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Curtailment as a mortgage performance indicator ☆

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Abstract

This paper studies the impact of mortgage curtailment behaviors on the subsequent default and prepayment performance. Although curtailment is not a popular event in the western countries, it is the dominant form of prepayment in Asia and other high saving rate regions. Using a sample of loan-level mortgage performance records from Taiwan, the results of the multinomial logit regressions indicate that curtailment is one of the most significant factors in predicting future default and prepayment probabilities of a seasoned mortgage pool. Mortgages with past curtailment are estimated to be 85% less likely to default and 23% more likely to prepay during the remaining life than a mortgage without any curtailment. Hence, ignorance of past curtailment records could lead to biased projection of default and prepayment and, hence, the pricing and hedging of a seasoned mortgage-backed security. By collecting and incorporating curtailment information, investors could more accurately estimate the fair market value, disclose risk-based capital, and perform effective hedging of a particular mortgage portfolio.

JEL classification: G1; G21

Keywords: Mortgage; Prepayment; Curtailment; Default; Securitization; Multinomial logit; Competing risk; Portfolio management; Seasoned MBS; Loan level data; Taiwan

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1. Introduction

A mortgage loan is a debt contract that is secured by a real property. The borrower obtains a loan at origination and promises to repay at a fixed amortization schedule. At each point in time during the amortization period, the borrower has the right to choose among four payment actions: scheduled payment, complete prepayment, default, and curtailment. Much of the literature in mortgage finance considers a mortgage contract embedded with default (put) and prepayment (call) options for mortgagors.[2] Default means that the borrower exercises the put option by selling the collateralized house to the lenders at the price equal to the unpaid principal balance (UPB) of the mortgage loan. The prepayment option gives the borrower the right to buy the loan back from the lender at the price equal to the UPB. The distinct difference is the fact that default only happens at the payment due dates, while a partial or complete prepayment can take place at any time before the maturity date.

The past delinquency history of the pool or similar loans is a key indicator of the future default risk. Pools with a high past delinquency rate are likely to experience high future default rates. As a result, the investor would require a higher return for pools with high delinquency records. When securitizing a seasoned pool of mortgages, an issuer would usually exclude loans with past delinquency records in order to realize a good sales price.

Curtailment refers to the event that a borrower makes a larger than scheduled periodical mortgage payment. The extra amount paid will be used to reduce the UPB of the mortgage.

Although curtailment may not be a popular event among the western countries, it is recognized as the dominant form of prepayment in some regions with high saving rates, such as the Asian markets. In those countries, when a household receives extra income, such as a year-end bonus, partially repaying the outstanding debts is always a high priority alternative way of using the money. This behavior is consistent with the high savings rate observed in these regions. As the mortgage market in those regions matures and becomes more competitive, there is an increasing need to understand the impact of curtailment.

The rest of the paper is organized as follows. The next section reviews relevant literature. Section 3 introduces the model and hypotheses regarding the impact of curtailment on future default and prepayment. The Section 4 presents the sample data and the econometric model applied in the empirical analysis. The Section 5 reports and interprets the regression results and their implications. The Section 6 concludes our study.

2. Literature review

Relative to the extensive academic literature on complete prepayments, the curtailment literature is limited. There are only four papers that document curtailment

[2] For example, Kau et al. (1995), Buist et al. (1998).

studies. Most literature focuses on the estimation of the probability of a curtailment to occur. Hayre and Lauterbach (1991) were the first to discuss the curtailment behavior and its unique features. To capture this effect, they add an average constant dollar amount each month in the prepayment model. However, they did not provide a detailed discussion of either the theoretical or empirical model underlying the curtailment behavior. Chinloy (1993) proposed a theoretical mortgage curtailment model based on consumer wealth maximization behavior and empirically tested the model using GNMA data. He clearly points out the difference between complete prepayment and curtailment, "In termination, the loan disappears from a pool, and the scheduled payment to investors in the pool is reduced. In curtailment, the loan survives, and the scheduled payment is unchanged but term is reduced." As a result, Chinloy contended that the pricing results at loan level on mortgage-related derivates would be biased if a full prepayment and curtailments were not specified separately.

Budinger and Fan (1995) studied the impact of curtailments on "jumbo loans." Their results show that curtailments reflect a small portion of the total amount prepaid in the early life of the mortgage, but can be a material contributor to prepayments as the pool ages. Abrahams (1997) discussed curtailments on prepayment modeling by formulating curtailments as a function of loan age. Fu (1997) provided an empirical analysis of the mortgage curtailment to extract expected correlates of the curtailment decision and concludes that previous curtailment is strongly predictive of future curtailment behavior.

This paper differs from the previous literature in that we examine the mortgage curtailment by focusing on the fact that the curtailment history should also provide valuable information about the future default behavior and the complete prepayment behavior of a MBS. By incorporating the curtailment records, one should be able to more accurately estimate the cash flows and the fair market value of a MBS or a pool of whole loans. The information provided by curtailment history is an important indicator of the heteroschadasticity among borrowers that is otherwise unobservable.[3] Using a loan level monthly transaction data set, we estimated the impact of past curtailment behavior on future mortgage performance. The results of the multinomial logit regressions indicate that the curtailment history has very strong predicting power over the future default and prepayment rates. The impact is sometimes even stronger than that of the more closely monitored delinquent history. This finding suggests that the secondary mortgage market should consider including curtailment history as an important disclosing element to allow the traders or the insurers/guarantors to price more efficiently.

3. Curtailment and mortgage performance

Typically, a residential mortgage borrower has the right to partially or in full prepay the mortgage loan prior to its maturity without penalty. Prepayment in full is

[3] Many researchers, such as Stanton (1995) and Deng et al. (2001), used advanced statistics tools to explain the heteroschadastic behavior among mortgage borrowers.

usually a result of the due-on-sale clause or triggered by refinancing. Most households in the western countries tend to have tenure in a house much shorter than the mortgage maturity term. Whenever a house is sold and the mortgage contains a due-on-sale clause, the remaining balance of the mortgage loan becomes due in full. Due-on-sale driven prepayment is usually realized as a result of job relocation, change of household size, or change of household income. There is relatively little correlation associated with this type of prepayment among households.

