Systemic Risk Estimation under Dynamic Volatility Matrix Models

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Abstract: This paper proposes a two-step procedure for systemic risk estimation under the stochastic volatility/correlation models. The first step utilizes Fourier transform method for dynamic volatility matrix estimation, and the second step develops efficient importance sampling estimators for extreme event probability. For the empirical analysis, we find that the systemic risk can be useful to measure the stability of financial system because it seems be able to provide early signs for institutions in U.S. during the 2008-2010 financial crisis. Moreover, it can serve as a predictor of the capital injections during the crisis. SRISK in China and Taiwan are also compared.

Keywords: dynamic volatility matrix model, Fourier transform method, importance sampling, systemic risk.

Section 1: Introduction

After the 2008 financial crisis, systemic risk has become a theme of studies for the financial stability. According to the definition from IMF, FSB, and BIS[1] in 2011, systemic risk is "a risk of disruption to financial services that is caused by an impairment of all or parts of the financial system, and can have serious negative consequences for the real economy." The European Central Bank (ECB) in 2010 defines the systemic risk as "a risk of financial instability so widespread that it impairs the functioning of a financial system to the point where economic growth and welfare suffer materially." Fouque and Langsam (2013) provide a working definition for systemic risk as "the risk of a disruption of the market's ability to facilitate the flows of capital that results in the reduction in the growth of the global GDP."

Bisias et al. (2012) provide a survey on the systemic risk measures and conceptual frameworks that have been developed over the past few years. There appear 31 quantitative measures of systemic risk in the economics and finance literature. These research methods are categorized by five systemic risk measures including probability distribution measures, contingent-claim and default measures, illiquidity measures, network analysis measures, and macroeconomic measures. These five categories and their related systemic risk measures are briefly described below.

Category Ⅰ Probability Distribution Measures: Perhaps one simple and direct measure of systemic risk is the total sum of negative outcomes of a collection of important financial institutions. Acharya et al. (2017) propose systemic expected shortfall (SES) to measure each financial institution's contribution to systemic risk. Furthermore, Acharya et al. (2012) introduces SRISK to measure the contribution of individual

[1] IMF: International Monetary Fund
FSB: Financial Stability Board
BIS: Bank of International Settlement

financial institution to the whole systemic risk. Several factors are considered into SRISK such as firm's size, leverage and conditional expected shortfall. All these factors are used to differentiate contribution level of financial institutions. Adrian and Brunnermeier's (2016) propose conditional value at risk (CoVaR) which measures the VaR of the system conditional on one institution being at its VaR with the same quantile. The CoVaR systemic risk measure can identify the risk contribution of an individual institution on the system.

Category Ⅱ Contingent-Claim and Default Measures: Based on Merton's structure model (1973), it is possible to construct measures of default likelihood for each institution and connect them either directly or indirectly through their joint distribution. This approach of contingent-claims analysis (i.e., derivatives pricing models) is taken by Capuano (2008), Gray and Jobst (2010), and Huang et al. (2009a).

Category Ⅲ Illiquidity Measures: Illiquidity is an example of a highly specific measure of systemic risk that often requires considerable structure. Khandani and Lo (2011) propose two distinct measures of equity market liquidity, one of which is the profitability of an equity mean-reversion strategy, and the other is a more direct measure of price impact based on Kyle (1985). Getmansky, Lo, and Makarov (2004) propose using serial correlation as a proxy for illiquidity. Aikman (2010) has developed its risk assessment model for systemic institutions to simulate the possibilities and point out that funding troubles can apply to both traditional intermediaries as well as shadow banks.

Category Ⅳ Network Analysis Measures: Kritzman et al. (2010) use principal components analysis to gauge the degree of commonality among a vector of asset returns, which is a way of network analysis measures. Using Granger-causality test statistics for asset returns to define the edges of a network of financial institutions are becoming popular. Billio et al. (2010) show that Granger-causality networks are highly dynamic and become densely interconnected prior to systemic shocks.

Category V Macroeconomic Measures: The opposite of the theoretical probability-distribution measures are the macroeconomic models of systemic risk. Reinhart and Rogoff (2009) provides comparisons of broad macroeconomic aggregates such as asset price indices (equities, housing, etc.), GDP growth rates, and public debt over many financial crises, and find a number of common patterns. Alessi and Detken (2009) construct simple yet early-warning indicators from a broad range of real and financial indicators including GDP and its components, inflation, interest rates, and monetary aggregates for 18 OECD[2] countries between 1970 and 2007. Borio and Drehmann (2011) propose a related approach, but with signals defined by simultaneous extreme values for pairs of property prices, equity prices, and credit spreads.

In this paper, our research is based on the systemic risk measure proposed by Acharya et al. (2012) in Category I Probability Distribution Measures. One innovation is applying the continuous-time dynamic volatility matrix models to estimate the capital shortfall and provide a systemic risk indicator for financial firms. The dynamic volatility matrix model (Malliavin and Mancino (2009), Mancino et al.(2017)) can represent the behavior of financial returns from the perspective from the non-parametric estimation. This facilitates the estimation of stochastic volatility/correlation models, which are known for capturing some stylized features of financial data (Fouque et al. (2011)). Those models have been intensively applied to derivative's pricing and hedging issues based on option price approximation. While these approximations are instrumental in reducing computational cost, there is still room of improvement for estimation accuracy.

Distinct from previous studies, we choose to value exact computation and incorporate the help of refined

[2] the Organisation for Economic Co-operation and Development

efficient simulation method for more accurate estimation to avoid approximation errors. We propose an improved procedure for systemic risk estimation with dynamic volatility matrix models under the historical (or physical) probability measure by the following two steps: (1) dynamic volatility matrix model estimation scheme by the Fourier transform method (Malliavin and Mancino (2009), Mancino et al.(2017)), and (2) enhanced importance sampling for estimating extreme capital shortfalls.

For the first step, the non-parametric Fourier transform method to estimate dynamic volatility matrix under continuous semi-martingale models is introduced. This method makes the estimation feasible and robust by relying on the integration of the time series based on the computation of Fourier coefficients of the variance process (Mancino (2017)). Some case studies for estimating instantaneous or spot volatility can be seen in Han (2013).

For the second step, importance sampling is developed for the estimation issues of the tail areas (McNeil et al. (2005)) when its problem is impossible to admit a close-form solution under complex dynamic volatility matrix models. Importance sampling can effectively improve convergence of sample means particularly in rare event simulation while direct Monte Carlo simulation suffers pitfalls like variance augmentation and slow convergence (Glasserman (2003), Lemieux (2009)). We further propose an enhanced version of importance sampling for estimating extreme event probability. The theoretical background of our proposed importance sampling combines the large deviation theory (Bucklew (2004)) which has the averaging effect on the homogenized system (Han (2012)) and provides a sharp estimate for the decay rate of small probabilities. This methodology is useful in handling the heavy (or fatter) tail distributions induced by stochastic volatility/correlation models.

We use this methodology to analyze the systemic risk of top financial firms in US, Taiwan and China between 2005 and 2016. The SRISK analysis shows that pre-crisis SRISK is a predictor of the capital injections performed by the Fed during the crisis and that an increase in aggregate SRISK provides an early warning signal of a decline in industrial production.

The organization of this paper is as follows. Section 2 introduces the parameter estimation for dynamic volatility matrix estimation. Section 3 reviews the Fourier transform method, one of the nonparametric approaches to estimate volatilities and correlations in time series. Section 4 discusses the construction of the efficient importance sampling estimators for extreme capital shortfall. Section 5 investigates backtesting results of systemic risk estimation of China, Taiwan, and U.S., then we conclude in Section 6.

Section 2: Systemic Risk Estimation under Dynamic Volatility Matrix Models

Acharya et al. (2012) introduce SRISK as the expected capital shortfall of a financial institution conditional on a prolonged market decline. The conditional capital shortfall(CS) is measured by the size of the firm, its degree of leverage, and its expected equity loss conditional on the market decline. This measure can be computed using the balance sheet information and an appropriate Long Run Marginal Expected Shortfall(LRMES) estimator. SRISK is used to construct rankings of systemically risky institutions: Firms with the highest SRISK are the largest contributors to the undercapitalization of the financial system in times of distress. The sum of SRISK across all firms is used as a measure of overall systemic risk in the entire financial system. It can be thought of as the total amount of capital that the government would have to provide to bail out the financial system in case of a crisis.

2.1 Conditional Capital Shortfall

To measure the shortage of firm's capital, capital shortfall is defined below as the firm's capital need subtracts current capital at time t:

$$CS_{i,t} = kA_{it} - w_{it} = k(D_{it} + w_{it}) - w_{it} \qquad (1)$$

where w_{it} is the market value of firm ith's capital, D_{it} is the book value of firm debt, A_{it} is the firm's total asset , k is the proportion of capital provision. According to Basel III[3], we choose k=8%. When capital shortfall is positive, this represents the capital amount needed for that firm to cover its risk exposure.

