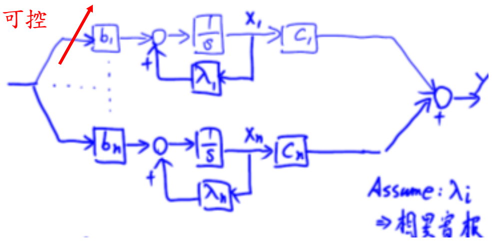
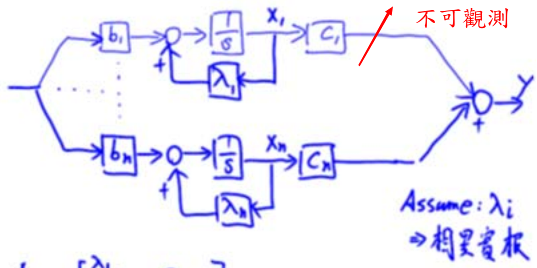




$$A_m = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}$$

$$\frac{Y}{u} = \frac{s+3}{s^2+7s+12} = \frac{s+3}{(s+4)(s+3)} = \frac{1}{s+4} + \frac{0}{s+3}$$

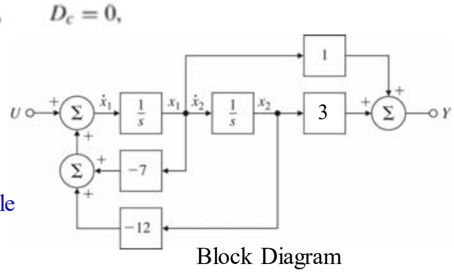
⇨ **Control Canonical Form**

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}_c\mathbf{x} + \mathbf{B}_c u, \\ y = \mathbf{C}_c\mathbf{x}, \end{cases} \Rightarrow \begin{cases} \mathbf{A}_c = \begin{bmatrix} -7 & -12 \\ 1 & 0 \end{bmatrix}, & \mathbf{B}_c = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ \mathbf{C}_c = [\ 1 \quad 2\ ], & D_c = 0, \end{cases}$$

⇨ **Controllability Matrix**

$$C = [B_c \quad A_cB_c] = \begin{bmatrix} 1 & -7 \\ 0 & 1 \end{bmatrix}$$

$\det(C) = 1$ ⇨ Controllable

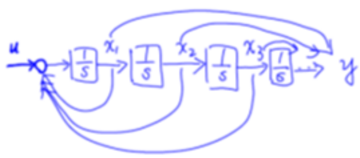


Block Diagram

⇨ **Observability Matrix**

$$O = \begin{bmatrix} C_c \\ C_cA_c \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ -4 & -12 \end{bmatrix}$$

$\det(O) = 0$ ⇨ Not Observable



$$\frac{Y}{u} = \frac{s+3}{s^2+7s+12} = \frac{s+3}{(s+4)(s+3)} = \frac{1}{s+4} + \frac{0}{s+3}$$

⇒ Observer Canonical Form

$$\begin{cases} \dot{x} = A_o x + B_o u \\ y = C_o x + D_o u \end{cases} \Rightarrow \begin{cases} \mathbf{A}_o = \begin{bmatrix} -7 & 1 \\ -12 & 0 \end{bmatrix}, & \mathbf{B}_o = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \\ \mathbf{C}_o = [\ 1 \quad 0\ ], & D_o = 0. \end{cases}$$

⇒ Controllability Matrix

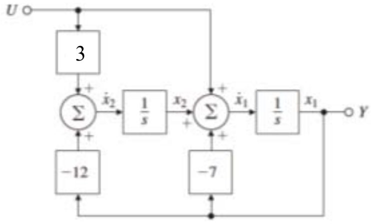
$$C = [B_o \quad A_o B_o] = \begin{bmatrix} 1 & -4 \\ 3 & -12 \end{bmatrix}$$

$\det(C) = 0 \Rightarrow$ Not Controllable

⇒ Observability Matrix

$$O = \begin{bmatrix} C_o \\ C_o A_o \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -7 & 1 \end{bmatrix}$$

$\det(O) = 1 \Rightarrow$ Observable



Block Diagram

State-Space Control Design — Prof. Cheng-Hsien Liu

$$\frac{Y}{u} = \frac{s+3}{s^2+7s+12} = \frac{s+3}{(s+4)(s+3)} = \frac{1}{s+4} + \frac{0}{s+3}$$

⇒ Modal Canonical Form

$$\begin{cases} \dot{\mathbf{z}} = \mathbf{A}_m \mathbf{z} + \mathbf{B}_m u, \\ y = \mathbf{C}_m \mathbf{z} + D_m u, \end{cases} \Rightarrow \begin{cases} \mathbf{A}_m = \begin{bmatrix} -4 & 0 \\ 0 & -3 \end{bmatrix}, & \mathbf{B}_m = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ \mathbf{C}_m = [\ 1 \quad 0\ ], & D_m = 0, \end{cases}$$

⇒ Controllability Matrix

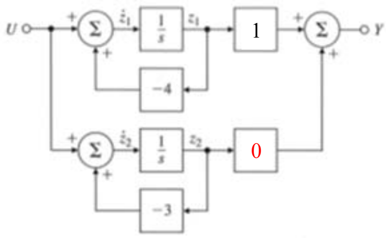
$$C = [B_m \quad A_m B_m] = \begin{bmatrix} 1 & -4 \\ 1 & -3 \end{bmatrix}$$

