Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
Journal of Housing Economics 14 (2005) 194–211
JOURNAL OF HOUSING ECONOMICS
www.elsevier.com/locate/jhe

Valuing US and Canadian mortgage servicing rights with default and prepayment

Richard J. Buttimer Jr. [a,*], Che-Chun Lin [b,1]

[a] *Department of Finance, Belk College of Business, University of North Carolina at Charlotte, 9201 University City Blvd, Charlotte NC 28227-0001, USA*
[b] *National Tsing Hua University, Department of Quantitative Finance, 101, Sec. 2, Kuang Fu Rd., Hsin Chu City 30013, Taiwan*

Received 7 July 2005
Available online 12 September 2005

Abstract

The increasing use of securitized mortgage throughout the world has led to the creation of an asset class, the mortgage servicing right (MSR). The typical MSR used in the United States and Canada is a contract between the owners of a pool of residential mortgages and a financial institution under which the institution provides retail-level services to borrowers. This paper presents an options-based MSR valuation mode that incorporates both prepayment and default. Numerical results indicate that the MSR value is sensitive to interest rate and housing volatility parameters, as well as to the correlation coefficient between those parameters.

JEL classification: G21

Keywords: Mortgage Servicing; MSR; Canada Servicing

1. Introduction

A growing trend throughout the world is to develop mortgage markets in which residential mortgages are collected into pools and sold as mortgage backed securities

* Corresponding author. Fax: +1 704 687 6219.
E-mail addresses: buttimer@email.uncc.edu (R.J. Buttimer Jr.), chclin@mx.nthu.edu.tw (C.-C. Lin).
1 Fax: +886 3 5742419.

doi:10.1016/j.jhe.2005.07.006

(MBS.) The development of these MBS markets will foster a need for a companion market—the market for mortgage servicing rights. For example, in the US, the vast majority of residential loans are either directly securitized by the GSEs, Freddie Mac and Fannie Mae, or are securitized by private companies through the Ginnie Mae guarantee program. Similarly in Canada, loans originated under Canada's National Housing Act (NHA) may be securitized by the originating lender into Mortgage Backed Securities that are then guaranteed by the Canadian Mortgage and Housing Corporation. Under both the US and Canadian securitization systems, the MBS are purchased by investors who then have the right to receive the majority of the borrower's monthly payments. The MBS investors do not have the desire, systems, or resources to interact directly with the mortgage borrowers. The MBS issuer, therefore will either provide those services themselves, or, frequently, will contract with a third-party to provide them.

A mortgage servicing right (MSR) is a contract between an MBS issuer and a financial institution. Under this contract, the financial institution provides retail-level services on behalf of the MBS-owners to their mortgage borrowers. These services include activities such as maintaining customer call-centers, providing balance and payment information to the borrowers, and insuring borrower escrow account compliance. In return for providing these services the financial institution is paid a monthly fee and is also granted the right to collect certain other fees and cash flows.

The MSR holder occupies an intermediate position between the mortgage borrower and the mortgage investor. Many of the same borrower decisions that affect the value of the mortgage also affect the value of the MSR, although not always with the same magnitude or even direction.

The purpose of this paper is to develop an options-based pricing model for Mortgage Servicing Rights in the US and Canada that is consistent with those of the modern mortgage pricing literature. We then use this model to examine the comparative statics of MSR investments. The paper is written as follows: in the next section we discuss the MSR contract and the relatively brief literature that surrounds it. We then discuss the MSR valuation model and the numerical technique that we use to solve the model. We then discuss the economic and contractual environment of the model, and then present the results our numerical results.

2. The MSR contract

The MSR contract is unusual in that unlike most fixed-income assets it is not a passive instrument: to earn their cash flows the MSR investor must perform real services and the more efficiently they can perform those services the higher their return on the MSR instrument. Lacour-Little (2000) notes it was a quest for efficiency that induced many servicers to invest heavily in automation and technology.[2] As a con-

[2] Lacour-Little credits the increased use of technology such as telephone voice-response systems and consumer-level electronic payment systems as major factors in reducing average per-loan servicing costs from $150 per year in the early 1990s to under $100 per year in the late 1990s.

tract the MSR is quite complex. It generates revenues and expenses for a variety of reasons. These different sources of income and expense generate incentives for the MSR holder that can be in conflict with those of either the MBS issuer or the MBS investor. Fully understanding those incentives requires a thorough understanding of the MSR contract, and the revenues and expenses that they generate.[3]

Mortgage servicers earn income from several sources. The most obvious source of income is the net servicing fee that they are paid. Typically this net servicing fee works out to be approximately 25 basis points per year or 2.03 basis points per month.[4] This net servicing fee is paid on the outstanding balance of the loan, so the cash received per month will decline as the mortgage ages and the principal balance declines. Note that technically the servicing fee is taken from the interest payment made by the borrower. One should also realize that on a per-mortgage basis, the net servicing fee does not generate much in revenue. Consider a hypothetical mortgage with a beginning of month balance of $300,000, 30 years maturity and a contract rate of 9%. In the first month of this mortgage's life, it will generate servicing revenue of $62.50. In contrast, if the loan were never prepaid, in its last month (i.e., month 360) it would only generate $0.31 in servicing revenue.

A second major source of servicer revenue comes from the float earned on payments collected from borrowers but not yet remitted to the MBS issuer.[5] Servicers collect payments from borrowers throughout the month, but only make a single remittance to the MBS Issuer, typically on the 20th of the month for Fannie/Freddie MBS and the 15th of the month for Canada Mortgage and Housing Corporation (CMHC) MBS. Under the assumption that mortgage due dates are distributed uniformly throughout the month this means that the servicer earns on average two weeks worth of interest on the amount they remit to the issuer.[6] Note that the servicer can earn float not only on the principal and interest that is due, but also on any property tax and hazard insurance escrow payments that are included in the payment. Indeed, it is the search for ways to increase the average period during which float is earned on these related payments that has led some servicers to offer their borrowers bi-weekly and weekly payment schemes.

A third source of revenues for the servicer is late fees and other miscellaneous fees. When a borrower does not make a payment by the due date, the servicer is allowed

[3] Aldrich et al. (2001) provide an excellent in-depth discussion of these revenue and expense categories. Van Drunen and McConnell (1988) also have a discussion of the basics of the servicing contract.

