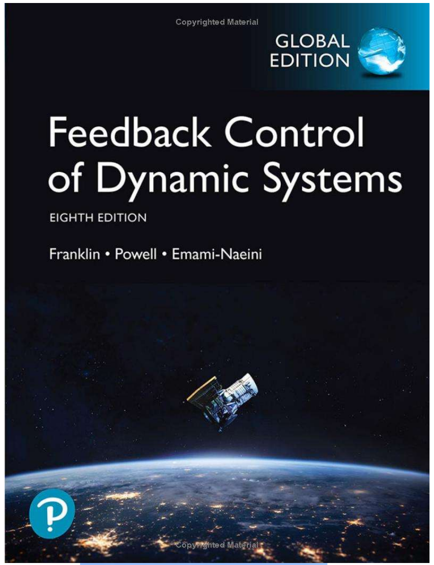
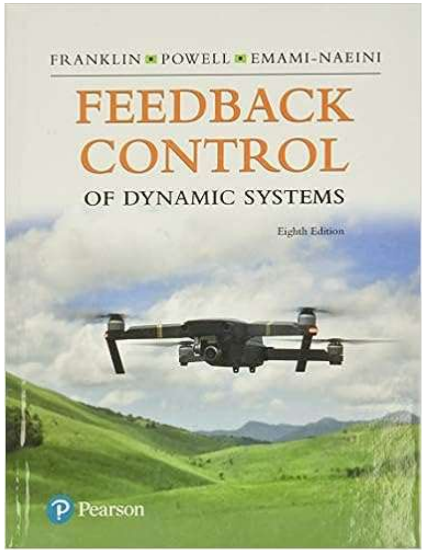



國立清華大學
National Tsing Hua University

# PME320701

# Feedback Control of Systems





## Lecture 2~4 (Chapter 2 & 3)

**Instructor :**

**Cheng-Hsien Liu**

劉 承 賢

助教：李千侑
王淳弘
蘇建翰

Chapter 1: An Overview and Brief History of Feedback Control

⇨Chapter 2: Dynamic Models

Chapter 3: Dynamic Response

Chapter 4: A First Analysis of Feedback

Chapter 5: The Root-locus Design Method

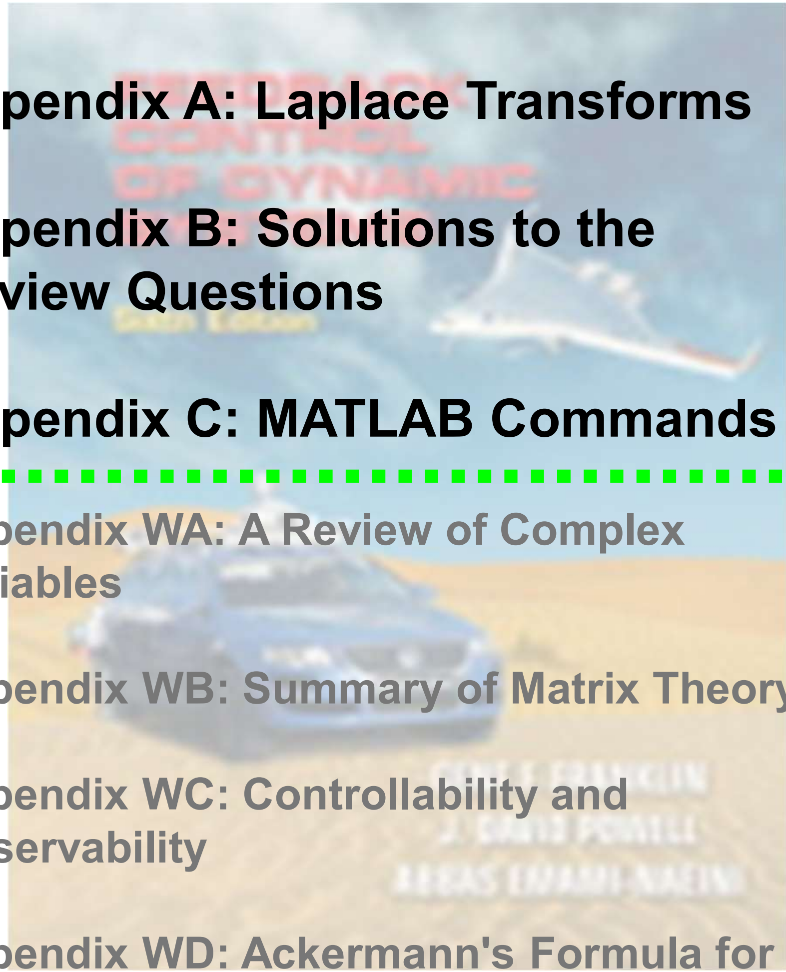
Chapter 6: The Frequency-response Design Method

Chapter 7: State-space Design

Chapter 8: Digital Control

Chapter 9: Nonlinear Systems

Chapter 10: Control Systems Design: Principles and Case Studies



Appendix A: Laplace Transforms

Appendix B: Solutions to the Review Questions

Appendix C: MATLAB Commands

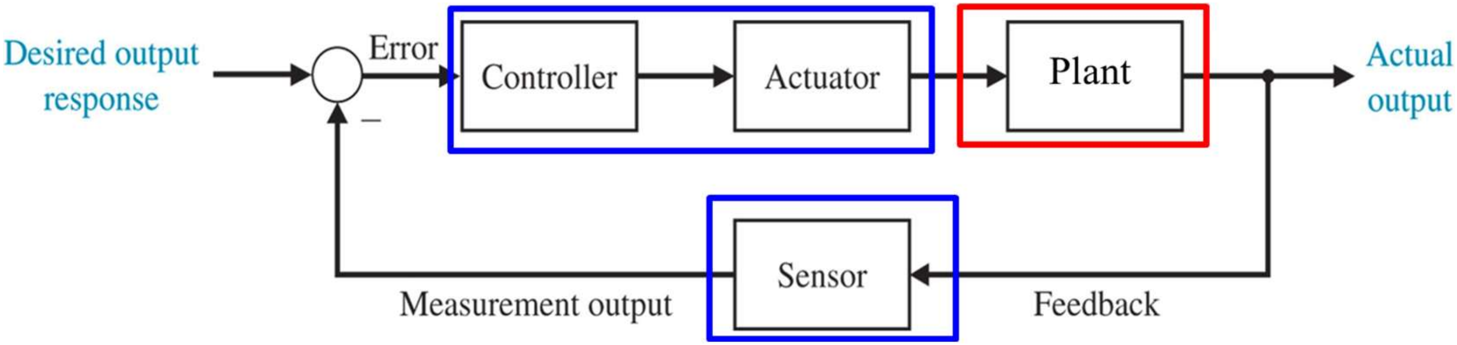
Appendix WA: A Review of Complex Variables

Appendix WB: Summary of Matrix Theory

Appendix WC: Controllability and Observability

Appendix WD: Ackermann's Formula for Pole Placement

Appendix W2.1.4: Complex Mechanical Systems



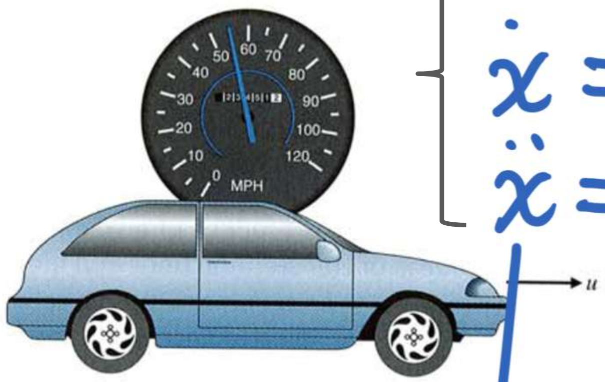

## Dynamic Modeling

(1) Mechanical System
- ① $F = ma$
- ② $Torque = I\alpha$

(2) Electrical System
- Kirchhoff' s voltage law
- Kirchhoff' s current law

機電整合系統 **(Mechatronics/ Electro Mechanical Systems…)**

**Figure 2.1**
Cruise control model

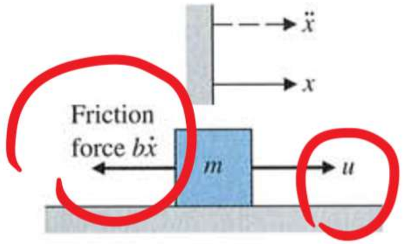



$x =$

$\dot{x} = v$

$\ddot{x} = a$

**Figure 2.2**
Free-body diagram for cruise control




阻尼 damper
避震器

$$u - b\dot{x} = m\ddot{x}, \tag{2.2}$$

$$\frac{dx}{dt}$$

time

$$\frac{V(s)}{U(s)} = \frac{\frac{1}{m}}{s + \frac{b}{m}}. \tag{2.7}$$

**transfer function**

<1> 訂座標 (方向)
<2> 寫平衡方程式

$x(t=0) = 0$
$\dot{x}(t=0) = 0$

**Figure 2.4**
Automobile suspension

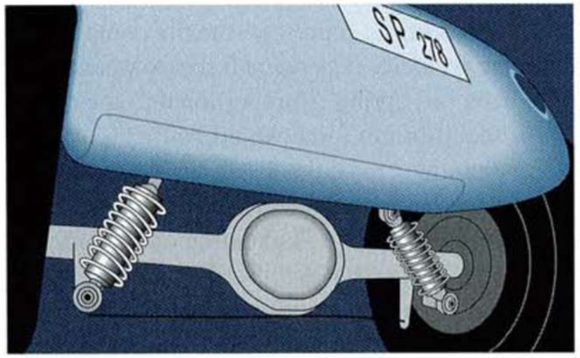
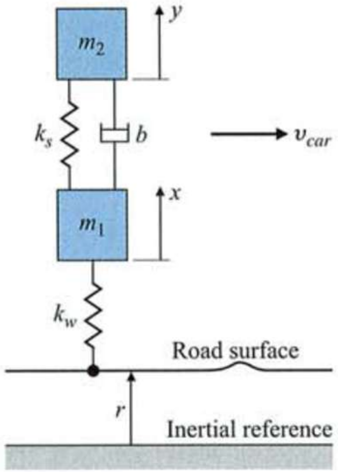
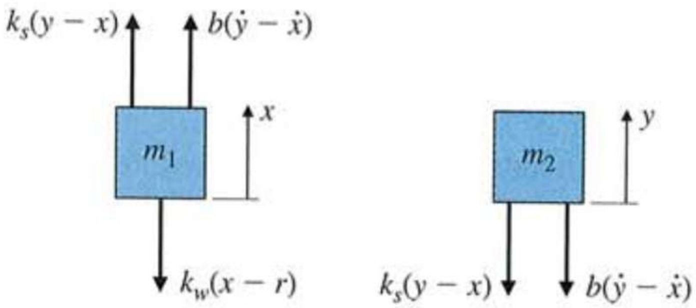



**Figure 2.5**
The quarter-car model




**Degrees of Freedom** (自由度 )- 系統中可以自由運動不受拘束的維度(dimension)。

**Free-Body Diagram**- diagram showing the relative magnitude and direction of all forces acting upon an object in a given situation.




$$b(\dot{y}-\dot{x})+k_s(y-x)-k_w(x-r)=m_1\ddot{x}, \tag{2.8}$$

$$-k_s(y-x)-b(\dot{y}-\dot{x})=m_2\ddot{y}. \tag{2.9}$$

**Equation of Motion**

$$\ddot{x}+\frac{b}{m_1}(\dot{x}-\dot{y})+\frac{k_s}{m_1}(x-y)+\frac{k_w}{m_1}x=\frac{k_w}{m_1}r, \tag{2.10}$$

$$\ddot{y}+\frac{b}{m_2}(\dot{y}-\dot{x})+\frac{k_s}{m_2}(y-x)=0. \tag{2.11}$$

$$\begin{cases} \ddot{x} + \dfrac{b}{m_1}(\dot{x} - \dot{y}) + \dfrac{k_s}{m_1}(x - y) + \dfrac{k_w}{m_1}x = \dfrac{k_w}{m_1}r, & (2.10) \\ \ddot{y} + \dfrac{b}{m_2}(\dot{y} - \dot{x}) + \dfrac{k_s}{m_2}(y - x) = 0. & (2.11) \end{cases}$$

Laplace → 把微分方程式转成代数方程式

inverse Laplace → 代数 → 微分方程式

$$\begin{cases} s^2X(s) + s\dfrac{b}{m_1}(X(s) - Y(s)) + \dfrac{k_s}{m_1}(X(s) - Y(s)) + \dfrac{k_w}{m_1}X(s) = \dfrac{k_w}{m_1}R(s), \\ s^2Y(s) + s\dfrac{b}{m_2}(Y(s) - X(s)) + \dfrac{k_s}{m_2}(Y(s) - X(s)) = 0, \end{cases}$$

**Transfer function** $\dfrac{Y(s)}{R(s)}$ (output / input) $= \dfrac{\dfrac{k_w b}{m_1 m_2}\left(s + \dfrac{k_s}{b}\right)}{s^4 + \left(\dfrac{b}{m_1} + \dfrac{b}{m_2}\right)s^3 + \left(\dfrac{k_s}{m_1} + \dfrac{k_s}{m_2} + \dfrac{k_w}{m_1}\right)s^2 + \left(\dfrac{k_w b}{m_1 m_2}\right)s + \dfrac{k_w k_s}{m_1 m_2}}.$

## 3.1 Review of Laplace Transforms

**Time response:** The dynamics of a system can be prescribed to MATLAB in terms of row vectors containing the coefficients of the polynomials describing the numerator and denominator of its transfer function. The transfer function for this problem is that given in part (a). In this case, the numerator (called num) is simply one number since there are no powers of $s$, so that num $= 1/m = 1/1000$. The denominator (called den) contains the coefficients of the polynomial $s + {}^{b}/_{m}$, which are

$$\text{den} = \begin{bmatrix} 1 & \frac{b}{m} \end{bmatrix} = \begin{bmatrix} 1 & \frac{50}{1000} \end{bmatrix}.$$

The step function in MATLAB calculates the time response of a linear system to a unit step input. Because the system is linear, the output for this case can be multiplied by the magnitude of the input step to derive a step response of any amplitude. Equivalently, num can be multiplied by the magnitude of the input step.