$\det(C) = 1 \Rightarrow$ Controllable

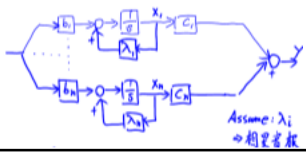
⇒ Observability Matrix

$$O = \begin{bmatrix} C_o \\ C_o A_o \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -4 & 0 \end{bmatrix}$$

$\det(O) = 0 \Rightarrow$ Not Observable



Block Diagram



$$\frac{Y}{u}=\frac{s+3}{s^2+7s+12}=\frac{s+3}{(s+4)(s+3)}=\frac{1}{s+4}+\frac{0}{s+3}$$

⇨ ~~Modal Canonical Form~~

$$\begin{cases}\dot{\mathbf{z}}=\mathbf{A}_m\mathbf{z}+\mathbf{B}_m u,\\ y=\mathbf{C}_m\mathbf{z}+D_m u,\end{cases} \Rightarrow \begin{cases}\mathbf{A}_m=\begin{bmatrix}-4 & 0\\ 0 & -3\end{bmatrix}, & \mathbf{B}_m=\begin{bmatrix}1\\ 0\end{bmatrix}\\ \mathbf{C}_m=[\,1\quad 1\,], & D_m=0,\end{cases}$$

⇨ **Controllability Matrix**

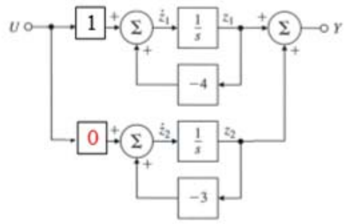
$$C=[B_m \quad A_m B_m]=\begin{bmatrix}1 & -4\\ 0 & 0\end{bmatrix}$$

$\det(C)=0$ ⇨ Not Controllable

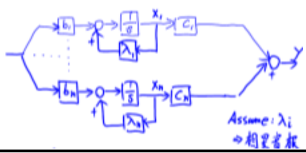
⇨ **Observability Matrix**

$$O=\begin{bmatrix}C_o\\ C_oA_o\end{bmatrix}=\begin{bmatrix}1 & 1\\ -4 & -3\end{bmatrix}$$

$\det(O)=1$ ⇨ Observable



Block Diagram



⇨ 若系統的Transfer Function 沒有分子分母對消，此系統一定可控(Controllable)且可觀(Observable)!

註：“可控/可觀”指的是對系統的每一個Eigenvalue可以藉由State-Feedback 移到任何希望的位置／每一個State Variable 都可以藉由Estimator/Observer估算出來

⇨ 若系統的Transfer Function有分子分母對消，同一個Transfer Function 的系統，可能會因為不同的State-Space Implementation，改變可控/不可控(Controllable)及可觀/不可觀(Observable)結果!

⇨ 若系統的State-Space Implementation是Control Canonical Form，此系統就是可控 (Controllable)!

若系統的State-Space Implementation是Observer Canonical Form，此系統就是可觀(Observable)!

⇨ 判斷系統可控/不可控(Controllable) ⇨ $\det(C)\neq 0$ 或 $=1$

判斷系統可觀/不可觀(Observable) ⇨ $\det(\mathrm{O})\neq 0$ 或 $=1$

⇨ 若系統的Transfer Function 沒有分子分母對消，此系統一定可控(Controllable)且可觀(Observable)！

註：“可控/可觀”指的是對系統的每一個Eigenvalue可以藉由State-Feedback 移到任何希望的位置／每一個State Variable 都可以藉由Estimator/Observer估算出來

Plant: $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$

$$u = -\mathbf{K}\mathbf{x} = -[\,K_1 \quad K_2 \quad \cdots \quad K_n\,]\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

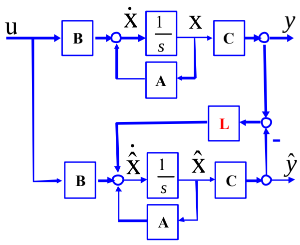
⇨ Closed-loop System

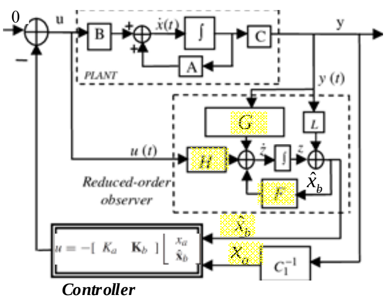
$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} - \mathbf{B}\mathbf{K}\mathbf{x}$$

⇨ The characteristic equation of this closed-loop system is

特徵方程式　$\det[s\mathbf{I} - (\mathbf{A} - \mathbf{B}\mathbf{K})] = 0.$

不可觀(Observable)　　*Transmission Zeros*





$$G(s) = \frac{Y(s)}{U(s)} = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + D$$

$$= \frac{\det\begin{bmatrix} (s\mathbf{I} - \mathbf{A}) & \mathbf{B} \\ -\mathbf{C} & \mathrm{D} \end{bmatrix}}{\det(s\mathbf{I} - \mathbf{A})} = 0$$

$$\frac{Y(s)}{U(s)} = G(s)$$