[4] Aldrich et al. (2001) note that the servicer collects the gross servicing fee but then pays a guarantee fee to the MBS originator. In a Fannie/Freddie MBS, the gross servicing fee is typically 50 basis points, and the Guarantee fee is 25 basis points, resulting in a net servicing fee of 25 basis points. Under GNMA the gross servicing fee is 50 basis points, but the servicing fee is 44 basis points. Under the Canada Mortgage and Housing Corporation (CMHC) MBS program, the typical net servicing fee is 25 basis points.

[5] In the US the float is earned until the aggregated payments are sent to the MBS issuer, i.e., Fannie, Freddie, or the issuer under the GNMA program. The CMHC MBS program requires issuers to use a single entity, the Central Payor and Transfer Agent (CPTA), to make payments to the MBS investors. Technically the issuer/servicer earns float only until the aggregated payments are made to the CPTA.

[6] There is some anecdotal evidence that more loans close during the last few days of the month and that this may result in a higher concentration of payments being due toward the end of each month. This does not, however, invalidate the basic point being made.

to charge and collect late fees on the unpaid amount. The servicer may also charge ancillary fees to the borrower under specific circumstances. For example, a servicer can typically charge a borrower for providing detailed payment history extending back more than a year, for providing additional copies of their tax or escrow statements, or for providing an updated amortization schedule.[7]

Of course the reason that the issuers allow the servicer to charge these fees is because they are compensation for real costs that the servicer must bear. The servicer must build a significant computing and customer-service infrastructure that will allow them to track and maintain customer accounts. This infrastructure has a high initial cost, but relatively low variable costs, and this drives servicers to seek large economies of scale. According to Mortgage Network News, the ten biggest servicers in the United States service approximately $3.77 trillion in mortgages. This corresponds to 81% of all conforming loans in the US.[8]

In addition to infrastructure development and maintenance costs, servicers must also stand ready to make advances on defaulted or delinquent loans. Typically the servicer must remit all scheduled payments for a month by the remittance date, regardless as to whether the payments have been received from the borrower. As a result, if a borrower is late on a payment, the servicer may have to make the payment on their behalf and then collect the payment from the borrower. Indeed under a typical servicing agreement, the servicer must make up to three months of payments before the loan is considered in default. Once the loan is in default the issuer will buy the loan out of the pool and the servicer will only be required to forward payments that it receives from the borrower; the servicer no longer has to advance scheduled but non-received payments. In most cases the servicer will then be tasked with foreclosing on the loan and will ultimately be reimbursed for the payments that they have advanced on the loan upon the sale of the property. If the loan does go into foreclosure the servicer faces significant costs associated with the foreclosure process. While these costs are ultimately repaid from the proceeds of the foreclosure sale, including any default insurance payouts, they are of concern to the servicer.

One of the most striking aspects of an MSR is the rather small degree to which the servicer can control the cash flows associated with it. Virtually the only aspects that the servicer can control are the costs associated with developing and maintaining the infrastructure. That is, they can limit the fixed and variable costs associated with the call centers, computer systems, and some customer-service related activities, and their ability to do this is really a function of their operational efficiency. Other costs, such as the opportunity cost associated with having to advance payments when borrowers are delinquent on their loans, cannot be directly controlled by the servicer. In essence the servicer holds an instrument that is equivalent to a long position in a

[7] Generally servicers will provide these services at no charge if the borrower will accept the data through the web or through other automated systems. These fees are typically charged if the borrower requires that the information be sent on paper via the mail.

[8] http://www.nationalmortgagenews.com/mortgagestats/freedata/serv.htm.

portfolio of bonds, but is short options on those bonds. Further, the payout that they receive from the bonds is a function of the servicer's operational efficiency.

Treating the MSR as a portfolio consisting of a bond and a series of short positions on that bond allows us to model the MSR in a contingent-claims framework. This was essentially the approach taken by Van Drunen and McConnell (1988) Their focus, however, was primarily on the effect of prepayment on the MSR value. As a result, their model only included stochastic interest and inflation rates. The model that we present incorporates both prepayment and default. A number of researchers (see, for example, Kau et al. (1992); Hilliard et al. (1998); Ambrose et al. (1997)) have shown that the interaction between these competing termination claims can have a significant impact on borrower behavior. Our a priori assumption is that this will also affect MSR value. As a result, we have elected to develop our MSR model in an economy with both stochastic interest rates and stochastic house prices. Developing that framework is the focus of the next section.

3. The model framework

In the model interest rates evolve stochastically through time following the square-root mean reverting process by Cox et al. (1985),

$$\mathrm{d}r = \gamma(\Theta - r)\mathrm{d}t + \sigma_{\mathrm{r}}\sqrt{r}\mathrm{d}z_{\mathrm{r}}, \tag{1}$$

where the increment $\mathrm{d}z_{\mathrm{r}}$ is a Wiener process, γ is the speed of adjustment toward the long-run steady-state interest rate, Θ is the long-term mean risk-free rate and σ_{r} is the volatility of interest rates.

House prices follow the stochastic process

$$\frac{\mathrm{d}H}{H} = (\alpha - s)\mathrm{d}t + \sigma_{\mathrm{H}}\,\mathrm{d}z_{\mathrm{H}}, \tag{2}$$

where $\mathrm{d}z_{\mathrm{H}}$ is the increment of a Wiener process, H is the nominal value of the house, α is the expected rate of total return of the house, s is the service flow of using the house and σ_{H} is the house price standard deviation. A security whose price f is contingent upon the underlying state variables expressed by Eqs. (1) and (2) is governed by the second-order partial differentiation equation:

$$\frac{\partial f}{\partial t} + (r - s)H\frac{\partial f}{\partial H} + \gamma(\Theta - r)\frac{\partial f}{\partial r} + \frac{1}{2}H^2\sigma_{\mathrm{H}}^2\frac{\partial^2 f}{\partial H^2} + \frac{1}{2}r\sigma_{\mathrm{r}}^2\frac{\partial^2 f}{\partial r^2} + \rho H\sqrt{r}\sigma_{\mathrm{H}}\sigma_{\mathrm{r}}\frac{\partial^2 f}{\partial H\partial r} = rf. \tag{3}$$

To determine the value of the MSR, we numerically solve Eq. (3), subject to boundary conditions developed below, using a bivariate-binomial lattice. As shown by Nelson and Ramaswamy (1990), one can use a binomial lattice to model stochastic processes such as those in Eqs. (1) and (2) by twice transforming the equations: the first transformation insures a constant volatility process and the second transformation removes any correlation between the processes. We use the specific transformations proposed by Hilliard et al. (1998).

