Calibration of Multifactor Heston Models to Credit Spreads

Chuan-Hsiang Han[1]
Department of Quantitative Finance,
National Tsing Hua University

Lei Shih
Department of Quantitative Finance,
National Tsing Hua University

[1] Corresponding author: 101, Section 2, Kuang Fu Rd., Taiwan, 30013, ROC. +886-3-5742224 chhan@mx.nthu.edu.tw This work is supported by MOST104-2115-M-007-009.

Abstract

This paper develops a modified closed-form formula for option prices under the multifactor stochastic volatility model by means of the Fourier transform method. We apply this result to evaluate credit spreads in the context of the structural-form modeling. Through numerical simulation, we observe that some model parameters are sensitive to the deformation of credit yields. This capability to generate various shapes of the credit spread term structure enables a further study of model calibration to corporate bond yields. With different investment grades, empirical results reveal that the two-factor Heston model is indeed superior to other models including the Black-Scholes model and the one-factor Heston model.

Keywords: model calibration, stochastic volatility, Fourier transform method, term structure, credit spread

Section 1: Introduction

The structural-form approach of credit risk modeling begins with the study of Merton (1974) that applied the option pricing theory developed by Black and Scholes (1973). Under the constant parameter assumption, Merton (1974) derived a formula for implied spreads which behaved too low in comparison with actual market credit spreads. Hull and White (1987) introduced one-factor stochastic volatility models to evaluate debt values in Merton's (1994) framework. However, it is documented that one-factor stochastic volatility model is often not sufficient to capture the complex structure of time variation and cross-sectional variation.

Multi-factor stochastic volatility models are suitable to provide flexibility to express return data such as some stylized effects, or fit the implied volatility surface. See Christoffersen et al. (2009), Fouque et al. (2011), Han et al. (2014), Han and Kuo (2017), and references therein. Also, in contrast to the enormous family of GARCH family of Engle (2009) in discrete time, stochastic volatility models defined in continuous time are natural to be applied for derivatives pricing and hedging under the paradigms of Black, Scholes and Merton's theory for modern finance.

Two-factor stochastic volatility models become accessible both theoretically and empirically in recent years. Christoffersen et al. (2009) extended Heston's (1993) one-factor model to a two-factor stochastic volatility model built upon square root processes. Effects of two driving volatilities can be understood in the following. One stochastic volatility determines the interaction between asset returns and the "fast-varying" variance process, whereas the other stochastic volatility determines the interaction between asset returns and the "slow-varying" variance process. Fast and slow varying processes may correspond to impacts of instant news and economic cycles, respectively.

Although two-factor stochastic volatility models are pronounced, it is surprising that these models have not yet been fully explored in the credit risk literature. To fill this gap, this article introduces a two-factor stochastic volatility specification within the structural Merton's (1974) model. Our contribution resides on the derivation of a modified close-form formula for the debt value under the two-factor Heston model, examine numerically effects of initial variances and long-run means of square root processes, and model calibration to actual credit spreads on different investment grades.

The rest of the paper proceeds as follows. Section 2 presents the derivation procedure of the closed-form formula for the two-factor Heston option pricing model. Section 3 demonstrate flexibility of the credit spread term structure. Section 4 explores several model calibrations to actual market credit spreads on different investment grades. We conclude in Section 5.

Section 2: Credit Spread Evaluation under Multi-factor Stochastic Volatility Models

Our objective here is to derive a modified characteristic function for the two-factor Heston model, and its use for the credit spread evaluation under the structural-form approach in the context of credit risk modeling.

2.1 A stylized model with stochastic volatility

Let $\{S_t \in \mathbb{R}, t > 0\}$ be the asset value process of a firm. It is defined in the probability space $(\Omega, (\mathcal{F}_t)_{t>0}, Q)$ under a risk-neutral probability (martingale) measure Q. Heston's (1993) one factor model is one of the most popular stochastic volatility models in the option pricing literature. It is given by

$$\mathrm{d}S_t = rS_t dt + \sqrt{V_t} S_t dZ_t \tag{1}$$

$$\mathrm{d}V_t = (a - bV_t)dt + \sigma\sqrt{V_t} dW_t, \tag{2}$$

where V_t denotes the variance process, Z_t and W_t are Wiener processes with the correlation ρ. The continuously compounded risk-free rate r is assumed constant.