The systemic event is defined as a market decline below a threshold C over a time horizon h. In this paper, C=10% and h=1 month are chosen to signify the event that market index falls down for more than 10% in one month. Capital shortfall of firm i conditional on systemic event's occurrence during the period from time t to $t+h$ can be further defined by the conditional expected capital shortage :

$$CS_{i,t:t+T} = E_t\left(kA_{i,t+T} - W_{i,t+h} \middle| Crisis_{t:t+h}\right)$$

$$= E_t\left(k\left(D_{i,t+T} + W_{i,t+h}\right) - W_{i,t+T} \middle| Crisis_{t:t+h} \right)$$

$$= kE_t\left(D_{i,t+h} \middle| Crisis_{t:t+h}\right) - (1-k)E_t\left(W_{i,t+h} \middle| Crisis_{t:t+h}\right).$$

It is assumed that firm's debt would not change in this short period of time such that $E_t\left(D_{i,t+h} \middle| Crisis_{t:t+h}\right) = D_{i,t}$. With definition of leverage $L_{i,t} = A_{i,t}/W_{i,t}$, we obtain

$$CS_{i,t:t+T} = \left[kL_{i,t} + (1-k)LRMES_{i,t:t+h} - 1\right]W_{i,t}, \qquad (2)$$

where $LRMES_{i,t:t+h}$ is firm i's contribution to the whole system:

[3] According to Basel III: A global regulatory framework for more resilient banks and banking systems: The minimum Common Equity Tier 1 and Tier 1 requirements will be phased in between 1 January 2013 and 1 January 2015. On 1 January 2015, banks will have to meet the 4.5% Common Equity Tier 1 and the 6% Tier 1 requirements. The total capital requirement remains at the existing level of 8.0% and so does not need to be phased in. The difference between the total capital requirement of 8.0% and the Tier 1 requirement can be met with Tier 2 and higher forms of capital.

$$LRMES_{i,t:t+h} = -E_t\left\{\frac{W_{i,t+h}}{W_{i,t}} - 1 \middle| Crisis_{t:t+h}\right\} \quad (3)$$
$$\approx \frac{1}{N}\sum_{s=1}^{N}\left[\frac{-\sum_{\#} r_{i,t,t+h}\ I\left(r_{M,t:t+h}<C\right)}{\sum_{\#} I\left(r_{M,t:t+h} \le C\right)}\right]$$

where $I(x) = 1$ when x is true and 0 otherwise, $r_{M,t:t+h}$ and $r_{i,t,t+h}$ are returns of the market index and firm i during the period from t to $t+h$, respectively. $LRMES_{i,t:t+h}$ is often estimated by Monte Carlo method.

Lastly, firm i's systemic risk is defined as a positive capital shortfall:

$$SRISK_{i,t:t+T} = (0, CS_{i,t:t+T})^{+} \ .$$

The aggregated systemic risk $SRISK_{M,t:t+T}$=$\sum_i (0, CS_{i,t:t+T})^{+}$ can be used as a measure of the injection from the government. Individual financial firm's contribution to the system is defined as the ratio below:

$$SRISK\%_{i,t:t+T} = {SRISK_{i,t:t+T}}\Big/{SRISK_{M,t:t+T}} \quad (4)$$

2.2 Parameter estimation for volatility models

In order to estimate LRMES, an appropriate model is in need to compute the expected market return and each firm's return. There are a number of methods that can be used to estimate. GARCH-DCC model use by Acharya et al. (2012) is a discrete-time parametric method to estimate correlation matrix of multi-asset returns. Similar dynamic models can be found in continuous time. For example, one can use Heston model to simulate stochastic volatility and assume that stochastic correlation follows Jacobi process.

2.2.1 Discrete-time model: GARCH-DCC

Dynamic conditional correlation model is a kind of multivariate model proposed by Engle (2002). It is defined as:

$$
\begin{aligned}
& r_t = u_t + \xi_t, \\
& \xi_t = H_t^{1/2} z_t, \\
& H_t = D_t R_t D_t, \\
& D_t^2 = diag\{\omega_i\} + diag\{\kappa_i\} \circ r_{t-1} r_{t-1}^{'} + diag\{\lambda_i\} \circ D_{t-1}^{2}, \\
& \varepsilon_t = D_t^{-1} \xi_t \sim N(0, R_t), \\
& Q_t = S \circ \left(\iota\iota^{'} - A - B\right) + A \circ \varepsilon_{t-1}\varepsilon_{t-1}^{'} + B \circ Q_{t-1}, \\
& R_t = diag\{Q_t\}^{-1} Q_t diag\{Q_t\}^{-1}
\end{aligned}
$$

Where ι is a vector of ones and $\circ$ is the Hadamard product of the matrices. ω_i,κ_i,λ_i are the GARCH parameters.

The other notations are as follow:

r_t: $\mathrm{n} \times 1$ vector of log returns of assets at time t.

ξ_t: $\mathrm{n} \times 1$ vector of mean-corrected returns of n assets at time t. $\mathrm{E}[a_t] = 0$. $\mathrm{Cov}[a_t] = H_t$

μ_t: $\mathrm{n} \times 1$ vector of the expected value of the conditional r_t, which can be modelled as a constant vector or a time series model.

H_t: $\mathrm{n} \times \mathrm{n}$ matrix of conditional variances of α_t at time t. $H_t^{1/2}$ may be obtained by a Cholesky factorization of H_t.

D_t: $\mathrm{n} \times \mathrm{n}$ diagonal matrix of conditional standard deviations of α_t at time t.

R_t: $\mathrm{n} \times \mathrm{n}$ conditional correlation matrix of α_t at time t.

z_t: $\mathrm{n} \times 1$ vector of i.i.d. errors such that $\mathrm{E}[z_t] = 0$, $\mathrm{E}[z_t z_t^T] = I$.

S: $\mathrm{n} \times \mathrm{n}$ unconditional covariance matrix of the standardized errors.

The GARCH-DCC model can be estimated in the following two step. In the first step, the conditional heteroscedasticity is estimated by GARCH model. The second step is to estimate the correlation parameters. The Maximum Likelihood Estimation(MLE) is used. With the parameters of GARCH-DCC, we can calculate the SRISK with the simulation returns.

2.2.2 Continous-time model: dynamic volatility matrix model

Let $\mathrm{S_t}$ denote an asset price at time t and $\sqrt{V_t}$ denote a volatility process. The Heston model is defined by (Heston,1993)

$$
\begin{cases}
dS_t = \mu S_t dt + \sqrt{V_t} S_t dW_t \\
dV_t = \kappa(\theta - v_t)dt + \xi\sqrt{V_t} dZ_t
\end{cases}
$$

where W_t and Z_t are the standard Brownian motions with a dynamic correlation ρ_t. Parameter κ denotes the mean reverting speed of volatility, θ denotes the long term level of volatility, and ξ denotes the volatility of V_t.

The correlation dynamics is assumed to follows the Jacobi process

$$d\rho_t = \alpha(m - \rho_t)dt + \beta\sqrt{1 - \rho_t{}^2}dX_t,$$

where m represents the long-term level of correlation, α the mean reverting rate, β the volatility of ρ_t, and X_t is an independent Brownian motion.

Our proposed dynamic volatility model consists of the Heston model and Jacobi process to simulate the dynamic process of stochastic volatility and stochastic correlation, respectively. The whole model is constructed as follows:

$$\begin{cases} dlnS_{m,t} = \left(\mu_m - \frac{V_{m,t}}{2}\right)dt + \sqrt{V_{m,t}}dW_{m,t} \\ dV_{m,t} = \kappa_m\left(\theta_m - V_{m,t}\right)dt + \xi_m\sqrt{V_{m,t}}dZ_{m,t} \\ dlnS_{i,t} = \left(\mu_i - \frac{V_{i,t}}{2}\right)dt + \sqrt{V_{i,t}}dW_{i,t} \\ dV_{i,t} = \kappa_i\left(\theta_i - V_{i,t}\right)dt + \xi_i\sqrt{V_{i,t}}dZ_{i,t} \\ d\langle W_m, Z_m\rangle_t = \rho_m dt \\ d\langle W_i \ , Z_i\rangle_t = \rho_i dt \\ d\langle W_m, W_i\rangle_t = \rho_{i,t} dt \\ d\rho_{i,t} = \alpha_i\left(m_i - \rho_{i,t}\right)dt + \beta_i\sqrt{1 - \rho_{i,t}^2}dX_{i,t}, \end{cases} \quad (5)$$

where $S_{m,t}$ and $S_{i,t}$ denote the market index and the firm i's stock price, which $V_{m,t}$ and $V_{i,t}$ denote their volatilities, $W_{m,t}$, $W_{i,t}$, $Z_{m,t}$, $Z_{i,t}$, and $X_{i,t}$ are standard Brownian motions.

Section 3: Volatility Matrix Estimation: Fourier Transform Method

This section concerns about the parameter estimation problem for stochastic volatility/correlation models under the historical probability measure. Han (2015) proposed an estimation method by means of (1) the Fourier transform method for observed stock prices and index prices, and (2) a maximum likelihood estimation (MLE) for estimated volatility/correlation time series.