Step response with MATLAB

The statements

```
num = 1/1000;               % 1/m
den = [1  50/1000];         % s + b/m
sys = tf(num*500, den);     % step gives unit step response, so num*500
                              gives u = 500.
step(sys);                  % plots the step response
```

**Skip /跳过**

**Matlab and Simulink**

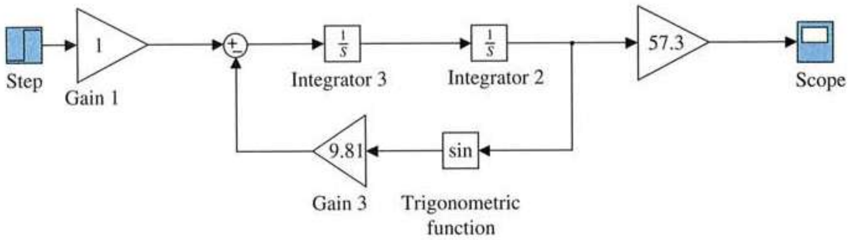



**Figure 2.17**
The SIMULINK block diagram representing the nonlinear equation (2.27)

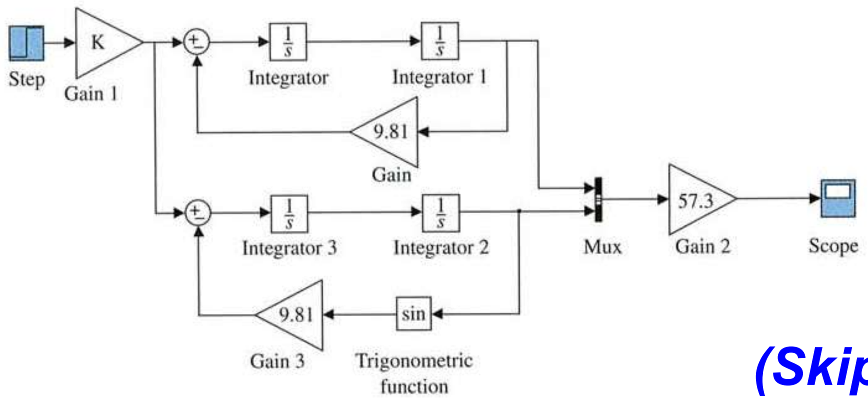
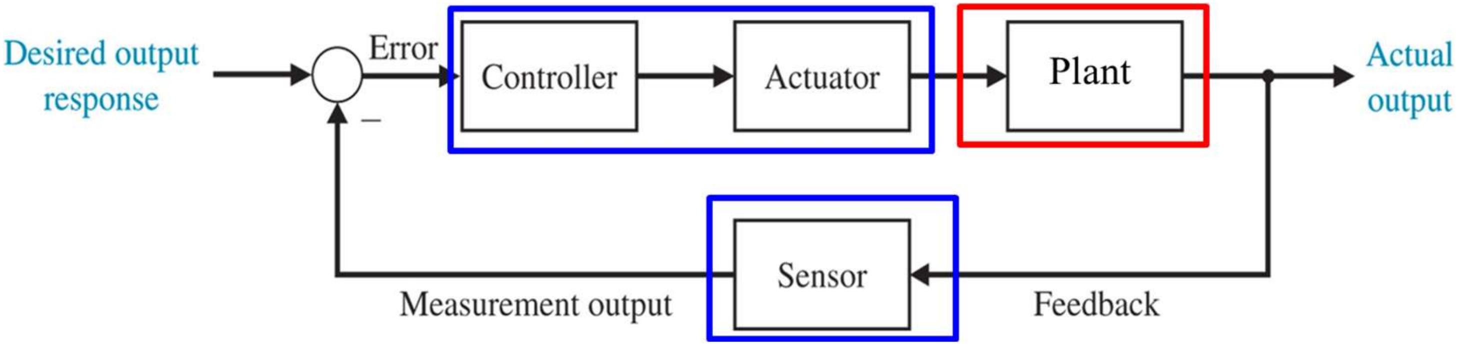



**Figure 2.18**
Block diagram of the pendulum for both the linear and nonlinear models

**(Skip)**




## Dynamic Modeling

(1) Mechanical System

① $F = ma$

② $\text{Torque} = I\alpha$

(2) Electrical System

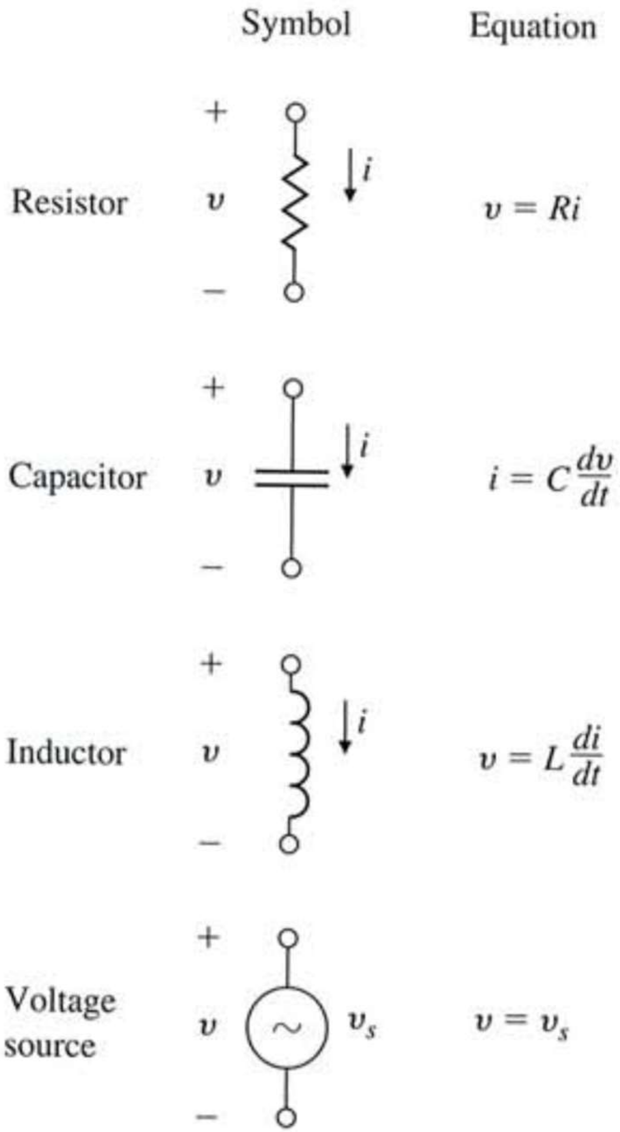
Kirchhoff' s voltage law

Kirchhoff' s current law

(3) 機電整合系統 **(Mechatronics/ Electro Mechanical Systems…)**

## 2.2 Models of Electric Circuits

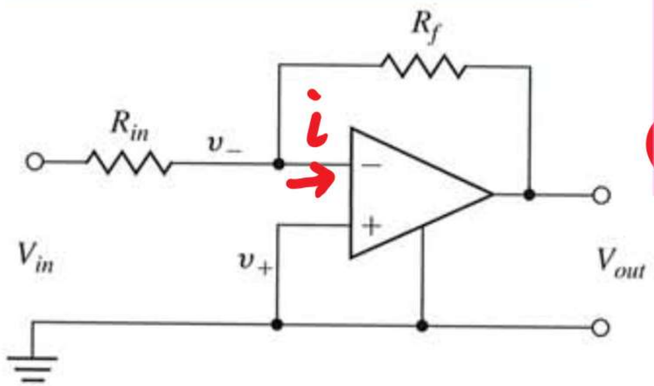
# Circuit Components




**Kirchhoff's current law (KCL)**

**Kirchhoff's voltage law (KVL)**

### Operational Amplifier




① $V_- = V_+$

② $i = 0$

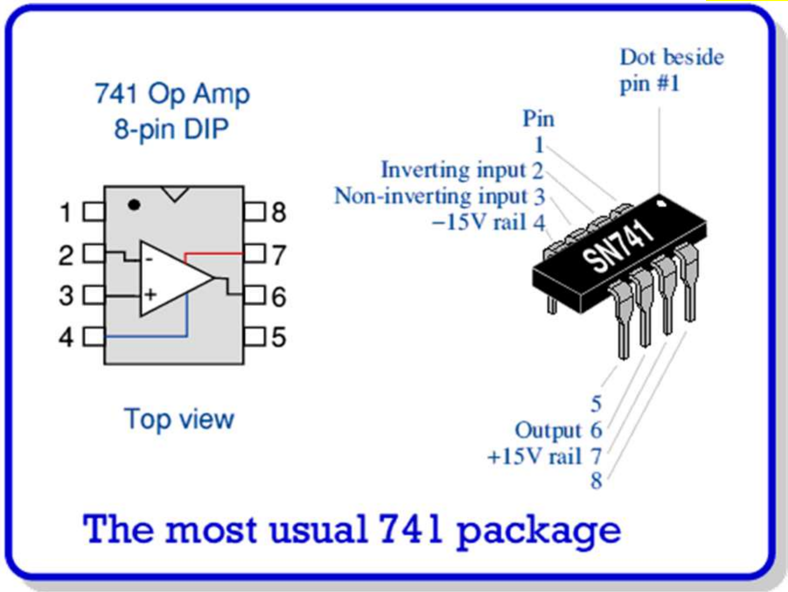
$$\begin{cases} \dfrac{V_- - V_{in}}{R_{in}} = \dfrac{V_{out} - V_-}{R_f} \\ V_- = V_+ = 0 \end{cases}$$

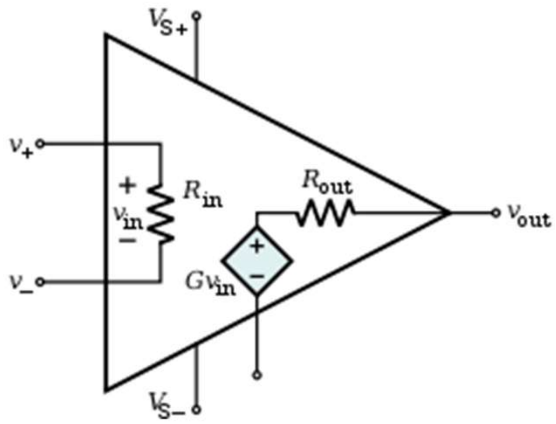
$$\frac{V_{out}}{V_{in}} = -\frac{R_f}{R_{in}}$$

