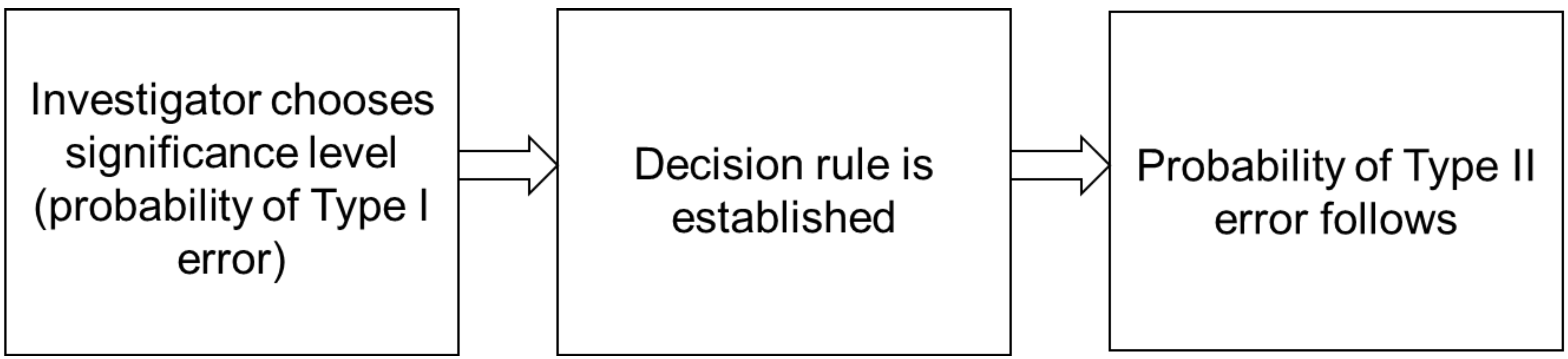
	Actual Situation	
Decision	H_0 True	H_0 False
Fail to Reject H_0	**Correct Decision** $(1 - \alpha)$	**Type II Error** (β)
Reject H_0	**Type I Error** (α)	**Correct Decision** $(1 - \beta)$

Key:
Outcome
(Probability)

$(1 - \beta)$ is called the power of the test

Consequences of Fixing the Significance Level of a Test

- Once the significance level α is chosen (generally less than 0.10), the probability of Type II error, β, can be found.



Type I & II Error Relationship

- Type I and Type II errors can not happen at the same time
  - Type I error can only occur if H_0 is true
  - Type II error can only occur if H_0 is false

If Type I error probability $(\alpha) \Uparrow$, then

Type II error probability $(\beta) \Downarrow$

Factors Affecting Type II Error

- All else equal,
  - $\beta \Uparrow$ when the difference between hypothesized parameter and its true value $\Downarrow$
  - $\beta \Uparrow$ when $\alpha \Downarrow$
  - $\beta \Uparrow$ when $\sigma \Uparrow$
  - $\beta \Uparrow$ when $n \Downarrow$

Power of the Test

- The power of a test is the probability of rejecting a null hypothesis that is false
- i.e., $$\text{Power} = P(\text{Reject } H_0 \mid H_1 \text{ is true})$$
  - Power of the test increases as the sample size increases

Hypothesis Tests for the Mean