First, we define the transformed variables R and S, $R = 2\sqrt{r}$ and $S = \ln(H)$, which have constant volatility. We then define two new, uncorrelated variables

$$X_1 = \sigma_r S + \sigma_H R, \tag{4}$$

$$X_2 = \sigma_r S - \sigma_H R. \tag{5}$$

We must make two final assumptions to insure that model is consistent with a risk-neutral world. First, we must assume that the Local Expectations Hypothesis holds. Second, we must assume that in a risk-neutral world the total returns to housing grow at the risk-free rate. To do this we modify Eq. (2):

$$\frac{\mathrm{d}\widehat{H}}{\widehat{H}} = (r - s)\mathrm{d}t + \sigma_H \mathrm{d}z_H. \tag{6}$$

We then construct a two-dimension lattice to model the evolution of X_1 and X_2 through time. Hilliard et al. (1998), and Ambrose et al. (1997) demonstrate that these processes follow

$$\mathrm{d}X_1 = \mu_1 \mathrm{d}t + \sigma_1 \mathrm{d}Z_1 \tag{7}$$

and

$$\mathrm{d}X_2 = \mu_2 \mathrm{d}t + \sigma_2 \mathrm{d}Z_2, \tag{8}$$

where

$$\mu_1 = \left(\sigma_r\left[(r - s) - \frac{\sigma_H^2}{2}\right] + \sigma_H\left[(\gamma(\Theta - r)r^{-1/2} - \frac{\sigma_r^2 r^{-1/2}}{4}\right]\right), \tag{9}$$

$$\sigma_1 = \sigma_r\sigma_H\sqrt{2(1+\rho)}, \tag{10}$$

$$\mu_2 = \left(\sigma_r\left[(r - s) - \frac{\sigma_H^2}{2}\right] - \sigma_H\left[(\gamma(\Theta - r)r^{-1/2} - \frac{\sigma_r^2 r^{-1/2}}{4}\right]\right), \tag{11}$$

and

$$\sigma_2 = \sigma_r\sigma_H\sqrt{2(1-\rho)}. \tag{12}$$

Given values of X_1 and X_2 we can recover the values of H and r at that point simply by inverting the transformations in Eqs. (4) and (5). Since the processes for $\mathrm{d}X_1$ and $\mathrm{d}X_2$ are uncorrelated, we can calculate their probabilities independently. From a given value of X_1 the probability of an up jump at the next time step is given by

$$p_1 = \frac{1}{2} - k_1 + \frac{\mu_1\sqrt{\Delta t}}{2\sigma_1}. \tag{13}$$

Similarly, for a given value of X_2 the probability of an up-jump at the next time step is given by

$$p_2 = \frac{1}{2} - k_2 + \frac{\mu_2\sqrt{\Delta t}}{2\sigma_2}. \tag{14}$$

The probability of a down jumps are given by $(1 - p_1)$ and $(1 - p_2)$, respectively.[9] Finally, the increment size for X_1 and X_2 are given by

$$\text{Step}_{X_1} = \sigma_1\sqrt{\mathrm{d}t} \tag{15}$$

and

$$\text{Step}_{X_2} = \sigma_2\sqrt{\mathrm{d}t}, \tag{16}$$

respectively.

After constructing the lattice and determining the relevant transition probabilities throughout it, we can price the MSR by working backwards through the lattice and applying the relevant boundary conditions. The next section presents those boundary conditions.

4. Boundary conditions

The cash flows received by the servicer at any point in time are determined by the actions of the mortgage borrower. The borrower can continue to make their scheduled payments, they can prepay the loan, or they can default on the loan.[10] As a result, the first step in determining the MSR boundary conditions is to identify the boundary conditions that determine when the mortgage borrower will exercise their various options. We can then translate those actions into costs and benefits for the MSR holder.

Standard mortgage boundary conditions have been presented in numerous papers including Kau et al. (1992); Ambrose et al. (1997); Hilliard et al. (1998), and we adopt those standard boundary conditions. These conditions can be thought of as terminal boundary conditions, i.e., those that apply at the scheduled maturity of the mortgage, and interior boundary conditions, i.e., those that apply at all time steps prior to the maturity of the mortgage. In all cases, the goal of the borrower is to maximize their wealth, which, since they are short the mortgage contract, is tantamount to minimizing the value of the mortgage.

[9] The variables k_1 and k_2 allow for the possibility of jumping up or down by more than one increment at a given time step. The k_1 and k_2 values are selected to insure meaningful probability values (Hilliard et al. (1996), or Hilliard et al. (1998)).

[10] Although some papers (Ambrose and Buttimer (2000)) note that the lag between default and foreclosure grants an implicit option to the borrower to reinstate the loan, in this paper we make the standard simplifying assumption that once default occurs, foreclosure is inevitable and immediate.

4.1. Terminal conditions

At the maturity date of the mortgage contract, the borrower has only two options: they can make their final payment and fulfill the contract, or they can default on the payment.[11] Let UMB_T represent the unpaid mortgage balance at maturity (i.e., at time $t = T$), V_T the value of the mortgage at time T and H_T the value of the house at time T.[12] The borrower will only pay the mortgage if the value of the house exceeds the unpaid balance of the mortgage, and so

$$V_T = \min(\text{UPB}_T, H_T). \tag{17}$$

The value of the MSR depends upon the action taken by the borrower. If the borrower makes the final payment, the value of the MSR is given by

$$\text{MSR}_T = \text{UPB}_T\left(\frac{\text{SF}}{12}\right) - \left(\frac{\text{SC}}{12}\right), \tag{18}$$

where SF is the annual servicing fee rate and SC is the annual cost to service the loan, expressed in dollars. If the borrower defaults, the value of the MSR will be

$$\text{MSR}_T = -\left(\frac{\text{SC}}{12}\right) - \text{DC} + \text{REC}_T, \tag{19}$$

where DC is the dollar cost to the servicer associated with the default, and REC_T is the present value of the dollars that will be recovered during the foreclosure process. We note that REC_T will be a function of the market value of the house plus whether the loan has default insurance on it or not. We can combine Eqs. (18) and (19) into a single equation

$$\text{MSR}_T = \begin{cases} \left[\text{UPB}_T\left(\frac{\text{SF}}{12}\right) - \left(\frac{\text{SC}}{12}\right)\right] & \text{if } H_T \geqslant \text{UMB}_T \\ \left[-\left(\frac{\text{SC}}{12}\right) - \text{DC} + \text{REC}_T\right] & \text{if } H_T < \text{UMB}_T \end{cases} \tag{20}$$

4.2. Interior boundary conditions

For non-terminal time steps the model must be ready to consider not only default and prepayment, but also the possibility that the borrower will prepay the mortgage.