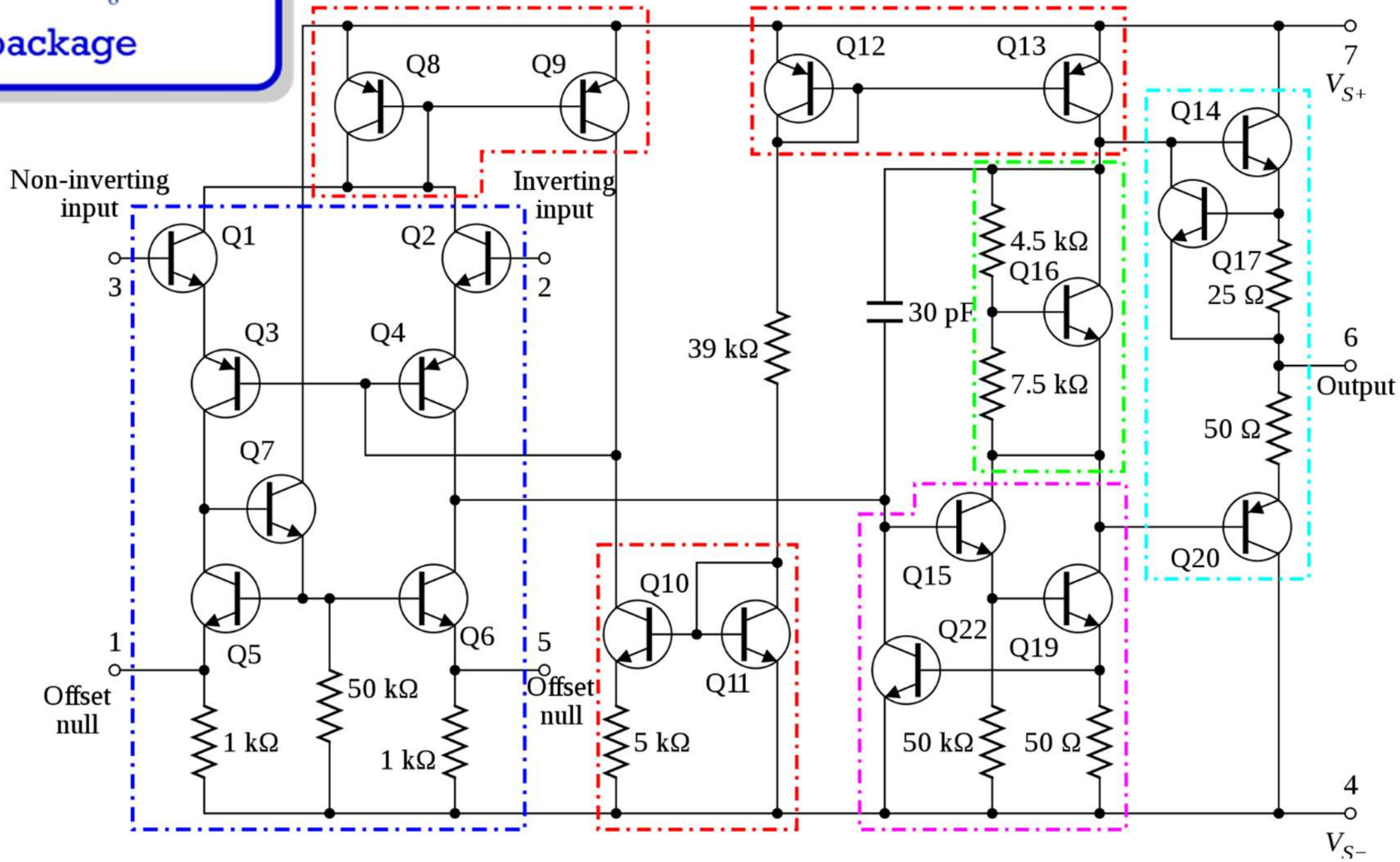
# Operational Amplifier



Reference-不考



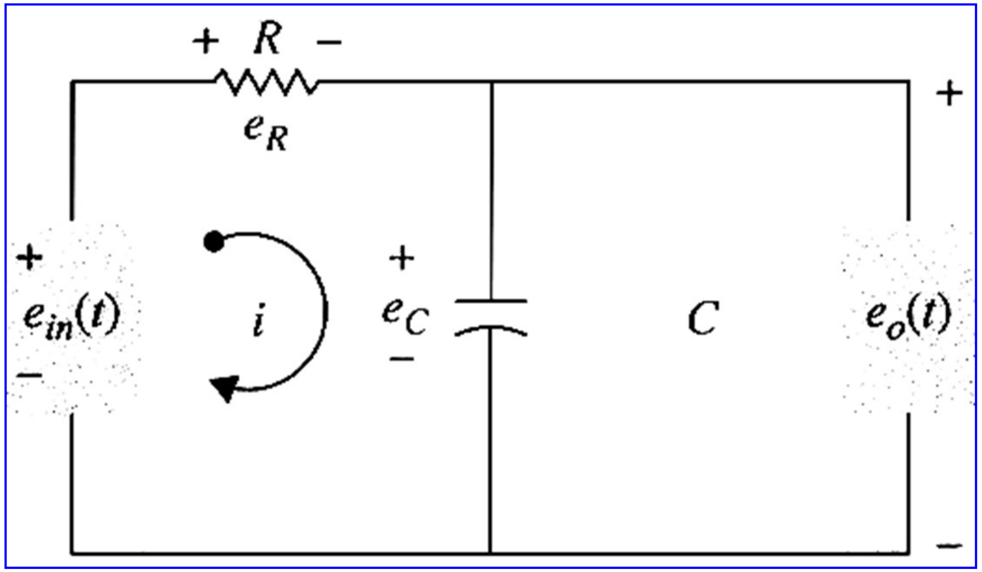

(Skip)

| Quantity | Units | Circuit symbol |
|---|---|---|
| Voltage | volt (V) | Voltage Source $+$ $v$ $-$ |
| Charge | 庫倫 coulomb Q | |
| Current | ampere (A) = Q/s | Current Source $i$ |
| Resistance | ohm (Ω) = V/A | $R$ |
| Capacitance 法拉 | farad (F) = Q/V | $C$ 法拉 |
| Inductance $\boxed{L\frac{dI}{dt}=V}$ | henry (H) = V · s/A | $L$ |
| Battery | — | $+$ $-$ |
| Ground | — | |

$$\boxed{C\frac{dV}{dt}=I}$$

## Find the differential equation of the system.

Simple electrical $RC$ circuit.




$$e_{in}(t) = e_R(t) + e_C(t)$$

$$e_R = iR$$

$$e_C(t) = \frac{1}{C}\int i dt$$

$$e_o(t) = \frac{1}{C}\int i dt = e_C(t)$$

$$\frac{i}{C} = \frac{d\, e_o(t)}{dt}$$

$$C\dot{e}_o(t) = i$$

$$e_{in}(t) = RC\dot{e}_o(t) + e_o(t)$$

$$\frac{E_0(s)}{E_{in}(s)} = \frac{1}{RCs + 1}$$

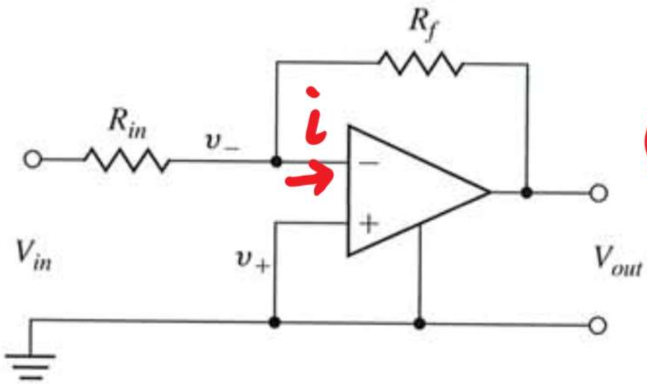
**3.1 Review of Laplace Transforms**

***(Skip)***

(2) Electrical System
$$\begin{cases} ① \text{ K.V} & \text{Kirchhoff's voltage law} \\ ② \text{ K.C.} & \text{Kirchhoff's current law} \end{cases}$$

## 2.2 Models of Electric Circuits

**Operational Amplifier**

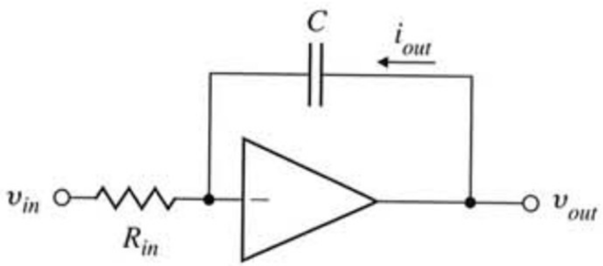



① $V_- = V_+$

② $i = 0$

$$\begin{cases} \dfrac{V_- - V_{in}}{R_{in}} = \dfrac{V_{out} - V_-}{R_f} \\ V_- = V_+ = 0 \end{cases}$$

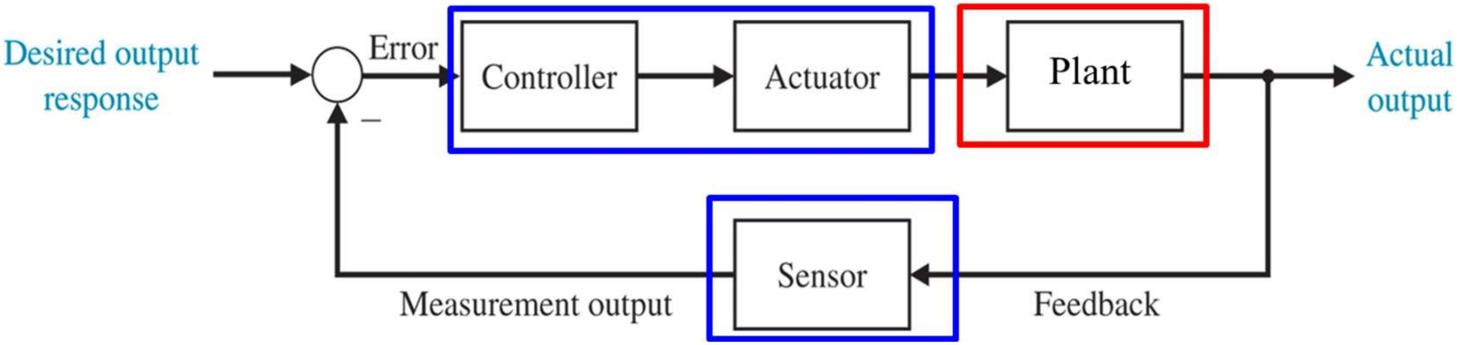
$$\frac{V_{out}}{V_{in}} = -\frac{R_f}{R_{in}}$$




$$\frac{V_{in} - 0}{R_{in}} = C\frac{d(0 - V_{out})}{dt}$$

## 3.1 Review of Laplace Transforms

$$V_{out}(s) = -\frac{1}{s}\frac{V_{in}(s)}{R_{in}C}.$$




## Dynamic Modeling

(1) Mechanical System

① $F = ma$ ←

② $Torque = I\alpha$ ←

(2) Electrical System

**Kirchhoff' s voltage law**

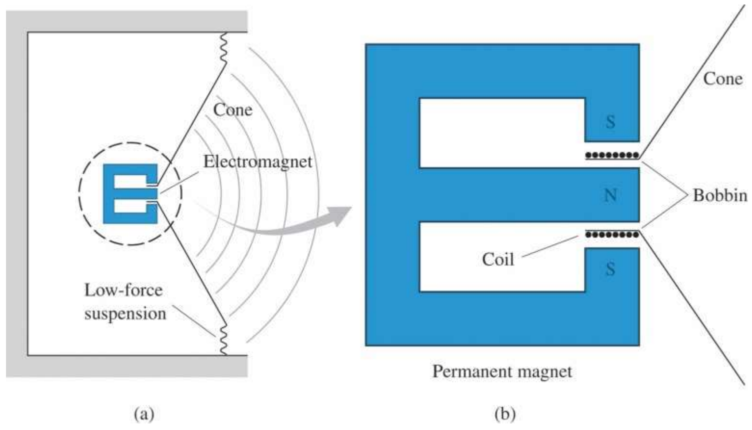
**Kirchhoff' s current law**

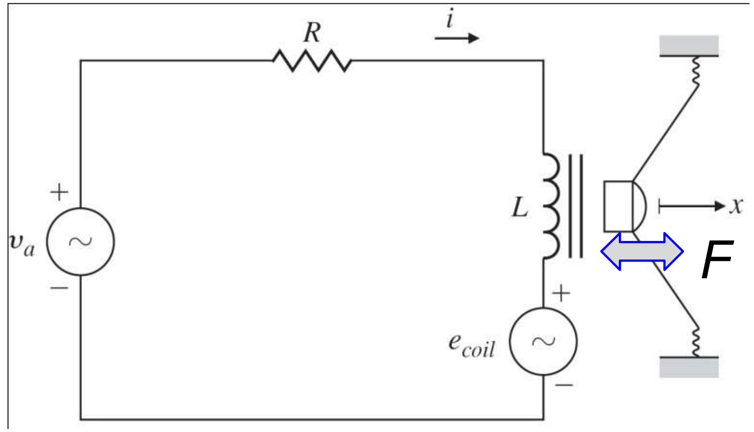
**機電整合系統 (Mechatronics/ Electro Mechanical Systems…)**