On the other hand, the refinancing driven prepayment can be highly correlated among all households in a country. Refinancing usually occurs when the market interest rate drops significantly below the interest rate on the existing mortgage. The borrower can apply for a new mortgage at the lower market rate and use new loan amount to pay off the old mortgage loan. Although default is not a relevant risk to conforming MBS investors, it is a significant risk to investors of the private label MBSs and to the mortgage insurance and guarantee providers. Default usually occurs when the value of the underlying house drops below the UPB. When default occurs, the lender receives the collateral house instead of the promised cash flows. As a result, in addition to the early retirement of the cash flows, like the prepayment condition, the default sensitive investors also suffer from loss of principal recovery. Although some lenders do not go after the borrowers' personal properties with deficient judgment, a typical borrower does not default simply because the house price decreases. Most mortgage defaults are triggered by two events: a borrower's income shock, making it difficult to continue the scheduled mortgage payments, and the decrease of the value of the house, making prepayment by selling the house not feasible.[4]

Numerous research studies have been published during the past three decades trying to explain and predict prepayment and default rate of mortgages and MBSs.[5] In the mortgage industry, some of the best econometricians have developed proprietary prepayment and default forecast models to support the trading in MBSs, which is now the single largest security type in the world. Typically, the prepayment and default rates are estimated based on four categories of explanatory variables: information related to the borrower, to the collateral property, to the loan contract features, and to the past performance of the loan. The first category includes payment-to-income ratio, job stability, credit history, etc. The second category includes loan-to-value ratio (LTV), geographic location, property condition, etc. The third category includes amortization term, interest rate adjustment feature, etc. The last and newest category variables are more difficult to capture and are only relevant to analyze seasoned mortgages. Some examples are burnout, delinquency status, and history. These variables require more comprehensive data regarding individual actual historical payments of a mortgage or a pool. Nevertheless, these variables tend to be very effective in explaining termination behaviors that previous models fail to capture.

In this paper, we argue that curtailment history of a seasoned mortgage should serve as valuable information in predicting future prepayment default terminations.

[4] See Buist et al. (1998).

[5] See Calhoun and Deng (2002) for a review.

3.1. Information conveyed in curtailment

According to Chinloy (1993), a borrower's behavior of prepay, default, curtail, and delinquent are direct results of the personal utility maximization choices. As a result, the outcome of a particular mortgage's performance reveals important information about the borrower's saving/consumption habits, income stability, true wealth, and other characteristics that are not observable or measurable by the mortgage lender. When complete prepayment or default happens, the mortgage loan terminates. Although important information about the borrower may be revealed from such an event, the existing lender has little flexibility to take advantage of this information.

Curtailment and delinquency also provide the additional information about the borrower's utility function. More importantly, since the mortgage loan will continue to exist after the event, the information obtained can serve as important indicators of the future performance of the particular mortgage loan. The impact of delinquency is quite intuitive and has received vast attention in the industry. However, the potential impact of curtailment has not been carefully studied. Most mortgage investors do not keep or use such information for business decisions.

One direct result of curtailment is that the UPB of the loan decreases faster than the amortization schedule. Other things being equal, curtailment makes the updated LTV lower. In addition, since the UPB is smaller, if the borrower continues the original scheduled periodical payment amount, the speed of UPB amortization will run at much faster rate, making the effective maturity term shorter. As the LTV ratio reduces, the probability of default in the future will naturally decrease.

In addition to the direct impact of the lower updated (or current) LTV, the borrower's curtailment behavior also provides otherwise unobservable characteristics of the borrower. First, the fact that the borrower is able to afford the extra payment is an indication that the borrower has income capacity in excess the minimum-underwriting requirement. This *excess debt coverage capacity* has implications to both default and prepayment risks. With higher than minimum repayment capacity, the borrower is able to absorb a temporarily reduction of income or loss of employment. Compared to their counterparts, borrowers that curtail should have lower default risk, after netting the current LTV effect. On the other hand, the excess debt coverage capacity also suggests that the borrower can easily obtain an alternative mortgage on the market. When market interest rates decline, these borrowers are more likely to take advantage of the opportunity to refinance the existing mortgage.

Second, the fact that the borrower is willing to save the extra income by reducing the outstanding debt indicates the household's utility of relief from debt is higher than that of additional consumption. If a household used excess cash flow to partially prepay the mortgage in the past, it is likely to do it again in the future. This habit will cause the effective life of the mortgage to be short, which is an undesirable feature by some MBS investors, especially the servicers.

Third, curtailment could indicate that the borrower is less likely to face income fluctuation. Liquidity is particularly important to a household with high-income volatility. By parking the excess money in a liquid asset, such as stocks or money market accounts, the household retains the immediate access to the cash when experiencing income difficulties. If the excess cash is used to partially payoff the mortgage debt, the household has to go through the trouble of cash-out refinancing to regain the immediate control of the money. Such a process would require transaction costs and could take a period of time longer than the household can afford. Therefore, the income stability of borrowers that curtail is likely to be better than their counterparts, such as the self-employed borrowers. With the higher income stability, the in ability-to-pay (default) is less likely to occur, making the overall default risk lower.

3.2. The hypotheses

Based on the above rationales, we contented that the curtailment history provides valuable information in predicting subsequent prepayment and default probabilities. In particular, borrowers that curtailed in the past tend to experience lower default rate and higher prepayment rate for the remaining life of the mortgage. To verify these hypotheses and to measure the potential size of such impacts, we conducted an empirical research using a loan-level monthly payment record sample from a commercial bank in Taiwan, where curtailment is frequently observed.

4. The empirical evidence

4.1. The model

With the loan-level performance information, we are able to adopt the multinomial logit regression model to capture the competing risk nature of the mortgage contracts. Quarterly conditional default and prepayment rates are estimated with the multinominal logit model introduced by Calhoun and Deng (2002). The multinomial logit model takes the form of:

$$\pi_D(t) = \frac{e^{\alpha_D + X_D(t)\beta_D}}{1 + e^{\alpha_D + X_D(t)\beta_D} + e^{\alpha_P + X_P(t)\beta_P}}, \tag{1}$$

$$\pi_P(t) = \frac{e^{\alpha_P + X_P(t)\beta_P}}{1 + e^{\alpha_D + X_D(t)\beta_D} + e^{\alpha_P + X_P(t)\beta_P}}, \tag{2}$$

$$\pi_A(t) = \frac{1}{1 + e^{\alpha_D + X_D(t)\beta_D} + e^{\alpha_P + X_P(t)\beta_P}}, \tag{3}$$

where $\pi_D(t)$, $\pi_P(t)$, and $\pi_A(t)$ are the single period default, prepayment, and active rates conditional on the loan being outstanding at the beginning of time period t. The explanatory variables $X_D(t)$ and $X_P(t)$ are associated with a particular loan during the exposure period t, and α_D, α_P, β_D, β_P are constant coefficients estimated by the regression.