3.1 Fourier Transform Method

Fourier transform method (Malliavin and Mancino (2009)) is a nonparametric method to estimate multivariate volatility process. Its main idea is to reconstruct volatility as time series in terms of sine and cosine basis under the following continuous semi-martingale assumption. Let u_{jt} be the log-price of an underlying asset price S_j at time t, so that $u_{jt} = \ln(S_{jt})$, and follow a diffusion process

$$du_{jt} = \mu_{jt} dt + \sigma_{jt} dW_{jt},$$

where μ_{jt} is the instantaneous growth rate, σ_{jt} is the instantaneous volatility, and W_{jt} is a one-dimensional standard Brownian motion. Note that the original time interval $[0,T]$ can always be rescaled to $[0,2p]$ so that the Fourier transform of $u_j(t)$ can be defined by $\forall\ k \in Z$,

$$\mathfrak{F}(u_j)(k) = \frac{1}{2\ \pi}\int_0^{2\pi} u_j(t) e^{-ikt}\, dt$$

$$= \frac{i}{k}\left[\frac{1}{2\pi}\left(u_j(2\pi) - u_j(0)\right) - \mathfrak{F}(du_j)(k)\right].$$

The last equality is derived from the integration by parts. The new notation $\mathfrak{F}(du_j)(k)$ is defined by

$$\mathfrak{F}(du_j)(k) \equiv \frac{1}{2\pi}\int_0^{2\pi} \exp(-ikt) du_{jt}.$$

Given two functions Φ and Ψ, the Bohr convolution product is defined as

$$(\Phi *_B \Psi)(k) = \lim_{N\to\infty} \frac{1}{2N+1}\sum_{s=-N}^{N} \Phi(s)\Psi(k-s).$$

Malliavin and Mancino (2009) proved that in frequency domain, the Fourier coefficient of the (i,j)-th instantaneous volatility matrix $\Sigma(t)$ is

$$\frac{1}{2\pi}\mathfrak{F}(\Sigma_{i,j})(k) = \left(\mathfrak{F}(du_i) *_B \mathfrak{F}(du_j)\right)(k), for\ \ all\ \ k \in Z(\text{In probability}) \qquad (6)$$

Therefore, the instantaneous volatility matrix $\Sigma(t)$ can be calculated by the inverse Fourier transform

$$\Sigma_{i,j}(t) = 2\pi\ \ \mathfrak{F}^{-1}\left(\left(\mathfrak{F}(du_i) *_B \mathfrak{F}(du_j)\right)(k)\right). \qquad (7)$$

Equation (7) reveals that given a set of spot price data $\{S_{it}, S_{jt}\}$ under the historical probability measure, its instantaneously volatility $\{\Sigma_{i,j}(t)\}$ can be estimated by the Fourier transform method. Taking the financial system described by equation (5) as an example : $\Sigma_{m,i}(t) = \sigma_m(t)\sigma_i(t)\rho_{i,t}$, where $\sigma_m(t) = \sqrt{V_{m,t}}$ and$\sigma_j(t) = \sqrt{V_{j,t}}$ Note that $\sigma_i^2(t) = \Sigma_{i,i}(t)$ denotes the instantaneous variance process of the i-th asset and

$2\pi\ \mathfrak{F}(\Sigma_{i,j})(0)=\int_0^{2\pi}\Sigma_{i,j}(t)\,dt$ is equal to the integrated cross-volatility (Mancino et al.(2017)).

3.2 Parameter estimation for stochastic volatility/correlation model

Once instantaneous volatility and correlation are estimated by Fourier transform method, we can further estimate parameters (κ, θ, ξ) of the volatility process and (α, m, β) of the correlation process via the MLE from the discretized stochastic differential equations (Han (2013)):

$$\hat{\kappa} = \frac{1}{\Delta_t}\left[1 - \frac{t\left(\sum_{i=1}^{t}\frac{V_{t+1}}{V_t}\right) - (\sum_{i=1}^{t}V_{t+1})(\sum_{i=1}^{t}\frac{1}{V_t})}{t^2 - (\sum_{i=1}^{t}V_t)(\sum_{i=1}^{t}\frac{1}{V_t})}\right]$$

$$\hat{\theta} = \frac{-1}{\hat{\kappa}\Delta_t}\left[\frac{\left(\sum_{i=1}^{t}\frac{V_{t+1}}{V_t}\right)(\sum_{i=1}^{t}V_t) - n(\sum_{i=1}^{t}V_{t+1})}{t^2 - (\sum_{i=1}^{t}V_t)(\sum_{i=1}^{t}\frac{1}{V_t})}\right]$$

$$\hat{\xi} = \sqrt{\frac{1}{n\Delta_t}\sum_{i=1}^{t}\frac{1}{V_t}\left[V_{t+1} - \left(\hat{\kappa}\hat{\theta}\Delta_t + (1-\hat{\kappa}\Delta_t)V_t\right)\right]^2}$$

Parameter estimation for Jacobi process can be derived in the same way:

$$\hat{\alpha} = \frac{1}{\Delta_t}\left[1 - \frac{\sum_{t=1}^{n-1}\frac{1}{1-\rho_t^2}\sum_{t=1}^{n-1}\frac{\rho_{t+1}\rho_t}{1-\rho_t^2} - \sum_{t=1}^{n-1}\frac{\rho_t}{1-\rho_t^2}\sum_{t=1}^{n-1}\frac{\rho_{t+1}}{1-\rho_t^2}}{\sum_{t=1}^{n-1}\frac{1}{1-\rho_t^2}\sum_{t=1}^{n-1}\frac{\rho_t^2}{1-\rho_t^2} - \left(\sum_{t=1}^{n-1}\frac{\rho_t}{1-\rho_t^2}\right)^2}\right]$$

$$\widehat{m} = \frac{1}{\hat{\alpha}\Delta_t}\left[\frac{\sum_{t=1}^{n-1}\frac{\rho_t}{1-\rho_t^2}\sum_{t=1}^{n-1}\frac{\rho_{t+1}\rho_t}{1-\rho_t^2} - \sum_{t=1}^{n-1}\frac{\rho_{t+1}}{1-\rho_t^2}\sum_{t=1}^{n-1}\frac{\rho_t^2}{1-\rho_t^2}}{\left(\sum_{t=1}^{n-1}\frac{\rho_t}{1-\rho_t^2}\right)^2 - \sum_{t=1}^{n-1}\frac{1}{1-\rho_t^2}\sum_{t=1}^{n-1}\frac{\rho_t^2}{1-\rho_t^2}}\right]$$

$$\hat{\beta} = \sqrt{\frac{1}{(n-1)\Delta_t}\sum_{t=1}^{n-1}\frac{[\rho_{t+1} - (\hat{\alpha}\ \widehat{m}\Delta_t + (1-\hat{\alpha}\Delta_t)\rho_t)]^2}{(1-\rho_t^2)}},$$

For a simulation study, we set model parameters to simulate the market index and the firm i's share price path, and then through the MLE method aforementioned to estimate those model parameters. Model parameters are set as follows:

[INSERT TABLE 1 ABOUT HERE]

Figure 1 shows the simulated market index, the simulated volatility of the individual stock and the volatility estimated by the Fourier method. Figure 2 shows that a simulated correlation in comparison with the correlation estimated by the Fourier transform method.

[INSERT FIGURE 1 ABOUT HERE]

[INSERT FIGURE 2 ABOUT HERE]

Section 4: Importance Sampling: Variance Reduction

When dealing with sparse observations in the tails, the basic Monte Carlo simulation suffers some undesirable properties such as large relative error and data clustering around the center, etc. Importance sampling is suitable for rare event simulation by improving the convergence of the basic Monte Carlo method. The fundamental idea behind importance sampling is relocating the original density function to incur more occurrences of rare events so that accurate estimates for small probabilities can be achieved. See Bucklew (2004) for discussions on importance sampling and extreme event probability estimation and Han et al. (2014) for applying importance sampling to the VaR/CVaR estimation problem under stochastic volatility model.

Based on the large deviation theory (Bucklew (2004)), an importance sampling estimator is asymptotically optimal or efficient when the decay rate of its variance is zero in an asymptotic sense. When a practical model appears to be complex, the asymptotic analysis doesn't seem a plausible approach. One can homogenize the system model (Han (2012)), then study the variance decay rate for the proposal importance sampling (Han et al. (2014)). If the asymptotic optimality of the homogenized system model is assured, the corresponding importance sampling is often effective to estimate capital shortfalls for complex systems.

4.1 Systemic Risk

We provide an application for estimating the systemic risk in the financial market. Define the LRMES from time t to $t+h$ by:

$$\begin{aligned}\text{LRMES}_{r,t:t+h} &= -E[r_{i,t:t+h}(t+h)|\text{Crisis}_{t:t+h}] \\ &:= -\frac{E[r_{i,t:t+h}(t+h)1(\inf_t r_{M,t:t+h} < C)]}{E[1(\inf_t r_{M,t:t+h} < C)]}\end{aligned} \tag{8}$$

where $r_{M,t:t+h}$ and $r_{i,t:t+h}$ are the return of market index and firm i during the period t to $t+h$.

In equation (8), Monte Carlo method can be used to simulate and the estimate LRMES. Since the market "Crisis" is considered as a rare event, importance sampling can be feasible to reduce sample variance. Following the same argument in Han et al. (2014), first homogenization then probability measure change, we can derive the same importance sampling algorithm for estimating LRMES by virtue of large deviation theory.

4.2 Numerical Examples

We estimate LRMES under the complex dynamic volatility matrix model described in section 2. The "barrier" style (hitting time) problem is consider and their numerical results are recorded on in Table 2.

[INSERT TABLE 2 ABOUT HERE]

From the numerical results, it is observable that the proposed importance sampling reduces the standard error dramatically so that estimation for LRMES and SRISK become more accurate. The variance reduction ratios for the constant cases and the stochastic volatility/correlation cases are 5~10 and 20~500 respectively.