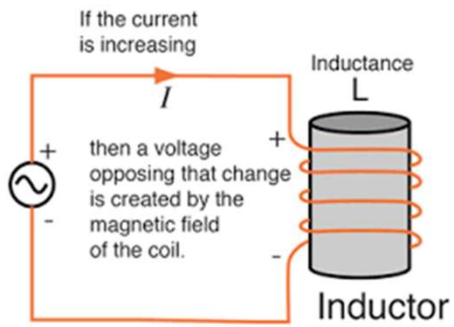
# 機電整合系統（Mechatronics）

## 2.3 Models of Electromechanical Systems

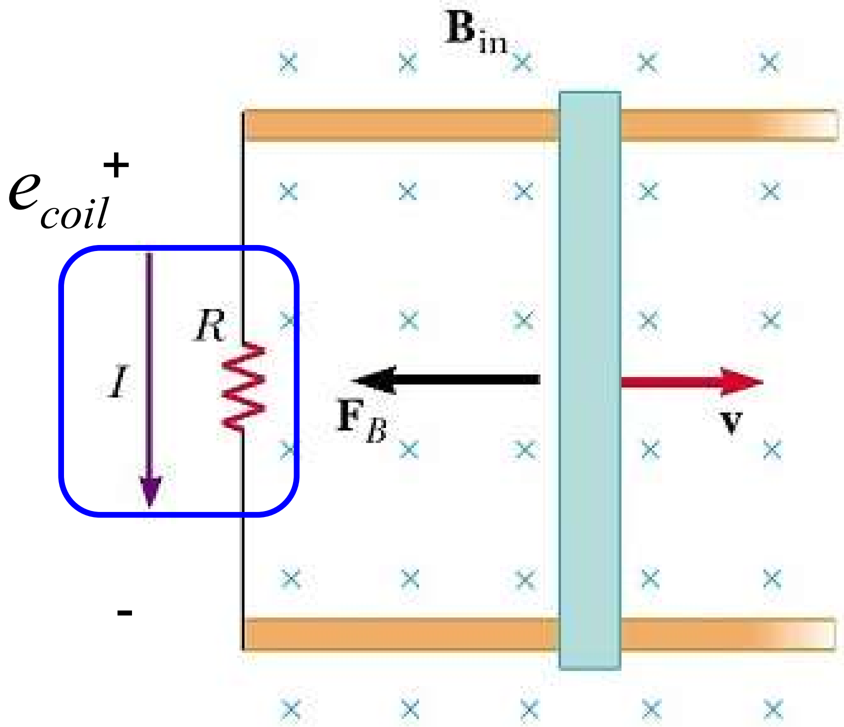
### EXAMPLE 2.11 Loudspeaker




## Lenz's Law



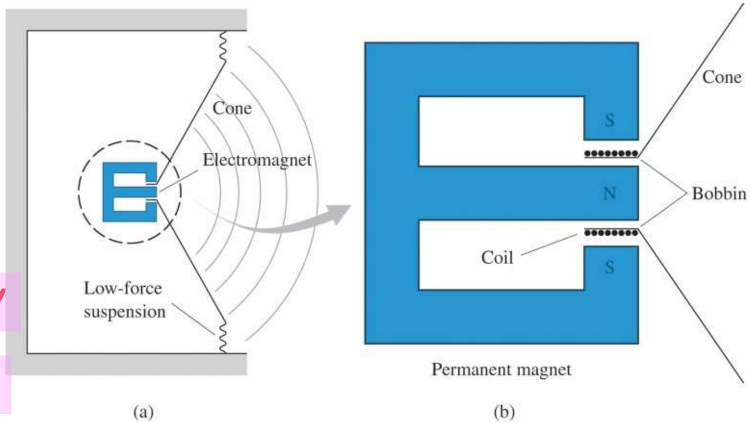

$$F_B = i\,\vec{l} \times \vec{B}$$

**back electro-motive force** $$e_{coil} = -\frac{d\Phi}{dt} = -\frac{d(Blx)}{dt} = -Bl\dot{x}$$

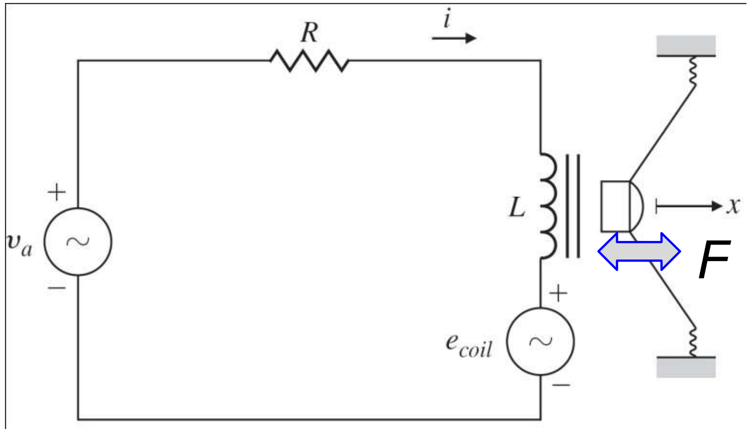
# 機電整合系統 (Mechatronics)

## 2.3 Models of Electromechanical Systems

### EXAMPLE 2.11 Loudspeaker




(a) (b)




magnet establishes a uniform field $B$ of 0.5 tesla and the bobbin has 20 turns at a 2-cm diameter.

$$l = 20 \times \frac{2\pi}{100} m = 1.26\ m$$

$$F = 0.5 \times 1.26 \times i = 0.63i\ \text{N} \leftarrow liB$$

law of generators.

a voltage across the coil $e_{coil} = Bl\dot{x} = 0.63\dot{x}$

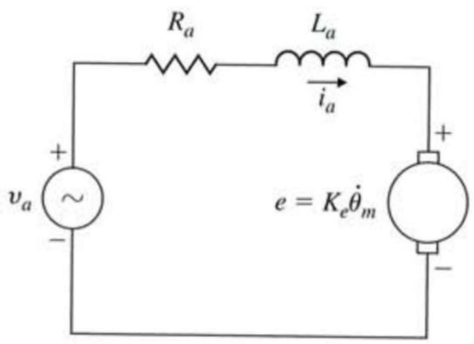
$$\begin{cases} M\ddot{x} + b\dot{x} = 0.63i, \\ L\dfrac{di}{dt} + Ri = v_a - 0.63\dot{x} \end{cases}$$

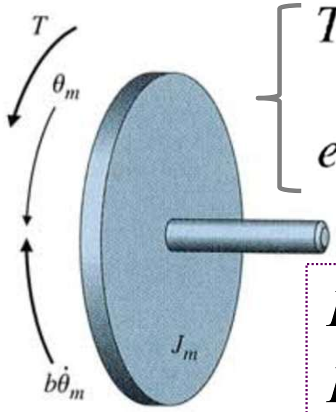
$$M\ddot{x} = 0.63i - b\dot{x}$$

$$\frac{X(s)}{V_a(s)} = \frac{0.63}{s\left[(Ms+b)(Ls+R)+(0.63)^2\right]}. \qquad (2.49)$$

## 3.1 Review of Laplace Transforms

## EXAMPLE 2.13 *Modeling a DC Motor*



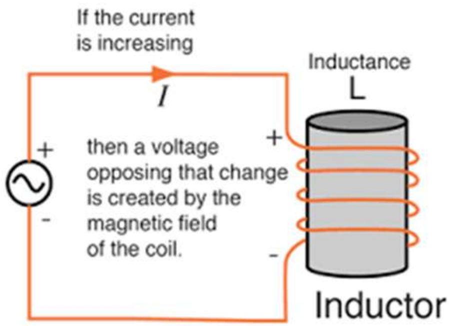



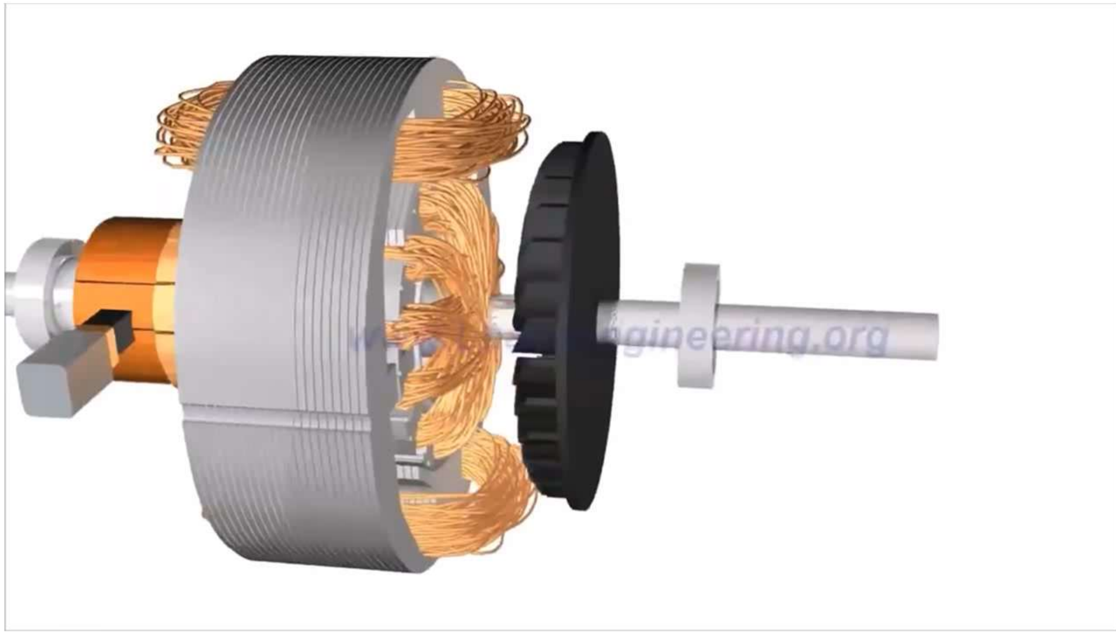

$$\begin{cases} T(t) = K_t i_a(t) \\ e(t) = K_e \dfrac{d\theta_m(t)}{dt} = K_e \omega_m(t) \end{cases}$$

$\Leftarrow \quad F_B = i\,\vec{l} \times \vec{B}$

$\Leftarrow \quad e_{coil} = -\dfrac{d\Phi}{dt} = -Bl\dot{x}$

$K_t$: torque constant
$K_e$: back-emf constant

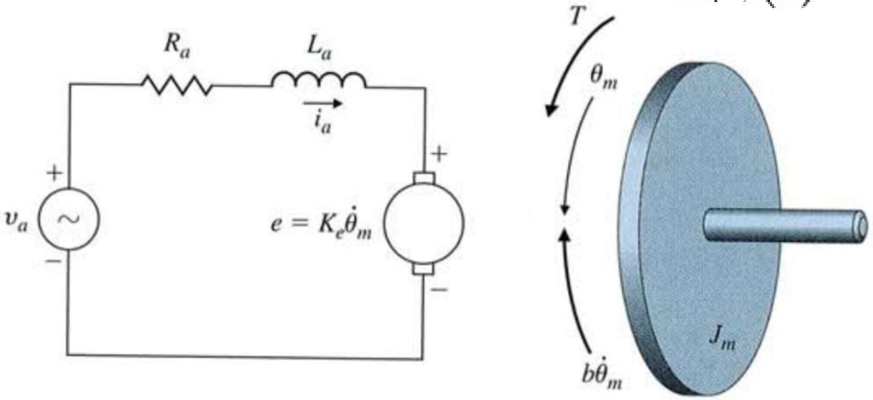


**EXAMPLE 2.13** *Modeling a DC Motor*

lb-ft/amp

$$T(t) = K_t i_a(t)$$




$$\begin{cases} J_m\ddot{\theta}_m + b\dot{\theta}_m = K_t i_a. & (2.52) \\ L_a\dfrac{di_a}{dt} + R_a i_a = v_a - K_e\dot{\theta}_m. & (2.53) \end{cases}$$

volt-sec/rad

$$e(t) = K_e\frac{d\theta_m(t)}{dt} = K_e\omega_m(t)$$

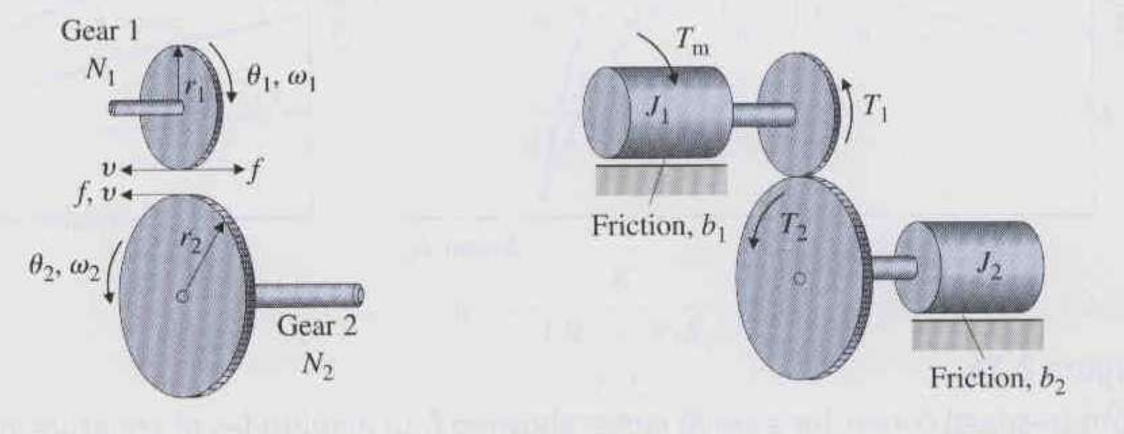
$K_t$: torque constant
$K_e$: back-emf constant

$$\frac{\Theta_m(s)}{V_a(s)} = \frac{K_t}{s[(J_m s + b)(L_a s + R_a) + K_t K_e]}. \quad (2.54)$$

