Control Experiments on a Shoe String

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When teaching control systems, it is important for students to implement controllers experimentally. Oftentimes off-the-shelf experimental systems are used to achieve this goal. These systems can have a number of advantages, such as repeatability, durability, and safety, and can be interesting and exciting for students. However, they have two distinct disadvantages:

- Cost: The high cost of an experimental system can prohibit the purchase of multiple experimental work stations. The use of only one or a small number of experimental work stations results in a) students having to work in large groups (reducing the time that each student interacts with the experiment), b) the course requiring multiple laboratory sections (which is costly in time for the instructor or teaching assistant), or c) the experiment being used only as a classroom demonstration.
- The experiments are a black box: These experiments typically have computer interfaces with enclosed control boxes and sensors, etc. Students enter the control parameters, press a button on a user interface, and watch the system run. Although perhaps an advantage for some learning objectives, well-trained control engineers also need to acquire a working knowledge of sensors, actuators, and control systems implementation, which can not necessarily be gained from black-box experiments.

This article describes an approach that allows a control systems instructor to enable students to obtain some hands-on experimental experience even when there are not enough resources for expensive experimental equipment. A low-cost, hands-on motor speed control experiment is presented where students build all parts of the experimental apparatus, including the motor. In addition to being inexpensive, the experiment is fun and achieves the goal of giving students a hands-on introduction to actuators, sensors, and control system implementation.

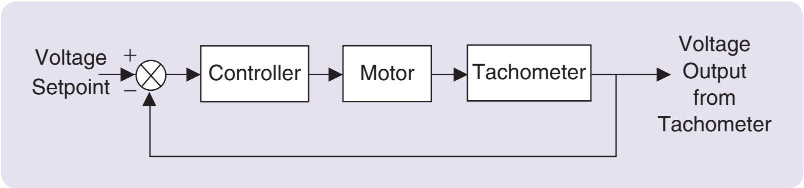

FIGURE 1 Block diagram showing the proposed motor control experiment.

EXPERIMENTAL APPARATUS

Motors are ubiquitous, being in everyday items such as children's toys, electric toothbrushes, and automobiles, as well as in manufacturing machinery [1], [2]. In many cases, the motor speed needs to be controlled [1], [3]. A typical motor speed control system is shown as a block diagram in Figure 1. In this article, each component of the closed-loop system is made from office/craft supplies and components available from any online electrical component store or your university's electrical engineering stores (see the parts lists in Tables 1 and 2). Standard electrical test equipment (dc power supply, oscilloscope, function generator, and multimeter) also must be available for troubleshooting and characterization. The remainder of this article provides an overview of the experimental setup and example results. Please visit the author's Web site [4] for more details concerning the experiment, such as electrical component value choice and assembly instructions, or to share your low-cost control systems/mechatronics experiments.

The Motor

There are many different commonly used motor configurations, including brushed or brushless dc permanent magnets or ac induction motors. For a breakdown of different motors, see [5, pp. 396], and for more detailed information see [6, Chapter 8]. In this article, the popular brushed permanent magnet dc motor (from here forward referred to as a dc motor) is used. DC motors consist of two main components: rotor and stator. The *rotor* (the spinning part of the motor) consists of coils and a commutator, and the *stator* (the stationary part of the motor) consists of brushes and magnets. When current in the coil flows through the magnetic field, a torque is generated on the rotor that causes the rotor to spin. An electrical connection is made between the spinning rotor and the stationary stator by passing current

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TABLE 1 Bill of materials for the homemade motor.

