

## 評估抵押債權事務性服務權利的價值 - (Option-Adjusted Spread) 的方法

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### 摘要

這項研究的主要目的是試圖運用一個(Option-Adjusted Spread)模型來評估抵押債權事務性服務權利的價值。此評價模型包括隨機利率途徑、外生的抵押債權提前清償架構、及預先假定事務性服務權利成本。根據前述三項，此模型可應用來計算事務性服務權利未來服務的淨收益和決定計算其現值的折現因子。然後運用情境分析法進一步探討在各種不同的經濟環境下，抵押債權事務性服務權利所面臨的風險。本文的實用價值在於提供與事務性服務權利相關的政策探討給抵押債權事務性服務權利機構作一參考。

**關鍵詞：**抵押債權事務性服務權利、提前償還

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Valuing Individual Mortgage Servicing Contracts:  
An Option-Adjusted Spread Approach

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Abstract

The study develops a model from which an option-adjusted spread approach is utilized for pricing individual mortgage servicing contracts. The pricing model is comprised of a stochastic interest rate process, an exogenous prepayment function and an assumed servicing cost, all of which jointly determines the contract's future net cash flows and the rate at which to discount these cash flows. Then a scenario analysis is employed to examine a myriad of risk exposures of servicing contracts under various economic environments. The implication of this paper is potentially useful for mortgage servicers to investigate the policy-related issues.

Key words: Mortgage servicing rights, option-adjusted spread, prepayment

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## 1. Introduction

Every mortgage loan must be serviced. Over the past twenty years, mortgage servicing has become a major and growing part of the mortgage banking business in that the secondary mortgage market created a good deal of standardized mortgage servicing contracts. According to mortgage industry practice, the originating institution of a portfolio of mortgages sells these “conforming” mortgages to such investors as Fannie Mae and Freddie Mac, typically the major players in the secondary mortgage market. During the course of the transaction stage, the servicer, either an originator or a third party, is selected to continue service the mortgage underlying the mortgage securities: mortgage-backed security or participation certificate.

Mortgage servicing rights, hereafter MSR, are contractual rights in return for the compensation to fulfill the following tasks including collecting mortgage monthly payments and forwarding the proceeds to the mortgage investors, managing escrow accounts for collecting and distributing property taxes and hazard insurance purpose, covering delinquent payments, or even initializing foreclosure proceedings on defaulted loans, if needed.

Next we turn to the valuation of a MSR. As indicated above, the originating institutions either retain the servicing rights to loans being sold off or may sell the servicing rights to outside parties. The valuation of a MSR is critical, regardless of whether the servicing rights is retained or sold. The value of the MSR can be explained as either the maximum purchasing price at which the outside parties are willing to pay to receive the servicing rights; or the maximum origination costs the mortgage originators are willing to spend to retain the servicing.

The value of a MSR is dependent upon the net cash flows that received from the servicing revenue minus the servicing costs in order to operate the servicing business. Put it differently, the value of the servicing rights is the present value of net income from servicing. On the revenue side, the servicing income is comprised of the servicing fee, a fixed percentage of the amortizing principal of mortgage loan being serviced; float on either the escrow accounts or on the time lag between receiving of the monthly mortgage payments and remitting these proceeds to the loan purchasers. On the cost side, expenses components include servicing costs, foreclosure costs, and principal and interest advances.

In this paper, we present a pricing framework for mortgage servicing rights that includes Option Adjusted Spread model and Office Thrift Supervision prepayment model.<sup>1</sup> The contribution of this work is to incorporate OTS dynamic prepayment model into the pricing process, which hasn't even been done in the real estate literature.

The layout of this paper is as follows. The next section provides a comprehensive review of related mortgage servicing literature. The third section delineates the methodology used in our paper. The fourth section reports our simulation results and discusses the implications implied by the results provided. The final section presents the conclusions on this servicing study and the recommendation for the research going forward.

## 2. Literature Review

Since the first MSR paper published in 1976 up to date, pricing servicing rights has not received much attention by academicians over the past twenty-six years. This could

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<sup>1</sup> Please see the OTS net present value model (Page 5A-6), risk management division, office of the thrift supervision.

be evidenced by the fact that there are just few papers that document MSR pricing in literature. Methodologies employed in the academic literature are essentially based on the discounted cash flow approach. McConnell (1976) is the first one to develop a static cash flow model for pricing a MSR. However, this paper ignores the property of interest rate term structure theory and the reflection of the critical path dependence of the value of a MSR on prepayments. Van Drunen and McConnell (1988) develop a two-state continuous time model from which their model allows for stochastic one-period short-term interest rates and inflation rates. Their model only explicitly values the borrower's prepayment decision, but ignores the impact of default that can have on MSR valuation.