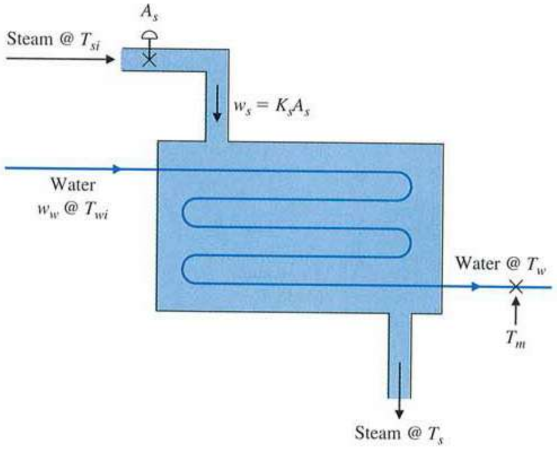
**3.1 Review of Laplace Transforms**

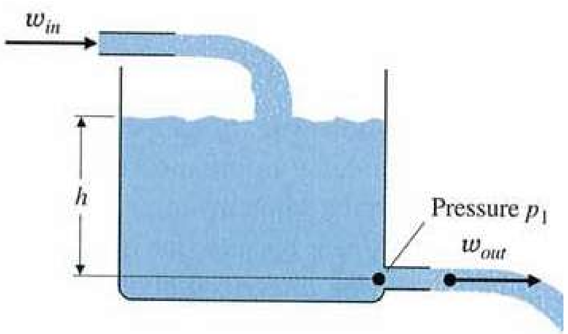
**Not covered in our class/ 跳過**

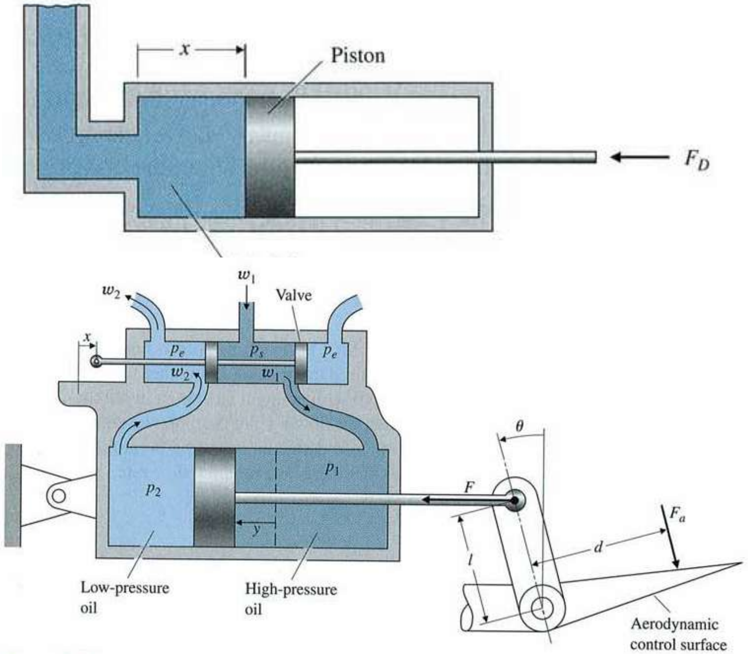
## △ 2.3.3 Gears




## △ 2.4 Heat and Fluid-Flow Models




**(Skip)**

**Chapter 1: An Overview and Brief History of Feedback Control**

**Chapter 2: Dynamic Models**

⇨**Chapter 3: Dynamic Response**

**Chapter 4: A First Analysis of Feedback**

**Chapter 5: The Root-locus Design Method**

**Chapter 6: The Frequency-response Design Method**

**Chapter 7: State-space Design**

**Chapter 8: Digital Control**

**Chapter 9: Nonlinear Systems**

**Chapter 10: Control Systems Design: Principles and Case Studies**

**Appendix A: Laplace Transforms**

**Appendix B: Solutions to the Review Questions**

**Appendix C: MATLAB Commands**

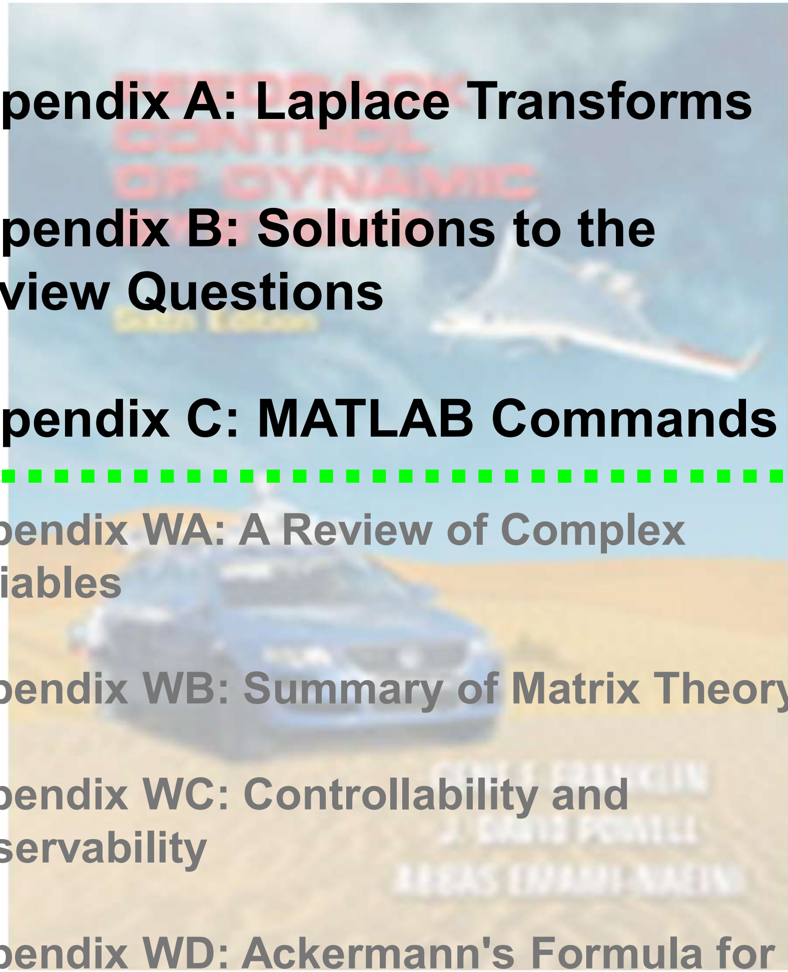
**Appendix WA: A Review of Complex Variables**

**Appendix WB: Summary of Matrix Theory**

**Appendix WC: Controllability and Observability**

**Appendix WD: Ackermann's Formula for Pole Placement**

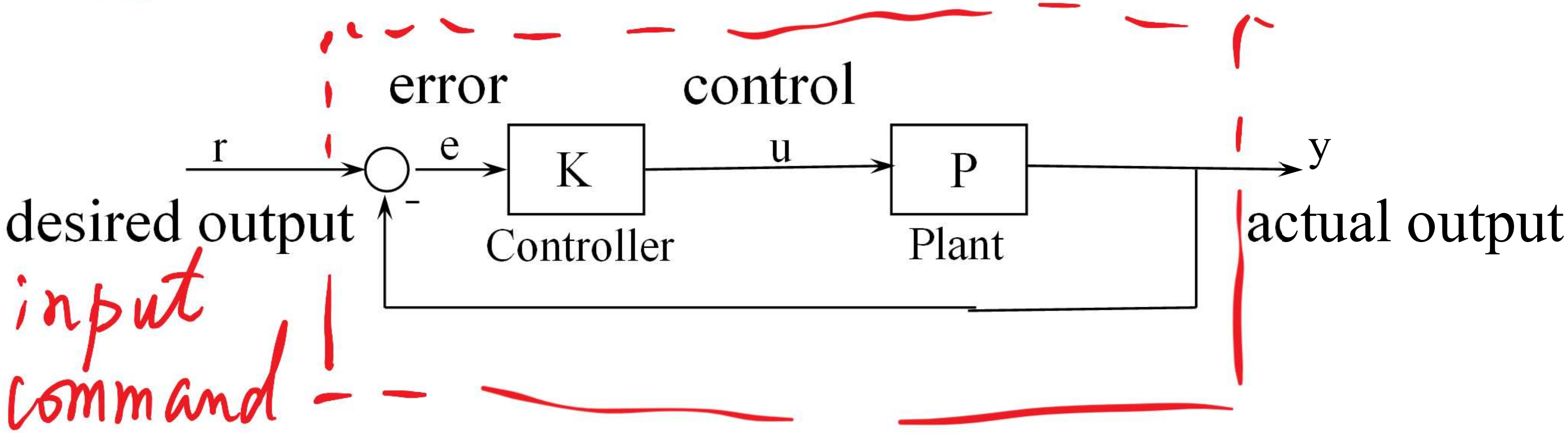
**Appendix W2.1.4: Complex Mechanical Systems**



# Chapter 3

## Dynamic Response

# 3 Dynamic Response




3.1 Review of Laplace Transforms

3.2.1 The Block Diagram

3.3 Effect of Pole Locations

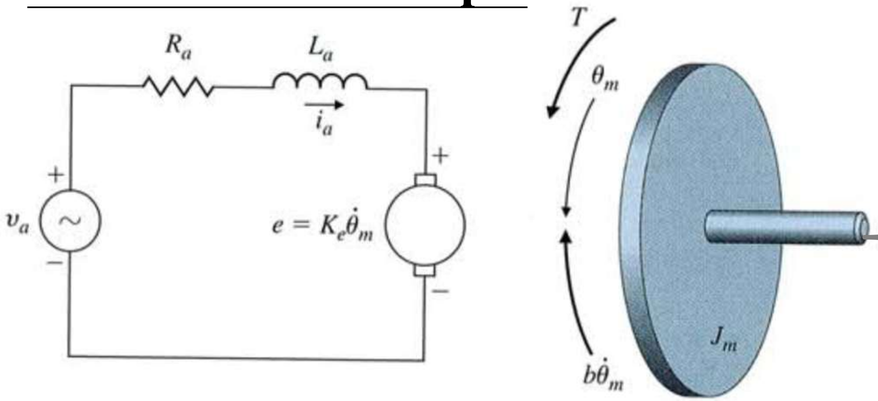
3.4 Time-Domain Specifications

3.5 Effects of Zeros and Additional Poles

3.6.3 Routh's Stability Criterion

# Linear Time Invariant Dynamic System

**Mechatronic Example**




*2nd order linear dynamic system*

$$J_m\ddot{\theta}_m + b\dot{\theta}_m = K_t i_a. \tag{2.52}$$

$$L_a\frac{di_a}{dt} + R_a i_a = v_a - K_e\dot{\theta}_m. \tag{2.53}$$

*1st order linear dynamic system*

n階 線性

For a general $n$th order linear dynamic system, represented by

parameter
variable

$$\frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}} + a_{n-2}\frac{d^{n-2}y(t)}{dt^{n-2}} + \cdots + a_1\frac{dy(t)}{dt} + a_0 y(t)$$
$$= b_m\frac{d^m f(t)}{dt^m} + b_{m-1}\frac{d^{m-1}f(t)}{dt^{m-1}} + \cdots + b_1\frac{df(t)}{dt} + b_0 f(t)$$

the coefficients $a_i, i = 0, 1, 2, \ldots, n-1$, and $b_i, i = 0, 1, 2, \ldots, m$, are assumed to be constant, which indicates that the given *system is time invariant.*

## Linear Dynamic System

parameter
variable

y: 因變數
t: 自變數

For a general $n$th order linear dynamic system, represented by

$$\begin{cases} \dfrac{d^n y(t)}{dt^n} + a_{n-1}\dfrac{d^{n-1}y(t)}{dt^{n-1}} + a_{n-2}\dfrac{d^{n-2}y(t)}{dt^{n-2}} + \cdots + a_1\dfrac{dy(t)}{dt} + a_0 y(t) \\ = b_m\dfrac{d^m f(t)}{dt^m} + b_{m-1}\dfrac{d^{m-1}f(t)}{dt^{m-1}} + \cdots + b_1\dfrac{df(t)}{dt} + b_0 f(t) \end{cases}$$

⇒ 線性方程式

⇒ 因變數不自乘 (1)
不相乘 (2)
不在 sin cos log || 內 (3)

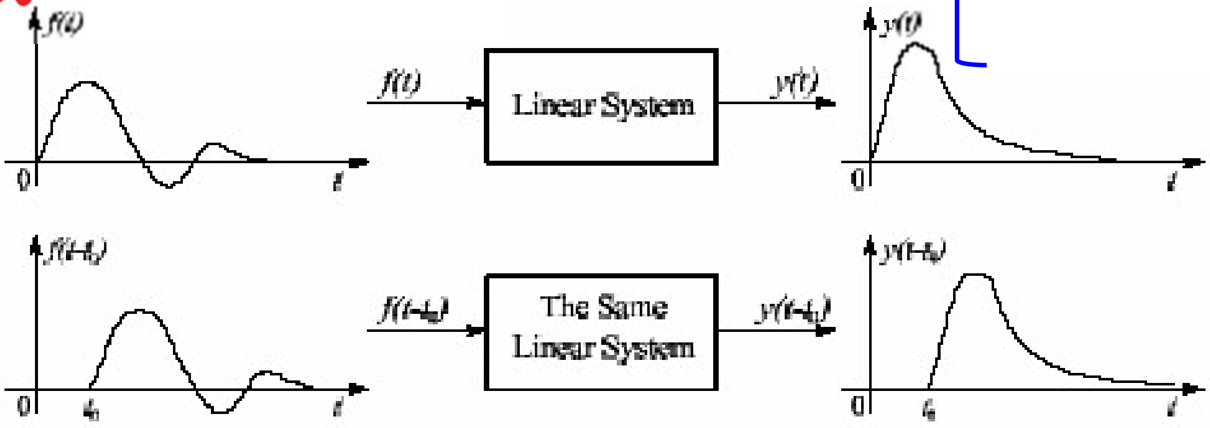
Linear Time variant Dynamic System

$a_i(t)$

LTI Dynamic System

Linear Time Invariant Dynamic System

$a_i = \text{constant}$




Graphical representation of system time invariance

# Transfer Functions

轉移函數

## LTI Dynamic System

$$\Rightarrow \quad T.F. \quad G(s) = \frac{Y(s)}{F(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

$$= K \frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}$$

Def.:

1. G(s) is said to be proper if G(∞) is a finite constant -> $n \geq m$
2. G(s) is said to be strictly proper if G(∞) = 0 -> $n > m$
3. Relative order : n-m
4. The order of system: n
5. Poles: $-p_1, -p_2, \ldots, -p_n$ 極點
6. Zeros: $-z_1, -z_2, \ldots, -z_n$ 零點