Since the model runs with several time steps per month it must consider the borrower's options both on dates when payments are due as well as for the period between payment dates. This is important since the borrower can prepay the loan at any time during the month.[13]

[11] Obviously the borrower does not have a prepayment option at the maturity date of the loan.

[12] To fully identify the value of the mortgage at a particular point in the lattice we must specify three parameters: t, the time-step of the lattice, i, the index for X_1 within the lattice, and j, the index for X_2 within the lattice. Thus the value of the mortgage would be denoted $V_{t,i,j}$. For ease of exposition, however, we omit the i and j designations when doing so will not create confusion.

[13] As noted by Kau et al. (1992), the borrower can only default on payment due dates.

As shown in a number of papers (Ambrose et al. (1997); Hilliard et al. (1998), Kau, et al.), the borrower will seek to minimize the lender's position.[14] From the borrower's perspective, if they default the lender receives the house (H_t). In reality the lender's position is less than H_t since there are costs associated with default, but these costs are irrelevant to the borrower's decision. Similarly, from the borrower's perspective if they prepay the loan the lender receives the unpaid mortgage balance UMB_T. Finally, if the borrower makes their scheduled payment, the lender's position is (again from the borrower's perspective) the present value of the remaining payments (A_t) less the value of the options to default and prepay in the future, D_t and C_t. Thus, the borrower will seek to minimize the lender's position, and so we can write the value of the mortgage contract in a single equation as

$$V_t = \min(A_t - C_t - D_t, H_t, \text{UMB}_t). \tag{21}$$

When the model is at time steps where payments are not due, the borrower will not default, and so they only select between continuation and prepayment. As a result, Eq. (21) simplifies to

$$V_t = \min(A_t - C_t - D_t, \text{UMB}_t). \tag{22}$$

Our concern, of course, is with the value of the servicing contract, but we have to be concerned with how the borrower's actions influence the value of the servicing. If the borrower makes their scheduled payment, the value of the servicing contract is the cash flow received this period less the cost of servicing in this period, plus the expected present value of the servicing contract in future periods. This can be written as

$$\text{MSR}_t = M_t + \text{UPB}_t\left(\frac{\text{SF}}{12}\right) - \left(\frac{\text{SC}}{12}\right), \tag{23}$$

where M_t is the (net) present value of the future MSR cash flows, UPB_t is the unpaid balance at time t, SF is the annual servicing fee rate and SC is the annual dollar cost of servicing the loan.

If the borrower defaults, there will be no future cash flows associated with the MSR, so the value of the MSR will essentially be the same as that of Eq. (19)

$$\text{MSR}_t = -\left(\frac{\text{SC}}{12}\right) - \text{DC} + \text{REC}_t. \tag{24}$$

Similarly, if the borrower prepays the loan, there will be no more future cash flows associated with the loan, and so the servicing right terminates. The value of the servicing right in that case will be

$$\text{MSR}_t = \text{UPB}_t \times \left(\frac{\text{SF}}{12}\right) - \left(\frac{\text{SC}}{12}\right) + \text{PC}_t, \tag{25}$$

where PC_t is the costs associated with prepayment operations. These costs may come from a number of sources, some of which are immediate costs and some of which are

[14] Of course the borrower is not concerned with the distinctions between the servicer, the MBS issuer, and the MBS investors. From their perspective, they are just different aspects of a single "lender."

longer-term costs. For example, in the short-run the servicer will have to notify the local government that the mortgage contract has been fulfilled and that the lien should be released (or, depending upon the state, the title returned to the borrower.) Similarly, the servicer may have to help the borrower close escrow accounts and recover monies deposited in them.

The longer run costs are more subtle. This model values a single servicing contract under the assumption that the borrower optimally exercises their prepayment and default options. Thus, in this model the servicer's portfolio prepays (or defaults) completely at once. In practice, however, all of the loans in a servicer's portfolio will not prepay simultaneously, even if the prepayment options are deeply in the money. As a result, the servicer cannot instantaneously shut down their servicing infrastructure and they will continue to bear at least the fixed costs of that infrastructure even if individual loans have prepaid. Modeling the "runoff" of a servicer's book of business is beyond the scope of this paper: our goal is to examine the pricing of a single MSR. As a result, we elect to use a single cost figure, PC_t, which incorporates all of the net present costs associated with the loan prepaying.

We can combine the borrower's options and the resulting MSR values into one equation:

$$\mathrm{MSR}_t = \begin{cases} M_t + \mathrm{UPB}_t\left(\frac{\mathrm{SF}}{12}\right) - \left(\frac{\mathrm{SC}}{12}\right) \text{ when } (A_t - C_t - D_t \leqslant H_t) \text{ and } (A_t - C_t - D_t \leqslant \mathrm{UMB}_t) \\ -\left(\frac{\mathrm{SC}}{12}\right) - \mathrm{DC} + \mathrm{REC}_t \text{ when } (H_t < A_t - C_t - D_t) \text{ and } (H_t < \mathrm{UMB}_t) \\ \mathrm{UPB}_t \times \left(\frac{\mathrm{SF}}{12}\right) - \left(\frac{\mathrm{SC}}{12}\right) + \mathrm{PC}_t \text{ when } (\mathrm{UMB}_t < A_t - C_t - D_t) \text{ and } (\mathrm{UMB}_t < H_t). \end{cases} \tag{26}$$

If the model is at a point where default cannot occur, i.e., it is not at a payment date, then Eq. (26) simplifies to

$$\mathrm{MSR}_t = \begin{cases} M_t \text{ when } (A_t - C_t - D_t \leqslant \mathrm{UMB}_t) \\ \mathrm{UPB}_t \times \left(\frac{\mathrm{SF}}{12}\right) - \left(\frac{\mathrm{SC}}{12}\right) + \mathrm{PC}_t \text{ when } (\mathrm{UMB}_t < A_t - C_t - D_t). \end{cases} \tag{27}$$

One can thus work backwards through the lattice to determine the value of the MSR at time $t = 0$.