Brown, et al. (1992) evaluate the value of a MSR followed by "Option Adjusted Spread" model. Unlike the traditional discounted cash flow approach, the purpose of their paper is to examine the effect that interest rate variations and the resulting changes in prepayments on MSR valuation. Their results indicate that the MSR exhibits the IO-like behavior. Langowski (1999) indicates that the servicing rights are similar to the interest-Only (IO) strips of a mortgage-backed security and thus this paper employs IO pricing techniques to price a MSR subject to various shocks. The results show that prepayment is the chief factor while pricing a MSR. A recent paper by Aldrich, Greenberg, and Payner (2001) point out that the cash flows of a MSR can be received from such various sources as servicing fees, the escrow account float and the principal and interest, float, and ancillary income and late charges. They find that pure servicing fee accounts for 70 to 80 percent of the value of a MSR.

### 3. Methodology

This paper utilizes an Option Adjusted Spread (OAS) model to price a MSR. Unlike the work by Brown, et al. (1992), which uses a static prepayment model, our pricing model uses Office Thrift Supervision (OTS) prepayment model, which captures the dynamic nature of borrower's prepayment behavior.

Referring to Brown, et al. (1992) and Aldrich, Greenberg, and Payner (2001), they view a MSR as a kind of IO-like instrument. As an industry practice, an IO instrument is evaluated by means of an OAS analysis. However, Buttimer and Lin (2002) indicate that an IO-style model may miss the incentive conflicts between mortgage servicer and investor because the MSR is not just a financial instrument. In spite of their viewpoints, our paper would still use an OAS analysis to price a MSR in that servicing exhibits the IO-like income stream. This is because our pricing model is primarily concerned with the valuation of a MSR from a pure capital markets perspective. It means that our paper focus mainly on interest rate risk and we treat prepayment and default as an exogenous event as well. From the viewpoints of the capital market, this is really the only viable solution for pricing in the market and that is why we still adopt the OAS approach. More importantly, compared with the traditional static cash flow pricing model of the MSR, our OAS pricing model would overcome the drawbacks to static cash flow methodology.<sup>2</sup>

The manipulation of an OAS analysis centers on pricing a financial instrument over an enormous number randomly interest rate paths. Given an OTS prepayment model, prepayments can be projected by feeding the interest rate paths into the OTS prepayment model. Thus, a corresponding monthly cash flow for a MSR on each interest rate path can

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<sup>2</sup> Two drawbacks are failure to incorporate the term structure of interest rates and failure to account for path-dependent nature of mortgage and MSR cash flows.

be computed. With taking the servicing costs and foreclosure costs into account, for each interest rate path, the present value of net cash flows from servicing can be calculated using the discount rate, which is determined from the term structure of the interest rates. Then we average total present value calculated from all interest rate paths to obtain the “fair” price of the MSR. If the fair price of the MSR equals to the one observed from the mortgage-servicing market, then the spread used is option-adjusted spread. If not, the process will be repeated until a new option-adjusted spread appears.<sup>3</sup>

Prior to describing the pricing model utilized, one thing needs to be clarified is as follows. As has been indicated in the prior studies, the cash inflows to a MSR include servicing fees, escrow earnings, float, and late payment charges and the cash outflows to a MSR include servicing costs, delinquency costs, and foreclosure costs. For the purpose of simplicity, our pricing model only takes servicing fees, the major source of income, servicing costs and foreclosure costs into account. This is because implementing all revenue and expense components creates the complexity of the simulation model used to price a MSR.

Take principal and interest float for example. These funds to be received from monthly mortgage payment and prepayment can be invested in the short-term money market until the remittance date. Another example is that mortgage servicers lose the interest due to prepayments or even curtailments since some portion of uncollected interest amount needs to be remitted to the investors when the amount is due. In addition, Fannie Mae and Freddie Mac both have different types of remittance program. It is, therefore, not trivial to account for all components of servicing revenue into the pricing model.