Note:
- T.F. is to describe the relationship between input and output
- T.F. is independent to the input and initial condition

## The $\mathcal{L}_-$ Laplace Transform

## Inverse Laplace Transform

Laplace transform of $f(t)$, denoted by $\mathcal{L}_-\{f(t)\} = F(s)$

$s = \sigma + j\omega,$

$$F(s) \triangleq \int_{0^-}^{\infty} f(t)e^{-st}\,dt. \tag{3.24}$$

Laplace

$$f(t) = \frac{1}{2\pi j}\int_{\sigma_c - j\infty}^{\sigma_c + j\infty} F(s)e^{st}\,ds, \tag{3.25}$$

inverse Laplace

## The $\mathcal{L}_-$ Laplace Transform

## Inverse Laplace Transform

**TABLE A.1** Properties of Laplace Transforms

| Number | Laplace Transform | Time Function | Comment |
|---|---|---|---|
| — | $F(s)$ | $f(t)$ | Transform pair |
| 1 | $\alpha F_1(s) + \beta F_2(s)$ | $\alpha f_1(t) + \beta f_2(t)$ | Superposition |
| 2 | $F(s)e^{-s\lambda}$ | $f(t-\lambda)$ | Time delay ($\lambda \geq 0$) |
| 3 | $\frac{1}{\lvert a\rvert} F\left(\frac{s}{a}\right)$ | $f(at)$ | Time scaling |
| 4 | $F(s+a)$ | $e^{-at}f(t)$ | Shift in frequency |
| 5 | $s^m F(s) - s^{m-1}f(0) - s^{m-2}\dot{f}(0) - \cdots - f^{(m-1)}(0)$ | $f^{(m)}(t)$ | Differentiation |
| 6 | $\frac{1}{s}F(s)$ | $\int_0^t f(\zeta)\,d\zeta$ | Integration |
| 7 | $F_1(s)F_2(s)$ | $f_1(t) * f_2(t)$ | Convolution |
| 8 | $\lim_{s\to\infty} sF(s)$ | $f(0^+)$ | Initial Value Theorem |
| 9 | $\lim_{s\to 0} sF(s)$ | $\lim_{t\to\infty} f(t)$ | Final Value Theorem |
| 10 | $\frac{1}{2\pi j}\int_{\sigma_c - j\infty}^{\sigma_c + j\infty} F_1(\zeta)F_2(s-\zeta)d\zeta$ | $f_1(t)f_2(t)$ | Time product |
| 11 | $\frac{1}{2\pi}\int_{-j\infty}^{+j\infty} Y(-j\omega)U(j\omega)\,d\omega$ | $\int_0^\infty y(t)u(t)\,dt$ | Parseval's Theorem |
| 12 | $-\frac{d}{ds}F(s)$ | $tf(t)$ | Multiplication by time |

**TABLE A.2** Table of Laplace Transforms

| Number | $F(s)$ | $f(t), t \geq 0$ |
|---|---|---|
| 1 | $1$ | $\delta(t)$ |
| 2 | $1/s$ | $1(t)$ |
| 3 | $1/s^2$ | $t$ |
| 4 | $2!/s^3$ | $t^2$ |
| 5 | $3!/s^4$ | $t^3$ |
| 6 | $m!/s^{m+1}$ | $t^m$ |
| 7 | $\frac{1}{s+a}$ | $e^{-at}$ |
| 8 | $\frac{1}{(s+a)^2}$ | $te^{-at}$ |
| 9 | $\frac{1}{(s+a)^3}$ | $\frac{1}{2!}t^2e^{-at}$ |
| 10 | $\frac{1}{(s+a)^m}$ | $\frac{1}{(m-1)!}t^{m-1}e^{-at}$ |
| 11 | $\frac{a}{s(s+a)}$ | $1-e^{-at}$ |
| 12 | $\frac{a}{s^2(s+a)}$ | $\frac{1}{a}(at-1+e^{-at})$ |
| 13 | $\frac{b-a}{(s+a)(s+b)}$ | $e^{-at}-e^{-bt}$ |
| 14 | $\frac{s}{(s+a)^2}$ | $(1-at)e^{-at}$ |
| 15 | $\frac{a^2}{s(s+a)^2}$ | $1-e^{-at}(1+at)$ |
| 16 | $\frac{(b-a)s}{(s+a)(s+b)}$ | $be^{-bt}-ae^{-at}$ |
| 17 | $\frac{a}{s^2+a^2}$ | $\sin at$ |
| 18 | $\frac{s}{s^2+a^2}$ | $\cos at$ |
| 19 | $\frac{s+a}{(s+a)^2+b^2}$ | $e^{-at}\cos bt$ |
| 20 | $\frac{b}{(s+a)^2+b^2}$ | $e^{-at}\sin bt$ |
| 21 | $\frac{a^2+b^2}{s[(s+a)^2+b^2]}$ | $1-e^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$ |

## The $\mathcal{L}_-$ Laplace Transform

## Inverse Laplace Transform

Laplace transform of $f(t)$, denoted by $\mathcal{L}_-\{f(t)\} = F(s)$ $s = \sigma + j\omega,$

$$F(s) \triangleq \int_{0^-}^{\infty} f(t)e^{-st}\,dt. \quad (3.24)$$

Laplace

$$f(t) = \frac{1}{2\pi j}\int_{\sigma_c - j\infty}^{\sigma_c + j\infty} F(s)e^{st}\,ds, \quad (3.25)$$

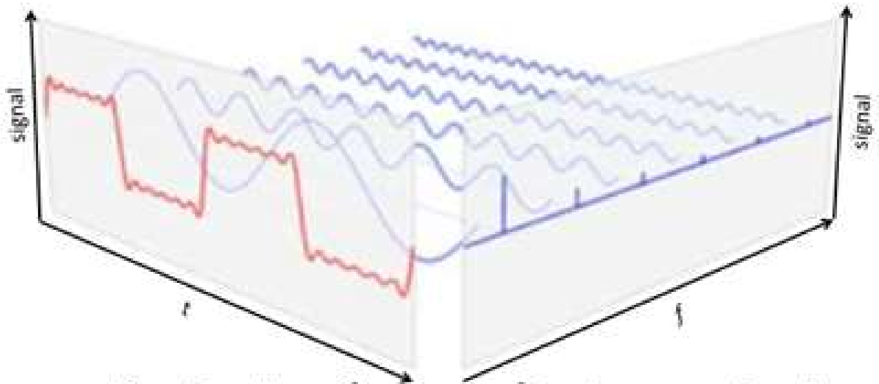
inverse Laplace

**Joseph-Louis Lagrange**
Italian 1736-1813
**Pierre-Simon Laplace**
French 1749-1827

**Any** periodic function can be rewritten as a weighted sum of Sines and Cosines of different frequencies

任何函數都可以展開為三角級數

Fourier Transform - Review




$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x)\, e^{-2\pi i \xi x}\, dx$$

**Fourier transform for functions *f***

$$f(x) = \sum_{n=-\infty}^{\infty} c_n\, e^{2\pi i (n/T) x} = \sum_{n=-\infty}^{\infty} \hat{f}(\xi_n)\, e^{2\pi i \xi_n x}\, \Delta\xi,$$

$$\begin{cases} c_n = \dfrac{1}{T} \displaystyle\int_{-T/2}^{T/2} f(x)\, e^{-2\pi i (n/T) x}\, dx. \\ c_n = (1/T)\hat{f}(n/T) \end{cases}$$

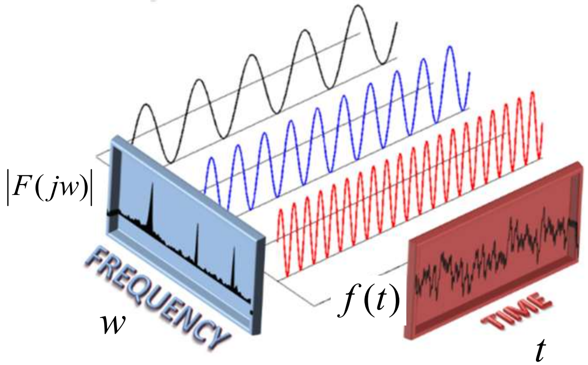
$$F(s) \triangleq \int_{0^-}^{\infty} f(t)e^{-st}\,dt.$$ Laplace

$s = j2\pi\xi$

$$f(t) = \frac{1}{2\pi j}\int_{\sigma_c - j\infty}^{\sigma_c + j\infty} F(s)e^{st}\,ds,$$ inverse Laplace

Fourier Transform

$$\begin{cases} F(\xi) = \int_{-\infty}^{\infty} f(x)e^{-j2\pi\xi x}\,dx \\ f(x) = \int_{-\infty}^{\infty} F(\xi)e^{j2\pi\xi x}\,d\xi \end{cases}$$



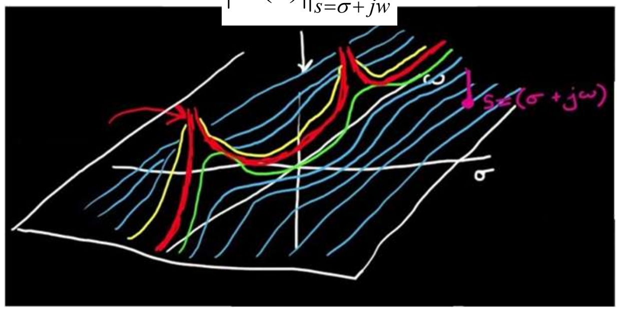

**Fourier Transform**

$$\hat{f}(w) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\boxed{e^{-iwx}}\,dx.$$

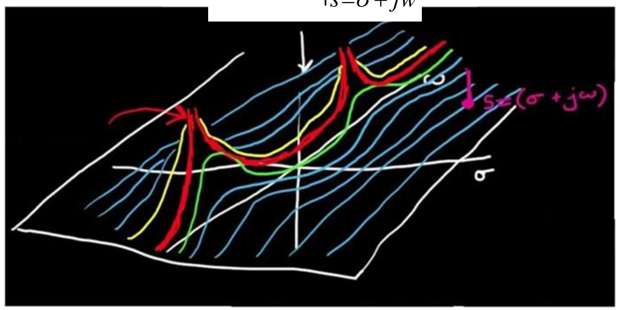
$$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \hat{f}(w)\boxed{e^{iwx}}\,dw$$

Laplace transform of $f(t)$, denoted by $\mathcal{L}_{-}\{f(t)\} = F(s)$ $s = \sigma + j\omega$,

$|F(s)|\big|_{s=\sigma+jw}$




$\angle F(s)\big|_{s=\sigma+jw}$



## The $\mathcal{L}_-$ Laplace Transform

## Inverse Laplace Transform

Laplace transform of $f(t)$, denoted by $\mathcal{L}_-\{f(t)\} = F(s)$

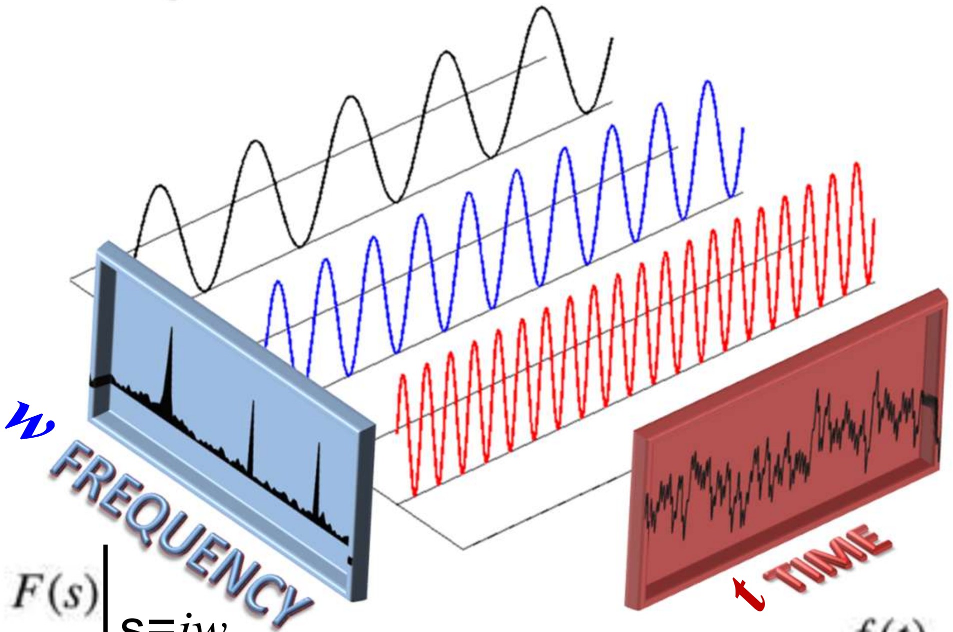
$s = \sigma + j\omega,$

$$F(s) \triangleq \int_{0^-}^{\infty} f(t)e^{-st}\,dt. \quad (3.24)$$

Laplace

$$f(t) = \frac{1}{2\pi j}\int_{\sigma_c - j\infty}^{\sigma_c + j\infty} F(s)e^{st}\,ds, \quad (3.25)$$