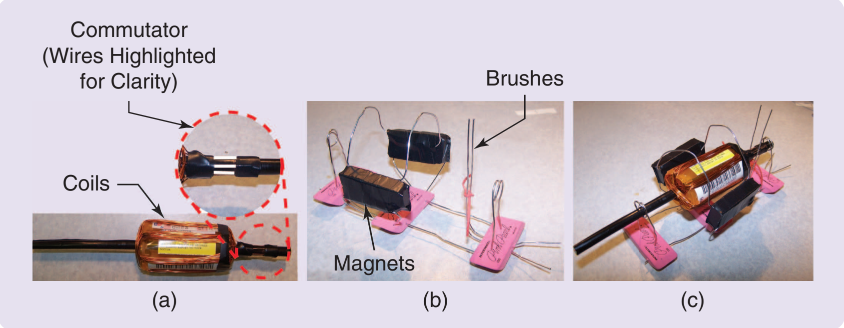
Item	Quantity	Notes
Drinking straw	1	Straight straws are preferred. Can replace with a pencil.
Erasers	3	
Cylinder: a pill bottle was used in this experiment	1	Can replace with other light cylindrical object approximately 1-in in diameter.
Insulated copper wire	1	Radio Shack: 315-ft magnet wire set model: 278-1345, cost US$6.99.* Can also use standard solid-core copper wire.
Magnets	2	Radio Shack: high-energy ceramic magnet model: 64-1877, cost US$1.99.*
Paper clips, metal, large		Do not use coated paper clips, bare metal only.
Thin rubber bands		
Tape		Electrical tape or similar.

*Prices are from www.digikey.com

TABLE 2 Bill of materials for the tachometer and controller.

Component	Part Number	Quantity	Price (US$)	Notes
Power transistor (NPN)	TIP33	1	1.73*	Other transistors that have similar power characteristics can be used.
Power transistor (PNP)	TIP34	1	1.49*	
Quad op amp	LM324	1	0.48*	Two 741 op amps can also be used
Frequency to voltage converter	LM2907	1	2.02*	The 14-pin configuration was used
IR emitter detector pair	Radio Shack part number: 276-142	1	3.69**	Can replace with any similar components
Resistor, capacitors, and potentiometers	Various	Various (see circuit diagram)		These are assumed to be on hand (or easily available).
		TOTAL	US$9.41	

*Prices are from www.digikey.com **Price is from www.radioshack.com



(a) (b) (c)

FIGURE 2 The homemade motor showing (a) the rotor with commutator (wires highlighted in white) and coils, (b) the stator with magnets and paperclip brushes, and (c) the fully assembled motor.

from the brushes (in the stator) to the commutator (in the rotor). The bushes/commutator configuration allows the current to be switched between different coils, which keeps the motor spinning. Readers are referred to [7] for further reading on dc motors.

The homemade motor is shown in Figure 2. Figure 2(a) shows the rotor consisting of coils wrapped around a pill bottle with a straw through its center. To construct the commutator, the ends of the coil are secured with tape at one end of the straw. Figure 2(b) shows the stator, which has paper clip bearings, magnet mounts, and brushes all supported on three erasers. A rubber band attached to the paper clip brushes keeps the brushes in contact with the commuter; see the fully assembled motor in Figure 2(c). In the author's experience, having students build these motors really solidifies their understanding of how dc motors work, especially the commutation, which seems to be a stumbling block for many students [8].

The Tachometer

To sense the speed of the motor, an encoder-based tachometer [5, pp. 346] will be built. A circuit diagram and breadboard implementation are shown on the left of Figure 3. An encoder wheel is cut from a piece of cardboard (see the final assembly in Figure 4) and fit to the motor shaft. The electrical side of the encoder is built from an infrared (IR) emitter/detector pair. When the motor spins, the encoder wheel interrupts the IR signal between the emitter/detector, creating a pulse train with frequency proportional to speed. The encoder in this experiment was constructed to yield $n = 8$ pulses per revolution. This pulse signal is converted to a voltage, v_{tach}, that is proportional to pulse frequency, f_{in}, using a frequency-to-voltage converter chip (LM2907), which is designed to require a minimal number of external components (two resistors and two capacitors). The relationship between voltage and frequency is given by $v_{\text{tach}} = VC_1R_3f_{\text{in}}$, where V, C_1,