This equation system captures the competing risk nature of the three possible outcomes of a mortgage at any point in time. As the conditional probability of default rises, the denominator of Eqs. (2) and (3) increases, making the conditional prepayment and active rates lower. Similarly, as the conditional prepayment rate rises, the denominator of Eqs. (1) and (3) increases, causing the conditional default and active rates to drop. It also ensures that the sum of the three possibilities equals one, making the three outcomes mutually exclusive.

With the loan-level quarterly performance data, we first estimate the coefficients using standard explanatory variables without incorporating the information of curtailment. Then the same regressions were estimated by adding the additional curtailment information observed at the end of different mortgage age. That is:

$$\pi'_D(t) = \frac{e^{\alpha_D + X_D(t)\beta_D + \mathrm{Curtail}(\tau)\gamma_D}}{1 + e^{\alpha_D + X_D(t)\beta_D + \mathrm{Curtail}(\tau)\gamma_D} + e^{\alpha_P + X_P(t)\beta_P + \mathrm{Curtail}(\tau)\gamma_P}}, \tag{4}$$

$$\pi'_P(t) = \frac{e^{\alpha_P + X_P(t)\beta_P + \mathrm{Curtail}(\tau)\gamma_P}}{1 + e^{\alpha_D + X_D(t)\beta_D + \mathrm{Curtail}(\tau)\gamma_D} + e^{\alpha_P + X_P(t)\beta_P + \mathrm{Curtail}(\tau)\gamma_P}}, \tag{5}$$

$$\pi'_A(t) = \frac{1}{1 + e^{\alpha_D + X_D(t)\beta_D + \mathrm{Curtail}(\tau)\gamma_D} + e^{\alpha_P + X_P(t)\beta_P + \mathrm{Curtail}(\tau)\gamma_P}}. \tag{6}$$

Note that Eqs. (4)–(6) differ from Eqs. (1)–(3) only by the additional term of Curtail(τ) multiplied by the regression coefficient, where Curtail(τ) represents the cumulative curtailment occurred up to age τ.

The multinomial logit regression results of Eqs. (4)–(6) enable us to derive two levels of knowledge about the curtailment impact on subsequent default or prepayment. First, the statistical and economical significance of the coefficients γ's can provide direct evidence in supporting or rejecting our hypotheses. Second, by observing the magnitudes and significance of the estimated γ's, we can understand the impacts of curtailment to the subsequent prepayment and default with respect to the characteristics of a loan.

Because curtailment information is only available for seasoned loans, the above analysis is performed for loans with age greater than 0. Since the sample contains loan performance up to 26 quarters, we chose to repeat the above analysis conditional on the curtail information of age being equal to 2, 4, 8, and 12 quarters. This allows for at least 14 quarters of subsequent performance observations. Comparing the significance of the coefficients of curtailment information of loans with different seasoning, we are able to tell if the effectiveness of the curtailment information increases or decreases with the age of the mortgage.

With the same research setting, we can also test for the effectiveness of using past delinquency information to differentiate loan performance. As mentioned earlier, the secondary mortgage industry always request past delinquency information when purchasing or insuring a seasoned mortgage pool. With this comparison, we would be able to quantify the effectiveness of this information. As a result, the final models being estimated are Eqs. (1)–(3) and Eqs. (7)–(9) below.

$$\pi'_{\mathrm{D}}(t) = \frac{e^{\alpha_{\mathrm{D}}+X_{\mathrm{D}}(t)\beta_{\mathrm{D}}+\mathrm{Curtail}(\tau)\gamma_{\mathrm{D}}+\mathrm{Delinquent}(\tau)\delta_{\mathrm{D}}}}{1+e^{\alpha_{\mathrm{D}}+X_{\mathrm{D}}(t)\beta_{\mathrm{D}}+\mathrm{Curtail}(\tau)\gamma_{\mathrm{D}}+\mathrm{Delinquent}(\tau)\delta_{\mathrm{D}}}+e^{\alpha_{\mathrm{P}}+X_{\mathrm{P}}(t)\beta_{\mathrm{P}}+\mathrm{Curtail}(\tau)\gamma_{\mathrm{P}}+\mathrm{Delinquent}(\tau)\delta_{\mathrm{P}}}}, \tag{7}$$

$$\pi'_{\mathrm{P}}(t) = \frac{e^{\alpha_{\mathrm{P}}+X_{\mathrm{P}}(t)\beta_{\mathrm{P}}+\mathrm{Curtail}(\tau)\gamma_{\mathrm{P}}+\mathrm{Delinquent}(\tau)\delta_{\mathrm{P}}}}{1+e^{\alpha_{\mathrm{D}}+X_{\mathrm{D}}(t)\beta_{\mathrm{D}}+\mathrm{Curtail}(\tau)\gamma_{\mathrm{D}}+\mathrm{Delinquent}(\tau)\delta_{\mathrm{D}}}+e^{\alpha_{\mathrm{P}}+X_{\mathrm{P}}(t)\beta_{\mathrm{P}}+\mathrm{Curtail}(\tau)\gamma_{\mathrm{P}}+\mathrm{Delinquent}(\tau)\delta_{\mathrm{P}}}}, \tag{8}$$

$$\pi'_{\mathrm{A}}(t) = \frac{1}{1+e^{\alpha_{\mathrm{D}}+X_{\mathrm{D}}(t)\beta_{\mathrm{D}}+\mathrm{Curtail}(\tau)\gamma_{\mathrm{D}}+\mathrm{Delinquent}(\tau)\delta_{\mathrm{D}}}+e^{\alpha_{\mathrm{P}}+X_{\mathrm{P}}(t)\beta_{\mathrm{P}}+\mathrm{Curtail}(\tau)\gamma_{\mathrm{P}}+\mathrm{Delinquent}(\tau)\delta_{\mathrm{P}}}}. \tag{9}$$

4.2. The data

This section describes technical details of econometric model utilized to test the hypotheses introduced earlier. To achieve reliable statistic results, a mortgage performance with high curtailment frequency is necessary. As a result, the sample data are extracted from a commercial bank in Taiwan.

The sample includes 46,440 loans originated between 1992 and 2003. However, complete electronic payment records are only available starting in January 1997. To avoid any sample bias caused by censoring problem, loans originated prior to 1997 are removed from the database. This criterion excluded 5546 loans from the sample. Loans with features outside of the reasonable ranges may be involved in non-standard lending situation. To avoid the potential bias of such non-standard terms, an additional 17,539 loans are excluded based on the following criteria: loan size greater than NT$50,000,000 or less than NT$100,000; initial interest rate greater than 13% or less than 2%; loan term greater than 30 years or less than 7 years; interest-only loans; and loans with missing origination dates.