inverse Laplace




$F(s)\big|_{s=jw}$

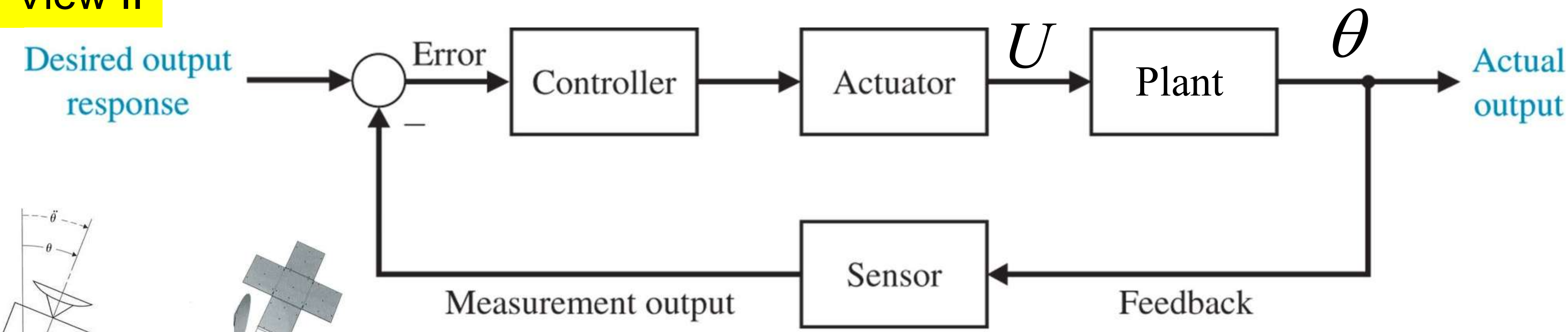
$f(t)$

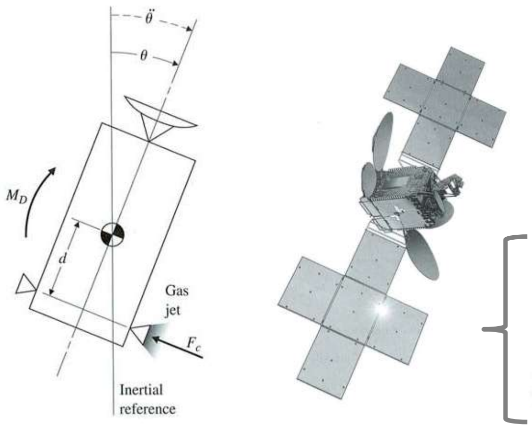
## View I

$$\begin{cases} M\ddot{x} + b\dot{x} = 0.63i, \\ L\dfrac{di}{dt} + Ri = v_a - 0.63\dot{x} \end{cases} \Rightarrow \begin{cases} 2\ddot{x} + 3\dot{x} = 0.63i \\ 4\dot{i} + 5i = 6\sin(2t) - 0.63\dot{x} \end{cases}$$

$$\Rightarrow \begin{cases} 2s^2X + 3sX = 0.63I \\ 4sI + 5I = 6\dfrac{2}{s^2+4} - 0.63sX \end{cases} \Rightarrow$$

## View II

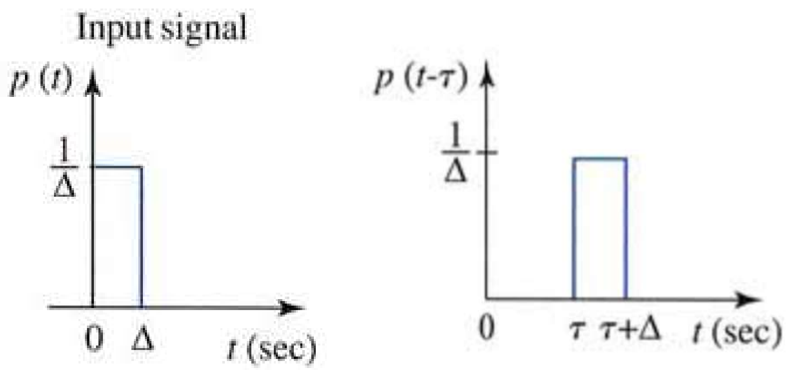



$$\begin{cases} F_c d + M_D = I\ddot{\theta}. \\ U = F_c d + M_D. \end{cases} \Rightarrow I\ddot{\theta} = U \Rightarrow \frac{\Theta(s)}{U(s)} = \frac{1}{I}\frac{1}{s^2}$$

**Def: 1**

**EXAMPLE 3.7** *Impulse Function Transform*

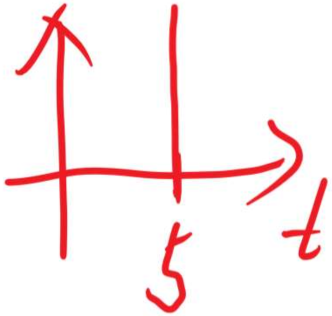



$$p_\Delta(t) = \begin{cases} \frac{1}{\Delta}, & 0 \le t \le \Delta \\ 0, & \text{elsewhere} \end{cases} \tag{3.3}$$

$$\lim_{\Delta \to 0} p_\Delta(t) = \delta(t), \qquad \textbf{impulse.} \tag{3.5}$$

$$\delta(t) = 0 \quad t \neq 0, \tag{3.8}$$

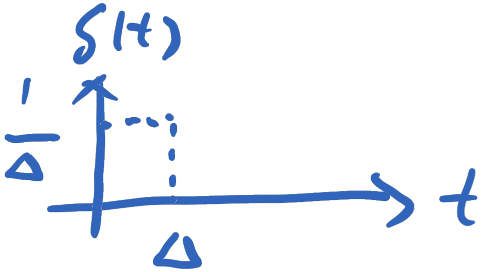
$\delta(t-5)$




$$\Rightarrow \int_{-\infty}^{\infty} \delta(t)dt = 1. \tag{3.9}$$

$$\Rightarrow \int_{-\infty}^{\infty} f(\tau)\delta(t-\tau)\,d\tau = f(t). \tag{3.10}$$

$$\int_{-\infty}^{\infty} \delta(t)\,dt = \lim_{\Delta \to 0} \int_{0}^{\Delta} \frac{1}{\Delta}\,dt = \lim_{\Delta \to 0} 1 = 1$$

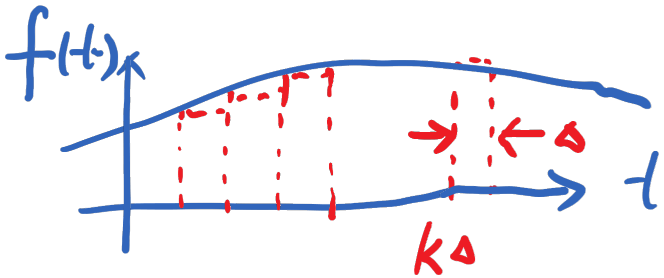



$$\int_{-\infty}^{\infty} f(\tau)\,\delta(t-\tau)\,d\tau = f(t)$$

$$\delta(t) = \begin{cases} \frac{1}{\Delta} & 0 < t < \Delta \\ 0 & \text{elsewhere} \end{cases}$$

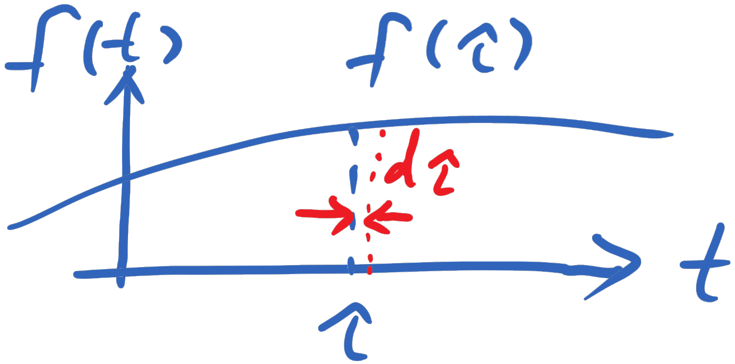
$$\delta(t-k\Delta) = \begin{cases} \frac{1}{\Delta} & k\Delta < t < (k+1)\Delta \\ 0 & \text{elsewhere} \end{cases}$$

**Proof:**




$$\hat{f}(t) = \sum_{k=-\infty}^{\infty} f(k\Delta)\,\underline{\delta(t-k\Delta)\cdot\Delta}$$

$$(\ \delta(t-k\Delta)\cdot\Delta = 1\ )$$




$$f(t) = \int_{-\infty}^{\infty} f(\tau)\,\delta(t-\tau)\,d\tau$$

$\delta(t) \Rightarrow$ ① $\int_{-\infty}^{\infty} \delta(t-a)\,dt = 1$

⇨ ② $\int_{b}^{c} \delta(t-a)\,dt = \begin{cases} 1 & a \in [b,c] \\ 0 & a \notin [b,c] \end{cases}$

⇨ ③ $\int_{-\infty}^{\infty} \delta(t-a)\,f(t)\,dt = f(a)$

⇨ ④ $\int_{b}^{c} \delta(t-a)\,f(t)\,dt = \begin{cases} f(a) & a \in [b,c] \\ 0 & a \notin [b,c] \end{cases}$

$$\mathcal{L}[\delta(t)] = \int_0^\infty \delta(t) e^{-st} dt$$

$$\Rightarrow \int_{-\infty}^{\infty} f(\tau)\delta(t-\tau)\, d\tau = f(t).$$

$$\parallel$$

$$\int_{-\infty}^{\infty} f(\hat{\tau})\, \delta(\hat{\tau}-t)\, d\hat{\tau} = f(t)$$

$$= \int_0^\infty e^{-s\hat{\tau}} \delta(\hat{\tau}-0)\, d\hat{\tau}$$

$$= e^{-s0} = 1$$

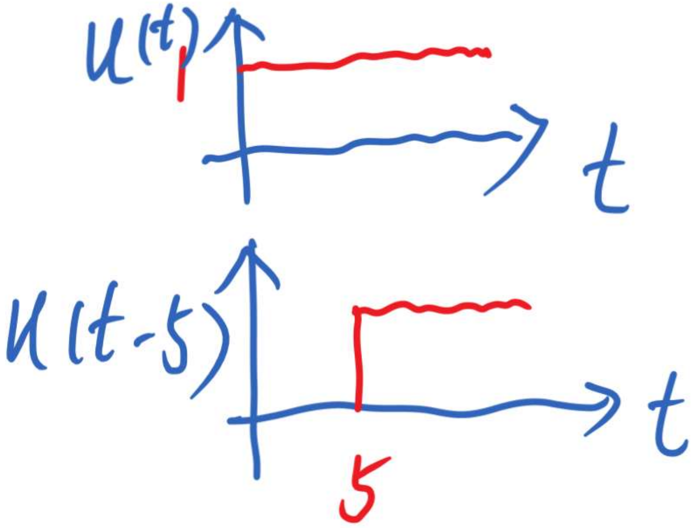
$$\Rightarrow \mathcal{L}[\delta(t)] = 1$$

Def 2 unit step function $u(t) \leftrightarrow 1(t)$

$$u_s(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$\Rightarrow$ $$u_s(t-a) = \begin{cases} 1 & t \geq a \\ 0 & t < a \end{cases}$$

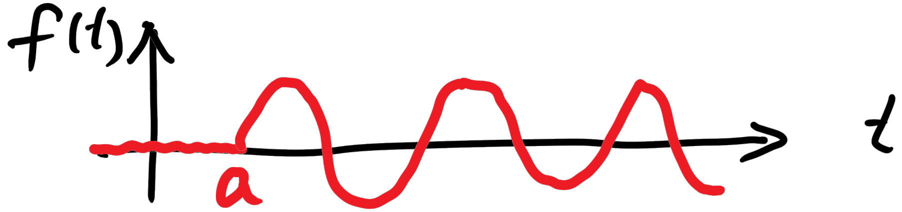
$\Rightarrow$ $$\delta(t) = \frac{d}{dt} u_s(t)$$



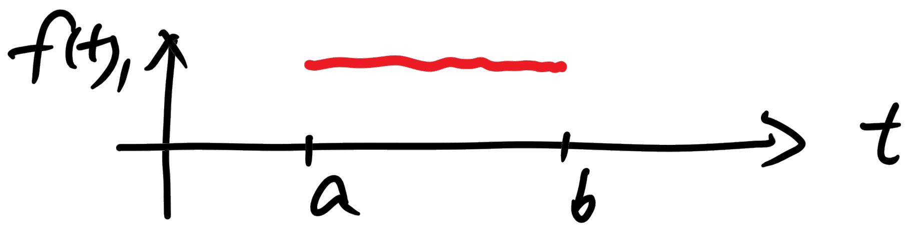
⊙ $f(t) = u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$

$$\mathcal{L}[u(t)] = \mathcal{L}[1] = \int_0^{\infty} e^{-st}\,dt = -\frac{e^{-st}}{s}\Big|_0^{\infty} = \frac{1}{s}$$

⊙ Unit function 具選段功能




$\Rightarrow f(t) = \sin(t-a) \times u(t-a)$




$\Rightarrow f(t) = u(t-a) - u(t-b)$

$$u'(t) = \delta(t)$$

$$\begin{cases} \mathcal{L}[u(t)] = \dfrac{1}{s} \\ \mathcal{L}[\delta(t)] = \displaystyle\int_0^\infty e^{-st}\,\delta(t)\,dt = e^{-st}\Big|_0 = 1 \ \# \end{cases}$$

$$\begin{cases} \mathcal{L}^{-1}[1] = \delta(t) \\ \mathcal{L}^{-1}\left[\dfrac{1}{s}\right] = u(t) \end{cases}$$