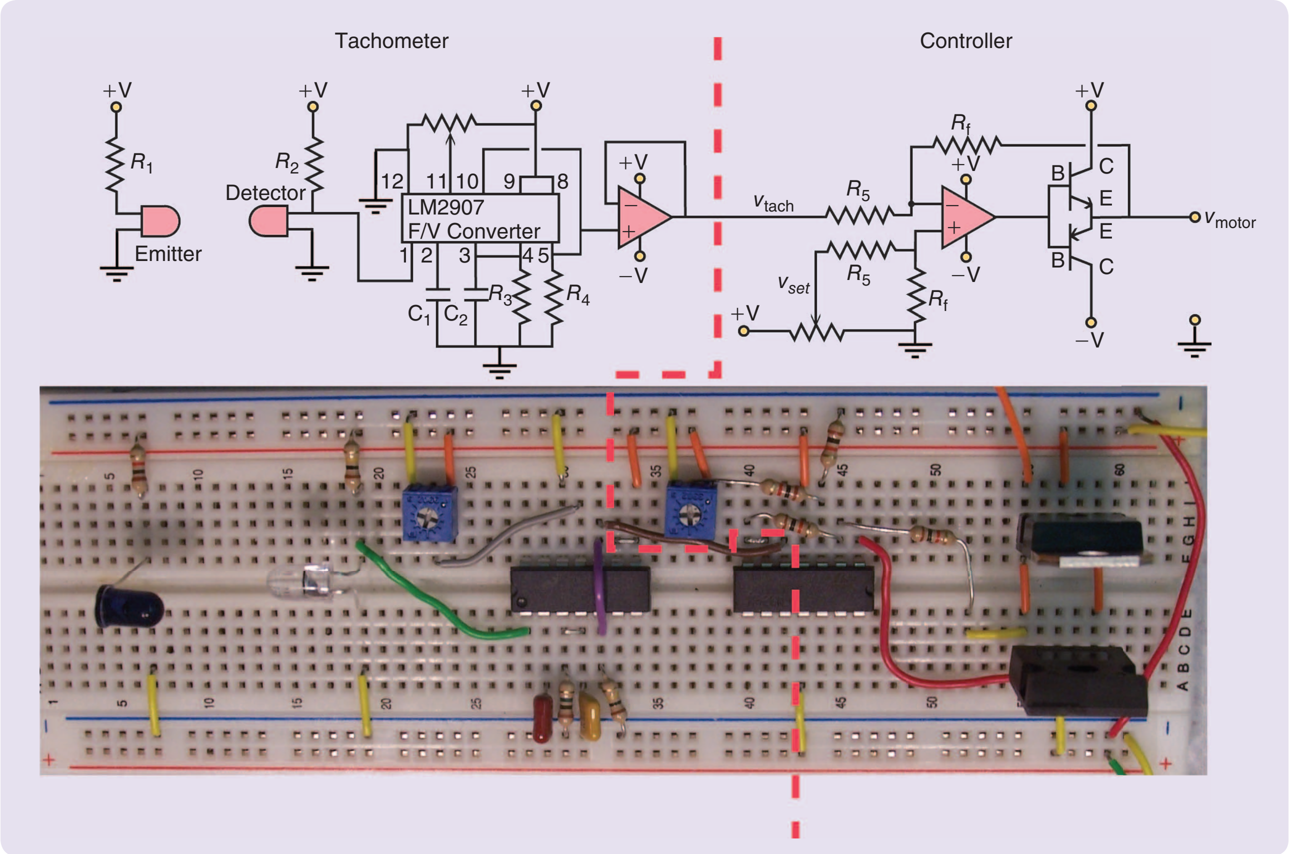


FIGURE 3 The tachometer (left) and controller circuitry (right), showing the circuit diagram (top) and the breadboard implementation (bottom).

and R_3 are shown in Figure 3. From this equation, the conversion factor from tachometer voltage to rotational speed of the motor (i.e., $\omega = k_{\text{tach}} v_{\text{tach}}$) is $K_{\text{tach}} = (1/n)VC_1R_3$. In the presented experiment, this value is $k_{\text{tach}} \approx 1$ revolution per V-s. Finally the tachometer voltage is passed through an operational amplifier buffer circuit (a voltage follower) to isolate the signal from downstream components.