Section 5: Data and SRISK measurement: US, Taiwan, and China

Our empirical analysis mainly focuses on financial institutions in the USA. Financial institutions which have market value larger than 5 billion dollars are included in our sample for the period from Jan. 4th, 2005 to Dec. 1st, 2016. Open data such as daily log price, market value of each firm, its quarterly book value of equity and debt are incorporated for calculating SRISK.

Sample firms are divided into four parts: Depositories (e.g., JP Morgen, Citi), Broker-dealers (e.g., Goldman Sachs, Morgen Stanly); Insurance (e.g., AIG); Others (e.g., Fannie Mae, Freddie Mac). Table 3 records the name and ticker of each selected firm. S&P 500 is chosen to represent the market index for measuring SRISK.

5.1 SRISK Measurement

We calculate daily SRISK for all sample companies from January 2005 to December 2016. SRISK in the form of moving window, using the calculation of previous half year's data, so our follow-up SRISK results are with no forward-looking error. To calculate LRMES, we need to first estimate each company's Heston model and the Jacobi process parameters with the relevance of the stock price. As a result of moving the form of the window, each company's parameter estimates will also change.

[INSERT FIGURE 3 ABOUT HERE]

5.2 SRISK's Evolution and Ranking

CASE 1: U.S.

Figure 4 illustrates the SRISK of each sector during the period from July 2005 to December 2016. Compared with S&P 500 index, one observes a strongly negative correlation between the total sum of SRISK and S&P 500 index.

[INSERT FIGURE 4 ABOUT HERE]

Table 4 specifies the proportion of the top ten financial systems with the highest systemic risk at the beginning of the third quarter of the sample period and the overall systemic risk. Among them, the 2014-2016 ranking has a 0% risk ratio, indicating that the market risk of the existence of the system is limited, and the overall systemic risk is low.

[INSERT TABLE 4 ABOUT HERE]

From July 2005 to July 2007, the overall financial system of the US is around $200 billion. In this figure we can observe that most of the systemic risk are from the Broker sector. Among this sector, the biggest contributors are Goldman Sachs, Morgan Stanly, Bear Stearns and Lehman Brothers. It is noteworthy that these companies have played an important role in the subsequent global financial crisis, and as early as 2005 their systemic risks emerge while S&P 500 index was still arising. At the same time, the second highest systemic risk comes from the 'Other' sector mainly contributed by Fannie Mae and Freddie Mac. Both contribute a sum of about 20% of the overall systemic risk.

Since July 2007, the impact of subprime mortgage crisis to the overall system becomes more and more apparent. Stock prices start to shake down and SRISK begins to accelerate. The composition of SRISK changes gradually since then. With the crisis's deepening and proliferation, the banking sector and the insurance sector have also becoming an important SRISK contributors. Some large commercial banks also appear on table 4(the TOP10 SRISK list), such as Bank of America, Citigroup, Wells Fargo and AIG.

In September 2008, the bankruptcy of Lehman Brothers raised the overall SRISK to achieve a peak. Stock prices began to plummet and at this point, the contribution of banks and insurance to the overall SRISK kept increasing. Top 5 SRISK contributors in the Q3 quarter of 2009 included the Bank of America, Citigroup, Wells Fargo, JP Morgan and AIG. Many of the companies with higher SRISK rankings were missing from the rankings. For example, Lehman Brothers went bankrupt in September 2008, Fannie Mae and Freddie Mac were government-managed, and Bear Stearns was acquired by JP Morgan in March 2008.

The financial system has been steadily stabilizing since the second half of 2009 by the aid of three consecutive quantitative easing (QE). SRISK has been reduced and the market has recovered. Although the European debt crisis during 2010-2011 is a bump along the way. Figure 5 shows the SRISK changing for some broker or depository companies.

[INSERT FIGURE 5 ABOUT HERE]

CASE 2: China and Taiwan

Except SRISK in US, we choose Taiwan and China as an alternative study case and compare their

SRISKs.[4]

We study a panel of financial institutions in the Taiwan stock market and China stock market from July, 2005 to March, 2017. Their daily log returns, their book value of debt and market value of equity are obtained from the Bloomberg database. As a proxy for the market's return, we use the Taiwan Stock Exchange Weighted Index and SSE Composite Index, respectively, in our analysis. Table 5 and Table 6 reports the full list of stock IDs, tickers and company names by industry groups. SRISK in these two regions are estimated by the same approach as before.

[INSERT TABLE 5 ABOUT HERE]

[INSERT TABLE 6 ABOUT HERE]

Figure 4 revels that the aggregate SRISK in China and Taiwan are quite different from US. We find that the structure of SRISK contribution from different sectors in both China and Taiwan is stable compared with high mobility among different sectors in U.S.. In Taiwan, financial holding companies dominate the contribution of SRISK and in China almost all the systemic risk is from banks.

Notice that SRISK in China is rising at a breathtaking pace in the past 10years and there is no clear evidence of significantly declining in recent years. This finding may correspond to a recent warning to restrain systemic risk from the Chinese government[5].

4 Although the Volatility Laboratory (V-Lab) of the NYU Stern School website provides China's and Taiwan's SRISK index, it only considers [1]limited financial institutions in China and Taiwan. V-Lab is a systemic risk measurement provider for U.S. and global financial firms. It is based at New York University Stern School of Business under the direction of NYU Stern Professor Robert Engle (see http://vlab.stern.nyu.edu/).

5 "Xi Stresses Reining in Systemic Risks as China's Leaders Gather" according to Bloomberg News： President Xi Jinping said China will continue "seeking progress while maintaining stability" this year and that better supervision is needed to control financial risk, according to the official Xinhua News Agency.

5.3 Compared with other Capital Shortfall Measures

We compared our systemic risk estimations, called SRISK, with other capital shortage measures, which include systemic expected shortfall (SES) and Engle-SRISK based on the GARCH-DCC model. SES measures the expected shortage of capital for individual companies in the event of a significant capital shortage in the system.

The outbreak of the financial crisis will have a long-term and deep impact on the financial system, and the risk of the financial system will have a very strong negative external effects. From the perspective of government regulation, by bailing out those companies with the most capital shortages can reduce effectively the systemic risk. In 2007-2009, the Federal Reserve implemented the Troubled Asset Relief Program (TARP) to capitalize on capital shortages. Among the sampled financial institution, 40 of them have received the plan. TARP's capital injection can be seen as a relatively accurate substitute for the company's capital shortage during the crisis, so we use the Tobit regression model to assess the predictive effect of SRISK on TARP capital injection.

$$\log T_{\mathrm{Capital}_i} * = \alpha_0 + \beta \log(1 + (\mathrm{SRISK}_i)_+) + \gamma b_i + \varepsilon_i$$

where ：$\log T_{\mathrm{Capital}_i} = \begin{cases} \log T_{\mathrm{Capital}_i} * , & \log T_{\mathrm{Capital}_i} * > 0 \\ 0 , & \text{others} \end{cases}$,T_{Capital_i} denotes the capital injection of firm i during crisis, b_i denotes some control variables, including sectors（dummy variable）, firm’s total asset, current volatility of stock, SES, Engle-SRISK based on GARCH-DCC model, and ε_i is i.i.d. shock.

Table 7 records the results of the Tobit model estimates. When the Tobit model contains only the sector classification, the log company's total assets and the company's stock price volatility, its interpretation of capital injection T_{Capital_i} is explained as 11.2%. The addition of SRISK makes the pseudo R-square up to 29.1%, and the SRISK coefficient is 0.63 being statistically significant. The pseudo R-square's contributed from other capital shortage indicators such as SES and Engle-SRISK are relatively small, respectively, 18.6% and 20.3%. Comparing all the indicators in the model, it can be seen that SRISK has a more significant statistical significance than other capital shortages. To sum up, SRISK has an effective support for capital injection and capital shortages.

[INSERT TABLE 7 ABOUT HERE]

5.4 Leverage and SRISK

Since SRISK can be a good proxy of systemic risk, we are interested in the effect of leverage weighted by firm's equity. We can observe a positive correlation between leverage and SRISK in both the U.S. and Taiwan markets. Their leverages are relatively high throughout the financial crisis. Systemic risk is usually descending with the deleveraging over the last period of crisis.

It is interesting that leverage of financial institutions in China is quite high before 2011. After financial crisis, Chinese government are consciously guiding financial institutions in deleveraging, which can be seen from Figure 6 that leverage declined from 16 to 13.5. From the government's recent report deleverage will remain a major driving force to control their systemic risk[6].

[INSERT FIGURE 6 ABOUT HERE]

Section 6: Conclusion

SRISK measures the expected capital shortfall of a financial institution conditional on a prolonged and severe market decline. We investigate SRISK estimation under dynamic volatility matrix models by proposing an improved procedure and comparing its estimation performance with major traditional methods. Fourier transform method to estimate the dynamic volatility matrix and enhanced importance sampling to estimate SRISK are crucial to provide accurate results.

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[6] http://www.pbc.gov.cn/english/130721/3401747/index.html

We use this methodology to analyze the systemic risk of top financial firms in US, Taiwan and China between 2005 and 2016. The systemic risk analysis is useful to measure the stability of financial system and it can provide early signs for the financial crisis. And we obseve that leverage can be an important driving force to systemic risk in cases of the U.S. , China and Taiwan..