Christoffersen, Heston and Jacobs (2009) extended Heston's (1993) model to the following two-factor stochastic volatility model:

$$\mathrm{d}S_t = rS_t dt + \sqrt{V_{1t}} S_t dZ_{1t} + \sqrt{V_{2t}} S_t dZ_{2t} \tag{3}$$

$$\mathrm{d}V_{1t} = (a_1 - b_1 V_{1t})dt + \sigma_1\sqrt{V_{1t}} dW_{1t} \tag{4}$$

$$\mathrm{d}V_{2t} = (a_2 - b_2 V_{2t})dt + \sigma_2\sqrt{V_{2t}} dW_{2t}, \tag{5}$$

where Z_{it} and W_{it} $(for\ i = 1{,}2)$ are Wiener processes with cross variation

$$dZ_{it}dW_{jt} = \begin{cases} \rho_i dt & \text{for } i = j \\ 0 & \text{for } i \neq j, \end{cases}$$

and W_{1t} and W_{2t} are uncorrelated. The log-return process is denoted by $Y_t = lnS_t$ and its dynamics is governed by

$$\mathrm{d}Y_t = \left[r - \frac{1}{2}(V_{1t} + V_{2t})\right]dt + \sqrt{V_{1t}}dZ_{1t} + \sqrt{V_{2t}}dZ_{2t}, \tag{6}$$

obtained by Ito's formula. For analytical convenience, equations (4) and (5) used to describe driving variances V_{1t} and V_{2t} are rewritten by

$$\mathrm{d}V_{1t} = \alpha_1(\bar{V}_1 - V_{1t})dt + \sigma_1\sqrt{V_{1t}}dW_{1t} \tag{7}$$

$$\mathrm{d}V_{2t} = \alpha_2(\bar{V}_2 - V_{2t})dt + \sigma_2\sqrt{V_{2t}}dW_{2t} \tag{8}$$

where $\bar{V}_i$ $(for\ i = 1{,}2)$ represents the long-run mean, α_i denotes the rate of mean reversion and σ_i the volatility of the variance process. One might further consider three-factor stochastic volatility models. According to Molina et al. (2010) and Han and Kou (2017), their results disclosed that the third factor's contribution is fairly limited and supported the use of two-factor models.

2.2 Calculate call option price by Fourier transform method

In the classical framework, the option price can be represented as an expectation under a risk-neutral probability distribution. Consequently, its value at time t = 0 is

$$C(S_0, T, K) = e^{-rT}\int_0^{\infty}(S_T - K)^+ f(S_T)dS_T, \tag{9}$$

where $f(S_T)$ is the risk-neutral density of the underlying asset S_T.

Conventionally it is easier to work with the log price rather than the price density itself. Re-expressing equation (9) by $Y_t = lnS_t$, it is straightforward to obtain the European call value function as

$$C(S_0, T, K) = e^{-rT} \int_0^{\infty} (e^{Y_T} - K)^+ f(Y_T) dY_T. \tag{10}$$

Here we abuse the use of notation $f(X)$ by denoting the density function of the random variable X.

The characteristic function $\Psi_{Y_T}(.)$ of a given stochastic process Y_t at time $t = T$ is the Fourier transform of its probability density function $f(Y_T)$

$$\Psi_{Y_T}(\omega) = E\left[e^{i\omega Y_T}\right] = \int_{-\infty}^{\infty} e^{i\omega Y_T} f(Y_T) dY_T. \tag{11}$$

Therefore, by applying the Fourier Inversion formula, the density function of the process Y_T is recovered in terms of its characteristic function $\Psi_{Y_T}(\omega)$

$$\mathrm{f}(Y_T) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\omega Y_T} \Psi_{Y_T}(\omega)\, d\omega. \tag{12}$$

Moreover, based on Kendall, et al. (2009), the cumulative density function of Y_T is

$$\mathrm{F}(x) = \frac{1}{2} - \frac{1}{\pi} \int_0^{\infty} Re\left[\frac{e^{-i\omega x} \Psi_{Y_T}(\omega)}{i\omega}\right] d\omega. \tag{13}$$

The option price (10) can be further expressed by characteristic functions. According to the main result of Heston (1993), the value of a European call option (10) can also be expressed by

$$C(S_0, T, K) = S_0 \Phi_1 - e^{-rT} K \Phi_2 \tag{14}$$

where Φ_1 and Φ_2 are two probability-related quantities. We can derive Φ_1 and Φ_2

in terms of (13) by rephrasing

$$C(S_0,T,K) = e^{-rT}\int_{lnK}^{\infty}(e^{Y_T}-K)\,f(Y_T)dY_T$$

$$= e^{-rT}\big(\int_{lnK}^{\infty} e^{Y_T} f(Y_T)dY_T - K\int_{lnK}^{\infty} f(Y_T)\,dY_T\big)$$

$$= e^{-rT}\Theta_1 - e^{-rT}K\Theta_2 \qquad (15)$$

Comparing (14) with (15), it can be seen that Φ_2 should be equal to Θ_2, while Φ_1 should be equal to $e^{-rT}\Theta_1/S_0$. We first derive Φ_2 which is simply the probability of the event defined by the log-stock price at maturity over the log-strike, i.e., $\ln S_T >$ lnK:

$$\Phi_2 = \Theta_2 = P(\ln S_T > \mathrm{lnK})$$

$$= 1 - P(\ln S_T \le \mathrm{lnK}) = 1 - \mathrm{F(lnK)}$$

$$= \frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} Re\left[\frac{e^{-i\omega \mathrm{lnK}}\Psi_{\ln S_T}(\omega)}{i\omega}\right]d\omega \qquad (16)$$

This is the derivation for Φ_2 in (14)