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<sup>3</sup> This paper uses an assumed spread to calculate the price of a MSR, instead of computing the spread.

The difficulty with our OAS technology is that it typically ignores the borrower's delinquency and default decisions. Due to this deficiency, we explicitly consider the costs associated with servicing. Like the general mortgage-related literature, our model also assumes that default is synonymous with the foreclosure. Based on this presumption, we follow the work by Brown, et al. (1992) to assume that the annual amount of foreclosure costs equals to a fixed percentage of the original loan. The pricing model is described below.

This model assumes that the stochastic process for risk-free interest rate is following square-root mean reverting process by Cox, Ingersoll and Ross (1985).<sup>4</sup> The mean-reverting process means that interest rates tend to be pulled back to some long-run average level over time.

$$dr = \gamma(\Theta - r)dt + \sigma_r \sqrt{r} dz_r \quad (1)$$

where the increment  $dz_r$  is a Wiener process,  $\gamma$  is the speed of adjustment toward the long-run steady-state interest rate,  $\Theta$  is the long-term mean risk-free rate and  $\sigma_r$  is the volatility of interest rates.

The following part introduces the version of the OTS prepayment model.

Concisely, the formulas needed are in the followings:

$$\text{Prepayment amount in particular month } I = \text{Unpaid Balance}_{\text{after monthly cash flow}} * \text{CPR} \quad (2)$$

$$\text{CPR} = 1 - (1 - \text{seasoning} * \text{seasonality} * \text{refinancing})^{1/12} \quad (3)$$

$$\text{Seasoning} = \min(1.0, 0.0333333 * i) \quad (4)$$

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<sup>4</sup> Of course, this process is one of many viable interest-rate processes that can be employed. This specific process is popular in academic mortgage pricing models. In addition, most of the papers cited are also using this process. Thus, our pricing model utilizes this process for demonstration purposes.



where seasoning is increasing at a rate of 0.03333 until month 30, then remaining one as a constant thereafter. Also,  $i$  denotes month  $i$ .

$$\text{Seasonality} = 1 + 0.2 * \sin \left[ \frac{1.571 * (\text{month} + i - 3)}{3} - 1 \right] \quad (5)$$

where month refers to the mortgage issuing month and  $\sin$  is the function of sine.

$$\text{Refinancing} = 0.1828 - 0.0892 * \arctan \left[ 4.776 * \left( 1.083 - \frac{c}{r + m} \right) \right] \quad (6)$$

where  $r$  is the risk-free rate,  $m$  is the spread of newly issued mortgages over the risk-free rate,  $c$  is the contract rate, and  $\arctan$  is the function of arctangent.<sup>5</sup>

There are three major factors that control mortgage pool prepayments within the OTS model. These are generally referred to as the seasoning of the mortgage pool, the seasonality of the mortgage pool, and the refinancing incentive. First, Seasoning indicates the fact that a newly mortgage pool tends to prepay at lower rate than the one issued previously, other things being equal. Next, Seasonality refers to the fact the level of mortgage payments is tied with the time of the year, holding all else constant. Finally, refinancing means that prepayments will increase when mortgage rates drop below the contract rates, all others remaining the same.

Equation (3) stands for the monthly prepayment rate. Given equation (3), the monthly prepayment amount can be calculated followed by equation (2). Then, the cash inflow to a MSR at each month is calculated by taking the unpaid mortgage balance minus the prepayment amount derived from equation (2). That is,

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<sup>5</sup> The OTS prepayment model has been fit employing data from the past experiences. Benchmark used is based on conventional 30-year fixed-rate mortgage with moderately seasoned loans. The estimated coefficients shown on equations (4), (5), and (6) were generated based on long-term prepayment rate forecasts of a number of Wall Street firms.

$$CF \text{ in month } I = \text{Unpaid mortgage balance} - \text{prepayment amount in month } I \quad (7)$$

$$\pi = CF * \frac{sf}{12} - \frac{sc}{12} - fc \quad (8)$$

where  $\pi$  stands for net cash flows from servicing,  $sf$  is the servicing fee,  $sc$  is the servicing cost and  $fc$  is the monthly foreclosure cost.

$$MSR = \frac{1}{N} \sum_{n=1}^N \sum_{t=1}^T \frac{\pi_{n,t}}{\prod_{t=1}^T (1 + r_{n,t} + oas)} \quad (9)$$

where  $N$  is the number of interest rate paths and  $oas$  is the spread over the risk-free rate.