Because of the loan size limitations of some government subsidized loan programs, it is a common practice that a borrower takes out more than one mortgage loans toward the purchase of more expensive housing. For these loans collateralized by the same property, they always prepay and default on the same date. As a result, they should be combined and viewed as one single large loan by the borrower. This loan consolidation process further reduces the sample size by 5710, leaving 17,645 loans in the final sample.

Among these loans, 269 or 1.5% loans defaulted and 9280 or 52.6% loans were prepaid during the sample period. At age equal to 4 quarters, 3289 loans were under delinquent and 2427 loans had previously been curtailed. For each loan in the final sample, quarterly performance records are constructed based on the monthly payment information. For each quarter prior to the maturity and the termination date of a mortgage, one observation is created. The dependent variable of each observation is set to be active, defaulted, or completely prepaid depending if default or complete prepayment is observed during one of the three months in the quarter. A total of 224, 853 quarterly loan-level performance observations are constructed by the 17,645 loans. These observations are used in the multinomial logit regression.

4.3. Sample variables

Table 1 provides the summary statistics of the sample. Like most countries in Asia, mortgage loans in Taiwan are predominantly adjustable rate mortgages (ARMs) with no annual or life rate caps. The typical term to maturity is 20 years with self-amortization. Each of the explanatory variables is described in detail below.

4.4. Curtailment

Curtailment is defined as any amount paid in excess of the scheduled monthly payment and these partial payments will be used toward the reduction of the UPB of the loan. For monthly payment transactions, the data only record actual monthly payments and thus scheduled payments must be inferred from the amortization schedule. To capture the curtailment behavior, we define the cumulated curtailment

Table 1
Descriptive statistics—sample used for estimation

Explanatory variable	Mean	SD	Min	Max
Cltv	0.693987	0.021025	0.481717	0.790388
Relative payment ratio	0.968427	0.073165	0.497059	1.411624
Relative contract rate	1.159342	0.189823	0.506446	2.032491
Relative loan size	1.262424	0.917989	0.036552	20.79369
Exp_yr02_q1	0.205214	0.403859	0	1
Spring	0.26154	0.439474	0	1
Summer	0.241207	0.427816	0	1
Fall	0.249861	0.432933	0	1
Winter	0.247393	0.431498	0	1
Term15	0.146287	0.353394	0	1
Term20	0.662686	0.472794	0	1
Term30	0.191027	0.393111	0	1
Age	8.104139	6.161939	0	25

Cltv: time-varying current LTV ratios, updated with Taiwan quarterly house price index assuming original LTV is 70% for each loan in the pool. Relative payment ratio: the ratio of mortgage payment amount at the end of each quarter over the payment amount at origination. Relative contract rate: the ratio of the original contract rate relative to the average for loans originated during the same origination year. Relative loan size: the ratio of the original loan amount relative to the average-sized loan originated during the same origination year. Exposure_yr02_q1: a variable to account for exposure year and quarter post to quarter 1, year 2002. Seasonality: spring, summer, fall, and winter. Term: maturity of loan. Age: age of loan in quarters. Agesquare: square of age of loan in quarters.

variable as the cumulated actual payment amounts divided by the cumulated scheduled payment amounts at the age of the loan in quarters. Since most of the loans are adjustable rates, the scheduled payment for each quarter is computed using the current UPB and the interest rate for that quarter. The actual payment amount is computed by the sum of the principal and interest amount paid during the three months of the quarter.

A curtailment ratio of greater than one indicates that cumulative curtailment is observed for the quarter. A curtailment ratio of less than one indicates that the loan has been previously delinquent. A ratio equal to one means the loan is actively following the scheduled payments. To avoid a rounding error effect, discrete ranges of the curtailment or delinquency are used in creating corresponding dummy variables to capture the effect of the curtailment. Five dummy variables are included: serious delinquent ($\text{ratio} < 0.50$), delinquent ($0.50 < \text{ratio} < 0.95$), normal ($0.95 < \text{ratio} < 1.05$), curtailment ($1.05 < \text{ratio} < 2$), serious curtailment ($2 < \text{ratio}$). We anticipate that curtailment should be positively related to prepayment but negatively related with default. Contrarily, delinquency is expected to be positively related to default but negatively related to prepayment.

4.5. Current loan-to-value ratio

In recognition of the impact of the trend of house prices on loan default and prepayment probabilities, we update each loan record with quarterly Taiwan House Price Indices to arrive at current LTV ratios (CLTV) . According to the general underwriting rule among all Taiwan banks, the down payment must be at least 30% of the value of the collateralized property. Due to lack of information on collateralized property values at mortgage origination, we assume the original LTV ratio to be 70% for each loan record in our study. Then this ratio is updated with current UPB and the house value is updated by the nationwide house price for each performance quarter. Higher values of CLTV could be attributed to house price depreciation, all else being equal.

Following the option theory and previous empirical literature, a borrower with a high CLTV is more likely to default because there is a greater likelihood that the borrower is in a negative equity position. Meanwhile, with a high CLTV, a borrower is less likely to prepay in that insufficient property value to qualify for a refinancing opportunity. Thus, the coefficient of the CLTV is expected to be positive in the default function and negative in the prepayment function.

4.6. Relative payment to income ratio

Loans included in our sample data are mostly ARMs, which means the payment amount could change over time. Since there are no annual or lifetime rate adjustment caps, these mortgages are basically free from rate refinancing risk. However, due to the interest rate adjustments, ARMs are subject to potential payment shocks. When payment shock happens, if the borrower's income does not increase as fast, the borrower may not have the ability to pay and this would lead to default. To capture the payment shock effect, a relative payment to income ratio variable is created as the updated scheduled payment amount divided by the scheduled payment amount at the mortgage origination. This ratio measures the payment shock faced by the borrower.

A relative payment factor with value greater than one indicates a higher payment burden compared to the original payment, which implies a higher probability of default. Similarly, prepayment is likely to increase when the relative payment burden is lower than origination because the relative household's ability to pay an extra amount in addition to scheduled mortgage payment becomes strong.

4.7. Relative contract rate

The credit quality of the individual borrower has a material impact on both default and prepayment rates. Unfortunately, there is no explicit information within our sample data to exactly delineate the credit quality of the loan records. However, the relative contract rate variable, defined as the contract rate at origination relative to the average rates of all loans originated during the same time period, can serve as a proxy to capture the credit quality of the loan. Typically, the borrower with worse credit quality would be charged a higher interest rate than other borrowers under the same market condition.