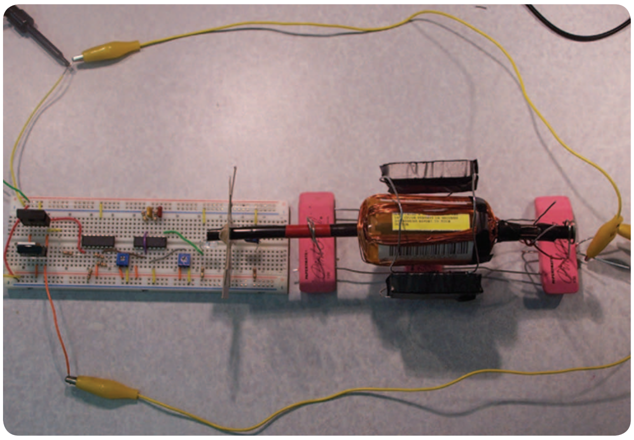


FIGURE 4 Final motor assembly. The circuit interacts with the motor through the encoder wheel (center) and the alligator clips (right and left).

The Controller

The controller circuit is a push-pull amplifier in a feedback loop as shown in Figure 3 (right). The operational amplifier compares the input coming from the tachometer (v_{tach}) with a desired setpoint voltage (v_{set} set using the potentiometer) and then amplifies the difference based on the resistances (R_5 and R_f). The push-pull stage (the two transistors) allows the circuit to drive the motor, which requires more current than the op amp can supply (safety note: the transistors may get hot when driving the motor). This circuit creates a proportional controller that is described by

$$v_{\text{motor}} = \frac{R_f}{R_5}(v_{\text{set}} - v_{\text{tach}}) = K_p e, \qquad (1)$$

where K_p is the proportional gain and e is the error. More advanced analog control such as proportional integral derivative controllers can be built using op amps [5, pp. 448], [9].

Final Assembly

The final motor/sensor/controller assembly is shown in Figure 4.

MODELING

Before a controller is implemented, it is useful to develop a model of the motor to design the controller and to explain the closed-loop results. A schematic of a typical electromechanical model of a motor is shown in Figure 5. This leads to two equations. One equation is for the electrical domain:

$$v_{\text{motor}} = Ri + L\frac{di}{dt} + v_{\text{emf}}, \tag{2}$$

where i is the current running through the coil, R is the terminal resistance of the motor, L is the inductance, and v_{emf} is the back EMF generated from the motor. Another equation is for the mechanical domain:

$$T = J\frac{d\omega}{dt} + c\omega, \tag{3}$$

where T is the torque produced by the motor, J is the rotor inertia, c is the damping (friction), and ω is the speed of the motor. The two equations are coupled using

$$T = k_t i \text{ and } v_{\text{emf}} = k_e \omega, \tag{4}$$

where k_t and k_e are motor constants that relate current to torque and speed to back EFM voltage, respectively.

Many assumptions can be made to simplify this model for control design. For example, the inductance (L) can often be neglected since it is very small compared to the resistance (this is true for the homemade motor because the inductor has an air core.) Also, the motor constants can be set equal to each other $(k = k_e = k_t)$, as is true in the ideal case and when using SI units [10]. With these assumptions, the open-loop transfer function model from the voltage input (v_{motor}) to the speed output (ω) is:

$$\frac{\Omega(s)}{V_{\text{motor}}(s)} = \frac{\frac{k}{RJ}}{s + \frac{c}{J} + \frac{k^2}{RJ}}, \tag{5}$$

where s is the Laplace variable. This simple first-order transfer function has a steady-state gain of $k/(Rc + k^2)$ and time constant of $RJ/(Rc + k^2)$.