Given an assumed spread, the fair price of the MSR can be calculated from equation (9). This simply means that for each interest rate path, the net cash flows from servicing can be determined from equation (2) to equation (8) and then we can arrive at the present value for each path. Next, we average out the total present value for all paths.<sup>6</sup>

To sum up, our pricing model involves the random sampling of each distribution of future interest rates to generate a large number of iterations. Under each of iterations, the price of a MSR is determined and the distribution of the price evaluated therefore reflects the true “fair” MSR price that could occur. However, the question arises as to how much iteration is needed to arrive at the mean price of the MSR under the simulated distribution. Theoretically speaking, the sampling distribution of a sample statistic computed from a sample size of  $n$  iterations is called the probability distribution of the statistic. In an attempt to make an inference about the population parameter in terms of a statistic, this paper aims to make the sampling distribution to center around the parameter and the standard error of estimate, the ratio of standard deviation to the square root of

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<sup>6</sup> The procedure is referred to as the Monte Carlo simulation.

number of iterations, as small as possible. Based on figure 1, we see that the standard error of estimate decreases as the number of iterations increases and levels off after 5,000 iterations and thus our pricing model selects 5,000 iterations as the breakeven point.

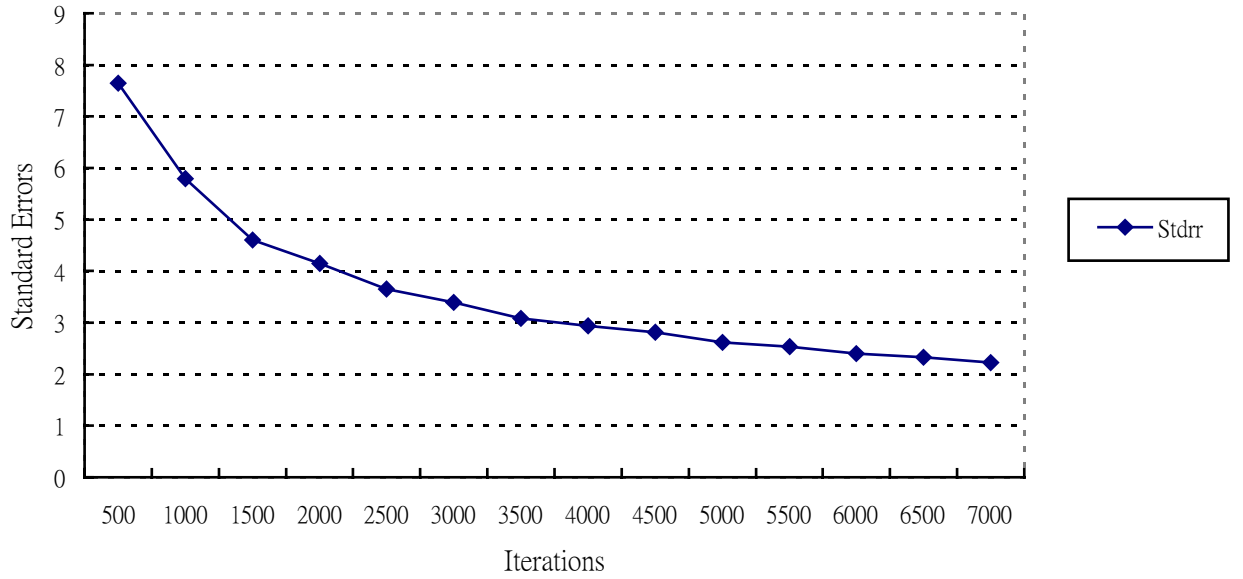


Figure 1. The relationship between the values of standard error of estimate and the number of iterations.

#### 4. Results

The economic parameters chosen are commonly used in mortgage pricing literature. These have been usually seen in Cox, Ingersoll and Ross (1979), Kau, et al (1994) and Ambrose, Buttimer and Capone (1997), Hilliard, Kau and Slawson (1998). As shown in table 1,  $r_0$  is the instantaneous short-term spot rate;  $\theta$  is the steady-state or long-term mean nominal interest rate;  $\gamma$  is an adjustment factor that governs the speed with which  $r_0$  tends to revert toward its mean level  $\theta$  at rate  $\gamma$ . The volatility of the interest rates is  $\sigma_r$ . The  $u$  is the spread of newly issued mortgage over the risk-free rate. For the

interest-rate model employed in this study, the interest-rate volatility decreases less than proportionately when the instantaneous short-term spot rate drops to ensure the low interest rate with less volatility. Also, this interest-rate model precludes negative interest rates and thus the interest-rate volatility goes to zero when  $r_0$  falls to zero.