5. Results

To understand the comparative statics of the model, we construct a series of simulations. Table 1 presents the basic parameters that we use for these simulations, and presents these parameters in two panels. Panel A presents the mortgage-specific and MSR-specific parameters. These include the initial house price, H_0 of \$300,000, the contract rate of 9%, the initial loan-to-value ratio of 90%, and a mortgage term of 30 years. The (net) servicing fee is 25 basis points a year, the cost to service the loan is \$44 per year, and the (net) foreclosure cost per loan is \$2,000. The servicing cost and foreclosure cost values were chosen to be representative of actual costs estimates used in the industry as reported by National Mortgage News, a data provider for

Table 1
Base case parameter values for simulations

Parameter	Value
Panel A: mortgage and mortgage servicing-related parameters	
Contract rates (C)	9%
Loan-to-value ratio (LTV)	90%
Original house values (H_0)	$300,000
Mortgage term	30 years
Loan type	Fixed-rate mortgage
Servicing income (SF)	25 Basis points
Servicing cost per loan (SC)	$44 per year.
Foreclosure cost per loan	$2000
Panel B: economic-related parameters	
Steady-state spot rate (θ)	10%
Interest rate volatility (σ_r)	10%
Housing price volatility (σ_H)	10%
Reversion coefficient (γ)	25%
House service flow (s)	8.5%
Original spot rate (r_0)	8%
Correlation coefficient ($\rho_{r,H}$)	0%

the servicing industry.[15] The $44 annual servicing cost is consistent with the trend in servicing costs noted by Lacour-Little (2000).

We outline our base economic parameters in panel B of Table 1. Where appropriate we have selected values that are consistent with those used in other mortgage papers. We assume an upward sloping yield curve with initial spot rate of 8% and long-run average rate of 10%. We assume that interest rate volatility is 10%, and that the speed of reversion factor (γ) is 25%. We assume housing volatility of 10%, and a housing service flow of 8.5%. Finally, we assume that the correlation coefficient between dz_r and dz_H is 0.

Based on these values, the value of the MSR would be $2699.62. Of course our primary interest is in understanding how the MSR value changes as the various parameters change. In Table 2 we present the MSR value for a variety of H_0, σ_r, and σ_H values. We first note that while the MSR value increases with H_0, it does not perfectly scale with it. The servicing cash flow in each period is a function of the house value, so that component of the MSR value should scale exactly with the value of the house. The reason that the overall MSR value does not scale with the house value is because the costs associated with servicing—the annual cost to service and the special costs associated with default—are not functions of the house price. As a result, increasing the initial house price 50% from $200,000 to $300,000 increases the value of the MSR by more than 50%. For example, with σ_r and σ_H both set to 10%, the MSR value when H_0 is $200,000 is $1592.83, but when H_0 is $300,000, the value of the MSR is $2699.62, a 69.5% increase in its value.

[15] National Mortgage News Annual Data Report, 2003.

Table 2
Mortgage servicing right values For various house and volatility values

σ_r	$H_0 = \$200{,}000$			$H_0 = \$300{,}000$		
	$\sigma_H = .05$	$\sigma_H = .10$	$\sigma_H = .15$	$\sigma_H = .05$	$\sigma_H = .10$	$\sigma_H = .15$
.05	2401.79	2082.41	1800.99	3809.92	3472.59	3138.47
.10	1674.25	1592.83	1459.06	2692.99	2699.62	2593.71
.15	1316.37	1287.78	1211.94	2121.53	2225.04	2203.85

Base case economic parameters include a 90 percent LTV with a house price house price of \$300,000, $\sigma_H = .10$, $S = .085$, $r = .08$, $\theta = 0.10$, $\gamma = .25$, $\sigma_r = .10$, and $\rho = 0$. The underlying mortgage has a 9% contract rate, a 30 year term, net servicing rate of 25 basis points, annual servicing costs of \$44, and foreclosure costs of \$2000.

Changing σ_r and σ_H results in some counter-intuitive results. Given that the servicer is, essentially, short the prepayment and default options, one might expect that as interest rate and housing volatility increased, that the value of those options would monotonically increase and the value of the MSR would fall. The values of the borrower's options do, in fact, increase, but this does not always translate into a decline in value to the MSR holder. There are two reasons for this. First, under some circumstances the value of the prepayment option may be in the money and if the value of the default option is low it may be optimal for the borrower to terminate the mortgage through prepayment. If σ_H were to increase, however, then the value of the default option would increase and this could take the borrower out of the optimal prepayment region. That is, even though the prepayment option was still in-the-money, it would not be optimal to terminate at that point in time through prepayment. The borrower's optimal action would be to preserve both the default and prepayment option, and so they would simply continue the mortgage.[16] This would lengthen the time over which the servicer would receive their servicing fees.

Second, when the borrower eventually does exercise the option, even though it is more deeply "in the money" this only affects the cost to the investor, not the servicer. To the servicer the cost of the borrower exercising one of the options is essentially fixed, it is not a function of the money-ness of option at the time the borrower exercises it. In this respect the servicer is short an option that is closer to a so-called "digital" option than a traditional American call or put. The only real costs to the servicer are the loss of future income, any extraordinary servicing costs associated with the prepayment or default, as well as any opportunity costs associated with continuing to maintain their servicing infrastructure. In contrast, to a mortgage investor the increased value of the option means that even though the timing of the exercise might be delayed and the investor may receive cash flows for a longer period of time, when the borrower does exercise their option the cost to the lender will exceed the benefits of the extra cash flow. For the servicer, however, the increased value of the options simply means that they will receive their servicing cash

[16] Kau et al. (1992), and Kau and Kim (1994) discuss how the interaction effects between the prepayment and default options affect the timing of the borrower's exercising of those options.

flow for a longer period of time, but the costs when the borrower eventually does exercise their options are invariant.