To derive Φ_1, we first calculate the integral Θ_1:

$$\Theta_1 = \int_{lnK}^{\infty} e^{Y_T} f(Y_T)dY_T = \frac{\int_{lnK}^{\infty} e^{Y_T} f(Y_T)dY_T}{\int_{-\infty}^{\infty} e^{Y_T} f(Y_T)dY_T}\int_{-\infty}^{\infty} e^{Y_T} f(Y_T)dY_T$$

$$= E(S_T)\int_{lnK}^{\infty}\left(\frac{e^{Y_T} f(Y_T)}{\int_{-\infty}^{\infty} e^{Y_T} f(Y_T)dY_T}\right)dY_T$$

$$= e^{rT}S_0\int_{lnK}^{\infty} f^*(x)\,dY_T. \qquad (17)$$

The last equation makes use of the martingale property of $e^{-rt}S_{t.}$ The Fourier transforms of $f^*(x)$ is given by

$$\Psi^*(\omega) = \int_{-\infty}^{\infty} e^{i\omega Y_T} f^*(Y_T)dY_T = \frac{\Psi(\omega - i)}{\Psi(-i)} \qquad (18)$$

Thus, using equation (13)

$$\Theta_1 = e^{rT} S_0 \left(\frac{1}{2} + \frac{1}{\pi} \int_0^{\infty} Re \left[\frac{e^{-i\omega \ln \mathrm{K}} \Psi_{\ln S_T}(\omega - i)}{i\omega \Psi_{\ln S_T}(-i)} \right] d\omega \right)$$

Since $\Phi_1 = \ e^{-rT} \Theta_1 / S_0$, we can derive

$$\Phi_1 = \frac{1}{2} + \frac{1}{\pi} \int_0^{\infty} Re \left[\frac{e^{-i\omega \ln \mathrm{K}} \Psi_{\ln S_T}(\omega - i)}{i\omega \Psi_{\ln S_T}(-i)} \right] d\omega \ . \tag{19}$$

This is the derivation for Φ_1 in (14)

In brief, we have derived the pricing formula of a European call option by obtaining (16) and (19), then substitute these results to (14).

2.3 Calculate characteristic function of the log-price $\boldsymbol{\Psi_{\ln S_T}(\omega)}$

Crisostomo (2014) applied the characteristic function method based on the process $\ln S_t$ for the one-factor Heston model. We extend this result to the two-factor Heston model and obtain a slightly different result from Christoffersen et al. (2009) in which the characteristic function $\ln \frac{S_t}{K}$ was used. Moreover, we extend Crisostomo's (2014) computational scheme to the two-factor case as well and find that numerical results are more stable.

The characteristic function $\Psi_{\ln S_t}(\omega)$ is shown follows. It can be easily checked by a change of variable from the result of $\Psi_{ln\frac{S_t}{K}}(\omega)$ derived in Christoffersen et al. (2009).

$$\Psi_{\ln S_T}(\omega) = e^{C_1(\tau,\omega)\overline{V}_1 + C_2(\tau,\omega)\overline{V}_2 + D_1(\tau,\omega)V_1 + D_2(\tau,\omega)V_2 + i\omega ln(S_t)} \tag{20}$$

$$C_j(\tau,\omega) = \frac{\alpha_j}{{\sigma_j}^2}\left[\left(\alpha_j - \rho_j\sigma_j i\omega - d_j\right)\tau - 2ln\left(\frac{1-e^{-d_j\tau}}{1-g_j}\right)\right]$$

$$D_j(\tau,\omega) = \frac{\alpha_j - \rho_j\sigma_j i\omega - d_j}{{\sigma_j}^2}\left[\frac{1-e^{-d_j\tau}}{1-g_j e^{-d_j\tau}}\right]$$

$$g_j = \frac{\alpha_j - \rho_j\sigma_j i\omega - d_j}{\alpha_j - \rho_j\sigma_j i\omega + d_j}$$

$$d_j = \sqrt{\left(\alpha_j - \rho_j\sigma_j i\omega\right)^2 + {\sigma_j}^2(i\omega + \omega^2)}$$

Then we can directly apply this result to the pricing framework presented in section 2.2 and calculate Φ_1 and Φ_2 as complements of two cumulative distribution functions.

2.4 Pricing credit spreads

We assume that one firm issues a zero coupon bond with a promised payment B at maturity $t = T$. In this case, default occurs only at maturity with debt face value B as the default boundary. Within the Merton's (1974) framework, the debt value corresponding the firm at time t=0 can be expressed as:

$$D_0 = S_0 - e^{-rT}\mathrm{E}_Q[(S_T - B)^+] \tag{21}$$

That is, the debt value is equal to the difference between a firm's asset value at time t=0 and the European call option on the firm's asset values with the strike price being equal to the debt payment at maturity. To evaluate D_0, one should evaluate a typical European call option as follow:

$$C(S_0, T, B) = e^{-rT}\mathrm{E}_Q[(S_T - B)^+]. \tag{22}$$

Taking into account the previous expressions, it is possible to express the time t=0 debt value as:

$$
\begin{aligned}
D_0 &= S_0 - C(S_0, T, B) \\
&= S_0 \\
&\quad - \left[S_0 \left(\frac{1}{2} + \frac{1}{\pi} \int_0^{\infty} Re \left[\frac{e^{-i\omega \ln B} \Psi_{\ln S_T}(\omega - i)}{i\omega \Psi_{\ln S_T}(-i)} \right] d\omega \right) \right. \\
&\quad \left. - e^{-rT} B \left(\frac{1}{2} + \frac{1}{\pi} \int_0^{\infty} Re \left[\frac{e^{-i\omega \ln B} \Psi_{\ln S_T}(\omega)}{i\omega} \right] d\omega \right) \right],
\end{aligned}
$$

where the last equation is derived from equations (14, 16, 19). When a characteristic function is chosen as (20), the two-factor Heston model is incorporated so that the credit spread can be given by:

$$CS_0 = -\frac{1}{T} ln\left(\frac{D_0}{B}\right) - r.$$

Section 3: Numerical Illustration

This section offers the numerical analysis and demonstrates how the two-factor Heston model can generate various shapes of the term structure. Model parameter specifications are mainly chosen from Romo (2014). We also provide a sensitivity analysis with respect to model parameters relate to the initial variances and long-run means.

Specifications I and II in Table 3.1 are taken from Romo (2014). Specification III is extrapolated from the first two specifications to represent a more volatile return process. The rest of parameters regard firm's accounting information such as the initial firm's asset value S_0 and the debt face value B. These parameters are used to determine firm's investment grades. Their values and the risk-free interest rate r are chosen from Zhang et al. (2009).

Note that Romo (2014) only provided numerical experiments for multi-factor stochastic volatility model specifications. Zhang et al. (2009) investigated the jump effects with one-factor stochastic volatility models. In order to make our numerical experiment more realistic, we choose the two-factor Heston model specifications from Romo (2014) and each firm's accounting information from Zheng et al. (2009).

Table 3.1: parameters specification for the two-factor Heston model

Parameter	Specification I	Specification II	Specification III
α_1	1.2017	1.5141	1.9077
α_2	0.3605	0.4542	0.5723
$\bar{V}_1$	0.0524	0.0660	0.0831
$\bar{V}_2$	0.0157	0.0198	0.0250
σ_1	0.8968	1.1300	1.4238
σ_2	0.2690	0.3390	0.4272
ρ_1	-0.5590	-0.7043	-0.8874
ρ_2	-0.1677	-0.2113	-0.2662
V_1	0.0581	0.0732	0.0922
V_2	0.0174	0.0220	0.0278

The choices of $r = 0.05$ and $S_0 = 1$ are fixed. With regard to different investment grades, the debt face value is chosen as $B = 0.43$ corresponding to rating category A in the study of Zhang et al. (2009); for the parametric specification II, the debt face value is chosen as B = 0.48 corresponding to rating category BBB of Zhang et al. (2009); whereas for the parametric specification III, the debt face value is chosen as B = 0.58 corresponding to rating category BB of Zhang et al. (2009).

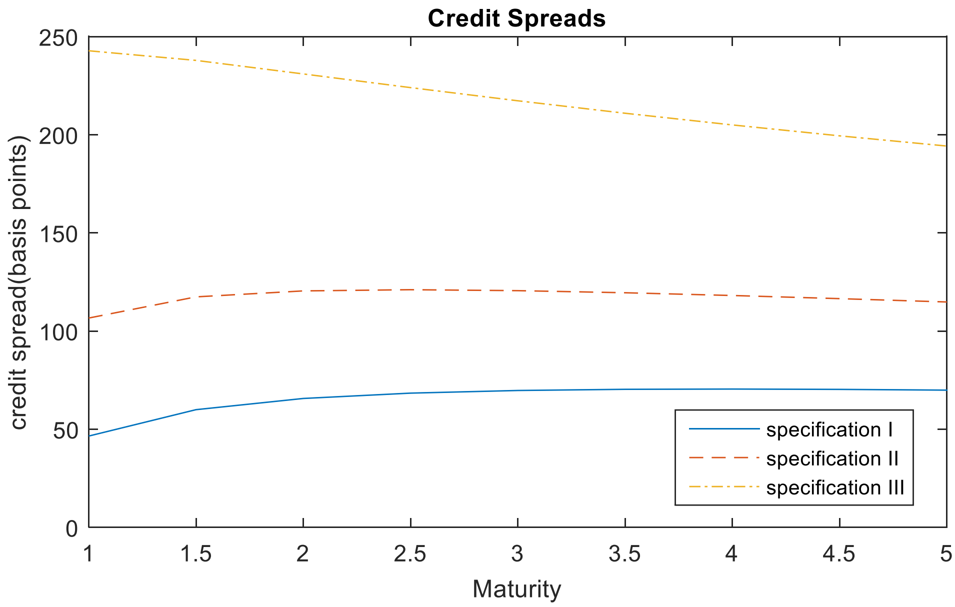

Figure 3.1: credit spreads term structure generated under the parametric specification of Table 3.1.