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Table 1: Parameters setting

Heston parameters							Jacobi parameters	
κ_m	θ_m	ξ_m	μ_m	ρ_m	S_{m0}	v_{m0}	α_i	m_i
3	0.5	1	0.1	-0.5	2000	0.5	10	0.3
κ_i	θ_i	ξ_i	μ_i	ρ_i	S_{i0}	v_{mi}	β_i	ρ_0
5	0.7	2	0.08	-0.6	400	0.7	1	0.3

Table 2: Simulation with Importance Sampling

Constant volatility

Hitting time style crisis

c	Prob(CMC)	s.e.	LRMES	Prob(IS)	s.e.	LRMES	V.V.R
-0.2	1.137E-02	3.35E-04	1.161E-01	1.144E-03	9.129E-05	1.109E-01	1.35E+01
-0.3	2.60E-04	5.098E-05	1.388E-01	2.113E-04	2.576E-06	1.623E-01	3.92E+02
-0.4	0	0	0	1.063E-07	1.936E-08	2.126E-01	0.00E+00
-0.5	0	0	0	1.373E-09	3.295E-09	2.63E-01	0.00E+00

V.V.R. stands for variance reduction ratio $\mathrm{V.V.R.} = \left({}^{s.e.(CMC)}\!/_{s.e.(IS)}\right)^2$

Stochastic volatility

Hitting time style crisis

c	Prob(CMC)	s.e.	LRMES	Prob(IS)	s.e.	LRMES	V.V.R
-0.2	2.706E-01	1.405E-03	9.609E-02	2.679E-01	1.041E-03	9.516E-02	1.822E+00
-0.3	1.091E-01	9.858E-04	1.321E-01	1.076E-01	5.668E-04	1.355E-01	3.025E+00
-0.4	1.730E-03	1.314E-04	2.616E-01	1.829E-03	1.968E-05	2.487E-01	4.458E+01
-0.5	6.000E-05	2.449E-05	2.783E-01	3.794E-05	5.165E-07	3.311E-01	2.248E+03

V.V.R. stands for variance reduction ratio $\mathrm{V.V.R.} = \left({}^{s.e.(CMC)}\!/_{s.e.(IS)}\right)^2$

Table 3: Classification for sample in US

Depositories(25)		Insurance(29)		Broker-Dealers(8)		Others(22)	
BAC	Bank of America	AET	Aetna	ETFC	E-TradeFinancial	ACAS	AmericanCapital
BBT	BB&T	AFL	Aflac	GS	GoldmanSachs	AMP	AmeripriseFinancial
BK	Bank of New York Mellon	AIG	AmericanInternationalGroup	LEH	LehmanBrothers	AMTD	TDAmeritrade
C	Citigroup	AIZ	Assurant	MER	MerrillLynch	AXP	AmericanExpress
CBH	Commerce Bancorp	ALL	AllstateCorp	MS	MorganStanley	BEN	FranklinResources
CMA	Comerica inc	WRB	W.R.BerkleyCorp	NMX	NymexHoldings	BLK	Blackrock
HBAN	Huntington Bancshares	BRK	BerkshireHathaway	SCHW	SchwabCharles	BOT	CBOTHoldings
HCBK	HUDSON City Bancorp	CB	ChubbCorp	TROW	T.RowePrice	CBG	C.B.RichardEllisGroup
JPM	JP Morgen Chase	CFC	CountrywideFinancial			CBSS	CompassBancshares
KEY	Key corp	CI	CIGNACorp			CIT	CITGroup
MTB	M&TBankCorp	CINF	CincinnatiFinancialCorp			CME	CMEGroup
NCC	NationalCityCorp	CNA	CNAFinancialcorp			COF	CapitalOneFinancial
NTRS	NorthernTrust	CVH	CoventryHealthCare			FITB	FifthThirdBancorp
NYB	NewYorkCommunityBancorp	FNF	FidelityNationalFinancial			FNM	FannieMae
PBCT	PeoplesUnitedFinancial	GNW	GenworthFinancial			FRE	FreddieMac
PNC	PNCFinancialServices	HIG	HartfordFinancialGroup			ICE	IntercontinentalExchange
RF	RegionsFinancial	HNT	HealthNet			JNS	JanusCapital
SNV	SynovusFinancial	HUM	Humana			MA	Mastercard
STI	SuntrustBanks	LNC	LincolnNational			LM	LeggMason
STT	StateStreet	MBI	MBIA			NYX	NYSEEuronext
USB	USBancorp	MET	Metlife			SEIC	SEIInvestmentsCompany
WFC	WellsFargo&Co	MMC	Marsh&McLennan			SLM	SLMCorp
WM	WashingtonMutual	PFG	PrincipalFinancialGroup				
WU	WesternUnion	PGR	Progressive				
ZION	Zion	PRU	PrudentialFinancial				
		TMK	Torchmark				
		TRV	Travelers				
		UNH	UnitedhealthGroup				
		UNM	UnumGroup				

Table 4: SRISK% Ranking

2005Q3	SRISK%	2006Q3	SRISK%	2007Q3	SRISK%
FNM	13.83%	FNM	14.13%	LEH US Equity	14.70%
FRM	12.06%	FRM	11.08%	FNM	12.35%
MS Equity	10.04%	LEH US Equity	10.73%	FRM	13.11%
BSC Equity	8.16%	GS US Equity	7.98%	WFC US Equity	12.37%
LEH US Equity	5.45%	MS	6.40%	GS US Equity	3.69%
PRU Equity	3.59%	WFC US Equity	5.17%	BSC Equity	3.57%
MMC US Equity	3.26%	CVH US Equity	3.15%	CVH US Equity	2.08%
WRB US Equity	2.42%	MMC US Equity	2.09%	MMC US Equity	1.89%
GS	2.26%	WRB US Equity	1.59%	WRB US Equity	1.18%
NCC US Equity	2.13%	SEIC US Equity	1.41%	SEIC US Equity	0.91%

2008Q3	SRISK%	2009Q3	SRISK%	2010Q3	SRISK%
C US Equity	13.53%	BAC	15.72%	PRU US Equity	26.60%
MER	11.91%	C US Equity	13.50%	C US Equity	18.34%
GS US Equity	8.45%	WFC US Equity	10.42%	AIG	15.33%
LEH US Equity	7.68%	JPM Equity	9.82%	MMC US Equity	4.01%
AIG	3.12%	AIG	2.29%	WRB US Equity	3.14%
MMC US Equity	2.38%	CVH US Equity	2.29%	STI US Equity	2.52%
CVH US Equity	2.14%	MMC US Equity	2.10%	NCC US Equity	2.07%
BSC Equity	2.03%	WRB US Equity	1.83%	WFC US Equity	1.76%
SEIC US Equity	1.98%	SEIC US Equity	1.53%	GS US Equity	1.41%
WRB US Equity	1.50%	HNT US Equity	1.17%	HNT US Equity	1.29%

2011Q3	SRISK%	2012Q3	SRISK%	2013Q3	SRISK%
MMC US Equity	16.56%	MMC US Equity	17.94%	MMC US Equity	20.84%
CVH US Equity	14.82%	CVH US Equity	13.14%	CVH US Equity	11.15%
SEIC US Equity	12.08%	SEIC US Equity	12.37%	STI US Equity	10.83%
C US Equity	6.58%	STI US Equity	8.89%	SEIC US Equity	9.87%
STI US Equity	5.59%	GS US Equity	8.23%	HNT US Equity	7.58%
NCC US Equity	5.48%	WRB US Equity	5.72%	GS US Equity	4.84%
GS US Equity	4.98%	CFC US Equity	4.66%	CFC US Equity	4.68%
CFC US Equity	3.48%	HNT US Equity	4.54%	WRB US Equity	4.18%
HNT US Equity	2.80%	FITB US Equity	2.76%	FITB US Equity	3.27%
CINF US Equity	0.79%	BBT US Equity	0.70%	BBT US Equity	0.37%

2014Q3	SRISK%	2015Q3	SRISK%	2016Q3	SRISK%
MMC US Equity	31.18%	MMC US Equity	27.48%	MMC US Equity	28.81%
CFC US Equity	19.39%	HNT US Equity	12.57%	HNT US Equity	15.63%
CVH US Equity	15.58%	HCBK US Equity	9.21%	CFC US Equity	12.91%
HNT US Equity	11.42%	CFC US Equity	8.77%	CVH US Equity	11.30%
FITB US Equity	5.83%	FITB US Equity	7.79%	FITB US Equity	8.68%
WRB US Equity	3.50%	CVH US Equity	7.02%	HCBK US Equity	3.89%
SEIC US Equity	2.60%	WRB US Equity	2.92%	BBT US Equity	3.51%
HCBK US Equity	2.44%	BBT US Equity	1.94%	WRB US Equity	0.48%
BBT US Equity	1.06%	SEIC US Equity	0.00%	WFC US Equity	0.00%
STI US Equity	0.00%	STI US Equity	0.00%	PRU US Equity	0.00%

Table 5: Classification for sample in Taiwan

FHCs (14)	
2880 TT Equity	HNFHC Hua Nan Financial Holdings Co., Ltd.
2881 TT Equity	FbFHC Fubon Financial Holding Co., Ltd.
2882 TT Equity	CFHC Cathay Financial Holdings Co., Ltd.
2883 TT Equity	CDFHC China Development Financial Holding Co., Ltd.
2884 TT Equity	ESFHC E.SUN Financial Holding Co., Ltd.
2885 TT Equity	YFHC Yuanta Financial Holding Co., Ltd.
2886 TT Equity	MFHC Mega Financial Holding Company
2887 TT Equity	TFHC Taishin Financial Holding Co., Ltd.
2888 TT Equity	SKFHC Shin Kong Financial Holding Co., Ltd.
2889 TT Equity	WFHC Waterland Financial Holding Co., Ltd.
2890 TT Equity	SFHC SinoPac Financial Holding Co., Ltd.
2891 TT Equity	CTBCFHC CTBC Financial Holding Co., Ltd.
2892 TT Equity	FiFHC First Financial Holding Co., Ltd.
5880 TT Equity	TCFH Taiwan Cooperative Financial Holdings Co., Ltd.