As higher relative initial interest rate implies poor credit quality, higher default rate is expected, making the expected coefficient of the default function to be positive. Meanwhile, as those borrowers originally penalized by high interest rate may improve their credit records over time, they may also be able to refinance into mortgages with lower credit risk premium. Such a phenomenon is widely observed among sub-prime borrowers in the US mortgage market. As a result, the coefficient of the relative contract rate variable is also expected to be also positive in the prepayment function.

4.8. Relative loan size

Comparing the original loan amount with the average-sized loan originated during the same origination year identifies relative loan size variable. There are two imperative measures implied by this variable. It can be viewed as a proxy to measure the relative income level of individual borrower. Higher income borrowers can qualify for a larger loan size, based on the maximum payment to income ratio allowed by the lender. It can also be viewed as a proxy to measure the relative value of the house. Larger loans are usually associated with more expensive houses on the market, based on the maximum LTV ratio constraint.

In the US residential market, the relative loan size variable is found to have a U-shaped effect on default. That is, default probability is high for both the largest and the smallest loans, but low for loans in the middle range. It would be interesting to learn if the same pattern can be found in a very different financial market of different consumption and borrowing culture.

4.9. Loan term

The loan term is categorized into 15-year, 20-year, and 30-year maturity. Loan term has a direct impact on a loan's default and prepayment risks. Typically, loans with shorter maturity are less risky than longer maturity loans since principal tends to be amortized quicker, making default less likely to occur. Meanwhile, borrowers with excess payment capability and/or strong preference of quickly repaying debts are more likely to self-select into the shorter-term mortgages, further making the default risk lower. Likewise, we expect that a borrower anticipating a change of residence or prepayment in the very near future will be more likely to take advantage of lower initial rates associated with a loan of shorter maturity.

Thus, the coefficient of the 15-year dummy variable is expected to be negative in the default function and positive in the prepayment function. However, only the most favorable customers of a bank are allowed to originate mortgages of maturity longer than 20 years. The default risk of these preferred customers is usually very low. Thus, the coefficient of the 30-year dummy is expected to be negatively related to the default and be positively related to the prepayment.

4.10. Exposure quarter market rates

Prior to the first quarter of 2002, Taiwan's ARMs were indexed to the bank's prime rate, which is not transparent and is heavily controlled by the bank. A different type of ARMs, whose coupon rates adjust periodically with the public underlying index, was introduced to the market during the first quarter of 2002. Due to its more transparent and stable, the new index-based product became very popular. Many borrowers chose to refinance into the new product, making the overall prepayment rate high after the first quarter of 2002. A dummy variable is included to capture this market regime shift.

4.11. Seasonality

In recognition of the impact of seasonal patterns on mortgage terminations, we have included a set of dummy variables to account for the current season of the year for each quarterly performance observation. To accurately capture the season effect that matches the Taiwan weather pattern: Spring is defined as from March to May, Summer is from June to August, Fall is from September to November, and Winter is from December to February.

4.12. Mortgage age

For each observation, the age of the mortgage is measured in number of quarters from the origination date. Both age and the square of age are included in the regression to allow for a quadratic relationship between mortgage age and the conditional prepayment and default rates.

5. The results and applications

Tables 2 and 3 report the multinomial logit coefficients estimated from the quarterly conditional probabilities of default and prepayment.

The results show that the CLTV ratio is positively related to both default and prepayment rates. The positive relationship with default is consistent with the expecta-

Table 2
Multinominal logit parameter estimates for subsequent quarterly conditional probabilities of default

Explanatory variables	Quarter 0	Quarter 4	Quarter 8	Quarter 12
Cltv	22.30905	21.410483	23.707036	24.736428
	7.05	6.63	6.76	6.17
Relative payment ratio	11.13695	11.091041	11.536372	11.820198
	14.19	13.64	13.44	12.09
Relative contract rate	1.835935	1.4354154	1.0469454	0.9890567
	4.16	3.26	2.25	1.85
Relative loan size	0.269138	0.2976787	0.3527844	0.367496
	5.59	6.16	6.78	5.03
Exp_yr02_q1	1.262873	1.2499617	1.3183071	1.5725903
	6.08	5.75	5.6	5.79
Term20	−0.02473	0.0586641	−0.030235	0.0163568
	−0.13	0.3	−0.15	0.07
Term30	−0.3198	−0.22447	−0.299066	−0.1025913
	−1.55	−1.07	−1.37	−0.45
Summer	−0.2097	−0.118025	−0.139245	−0.1464318
	−1.12	−0.62	−0.67	−0.63
Fall	−0.30887	−0.224009	−0.155178	−0.2255703
	−1.71	−1.21	−0.8	−1.02
Winter	0.208742	0.2852627	0.286178	0.2702516
	1.33	1.78	1.68	1.43
Age	0.288889	0.2539701	0.4039769	0.4481511
	6.2	3.9	3.52	1.94
Age square	−0.00343	−0.002115	−0.006439	−0.0072104
	−1.98	−0.95	−1.84	−1.13
0.05 < cum_delin <= 0.50		0.6882586	0.9492671	0.9958295
		2.67	3.89	4.01
0.50 < cum_delin < 0.95		1.0628098	1.1682536	1.6820666
		4.65	6.04	9.33
1.05 < cum_curtail <= 2		0.8641299	−0.231204	−0.1388122
		2.37	−0.51	−0.33
Cum_curtail > 2		−4.34	−4.66	−3.77
Constant	−38.6652	−37.47184	−40.3824	−42.34389
	−16.25	−14.86	−13.43	−10.39
Statistics				
N (number of observations)	224853	158144	106890	65257
χ^2 (likelihood-ratio χ^2)	12837.98	6962.7406	5380.8005	3699.7389
r2_p (pseudo R^2)	0.157583	0.1223244	0.1214396	0.1181807

Notes. The t values are displayed below the estimated coefficients. The values in the Quarter 0 column are the estimates obtained from the model without accounting for cumulated historical curtailment and delinquency experiences. The values under the Quarter 4, Quarter 8, and Quarter 12 are the estimates obtained from the model with historical cumulated curtailment and delinquency experiences included.