Table 1. Base-case parameters for numerical modeling

Parameters	Value
The Mortgage Servicing Contract	
Contract Rates ( $C$ )	9%
Mortgage Term	30 years
Loan Type	Fixed-Rate Mortgage
Servicing Fee ( SF )	0.0025 (Fannie/Freddie Investor)
Servicing Cost Per Loan ( SC )	\$44 (Fannie/Freddie Investor)
Foreclosure Cost Per Loan	2 % of the original loan amount
The Economic Parameters	
Steady-State Spot Rate ( $\theta$ )	10%
Interest Rate Volatility ( $\sigma_r$ )	10%
Reversion Coefficient ( $\gamma$ )	25%
Spread (u)	2.5%
Original Spot Rate ( $r_0$ )	8%

Note: The foreclosure cost assumed is 2 percent of the original loan amount, which is the idea from the foreclosure cost, \$2000 per loan from Fannie Mae and Freddie Mac type of mortgage. The spread  $u$  assumed is 2.5 percent, which is close to a fair OAS spread, 2.0 percent, shown on the work by Brown et al. (1992).

Given a textbook servicing example by Clauretie and Sirmans (1996), they simply assume that the servicing income comes from servicing fees only, which are 37.5 basis points, servicing costs in the first year equal to \$30 plus 0.0002 percent of the ending balance and grow 4 percent annually thereafter, the term is 30-year loan and the contract rate is 10 %. Followed by the 100 percent PSA model, the value of a MSR is \$1,588 per \$100,000 in mortgage. In our base-case simulation based on the parameters shown in table 1 except for servicing fee, 0.00375, the value of a MSR is \$1,156. The \$432 dollar off can be attributed to a dynamic OTS prepayment model used, instead of PSA 100%,

and a relative high servicing cost selected by our pricing model. Another example by Brown, et al. (1992), the price of a MSR portfolio equals to 1.4 percent of the original loan amount assuming that the servicing fee is 0.5 percent, servicing cost is \$85 per loan, other fees \$30 and escrow earnings are \$24 per loan per year, and the weighted average coupon rate is 10.25%. The price of a MSR portfolio is \$4,200,000 out of 5,000 number of loans. Following their assumptions conducted, our pricing model demonstrates that the price of a MSR portfolio is, \$4,602,400, 1.53 percent of the original mortgage balances. The difference could be explained by two reasons: First, they use PSA 159 % for prepayment assumption and secondly they assume the servicing costs increase with the inflation rate every year in tandem. According to the above two examples, it appears to make sense that our pricing model is plausible in its design.

#### 4.1 Interest Rate Volatility

Ambrose and Buttimer (2000) document that increasing the term structure volatility leads to an increase in the probability of prepayment in that it provides greater opportunities for interest rates to decline and thus prepayment is induced. As previously indicated, prepayments cause decrease in servicing income, which is a function of the unpaid mortgage balance, thereby reducing the price of a MSR. As shown in table 2, it is indeed that the price of a MSR declines from \$684.12 (\$787.67) to \$577.48 (\$684.12) as  $\sigma_r$  increases from 10 (5) percent to 15 (10) percent, holding all else constant. The results indicate that the higher the interest rate volatility, the lower will be the price of a MSR. This also implies that, during periods of high interest rate volatility, servicers should pay much less to purchase the servicing rights or spend much less in order to retain the servicing rights.

## 4.2 Speed of Adjustment

Next, we investigate the adjustment factor that governs the speed with which  $r_0$  tends to revert toward its mean level  $\theta$  at rate  $\gamma$ . Intuitively, the larger the speed of the adjustment factor, the faster will be  $r_0$  tends to revert toward its long-term mean level  $\theta$ . In other words, the length of time for varying interest rates is short when the adjustment factor  $\gamma$  is large. Thus, the price of a MSR decreases when the adjustment factor is small. As also exhibited in table 2, we see that the price of a MSR increases from \$684.12 to \$749.25 and also from \$749.25 to \$773.69 when  $\gamma$  goes up from 0.25 to 0.50 and from 0.50 to 0.75, respectively. This indicates that price of a MSR varies with the time during which  $r_0$  that tends to revert toward its mean level  $\theta$  is not fixed. It is not surprising that the lowest price of a MSR, \$577.48, is located at the area where  $\gamma$  is 0.25 and  $\sigma_r$  is 0.15 because there is a greater chance for the mortgage prepayment to be induced.