Banks(8)	
2801 TT Equity	CHB Chang Hwa Bank
2809 TT Equity	KTB King's Town Bank Co., Ltd.
2812 TT Equity	TCB Taichung Commercial Bank Co., Ltd.
2834 TT Equity	TBB Taiwan Business Bank
2836 TT Equity	BOK Bank of Kaohsiung
2838 TT Equity	UBT Union Bank of Taiwan
2845 TT Equity	FEIB Far Eastern International Bank
2849 TT Equity	ECB EnTie Commercial Bank

Insurance(6)	
2816 TT Equity	UIC Union Insurance Co., Ltd.
2823 TT Equity	CLIC China Life Insurance Co., Ltd.
2832 TT Equity	TFMIC Taiwan Fire & Marine Insurance Co., Ltd.
2850 TT Equity	SKIC Shingkong Insurance Co., Ltd.
2851 TT Equity	CRC Central Reinsurance Co., Ltd.
2852 TT Equity	TFIC The First Insurance Co., Ltd.

Table 6: Classification for sample in China

Banks(15)	
600000 CH Equity	Shanghai Pudong Development Bank Co Ltd
600015 CH Equity	Hua Xia Bank Co.
600016 CH Equity	CHINA MINSHENG BANKING
600036 CH Equity	China Merchants Bank Co., Ltd
600919 CH Equity	Bank Of Jiangsu Co Ltd
600926 CH Equity	Bank of Hangzhou Co Ltd
601009 CH Equity	Bank Of Nanjing Co.,Ltd
601128 CH Equity	Bank Of Changshu Co.,Ltd
601166 CH Equity	Industrial Bank Co Ltd
601169 CH Equity	Bank Of Beijing Co.,Ltd
601229 CH Equity	Bank of Shanghai Co Ltd
601288 CH Equity	Agricultural Bank of China Ltd
601328 CH Equity	Bank of Communications Co Ltd
601398 CH Equity	Industrial & Commercial Bank of China Ltd
601818 CH Equity	China Everbright Bank Co Ltd
601939 CH Equity	China Construction Bank Corp
601988 CH Equity	Bank of China Ltd
601997 CH Equity	Bank of Guiyang Co LTD
601998 CH Equity	China CITIC Bank Corp Ltd
603323 CH Equity	Wujiang Bank
000001 CH Equity	Ping An Bank Co Ltd
002142 CH Equity	Bank of Ningbo Co Ltd
002807 CH Equity	Jiangyin Rural Commercial Bank Co Ltd
002839 CH Equity	Jiangsu Zhangjiagang Rural Commercial Bank Co Ltd
600816 CH Equity	Anxin Trust Co Ltd
000563 CH Equity	Shaanxi International Trust Co Ltd
Brokers(14)	
600030 CH Equity	CITIC Securities Co Ltd
600109 CH Equity	Sinolink Securities Co Ltd
600369 CH Equity	Southwest Securities Co Ltd
600837 CH Equity	Haitong Securities Co Ltd
600909 CH Equity	Huaan Securities Co Ltd
600958 CH Equity	Orient Securities Co Ltd/China
600999 CH Equity	China Merchants Securities Co Ltd
601099 CH Equity	Pacific Securities Co Ltd
601198 CH Equity	Dongxing Securities Co Ltd
601211 CH Equity	Guotai Junan Securities Co Ltd
601375 CH Equity	Central China Securities Co Ltd
601377 CH Equity	Industrial Securities Co Ltd
601555 CH Equity	Soochow Securities Co Ltd
601688 CH Equity	Huatai Securities Co., Ltd.
601788 CH Equity	Everbright Securities Co Ltd
601881 CH Equity	China Galaxy Securities Co Ltd
601901 CH Equity	Founder Securities Co Ltd
000166 CH Equity	Shenwan Hongyuan Group Co Ltd
000686 CH Equity	Northeast Securities Co Ltd
000728 CH Equity	Guoyuan Securities Co Ltd
000750 CH Equity	Sealand Securities Co Ltd
000776 CH Equity	GF Securities Co Ltd
000783 CH Equity	Changjiang Securities Co Ltd
002500 CH Equity	Shanxi Securities Co Ltd
002673 CH Equity	Western Securities Co Ltd
002736 CH Equity	Guosen Securities Co Ltd
002797 CH Equity	First Capital Securities Co Ltd
Insurances(5)	
600291 CH Equity	Xishui Strong Year Co Ltd Inner Mongolia
601318 CH Equity	Ping An Insurance (Grp) Co
601336 CH Equity	New China Life Insurance Co Ltd
601601 CH Equity	China Pacific Insurance Group Co Ltd
601628 CH Equity	China Life Insurance Co Ltd

Table 7： TARP Capital Injection

Const	8.10 ***	1.19	-2.92	-1.26	5.84
	1.542	4.792	5.342	3.144	5.3
Broker	-10.69 ***	-8.54 ***	-11.9 ***	-10.3 ***	-7.84 ***
	2.32	2.241	3.895	4.362	1.867
Insurance	-4.03	-1.57	-2.75	-1.97	-1.72
	2.82	2.458	2.192	2.399	2.369
Others	-12.71 ***	-9.03 ***	-11.5 ***	-10.5 ***	-8.92 ***
	3.12	2.691	4.125	4.472	2.005
Asset	1.60 ***	1.58 ***	1.83 ***	1.78 ***	1.14 **
	0.38	0.433	0.699	0.824	0.586
Volatility	-0.16	-0.31 ***	-0.21 **	-0.19 **	-0.34 ***
	0.12	0.109	0.103	0.087	0.112
SES		6.21 ***			2.49
		2.829			2.167
Engle_SRISK			0.51 ***		0.4 *
			0.192		0.228
SRISK				0.63 ***	0.46 **
				0.228	0.207
pseudo R2	**11.2%**	**18.6%**	**20.3%**	**29.1%**	**31.5%**