Table 3
Multinominal logit parameter estimates for subsequent quarterly conditional probabilities of prepayment

Explanatory variables	Quarter 0	Quarter 4	Quarter 8	Quarter 12
Cltv	18.91772	9.809399	5.2302792	−0.855131
	31.43	13.71	6.71	−0.91
Relative payment ratio	1.074856	−1.11568	−1.978945	−2.948848
	6.32	−6.04	−9.75	−12.19
Relative contract rate	0.694699	1.424264	1.1378259	1.179379
	10.92	17.44	12.63	11.02
Relative loan size	0.16538	0.086011	0.0867395	0.088599
	17.5	6.12	5.12	4.12
Exp_yr02_q1	1.264556	0.805073	0.8568912	0.759429
	36.69	18.21	17.99	14.28
Term20	−0.371	−0.11508	−0.011718	0.100581
	−11.79	−2.92	−0.27	1.99
Term30	−0.60437	−0.34615	−0.258558	−0.137024
	−15.67	−7.8	−5.39	−2.56
Summer	−0.41543	−0.30813	−0.347002	−0.436179
	−15.09	−9.24	−9.32	−9.95
Fall	−1.12292	−0.91379	−1.007026	−1.156837
	−34.18	−23.42	−22.63	−21.44
Winter	−1.46133	−1.13061	−1.235521	−1.40076
	−40.74	−28.03	−26.99	−25.53
Age	0.010209	0.087277	0.0372389	0.258113
	1.71	7	1.57	5.01
Age square	−3.2E−05	−0.00129	0.0004164	−0.004403
	−0.12	−2.87	0.56	−3.06
0.05 < cum_delin < = 0.50		−0.54497	−0.424744	−0.234534
		−5.02	−3.8	−2.53
0.50 < cum_delin < 0.95		−0.10575	−0.011067	−0.030736
		−1.27	−0.14	−0.39
1.05 < cum_curtail < = 2		0.215699	0.1092717	0.125449
		2.21	1.36	1.64
Cum_curtail > 2		0.398796	0.4825	0.519113
		11.68	13.7	12.64
Constant	−18.1296	−11.3787	−6.836681	−4.117104
	−37.1	−19.1	−9.83	−4.29
Statistics				
N (number of observations)	224853	158144	106890	65257
χ^2 (likelihood-ratio χ^2)	12837.98	6962.7406	5380.8005	3699.7389
r2_p (pseudo R^2)	0.157583	0.1223244	0.1214396	0.1181807

Notes. The t values are displayed below the estimated coefficients. The values in the Quarter 0 column are the estimates obtained from the model without accounting for cumulated historical curtailment and delinquency experiences. The values under the Quarter 4, Quarter 8, and Quarter 12 are the estimates obtained from the model with historical cumulated curtailment and delinquency experiences included.

tion that borrowers will be more likely to default on their mortgages in declining housing markets. Quite interestingly, Table 3 shows that the Taiwan data demonstrate a relationship between CLTV and prepayment that is opposite to prior prepayment studies of mortgages with US data.[6] One possible reason could be attributed to the "accelerate amortization clause" popular to Taiwan mortgage contracts, which forces the borrowers to repay their debt obligation by selling the property when the loan becomes 180 days delinquent. Therefore, the loans with extremely high CLTV are most likely to experience delinquency and be forced to completely prepay.

Examining the impact of the relative payment ratio on mortgage termination rates, we find that the estimated coefficients are significantly positive in the default function and significantly negative in the prepayment function. This indicates that the probability of default of an ARM would rise when the borrower experiences payment shock due to the rising interest rate. On the other hand, the decrease of payment burden from the original payment to income ratio allows the same borrower to qualify for a larger loan amount with the same income level, suggesting cash-out refinancing is more likely to occur with a lower payment to income ratio.

Relative interest rate is a proxy that captures the credit quality of the loan. A loan to a borrower of poor credit quality is usually charged a higher interest rate than other loans originated at the same time. The empirical results show that default and prepayment are positively related to the relative interest rate variable. The fact is that the borrowers who have impaired credit histories are prevented from qualifying for lower interest rate loans, thereby making default more likely to occur. On the other hand, as these borrowers could gradually improve their credit quality through time. A few years later, they could be qualified for mortgages of lower interest rate. This is similar to the high early prepayment rates observed in the US subprime mortgage market.

The relative loan size effect has a significant impact on prepayment rates. From Table 3, larger loans tend to experience slightly faster prepayment. This could be due to the fact that the larger loans are often associated with borrowers with higher income, making refinancing or a cash-out refinance more possible. One interesting finding is that the default rate is higher for larger loans. With the same initial LTVs, the larger loans are likely to be collateralized by more expensive houses. Housing is very expensive in Taiwan relative to the average household income. Very limited households can afford the most expensive housing, making the high price housing market illiquid. When borrowers of these expensive properties face income difficulty, they could liquidate the house to avoid default on the mortgage loan. However, with few potential buyers, they are usually unable to sell the house at the full potential value. Instead, a large liquidity premium would be lost. If the highest offering price falls below the unpaid balance of the loan, the borrower would be forced to default. The most expensive properties on the market tends to be illiquid and could suffer the most from such quick sale necessity, making the default rates higher than the mortgages collateralized by average quality properties. Similar results are also found in the US jumbo mortgage market.

In Taiwan, owner occupied residential loans usually have a term between 15 and 20 years. For non-owner occupied (investor) mortgages, the terms are limited to be

[6] See Bennet et al. (2000) and Archer et al. (1997).

10 years or shorter. On the long end, only the most favorable customers of a bank could obtain a loan of maturity longer than 20 years. These borrowers either have demonstrated an outstanding credit record to the bank or they have another investment relationship with the bank. Thus, the term to maturity in the Taiwan mortgage market actually serves as a measure of distinguished borrower segments, instead of being a reflection of the borrowers' self-selection behavior, like the US market. Table 2 shows that the default rate of the longer than 20 year maturity loans is lower than the other two categories. According to Table 3, the prepayment rate is negatively related to the term to maturity, reflecting that borrowers with a shorter expected tenure in the house would self-select into the shorter maturity loans to take advantage of the lower interest rates.

Table 3 also indicates Taiwan's prepayments follow a strong seasonal pattern. The prepayment rate is highest in the Spring and lowest in the Winter. But no significant seasonality was found in the default rates.

The exposure quarter variable (2002: Q1) is introduced to capture the switch of the ARM index from individual banks' prime rates to a more robust publicly observable index. Because of the lower volatility and the more transparency of the new index, many borrowers refinance to take advantage of the new product. Surprisingly, this variable also has direct positive impact on the probability of default. With only 5 exposure quarters in the new index era, the higher default rate could simply be a reflection of the market condition of this short time period.