This effect can be most easily seen in by examining the MSR values in Table 2 where $H_0 = \$300{,}000$. When interest rate volatility is low, i.e., when $\sigma_r = .05$, increasing housing volatility reduces the value of the MSR. For example when $\sigma_H = .05$ the value of the MSR is \$3,809.92, but when $\sigma_H = .15$, the value of the MSR falls to \$3138.47. In this low interest rate volatility environment the increase in housing volatility is sufficient to raise the value of default to the point where the borrower will exercise the default option and forgo their prepayment option. In contrast, when interest rate volatility is high, i.e., when $\sigma_r = .15$, increasing the housing volatility can actually *increase* the MSR value. For example raising σ_H from .05 to .10 increases the value of the MSR from \$2121.53 to \$2225.04. This is because as σ_H increases the value of the default increases, but, since the value of the prepayment was already high due to the relatively high σ_r value the borrower will delay exercising either option. In the low housing volatility environment the borrower would exercise their prepay option sooner than they would exercise either option in the higher housing volatility environment. As a result, the expected termination time of the mortgage increases, and this increases the value of the MSR. If one increases σ_H even further to .15, however, the value of the MSR will start to decline (to \$2203.85.) The reason is that the default option begins to become sufficiently in the money that the borrower will begin to exercise it at earlier points in time, and thus the expected termination date of the mortgage begins to shorten, lessening the fees the servicer receives.[17]

To better illustrate this point we examine MSR values using our base economic parameters ($H_0 = \$300{,}000$, $r_0 = 8\%$, $\theta = 10\%$, $\sigma_H = 10\%$, and $\sigma_r = 10\%$) but with a variety of LTV values and coupon rates. Specifically in Table 3 we examine the MSR values for LTV ratios of 70, 80, 90, and 100%, and contract rates of 6, 6.5, 7, 7.5, 8, 8.5, 9, 9.5, and 10%. By examining the MSR value under these different LTV and contract rate combinations we are able to see how the prepayment and default options, as well as their interaction, affect the MSR value. For example, when the initial LTV is set to 70%, the default option has very little value, but when the LTV value is 100%, the default option is quite valuable. Similarly, when the contract rate is set to 6%, the prepayment option is not very valuable but when it is set to 10% it is very valuable.

Table 3 presents both the dollar value of the MSR for each LTV/contract rate combination and the value of the MSR expressed as a percentage of the original loan amount. Expressing the MSR in percent of original value terms is particularly useful when comparing across LTV values since the absolute dollar amounts change with the size of the initial loan.

[17] We note that this does not happen in the example where $H_0 = \$200{,}000$. The reason for this is because we have assumed that default costs to the servicer are \$2000. Although the default and prepayment options still behave in the same way as in the \$300,000 case, since the servicing income is reduced by 50% (due to the reduction in the loan size), the default costs are large enough to overcome any benefit derived from the increased servicing receipts.

Table 3
Mortgage servicing right values for various coupons and LTV values

Coupon (%)	LTV = 70%	LTV = 80%	LTV = 90%	LTV = 100%
10	1231.15 (0.586%)	1433.31 (0.597%)	1653.09 (0.612%)	1190.15 (0.397%)
9.5	1774.94 (0.845%)	2022.69 (0.843%)	2156.01 (0.799%)	1799.55 (0.600%)
9.0	2044.59 (0.974%)	2496.55 (0.974%)	2699.62 (1.00%)	2436.65 (0.812%)
8.5	2602.18 (1.239%)	3062.00 (1.276%)	3279.42 (1.215%)	3076.43 (1.025%)
8.0	3161.74 (1.506%)	3596.74 (1.499%)	3806.87 (1.410%)	3596.74 (1.199%)
7.5	3582.08 (1.706%)	4003.32 (1.668%)	4230.00 (1.567%)	4172.31 (1.391%)
7.0	3814.24 (1.816%)	4244.05 (1.768%)	4517.71 (1.673%)	4575.22 (1.525%)
6.5	3893.84 (1.854%)	4358.98 (1.816%)	4698.80 (1.740%)	4859.12 (1.620%)
6.0	3912.32 (1.863%)	4412.00 (1.838%)	4811.07 (1.782%)	5069.83 (1.690%)

Base case economic parameters include an initial house price house price of \$300,000, $\sigma_H = .10$, $S = .085$, $r = .08$, $\theta = 0.10$, $\gamma = .25$, $\sigma_r = .10$, and $\rho = 0$. The underlying mortgage has 30 years term, net servicing rate of 25 basis points, annual servicing costs of \$44, and foreclosure costs of \$2000. Numbers in parentheses are values expressed as percentage of the loan amount.

The results in Table 3 demonstrate the effects that the options have on the MSR value. The MSR takes on its maximal value (in percentage terms) when the LTV of the loan is 70% and the coupon is 6%. This corresponds to the case where the borrower is least likely to default or prepay the loan (since the equilibrium loan rate would be around 8.6% in this case.) In contrast, the lowest value of the MSR, both in percentage and dollar terms, occurs when the coupon rate is 10% and the LTV is 100%. In essence the borrower's options are both deeply in the money, and it takes only a slight change in either the value of the house or the interest rate to trigger a default or prepayment.

What is interesting, however, is to examine how the MSR value changes as the LTV value changes for a constant contract rate. Consider first when the contract rate is held steady at 10%. In this scenario the prepayment option is initially in the money and increasing the LTV from 70% to 90% increases the percentage value of the MSR. Once again, what is happening is that as the LTV increases, the value of the default option increases. This increased default value increases the opportunity cost to the borrower of immediately prepaying, and so the borrower waits longer to exercise their prepayment option. The result is a longer stream of MSR cash flows, and hence a larger MSR value. When the LTV value is set at 100%, however, both options are so deeply in the money that it takes only a slight change in either the interest rate or house value environment to trigger a prepayment or default. As a result the time to mortgage termination is relatively short, and the value of the MSR falls.