Table 2: Prices of MSR under various interest rate volatilities  $\sigma_r$  and the speed of adjustment factors  $\gamma$ . Servicing fee is 0.25 percent annually, and servicing cost is \$44 annually. The original loan amount is \$100,000. The foreclosure cost is 2 percent of the original loan amount per loan per year.

Component	$\sigma_r = 0.05$	$\sigma_r = 0.10$	$\sigma_r = 0.15$
$\gamma = 0.25$	787.67	684.12	577.48
$\gamma = 0.50$	808.25	749.25	671.21
$\gamma = 0.75$	815.27	773.69	711.29

Note: The analysis conducted assumes that mortgage term is 30 years, contract rate is 9 percent, the initial short term rate is 8 percent, long-term mean reversion rate is 10 percent, adjustment factor is 25 percent and the spread is 2.5percent.

### 4.3 Impact of Loan Amount

This section is intended to calculate the price of a MSR as the size of the loan amount differs. As shown in table 3, the price of the MSR increases when the notional loan amount increases, but it is increased more than proportionately to the loan amount being serviced, either in periods of high or low interest rate volatility. This could be attributed to the fact that the servicing income received is increasing once the size of the loan increases in that it is tied to a fixed percentage of the unpaid mortgage balance, while the servicing cost is independent of the size of the mortgage being serviced. It is suggested that mortgage servicers ought to service a large amount of mortgage than to service the mortgage loan with small dollar amount.

Table 3: Prices of MSR with different loan amounts. Servicing fee is 0.25 percent, servicing cost is \$44, both of which are on the annual basis. The foreclosure cost is 2 percent of the original loan amount per loan per year.

Component	$\sigma_r = 0.05$	$\sigma_r = 0.10$	$\sigma_r = 0.15$
Loan Amount			
100,000	787.67	684.12	577.47
200,000	1,932.55	1,744.38	1,581.87
300,000	3,070.47	2,801.83	2,558.85

Note: The analysis conducted assumes that mortgage term is 30 years, contract rate is 9 percent, the initial short term rate is 8 percent, long-term mean reversion rate is 10 percent, adjustment factor is 25 percent, and the spread is 2.5percent.

### 5. Concluding Remarks

This paper presents a model that combines an “Option Adjusted Spread” analysis with “Office Thrift Supervision” prepayment model for pricing mortgage servicing rights, which has not been done in the real estate literature. The literature is often based on a

static prepayment model instead of a dynamic one. The pricing model provides important perspectives to investigate the impact on the price of a MSR under various economic conditions.

Facing volatile interest rate environment, the results present that the price of a MSR declines as a large  $\sigma_r$  provides the greater opportunity for market interest rates drops further to cause prepayment. This is because mortgage prepayments eliminate the majority of future servicing income associated with the terminated loan and a servicing cost for the remaining amount may continue, however. This also provides an economic incentive for the servicers to examine the impact that changing interest rate volatility can have on MSR valuation.

This paper addresses another issue associated with the adjustment factor and the results show that the price of a MSR increases when the speed of an adjustment factor goes up. As the length of time allowed at which  $r_0$  tends to revert toward its mean level  $\theta$  at rate  $\gamma$  is short, there is little chance for interest rates to vary and thus it reduces the prepayment amount.

Moreover, the results in table 3 show that the price of a MSR increases as the size of the original loan increases. Particular attention should be paid on that it is increased more than proportionately to the mortgage balance being serviced. Therefore, a large amount of mortgage to be serviced is suggested for mortgage servicing industry. For a further research going forward, it is required that this work values more precisely the mortgage servicing rights by including all components of servicing income and delinquency costs into the pricing model.



Besides, the valuation model developed in this paper could be adopted to evaluate other types of securitized instruments by specifying the appropriate cash flows for those. These include credit card receivable, auto loan-backed securities, home equity loan securitization, and commercial mortgage-backed securities.

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