Figure 1: Simulation of market index & firm's volatility

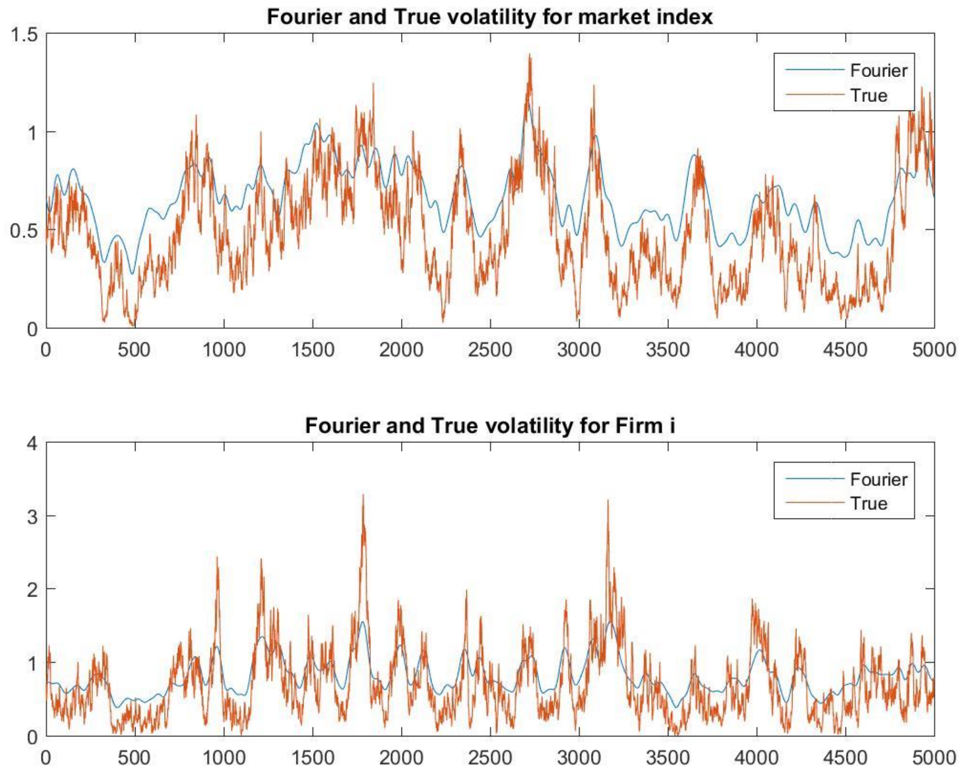


Figure 2: Simulation of market index & firm's correlation

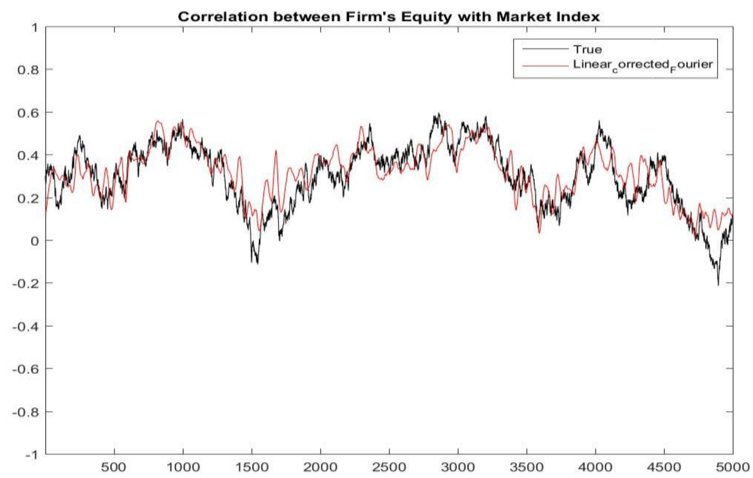


Figure 3: Time series changes of all company average Heston parameter

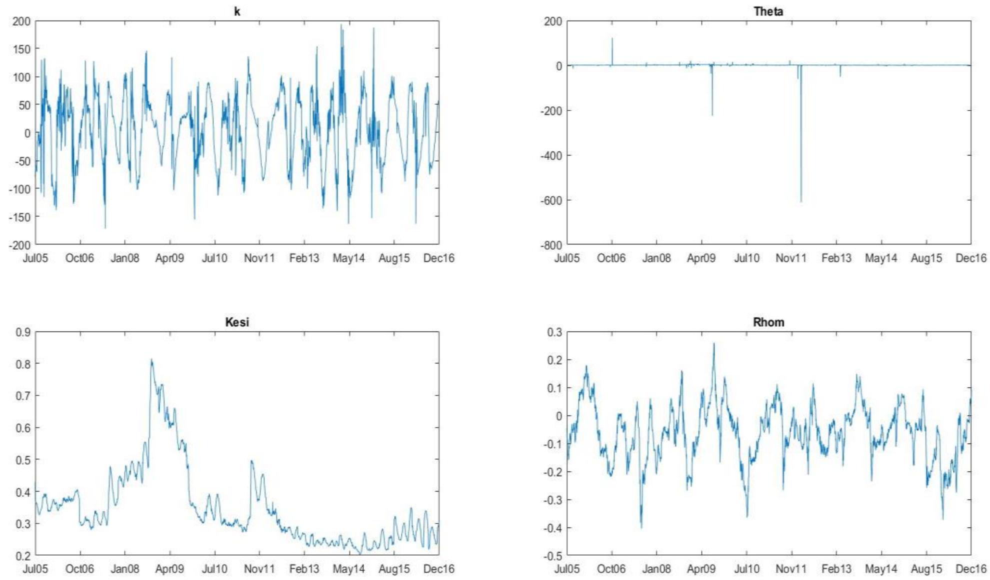
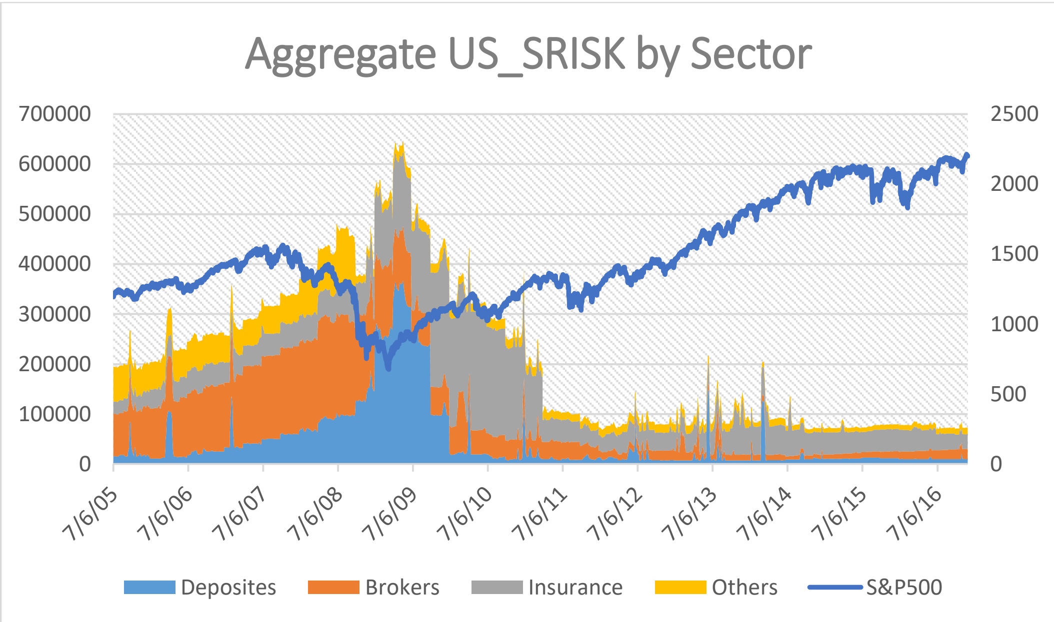
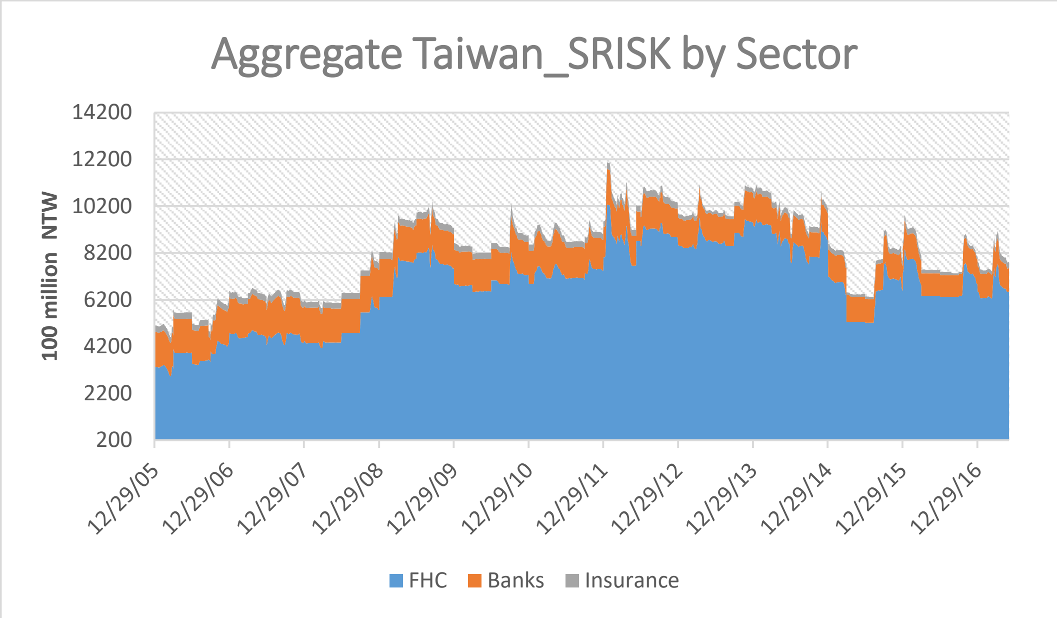


Figure 4: Aggregate SRISK with Importance Sampling by Sector in US





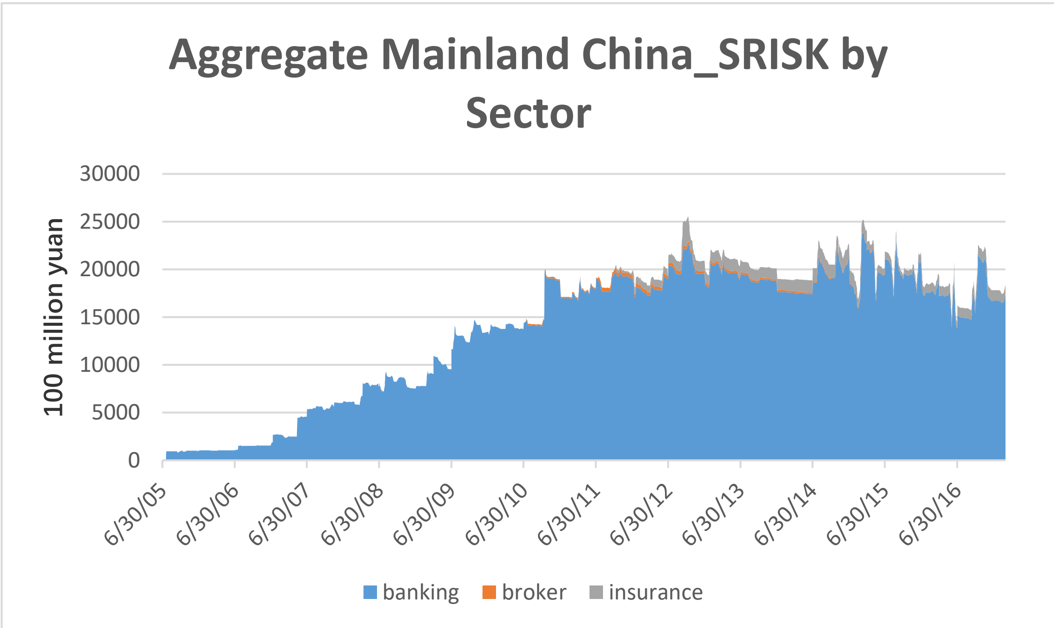
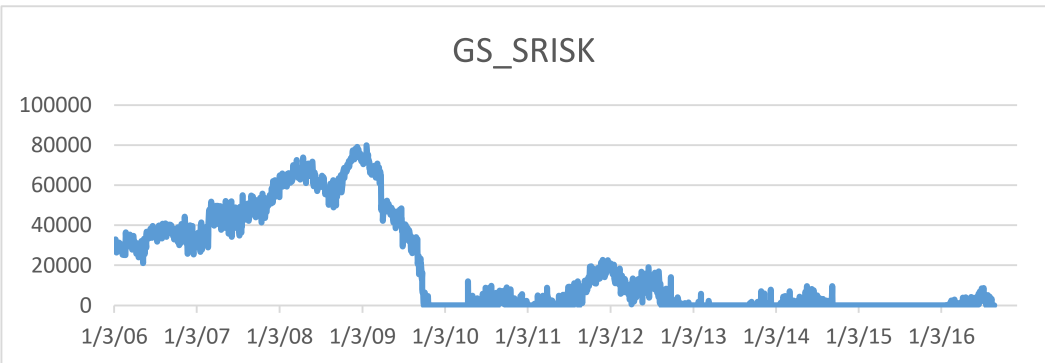