CONTROL

Experimental results show that the motor speed is indeed well controlled using a proportional controller from (1), which gives the closed-loop transfer function of the motor as

$$\frac{\Omega(s)}{\Omega_{\text{ref}}(s)} = \frac{\frac{k}{RJ}K_p}{s + \frac{c}{J} + \frac{k^2}{RJ} + \frac{k}{RJ}K_p}, \tag{6}$$

which has a steady-state gain of $kK_p/(Rc + k^2 + kK_p)$ and a time constant of $RJ/(Rc + k^2 + kK_p)$.

The dc gain of the system is not unity and depends on the motor components. An increase in the controller gain K_p results in a dc gain becoming closer to the ideal value of one and a faster closed-loop time constant. Choosing a controller gain of $K_p = 1$ (equal to R_5 and R_f) yielded the experimental step response in Figure 6(a), which shows

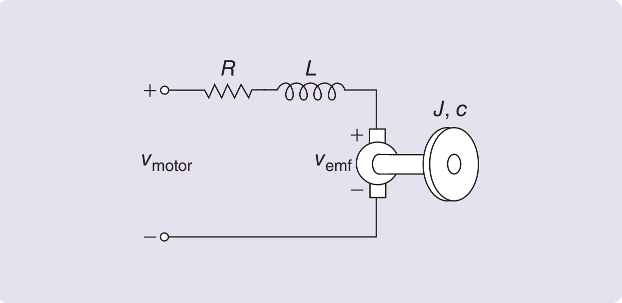
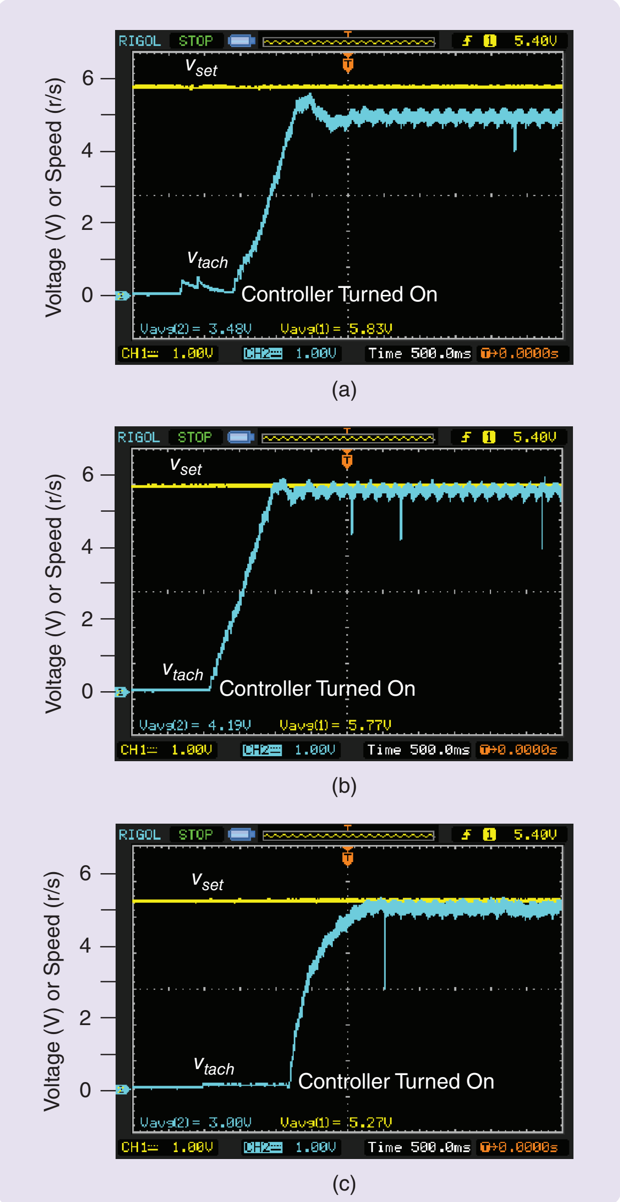


FIGURE 5 Schematic diagram of a motor.