When the coupon is held steady at 6%, however, increasing the LTV strictly reduces the percentage value of the MSR. At a 6% contract rate, the prepayment option is deeply out of the money, and so increasing the LTV value increases the default option value. Since the prepayment option is not very valuable, giving it up via default is not a major factor to the borrower. As a result, increasing the LTV increases the probability of the borrower defaulting, and thus reduces the time during which the servicer will receive their fee. Thus, increasing the LTV reduces the percentage value of the MSR. This result holds for each of the coupon rates that are less than 10%.

Fig. 1 graphs a surface of MSR values for each of the LTV and contract rate combinations. A point which can be see most readily from this graph is that although the MSR value is increasing in contract rate for all LTV values, the rate of increase is not uniform. This is a direct result of the interaction of the default and prepayment options altering the borrower's termination options.

In Table 4 we examine the effect that the term structure has on the MSR value. In particular, we examine the MSR value for three different term structure scenarios: the standard upward sloping yield curve ($r_0 = 8\%$, $\theta = 10\%$) that we used in Tables 2 and 3, a flat yield curve ($r_0 = 9\%$, $\theta = 9\%$), and a downward sloping yield curve ($r_0 = 10\%$, $\theta = 8$ %.) For each yield curve we price the MSR with several interest rate volatility (σ_r), housing volatility (σ_H), and correlation coefficient (ρ) values.

One can see a number of results from Table 4. In particular, as the yield curve flattens and then inverts, the value of the MSR drops for all volatility and correlation coefficient scenarios. The reason for this, of course, is that since the contract rate is held steady under each of the yield curve scenarios, the prepayment option is likely to become more deeply in the money quickly, and the borrower becomes correspondingly more likely to exercise it sooner in the life of the option. Similarly, as σ_r increases the value of the MSR drops because the prepayment option can become

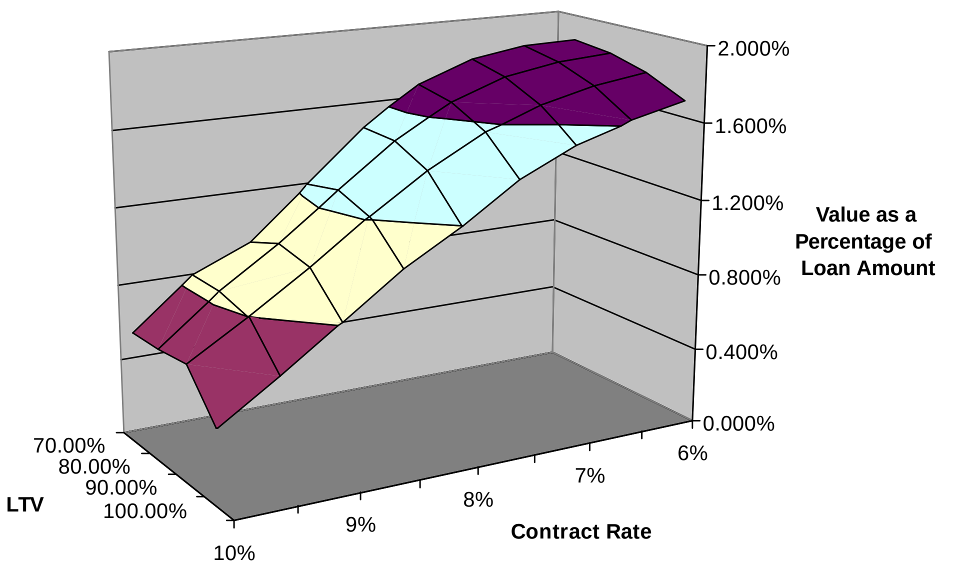

Fig. 1. MSR value as a percentage of initial loan amount for a variety of contract rate and LTV values. Base case economic parameters include a house price of \$300,000, $\sigma_H = .10$, $S = .085$, $r = .08$, $\theta = 0.10$, $\gamma = .25$, $\sigma_r = .10$, and $\rho = 0$. The underlying mortgage has 30 years term, net servicing rate of 25 basis points, annual servicing costs of \$44, and foreclosure costs of \$2000.

Table 4
Mortgage servicing right values for various term structures, volatilities, and correlation coefficients

σ_r	$\rho = -10\%$			$\rho = 0\%$			$\rho = 10\%$		
	$\sigma_H = .05$	$\sigma_H = .10$	$\sigma_H = .15$	$\sigma_H = .05$	$\sigma_H = .10$	$\sigma_H = .15$	$\sigma_H = .05$	$\sigma_H = .10$	$\sigma_H = .15$
Panel A: upward sloping term structure ($r_0 = .08$, $\theta = .10$)									
.05	3778.51	3433.91	3108.09	3809.92	3472.59	3138.47	3829.96	3498.88	3167.57
.10	2691.38	2667.22	2547.01	2692.99	2699.62	2593.71	2815.35	2785.89	2657.93
.15	2136.88	2213.35	2165.51	2121.53	2225.04	2203.85	2122.18	2301.72	2288.35
σ_r	$\rho = -10\%$			$\rho = 0\%$			$\rho = 10\%$		
	$\sigma_H = .05$	$\sigma_H = .10$	$\sigma_H = .15$	$\sigma_H = .05$	$\sigma_H = .10$	$\sigma_H = .15$	$\sigma_H = .05$	$\sigma_H = .10$	$\sigma_H = .15$
Panel B: flat term structure ($r_0 = .09$, $\theta = .09$)									
.05	2665.62	2531.77	2469.47	2730.75	2631.41	2530.77	2688.49	2645.32	2571.67
.10	2310.76	2253.10	2188.88	2392.44	2320.36	2247.36	2430.05	2343.74	2289.75
.15	1923.09	1965.42	1938.70	1865.25	1985.87	1971.07	1968.49	2078.16	2036.13
σ_r	$\rho = -10\%$			$\rho = 0\%$			$\rho = 10\%$		
	$\sigma_H = .05$	$\sigma_H = .10$	$\sigma_H = .15$	$\sigma_H = .05$	$\sigma_H = .10$	$\sigma_H = .15$	$\sigma_H = .05$	$\sigma_H = .10$	$\sigma_H = .15$
Panel C: downward sloping term Structure ($r_0 = .10$, $\theta = .08$)									
.05	2019.88	2034.79	2007.92	1960.50	2051.53	2066.05	2143.98	2127.71	2117.18
.10	2060.36	1994.90	1940.97	2072.86	2008.95	1965.99	2024.60	1994.53	1995.30
.15	1865.13	1820.61	1762.72	1985.66	1891.41	1832.04	1960.24	1850.95	1815.72

Base case economic parameters include a 90 percent LTV with a house price house price of $300,000, $S = .085$, $\gamma = .25$. The underlying mortgage has a contract rate of 9%, 30 years term, net servicing rate of 25 basis points, annual servicing costs of $44, and foreclosure costs of $2000.

sufficiently deep in the money to induce the borrower to exercise it early. This is true for all yield curve, housing volatility, and correlation coefficient combinations.