5.1. Curtailment and delinquency effects

As to the key variables of this paper, the coefficients of the cumulated curtailment variable on mortgage termination rates strongly supports our hypotheses. The empirical results show that conditional probabilities of prepayment are positively related to the past curtailment and negatively related to the delinquency. On the other hand, the conditional probabilities of default are negatively related to the cumulated curtailment and positively related to the delinquency. By comparing the coefficients and the corresponding standard errors of the curtailment and delinquency variables, we find that the signs of the estimates are very significant, indicating these relationships are confirmed at high confidence levels.

The literature documented the adverse impact of full prepayment on the quality of the loans remaining in a pool following a major refinancing opportunity. The fact is that the borrowers who can afford to refinance have already done so and thus the borrowers remaining in the pool tend to be the ones unable to refinance due to insufficient income or poor property value affecting their ability to qualify for a new mortgage.

On the contrary, curtailments tend to have a positive impact on the overall quality of a mortgage pool. With the reduction of UPB of individual loans, the average LTV ratio decreases, making the loans less likely to experience negative equity. The fact that the borrowers are able to pay an extra amount indicates the excess repayment capability, making an ability-to-pay problem less likely. Both situations lead to lower subsequent default rates. In addition, the borrowers who make additional payments on the loans regularly have excess income capacity, making refinancing more likely in periods of falling rates or a cash-out refinance more likely in a demand of improving house values. Both cases result in higher subsequent prepayment rates.

In the case of delinquency, the explanations previously mentioned are reversed in that the borrowers who have been delinquent are more likely to default and less likely to prepay possibly due to experiencing an ability-to-pay problem. Such relationships are intuitive and generally recognized and well managed by the industry. One could think of delinquency as negative curtailment. That is, curtailment is making payments in excess of the scheduled amount, while delinquency is making payment short of the scheduled amount. This interpretation is consistent with the exact opposite results of these two variables.

Table 3 also suggests that the inclusion of curtailment/delinquency information into the prepayment function significantly reduces the significance of several other explanatory variables, such as current LTV, relative loan size, exposure policy year, and the maturity term. Without incorporating the curtailment information, the model for age equal to Quarter 0 suggests that the probability of prepayment is positively related to the payment ratio. This effect is reversed when the curtailment information is included, confirming the hypothesis that when the market interest rate decreases, a borrower has a better chance to qualify for a cash-out refinance by the same income level.

While the cross effect of delinquency on subsequent prepayment diminishes as a mortgage ages, the curtailment effect remains to be a strong indicator of subsequent prepayment even 3 years following the origination. This indicates that by examining the up to date curtailment information, one could successfully differentiate fast prepayment mortgage pools from slow prepayment pools. Such information is particularly valuable for interest-only mortgage strip investors, such as mortgage servicers.

With respect to the indication of the subsequent default rate, Table 2 also suggests that the significance of other variables weakened when including curtailment/prepayment information. The impact of delinquency on subsequent default grows stronger in both the size and the significance over time. Except for the marginal curtailment in the Quarter 4 equation, the cross effect of curtailment on default remains strongly negative, suggesting curtailed loans are less likely to default, after controlling for the updated LTV ratio.

5.2. Magnitude of the curtailment impact

The empirical results provide strong support for the importance of curtailment information on the future performance of seasoned mortgage loans. That is, by reviewing historical cumulative delinquency together with cumulative curtailment, one would be able to more accurately project the future prepayment pattern and default probability of mortgages in an existing MBS pool. To obtain insights to the effect of the magnitude of the curtailment history has on future performance, the model coefficients are applied to a typical mortgage to estimate the prepayment and default rates during its remaining life. The typical mortgage has the following characteristics: current LTV = 65%, relative payment ratio = 1.1, relative contract rate ratio = 1.2, relative loan size ratio = 1.5, current season = summer, term = 20 years, current exposure year = 2003, and current age = 8 quarters. Figs. 1 and 2 plot the conditional default and prepayment rates of this typical loan over the remaining life. Three curves are presented in each figure. The Normal curves represent the cumulative default and prepayment rates of the loan starting from quarter 9, assuming there was no curtailment or default up to the end of quarter 8. The Delin curves represent the conditional default and complete prepayment rates conditional on the mortgage is slightly in delinquent (0.50 < cum_delin < 0.95). The Curtail curves depict the rates of a loan with a history of curtailment (cum_curtail > 2.0).

Figs. 1 and 2 reveal several practical insights. First, they confirm the industry intuition that delinquency is a strong indication of future default. Fig. 1A further dem-

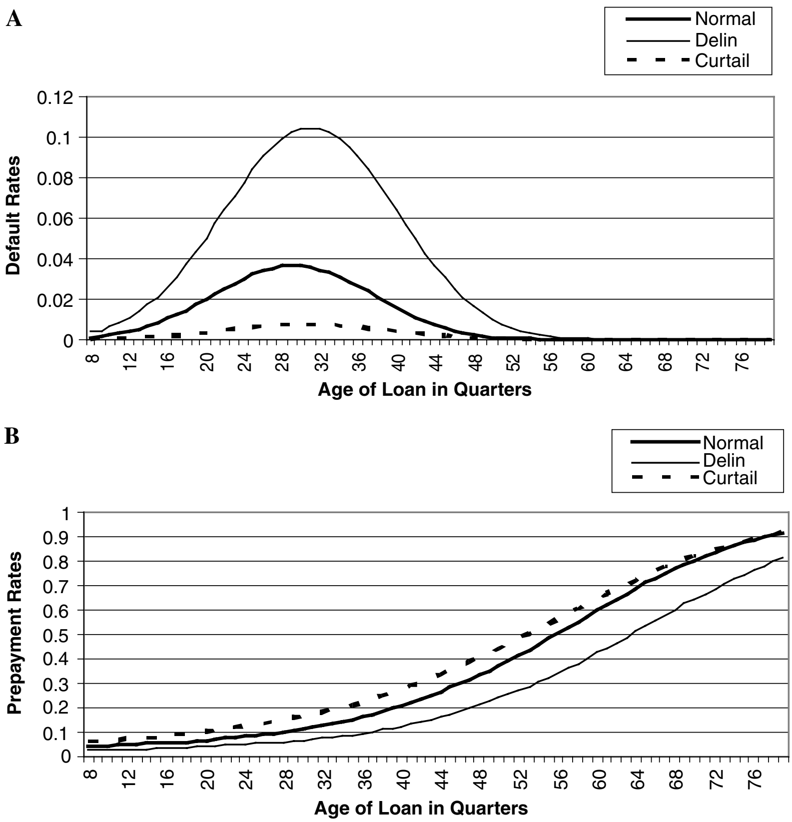

Fig. 1. (A) A typical loan conditional probability of default. Normal is the probability default curve without historical cumulated curtailment and delinquency experiences. Delin is the probability default curve with historical cumulated delinquency experience included. Curtail is the probability default curve with historical cumulated curtailment included. (B) A typical loan conditional probability of prepayment. Normal is the probability prepayment curve without historical cumulated curtailment and delinquency experiences. Delin is the probability prepayment curve with historical cumulated delinquency experience included. Curtail is the probability prepayment curve with historical cumulated curtailment included.