The credit spreads term structure generated by the two-factor Heston model for specification I, II, III have the similar shape to the average credit spread term structure displayed by Zhang et al. (2009) in their sample of firms associated with the A, BBB , BB rating categories.

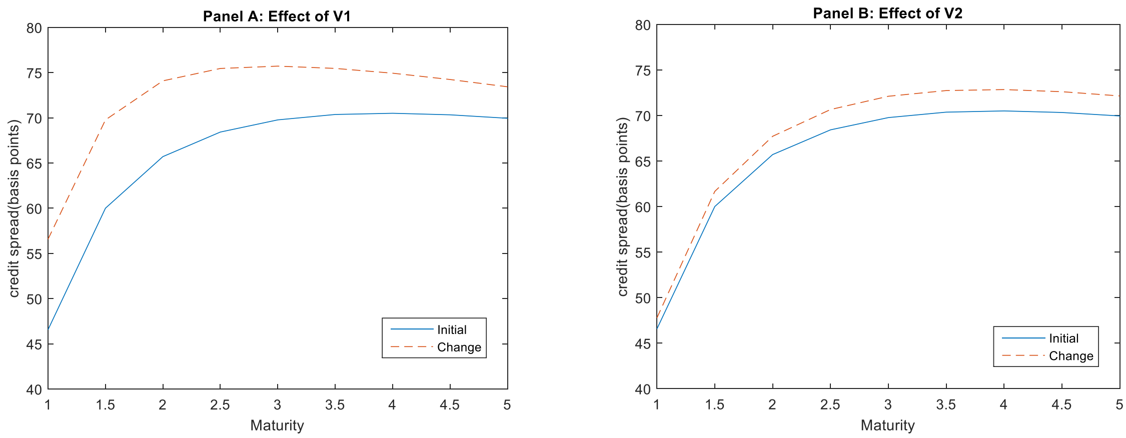

Figure 3.2: effect of variance factor V_1 and V_2 generated by specification I of Table 3.1

Figure 3.2 displays the sensitivities associated with the variance factor $V_1 = V_{10}$ and $V_2 = V_{20}$, representing the initial variances. That is, how the change of these initial variances reshapes the credit spread curves. One can observe that the effect of V_1 on the change of short-term credit spreads is more significant than the change of long-term credit spreads. On the other hand, the effect of V_2 on the change of long-term credit spreads is more significant than the change of short-term credit spreads. These numerical examples reveal how the introduction of two volatility factors can generate a wide range of combinations associated with short-term and long-term patterns corresponding to credit spreads. In this sense, multifactor stochastic volatility specifications are eligible to provide more flexibility than single-factor models so that they can capture a wide range of shapes associated with the term structure of credit spreads. Figure 3 displays that both $\overline{V}_1$ and $\overline{V}_2$ increase the credit spread, especially in the long-term, but the effect of $\bar{V}_1$ is more sensitive.

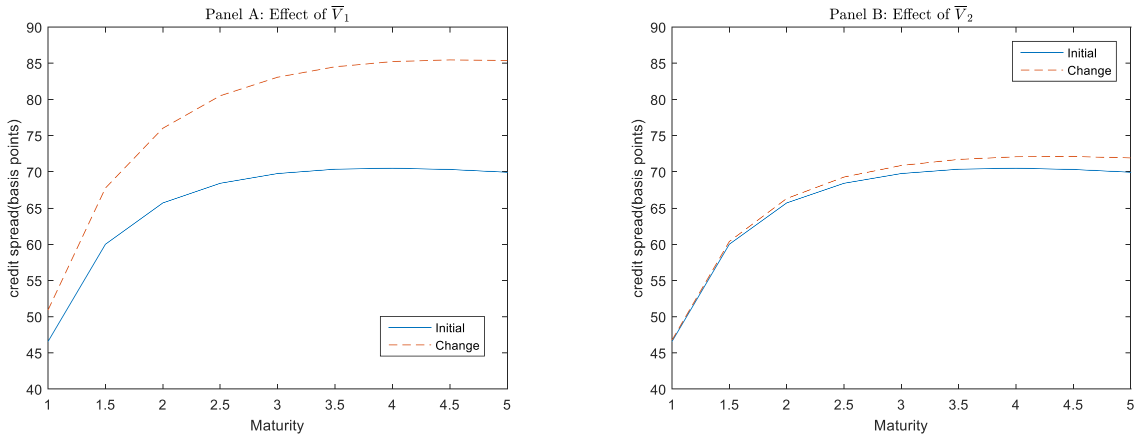

Figure 3.3: effect of long-run means $\bar{V}_1$ and $\bar{V}_2$ generated by specification I of Table 3.1

4. Calibration to market credit spreads

We have seen in the last section that the two-factor Heston model can produce various term structures for credit spreads. In addition, the existence of a closed-form formula is particularly useful for model calibration to actual observed market credit spreads. The problem of model calibration aims to obtain the optimal model parameters that are able to reproduce market credit spreads. Solving such an optimization problem with the high dimensional parametric space is challenging. Thus, an accurate and efficient pricing formula such as the Fourier transform are crucial in order to obtain reliable results within a reasonable computing timeframe.