(a)

(b)

(c)

FIGURE 6 An oscilloscope shows the speed set-point (yellow, adjusted using a potentiometer) and speed output (blue) for three different controllers: (a) proportional controller with gain of $K_p = 1$, (b) proportional controller with gain of $K_p = 10$, and (c) integral controller with gain of $K_i = 100{,}000$.

the setpoint voltage v_{set} (yellow) and the tachometer output v_{tach} (blue). The voltages shown in these step responses correspond directly to rotation in revolutions per second, as discussed earlier. The motor speed follows changes in the setpoint, with some transient response and steady-state error. From (6) it is clear that the dc gain approaches one as the proportional gain K_p is increased. Figure 6(b) shows the expected reduction in the steady-state error in the experimental step response for $K_p = 10$ (R_f is ten times greater than R_5).

The steady-state error seen in Figure 6(a) and (b) can be overcome by implementing an integral controller. This is achieved by replacing the feedback resistors (R_f) in Figure 3 with capacitors (C), resulting in

$$v_{\text{motor}} = \frac{1}{R_5 C s}(v_{\text{set}} - v_{\text{tach}}) = \frac{K_i}{s} e, \tag{7}$$

and a closed-loop transfer function:

$$\frac{\Omega(s)}{\Omega_{\text{ref}}(s)} = \frac{\frac{k}{RJ}K_i}{s^2 + \left(\frac{c}{J} + \frac{k^2}{RJ}\right)s + \frac{k}{RJ}K_i}, \tag{8}$$

where K_i is the integral control tuning parameter. The dc gain of the closed-loop system is one, so the closed-loop system is predicted to have zero steady-state error to a step in the setpoint. This integral controller was implemented to produce the experimental step response in Figure 6(c). These experiments show low steady-state error as expected. A small closed-loop error is seen, which is due to slight differences between the dynamics of the real system and its simplified model [see the text above (5)]. It should be noted that the implementation of pure integral control can result in *integrator windup* [11]; industrial implementations typically have a reset term that minimizes excessive control actions when the constraints are active for a significant amount of time.

DISCUSSION

Modifications

The proposed experiment can be modified to suit the desired learning outcomes for a particular course. For example, the homemade motor can be replaced with an off-the-shelf motor or the controller can be replaced by a data acquisition card (an amplifier would still be needed).

Potential Instructor Pitfalls

When implementing these types of low-cost experiments, the following are important:

- Try the experiment a number of times, preferably with student help, before implementing the presented experiment or any similar experiment in class. Many students become very upset when an experiment that does not work, which is directed as frustration and students can lose confidence in the instructor. In the author's experience, the potential benefits outweigh these potential costs because, when successful, students enjoy these types of exercises and learn a great deal from them.
- Have a clear idea of how long such experiments will take.
- Be ready to think on your feet. Students are quite good at making mistakes that the instructor did not expect.
- Have fun with these experiments and keep notes of potential changes that could be made to improve the experiments in the future.

CONCLUSIONS

In this article, an inexpensive hands-on motor speed control experiment was presented in which the experimental apparatus is entirely home built. In the author's experience, these types of experiments offer an alternative to more expensive off-the-shelf controls experiments that are more commonly used. For more information on this experiment, please visit the author's Web site [4]. The author also encourages the sharing of your low-cost controls/mechatronics experiments.

AUTHOR INFORMATION

Garrett M. Clayton received his B.S. from Seattle University in 2001 and his M.S. and Ph.D. from the University of Washington in 2003 and 2008, respectively, all in mechanical engineering. He is currently an assistant professor of mechanical engineering and a member of the Center for Nonlinear Dynamics and Controls (CENDAC) at Villanova University. His research interests lie in the fields of dynamics, controls, and mechatronics, with specific projects focused on modeling and control for high-speed nanopositioning systems and autonomous vehicles.

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