Figure 5: SRISK of Goldman Sachs, Citi and Lehman Brothers



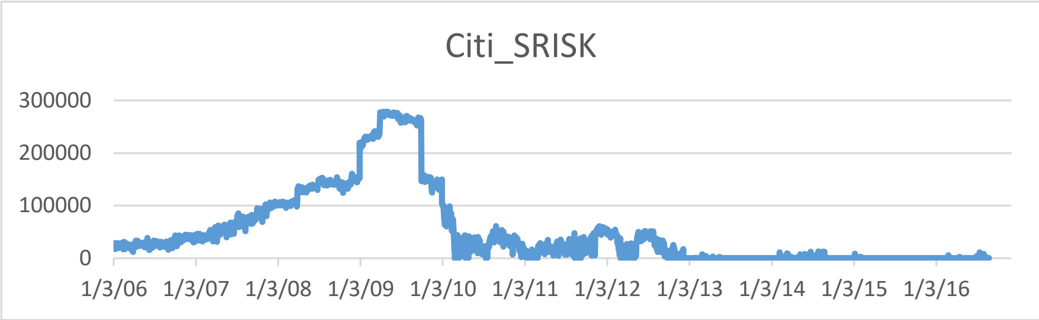
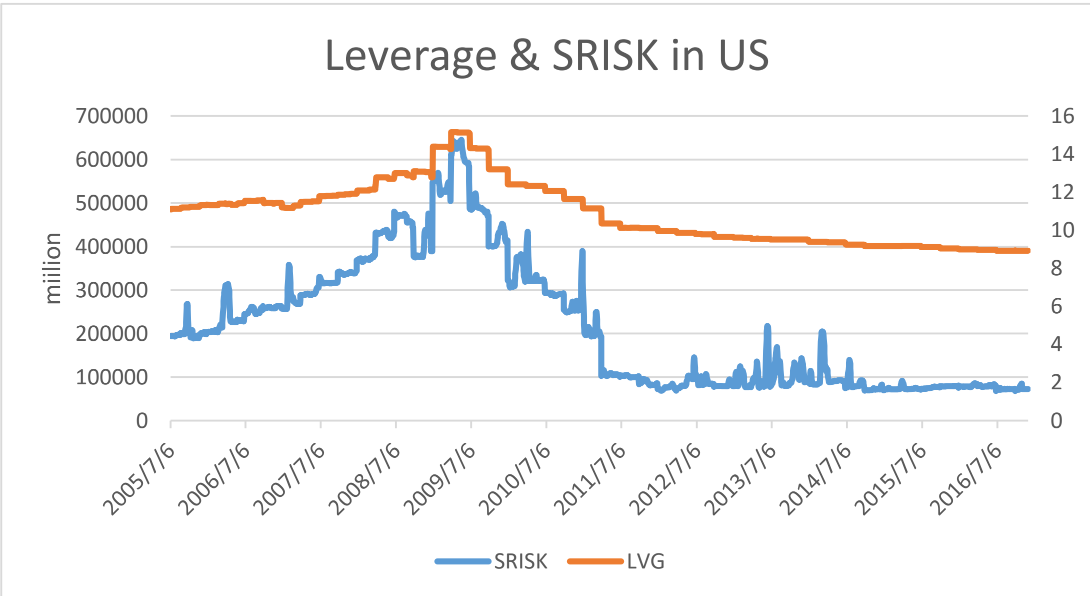
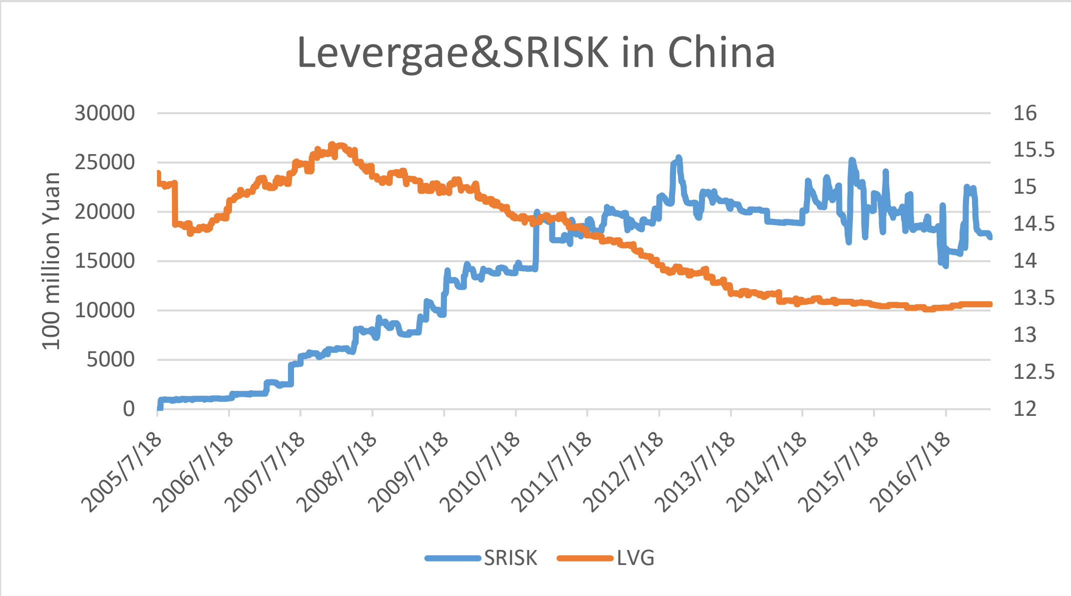


Figure 6: Leverage in SRISK




Levergae&SRISK in China
100 million Yuan
30000
25000
20000
15000
10000
5000
0
16
15.5
15
14.5
14
13.5
13
12.5
12
2005/7/18
2006/7/18
2007/7/18
2008/7/18
2009/7/18
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2012/7/18
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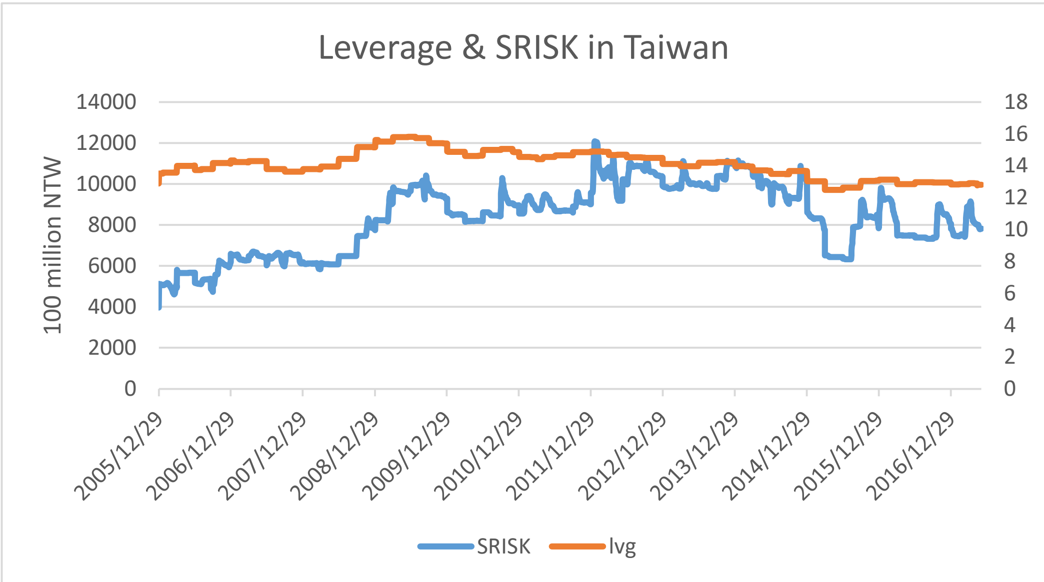
2014/7/18
2015/7/18
2016/7/18
SRISK
LVG


Leverage & SRISK in Taiwan
100 million NTW
14000
12000
10000
8000
6000
4000
2000
0
18
16
14
12
10
8
6
4
2
0
2005/12/29
2006/12/29
2007/12/29
2008/12/29
2009/12/29
2010/12/29
2011/12/29
2012/12/29
2013/12/29
2014/12/29
2015/12/29
2016/12/29
SRISK
lvg