These interest rate-related parameters have, by far, the biggest influence on the MSR value. They are also the risks against which a servicer can most easily hedge. For example, a servicer could hedge against changes in the shape or level of the yield curve using swaps, interest rate caps, or interest rate floors. Similarly, the servicer could hedge against changes in σ_r through swaptions contracts.

Table 4 also demonstrates that increasing σ_H tends, in general, to decrease the value of the MSR. The reason for this is that increasing σ_H increases the value of the default option and under most circumstances this reduces the MSR value. As was the case in Table 2, however, under some circumstances increasing σ_H can increase the value of the MSR, since increasing the value of the default option increases the benefit to the borrower of delaying prepayment.

The MSR value is much less sensitive to changes in the correlation coefficient than it is to changes in the shape of the yield curve or to either of the volatility parameters. Although the MSR value will change when one changes ρ, the scale of the changes is much smaller than when changing the other parameters.

When the yield curve is upward sloping, increasing ρ from 0 to 10% increases the value of the MSR for all values of σ_r and σ_H. This is, once again, because of the interaction of the default and prepayment options. Similarly, decreasing ρ from its base value of 0 to -10% reduces the MSR value for all values of σ_r and σ_H.

In contrast, when the terms structure is flat or downward sloping, the effect of changing ρ depends upon the relative values of σ_r and σ_H. This is because the MSR value is largely determined by the timing of the borrower's decision to default or prepay: the further into the life of the mortgage that this happens, the more valuable the MSR. Under the flat and downward sloping scenarios the borrower generally has a much higher propensity to prepay than under the upward sloping yield curve scenario. This results in the generally lower MSR values in panels B and C of Table 4. As a result, most of the differences in the MSR values within panels B and C are the result of differences in when the borrower exercises their prepayment option: when the borrower delays exercising that option, the value of the MSR tends to increase. The default option still has value, and the borrower does exercise it occasionally, but it is dominated by the prepayment option.

In panels B and C of Table 4, changing ρ has the most interesting effects when σ_H and σ_r are unequal. For example, when the term structure is flat and σ_H is 5% but σ_r is 15%, increasing ρ from 0 to 10% will increase the value of the MSR. Surprisingly, decreasing ρ from 0 to -10% will also increase the value of the MSR, although for different reasons. What is happening is that the value of the default option is decreasing as ρ decreases and the value of the prepayment option is increasing as ρ, but they are doing so at different rates. As a result, as ρ increases from 0 to 10%, the default option gains more in value than the prepayment option decreases, and so the borrower has an incentive to delay exercising either option when compared to the $\rho = 0$ case. When ρ decreases from 0 to -10%, the prepayment option gains more in value than the default option loses. Once again, the borrower has an incentive to delay either option when compared to the $\rho = 0$ case.

6. Conclusion

In this paper we have presented an option-based pricing model for pricing Mortgage Servicing Rights that is consistent with the servicing contracts used in both the US and Canada. We derive the boundary conditions for the MSR and then implement the pricing model within an economy that has both stochastic interest rates and house prices. Our numerical results demonstrate that the value of the MSR contract is sensitive to changes in interest rate and housing volatility, as well as to the correlation coefficient between these volatilities.

Our results also indicate that the interaction between the default and prepayment options can, under certain conditions, increase the value of the MSR, by increasing the value of the borrower's option to delay termination. While this would normally result in a lower value of the MBS holder, since the borrower's options are monotonically more valuable. They can increase the MSR holder's value, however, because they increase the time during which the MSR holder receives their servicing fee, they delay the realization of costs associated with default or prepay, and the MSR holder's position is invariant to the "moneyness" of the borrower's options.

References

Aldrich, S.P.B., Greenberg, W.R., Payner, B.S., 2001. A capital markets view of mortgage servicing rights. J. Fixed Income 11 (1), 37–54.

Ambrose, B.W., Buttimer, R.J., Capone, C.A., 1997. Pricing Mortgage Default and Foreclosure Delay. J. Money, Credit, Banking 29 (3), 314–323.

Ambrose, B.W., Buttimer, R.J., 2000. Embedded options in the mortgage contract. J. Real Estate Finance Econ. 21 (2), 95–111.

Cox, J.C., Ingersoll Jr., J.E., Ross, S.A., 1985. A theory of the term structure of interest rates. Econometrica 53 (2), 385–407.

Hilliard, J.E., Schwartz, A.L., Tucker, A.L., 1996. Bivariate binomial options pricing with generalized interest rate processes. J. Finan. Res. 19 (4), 585–602.

Hilliard, J.E., Kau, J.B., Slawson, V.C., 1998. Valuing prepayment and default in a fixed-rate mortgage: a bivariate binomial options pricing technique. Real Estate Econ. 26 (3), 431–468.

Kau, J.B., Keenan, D.C., Muller, W.J., Epperson, J.F., 1992. A generalized valuation model for fixed-rate residential mortgages. J. Money, Credit, Banking 24 (3), 279–299.

Kau, J.B., Kim, T., 1994. Waiting to default: the value of delay. J. Am. Real Estate Urban Econ. Assoc. 22 (3), 539–551.

Lacour-Little, M., 2000. The evolving role of technology in mortgage finance. J. Housing Econ. 11 (2), 173–205.

National Mortgage News 2003 Annual Data Report, Washington, DC.

Nelson, D.B., Ramaswamy, K., 1990. Simple binomial processes as diffusion approximations in financial models. Rev. Finan. Stud. 3 (3), 393–430.

Van Drunen, L.D., McConnell, J.J., 1988. Valuing mortgage loan servicing. J. Real Estate Finan. Econ. 1 (1), 5–22.