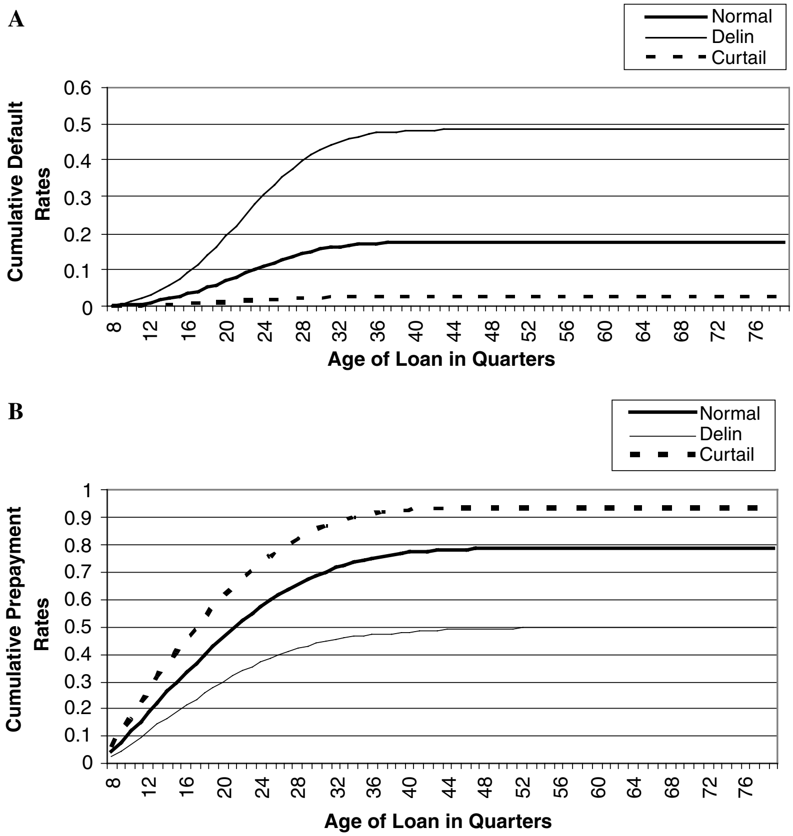

Fig. 2. (A) A typical loan lifetime cumulative probability of default. Normal is the probability default curve without historical cumulated curtailment and delinquency experiences. Delin is the probability default curve with historical cumulated delinquency experience included. Curtail is the probability default curve with historical cumulated curtailment included. (B) A typical loan lifetime cumulative probability of prepayment. Normal is the probability prepayment curve without historical cumulated curtailment and delinquency experiences. Delin is the probability prepayment curve with historical cumulated delinquency experience included. Curtail is the probability prepayment curve with historical cumulated curtailment included.

onstrates that for a typical mortgage, past delinquent experience implies that the future conditional default rate to about double. On the prepayment side, Fig. 1B shows that the delinquent loans are about 40% less likely to prepay than the normal loans without either delinquency or curtailment. However, the business impact is even stronger. Because the delinquent loans are prepaid much slower, they would have a higher survival rate. When more loans remaining in the pool are exposed to default risk, the higher conditional default rates in the future will be have a compounding effect. Fig. 2A shows that the cumulative default rate of loans with delinquent history is about 2.74 times of the level of the normal loans.

Second, the generally ignored curtailment history information has an almost equally significant impact on the future default risk. Fig. 1A shows that the loans with strong curtailment history are 80% less likely to default than the non-curtailed loans. From Fig. 1B, we understand the previously curtailed loans are 25% more likely to be completely prepaid in the future. Combining these two impacts together, Fig. 2A shows that the cumulative default rate of a previously curtailed loan is only about 10% of an otherwise identical loan with no curtailment history. On the other hand, these curtailed loans will experience about 23% higher cumulative prepayment rates.

6. Conclusions

This paper contends that the mortgage curtailment reveals critical information regarding the future performance of the mortgage during its remaining life. The empirical study using a Taiwan mortgage sample provides strong support to our hypotheses. That is, past curtailment indicates low subsequent default rates and high subsequent prepayment rates. The magnitude of the impact appears to be quite substantial.

The differential performance indicated by the curtailment can substantially affect the mortgage security's value and risk. For transaction purposes, mortgages with past curtailment tend to have shorter expected effective remaining life and much lower default losses. Such loans favor the PO strip investors while providing lower value to the IO strip investors. As the design of mortgage securities is getting more complicated, the additional accuracy in the estimation of prepayment and default becomes more critical for mortgage portfolio investors. Although curtailment may not be a popular action of US borrowers, it is frequently observed among Asian mortgages. With relatively long housing tenure, curtailment is the dominant form of prepayment in those regions, such as Taiwan. As there is an increase in the volume of securitization activities from these regions, there needs to be increasing attention by global mortgage securities investors to the information conveyed by historical curtailment behavior.

Our findings also have significant implication associated with the newly released Basel II risk-based capital rule. It indicates that internal rating based minimum capital could be significantly biased in the absent of the curtailment information. For mortgage portfolio mangers, the accuracy of the hedging strategies would also be heavily affected by the curtailment history. For this type of application, the impact of curtailment would be relevant to even the deeply seasoned mortgage pools.

In summary, this paper is the first attempt to investigate the effect of curtailment information embedded in seasoned mortgages. As more and more MBSs issued from the regions with high curtailment rates are traded on the global capital market, more in depth research towards this issue using data from different locations and different mortgage product types is highly encouraged.

To gain more comprehensive insights into the curtailment behavior, this paper can be extended in at least two directions. First, following Fu (1997) more research about the predictability of the probability and size of future curtailment can further enhance the richness of the mortgage performance and pricing model. It would allow investors to differentiate borrowers with different tendency to curtail at the time of mortgage origination and more accurately risk price the contract. Second, following Chinloy (1993), further attempt to investigate the reason behind borrower's incentive to curtail can add a lot value to the mortgage industry. By collecting the average curtailment rates of different cities or countries, one would be able to investigate into the incentives why borrowers curtail. Being able to understand the reason why people curtail their mortgages could greatly help the lenders better customize mortgage products to the specific borrowers.

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