4.1 Calibration Procedure

Throughout this paper, we choose to use the bond yield minus the risk-free interest rate as a direct measure of credit spreads,

$$CS_i = yield_i - r.$$

We calibrate a series of models — Black and Scholes, one factor (Heston) stochastic volatility, two factor (Heston) stochastic volatility — across rating categories of high investment grade (A+), low investment grade (AA-), and speculative grade (BBB).

The goal of calibration is to search for the optimal parameter set that minimizes the distance measure between model predictions and observed market prices. Under a risk-neutral measure, the two-factor Heston model is equipped of ten unknown parameters $\Omega = \{\alpha_1, \alpha_2, \bar{V}_1, \bar{V}_2, \sigma_1, \sigma_2, \rho_1, \rho_2, V_1, V_2\}$. The procedure of model calibration has two folds. Firstly, define a measure to quantify the distance between theoretical model values and observed market prices. Secondly, execute an optimization scheme to determine the parameter values that minimize such a distance measure. Our distance measure is defined by the mean sum of squared error ratios

$$\mathrm{I}(\Omega) = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{CS^{\Omega}{}_i(B,T) - CS^{\mathrm{mkt}}{}_i(B,T)}{CS^{\mathrm{mkt}}{}_i(B,T)}\right]^2,$$

where $CS^{\Omega}{}_i(B,T)$ denotes the *i*-th theoretical model credit spread using the parameter set Ω, $CS^{\mathrm{mkt}}{}_i(B,T)$ denote the *i*-th observed market credit spreads.

4.2 Calibration Data and Results

We take yield spreads from market prices of corporate bonds for three firms including IBM on 27 May 2016, when it was rated A+, and HP, a firm rated BBB, on 12 June 2016, and Chevron, an energy firm rated AA-, on 14 May 2016. The spreads are obtained from Morningstar (www.morningstar.com.)

A yield is a value to describe a zero coupon bond in the bond market. Since the market data consists of corporate coupon bonds, we need to employ the bootstrapping method to construct a zero-coupon fixed-income yield curve from those coupon-bonds. This method is based on the assumption that the theoretical price of a bond is equal to the sum of the cash flows discounted at the zero-coupon rate of each flow. Its implementation is given below.

Zero-coupon bond yields can be retrieved from the following formula (for the bond that pays dividends):

$$P = \frac{N}{(1 + ytm_n)^T} + \sum_{i=1}^{n-1} \frac{C_i}{(1 + ytm_i)^{t_i}},$$

where C_i is the zero coupon bond price, ytm_i is the zero coupon rate of t_i-year bond, N is face value. The risk-free rate data used in our calibration are the U.S treasury yields for three months, six months, one year, to ten years.

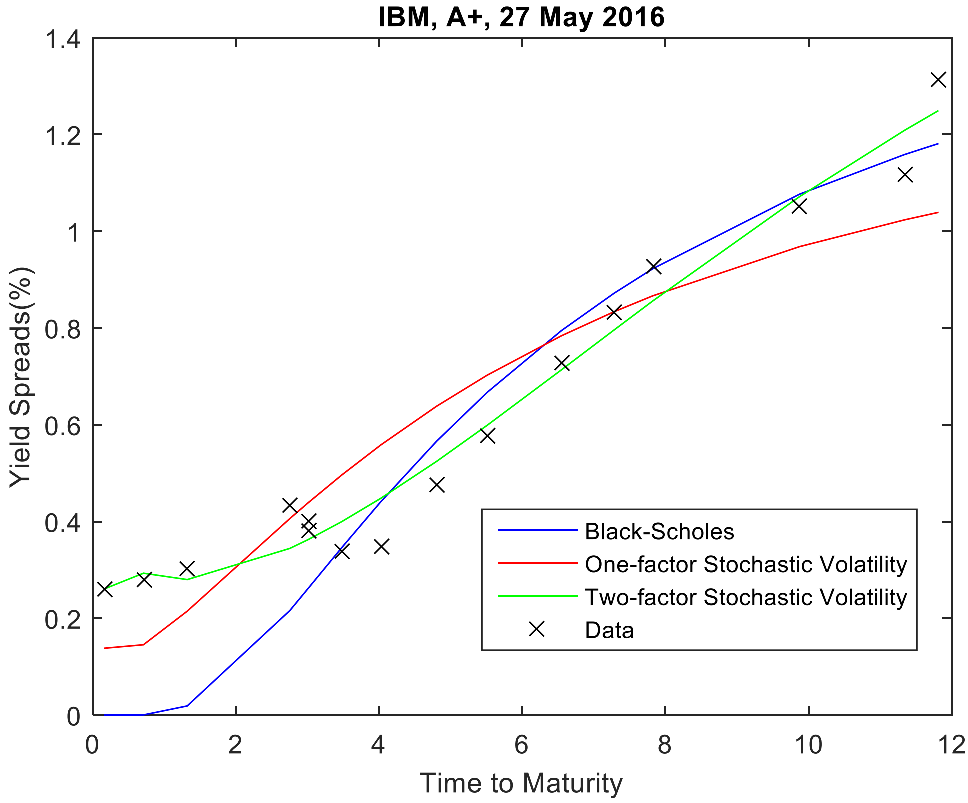

Figure 4.1: Comparisons of fitting IBM yield spreads by Black-Scholes, one-factor and two-factor stochastic volatility models. The short rate is fixed at $r = 0.0025$. Debt-to-asset ratio is B/S_0=0.36.

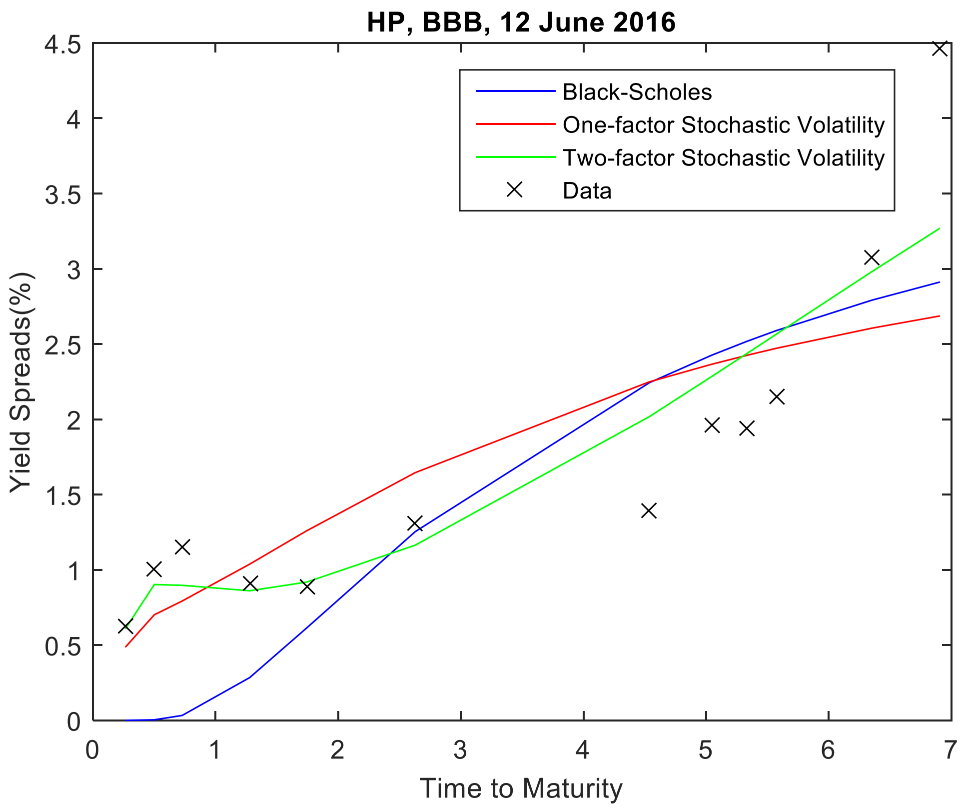

Figure 4.2: Comparisons of fitting HP yield spreads by Black-Scholes, one-factor and two-factor stochastic volatility models. The short rate is fixed at $r = 0.0025$. Debt-to-asset ratio is B/S_0=0.27.

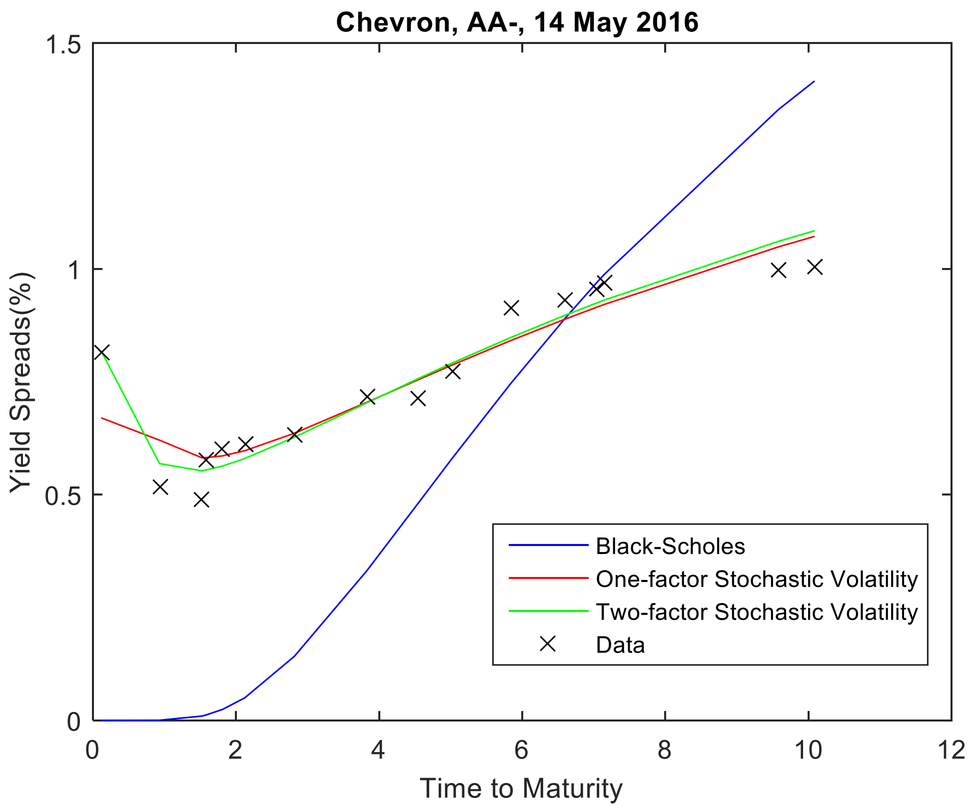

Figure 4.3: Comparisons of fitting Chevron yield spreads by Black-Scholes, one-factor and two-factor stochastic volatility models. The short rate is fixed at r = 0.0025. Debt-to-asset ratio is B/S_0=0.16.

Figure 4.1, 4.2, and 4.3 show the curve fitting of the actual credit yield spreads by three different models. The two-factor Heston model is visualized to produce various term structures that are closest to actual market data. Tables 4.1-4.3 records estimated model parameters. Table 4.4 describes the fitting error (MSE) of three models. For each firm, MSE of the two-factor stochastic volatility model is smallest.

However, we should point out related problems such as over fitting and parameter identification. The usual way to treat the over-fitting problem is to perform sensitivity analysis and/or robustness check for out-of-sample test. We have done some sensitivity analysis in section 3 and seen that credit spread curves can be sensitive to the change of some volatility model parameters such as long-run mean and initial variances. These are done through numerical experiments. As for empirical test, we encounter a problem about data. Since the corporate bond prices in the whole term structure are not daily observable in our database, it becomes difficult to perform an out-of-sample test. Zhang et al. (2009) only compared several models via calibration to CDS prices and didn't perform the out-of-sample test as well. As for the identification problem, Han and Kuo (2017) discovered such problem under a two-factor exponential OU (Ornstein-Uhlenbeck) stochastic volatility model. It happened that not every model parameter can be uniquely identifiable in such high-dimensional and complex model structure. Nevertheless, most literature recognize that multi-factor stochastic volatility models are essential to account for economic or business cycles of short-term and long-term change of the market risk.

Table 4.1: parameters specification of the Black-Scholes model for each firm's credit rating

Parameter	IBM(A+)	HP(BBB)	Chevron(AA-)
V	0.301295	0.500082	0.444667

Table 4.2: parameters specification of one-factor stochastic volatility model for each firm's credit rating

Parameter	IBM	HP	Chevron
V	2.742524	2.433317	5
$\bar{V}$	0.074364	0.202235	0.129696
σ	1.778405	1.700618	2.033256
ρ	0.36894	0.525417	-1
α	21.26858	7.150363	15.93781

Table 4.3: parameters specification of two-factor stochastic volatility model for each firm's credit rating

Parameter	IBM	HP	Chevron
V_1	0.122286	0.077631	3.93657
V_2	1.557314	1.559366	2.904554
$\bar{V}_1$	0.103331	0.340766	0.019903
$\bar{V}_2$	0.002996	0.009473	0.108046
σ_1	0.366838	0.637395	0.578341
σ_2	0.281581	0.331587	1.824795
ρ_1	0.998741	0.966932	-0.85514
ρ_2	-0.9905	-0.93536	-0.999

α_1	0.651262	0.616602	17.99047
α_2	13.32973	5.891756	17.99455

Table 4.4: mean square error specification of three models for each firm's credit rating

MSE	IBM(A+)	HP(BBB)	Chevron(AA-)
Black-scholes	0.0224	0.5886	0.1833
One-factor	0.0161	0.4261	0.0039
Two-factor	0.0029	0.2037	0.0019

Section 5: Conclusion

This paper provides a modified closed-form formula by means of the Fourier transform method for the multifactor stochastic volatility option pricing model. Through numerical experiments, the two-factor Heston model demonstrates flexibility to model the credit spread term structure. It is peculiarly observed that some model parameters are sensitive to the deformation of short-term credit spread, while others are sensitive to long-term credit spreads.

Two aforementioned advantages, including (1) accurate and fast calculation by the closed-form solution, and (2) flexible shapes of the credit spread term structure, enables the study of model calibration Within the diffusion family, three models—Black and Scholes (BS), one factor stochastic volatility (1SV), two factor stochastic volatility (2SV)— are introduced to distinct rating categories of high investment grade (A+), low investment grade (AA-), and speculative grade (BBB). Empirical studies confirm that two-factor Heston model best fit to the credit yields across these corporate bonds.

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