背

$$\longleftrightarrow \begin{cases} \mathcal{L}^{-1}[s] = \delta'(t) \\ \mathcal{L}^{-1}[s^n] = \delta^{n}(t) \end{cases}$$

$$\mathcal{L}[\delta(t-a)] = e^{-sa}$$

$$\int_{-\infty}^{\infty} \delta'(t) f(t)\, dt = \delta(t) f(t) \Big|_{-\infty}^{\infty}$$

$$- \int_{-\infty}^{\infty} \delta(t) f'(t)\, dt$$

$$= (-1) f'(t) \Big|_{t=0}$$

$$\Rightarrow \int_{-\infty}^{\infty} \delta^{(n)}(t-a) f(t)\, dt = (-1)^n f^{n}(t) \Big|_{t=a}$$

$$\mathcal{L}[t^a] = \int_0^\infty e^{-st}\, t^a\, dt$$

令 $st = y \Rightarrow dt = dy/s$

$$= \int_0^\infty \frac{y^a}{s^a} e^{-y} \frac{dy}{s} = \frac{1}{s^{a+1}} \int_0^\infty y^a e^{-y} dy$$

$$= \frac{1}{s^{a+1}} \left[ -y^a e^{-y} \Big|_0^\infty + (a) \int_0^\infty y^{a-1} e^{-y} dy \right]$$

$$= \frac{a}{s^{a+1}} \int_0^\infty y^{a-1} e^{-y} dy$$

$$= \cdots = \frac{a!}{s^{a+1}} \int_0^\infty e^{-y} dy$$

$$= \frac{a!}{s^{a+1}}\left[-e^{-y}\Big|_0^{\infty}\right]$$

$$= \frac{a!}{s^{a+1}}\left[-0+1\right] = \frac{a!}{s^{a+1}} \quad \text{得證}$$

$$\left\{\begin{array}{l} \mathcal{L}[\sin wt] \\ \mathcal{L}[\cos wt] \end{array}\right. \rightarrow \quad e^{iwt} = \cos wt + i \sin wt$$

$$\mathcal{L}[e^{iwt}] = \int_0^\infty [e^{iwt} e^{-st}]\, dt$$

$$= \int_0^\infty e^{(iw-s)t}\, dt = \frac{1}{iw-s} e^{(iw-s)t}\Big|_0^\infty$$

$$= \cdots = \frac{s+iw}{s^2+w^2}$$

$$\Rightarrow \left\{\begin{array}{l} \mathcal{L}[\sin wt] = \dfrac{w}{s^2+w^2} \quad \text{背} \\ \mathcal{L}[\cos wt] = \dfrac{s}{s^2+w^2} \quad \text{背} \end{array}\right.$$

$$\mathcal{L}[e^{at}] = \int_0^\infty e^{at} e^{-st}\, dt = \frac{-e^{-(s-a)t}}{s-a}\Big|_0^\infty$$

$$= \frac{1}{s-a}$$ # 背

**TABLE A.2** Table of Laplace Transforms

Mentioned So far!

| Number | $F(s)$ | $f(t), t \geq 0$ | |
|---|---|---|---|
| 1 | $1$ | $\delta(t)$ | ← p. 20 |
| 2 | $1/s$ | $1(t)$ | ← p. 22/ p. 28 |
| 3 | $1/s^2$ | $t$ | ← p. 25-26 |
| 4 | $2!/s^3$ | $t^2$ | ← p. 25-26 |
| 5 | $3!/s^4$ | $t^3$ | ← p. 25-26 |
| 6 | $m!/s^{m+1}$ | $t^m$ | ← p. 25-26 |
| 7 | $\frac{1}{s+a}$ | $e^{-at}$ | ← p. 28 |
| 8 | $\frac{1}{(s+a)^2}$ | $te^{-at}$ | |
| 9 | $\frac{1}{(s+a)^3}$ | $\frac{1}{2!}t^2e^{-at}$ | |
| 10 | $\frac{1}{(s+a)^m}$ | $\frac{1}{(m-1)!}t^{m-1}e^{-at}$ | |
| 11 | $\frac{a}{s(s+a)}$ | $1-e^{-at}$ | |
| 12 | $\frac{a}{s^2(s+a)}$ | $\frac{1}{a}(at-1+e^{-at})$ | |
| 13 | $\frac{b-a}{(s+a)(s+b)}$ | $e^{-at}-e^{-bt}$ | |
| 14 | $\frac{s}{(s+a)^2}$ | $(1-at)e^{-at}$ | |
| 15 | $\frac{a^2}{s(s+a)^2}$ | $1-e^{-at}(1+at)$ | |
| 16 | $\frac{(b-a)s}{(s+a)(s+b)}$ | $be^{-bt}-ae^{-at}$ | |
| 17 | $\frac{a}{s^2+a^2}$ | $\sin at$ | ← p. 27 |
| 18 | $\frac{s}{s^2+a^2}$ | $\cos at$ | ← p. 27 |
| 19 | $\frac{s+a}{(s+a)^2+b^2}$ | $e^{-at}\cos bt$ | |
| 20 | $\frac{b}{(s+a)^2+b^2}$ | $e^{-at}\sin bt$ | |
| 21 | $\frac{a^2+b^2}{s[(s+a)^2+b^2]}$ | $1-e^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$ | |

# Appendix A Laplace Transforms

### 4. Shift in Frequency

Multiplication (modulation) of $f(t)$ by an exponential expression in the time domain corresponds to a shift in frequency:

$$F_1(s) = \int_0^\infty e^{-at} f(t) e^{-st}\, dt = \int_0^\infty f(t) e^{-(s+a)t}\, dt = F(s+a). \quad \text{(A.9)}$$

**TABLE A.2** Table of Laplace Transforms

Mentioned So far!

$$\int_0^\infty e^{-at} f(t) e^{-st}\, dt = \int_0^\infty f(t) e^{-(s+a)t}\, dt = F(s+a)$$

| Number | $F(s)$ | $f(t), t \geq 0$ | |
|---|---|---|---|
| 1 | $1$ | $\delta(t)$ | p. 20 |
| 2 | $1/s$ | $1(t)$ | p. 22/ p. 28 |
| 3 | $1/s^2$ | $t$ | p. 25-26 |
| 4 | $2!/s^3$ | $t^2$ | p. 25-26 |
| 5 | $3!/s^4$ | $t^3$ | p. 25-26 |
| 6 | $m!/s^{m+1}$ | $t^m$ | p. 25-26 |
| 7 | $\frac{1}{s+a}$ | $e^{-at}$ | p. 28 |
| 8 | $\frac{1}{(s+a)^2}$ | $te^{-at}$ | p. 28 + p. 25-26 |
| 9 | $\frac{1}{(s+a)^3}$ | $\frac{1}{2!}t^2 e^{-at}$ | p. 28 + p. 25-26 |
| 10 | $\frac{1}{(s+a)^m}$ | $\frac{1}{(m-1)!}t^{m-1}e^{-at}$ | p. 28 + p. 25-26 |
| 11 | $\frac{a}{s(s+a)}$ | $1-e^{-at}$ | |
| 12 | $\frac{a}{s^2(s+a)}$ | $\frac{1}{a}(at-1+e^{-at})$ | |
| 13 | $\frac{b-a}{(s+a)(s+b)}$ | $e^{-at}-e^{-bt}$ | |
| 14 | $\frac{s}{(s+a)^2}$ | $(1-at)e^{-at}$ | |
| 15 | $\frac{a^2}{s(s+a)^2}$ | $1-e^{-at}(1+at)$ | |
| 16 | $\frac{(b-a)s}{(s+a)(s+b)}$ | $be^{-bt}-ae^{-at}$ | |
| 17 | $\frac{a}{s^2+a^2}$ | $\sin at$ | p. 27 |
| 18 | $\frac{s}{s^2+a^2}$ | $\cos at$ | p. 27 |
| 19 | $\frac{s+a}{(s+a)^2+b^2}$ | $e^{-at}\cos bt$ | p. 28 + p.27 |
| 20 | $\frac{b}{(s+a)^2+b^2}$ | $e^{-at}\sin bt$ | p. 28 + p.27 |
| 21 | $\frac{a^2+b^2}{s[(s+a)^2+b^2]}$ | $1-e^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$ | |

$$\int_0^\infty e^{-at} f(t) e^{-st}\, dt = \int_0^\infty f(t) e^{-(s+a)t}\, dt = F(s+a)$$

**The $\mathcal{L}_-$ Laplace Transform** Laplace transform of $f(t)$, denoted by $\mathcal{L}_-\{f(t)\} = F(s)$

$$F(s) \triangleq \int_{0^-}^{\infty} f(t)e^{-st}\,dt$$

**Inverse Laplace Transform**

$$f(t) = \frac{1}{2\pi j}\int_{\sigma_c - j\infty}^{\sigma_c + j\infty} F(s)e^{st}\,ds$$

$$\mathcal{L}[y'(t)] = \int_0^\infty y'(t)e^{-st}dt = [y(t)e^{-st}]_0^\infty - \int_0^\infty y(t)e^{-st}dt$$

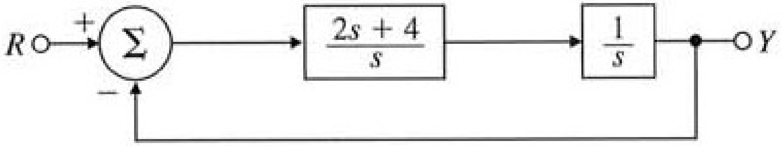
$$= 0 - y(0) + s\int_0^\infty y(t)e^{-st}dt$$

$$= sY(s) - y(0)$$

$$\mathcal{L}\left\{\frac{df}{dt}\right\} = \int_{0^-}^{\infty}\left(\frac{df}{dt}\right)e^{-st}\,dt = -f(0^-) + sF(s)$$

$$\mathcal{L}\{\ddot{f}\} = s^2F(s) - sf(0^-) - \dot{f}(0^-)$$

$$\mathcal{L}\{f^m(t)\} = s^mF(s) - s^{m-1}f(0^-) - s^{m-2}\dot{f}(0^-) - \cdots - f^{(m-1)}(0^-)$$






# Convolution

$$\mathcal{L}\{f_1(t)\} = F_1(s) \text{ and } \mathcal{L}\{f_2(t)\} = F_2(s)$$

$$\Rightarrow \mathcal{L}\{f_1(t) * f_2(t)\} = \int_0^\infty f_1(t) * f_2(t) e^{-st}\, dt = \int_0^\infty \left[ \int_0^t f_1(\tau) f_2(t-\tau)\, d\tau \right] e^{-st}\, dt$$

$$= \int_0^\infty \int_\tau^\infty f_1(\tau) f_2(t-\tau) e^{-st}\, dt\, d\tau$$

Multiplying by $e^{-s\tau} e^{s\tau}$

$$= \int_0^\infty f_1(\tau) e^{-s\tau} \left[ \int_\tau^\infty f_2(t-\tau) e^{-s(t-\tau)}\, dt \right] d\tau$$

change variables $t' \triangleq t - \tau$

$$= \int_0^\infty f_1(\tau) e^{-st}\, d\tau \int_0^\infty f_2(t') e^{-st'}\, dt' \quad = F_1(s) F_2(s)$$

$$\Rightarrow \mathcal{L}^{-1}\{F_1(s) F_2(s)\} = f_1(t) * f_2(t)$$

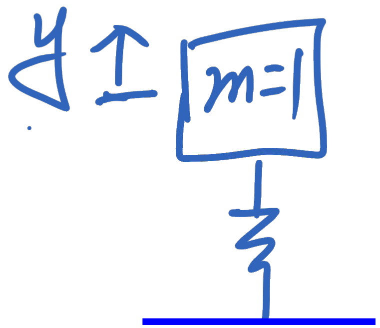
# Convolution (My viewpoint)

$$f(t) \;\xrightarrow{\quad}\; \boxed{\frac{1}{S^2+1}} \;\xrightarrow{\quad}\; y(t)$$

$F(s)$ → $g(t)$, $G(s)$ → $Y(s)$

$$S^2 Y + Y = F$$

物理系統




方程式
$$\begin{cases} \ddot{y} + y = f(t) \\ F \cdot \dfrac{1}{S^2+1} = Y \end{cases}$$

Initial Condition

$$\begin{cases} y(t=0) = 0 \\ y'(t=0) = 0 \end{cases}$$

$\Rightarrow$ $f(t) * g(t) = y(t)$

convolution

$$\ddot{y} + y = \delta(t)$$

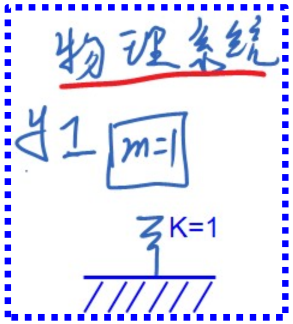
Laplace

$$s^2 Y + Y = 1$$

$$Y = \frac{1}{s^2+1}$$

$\mathcal{L}^{-1}$aplace

$$y = \sin t \ \#$$




$$\mathcal{L}[\delta(t)] = \int_0^\infty \delta(t)\, e^{-st}\, dt$$

$$\Rightarrow \int_{-\infty}^{\infty} f(\tau)\delta(t-\tau)\,d\tau = f(t).$$

$$\|$$

$$\int_{-\infty}^{\infty} f(\tau)\,\delta(\tau - t)\,d\tau = f(t)$$

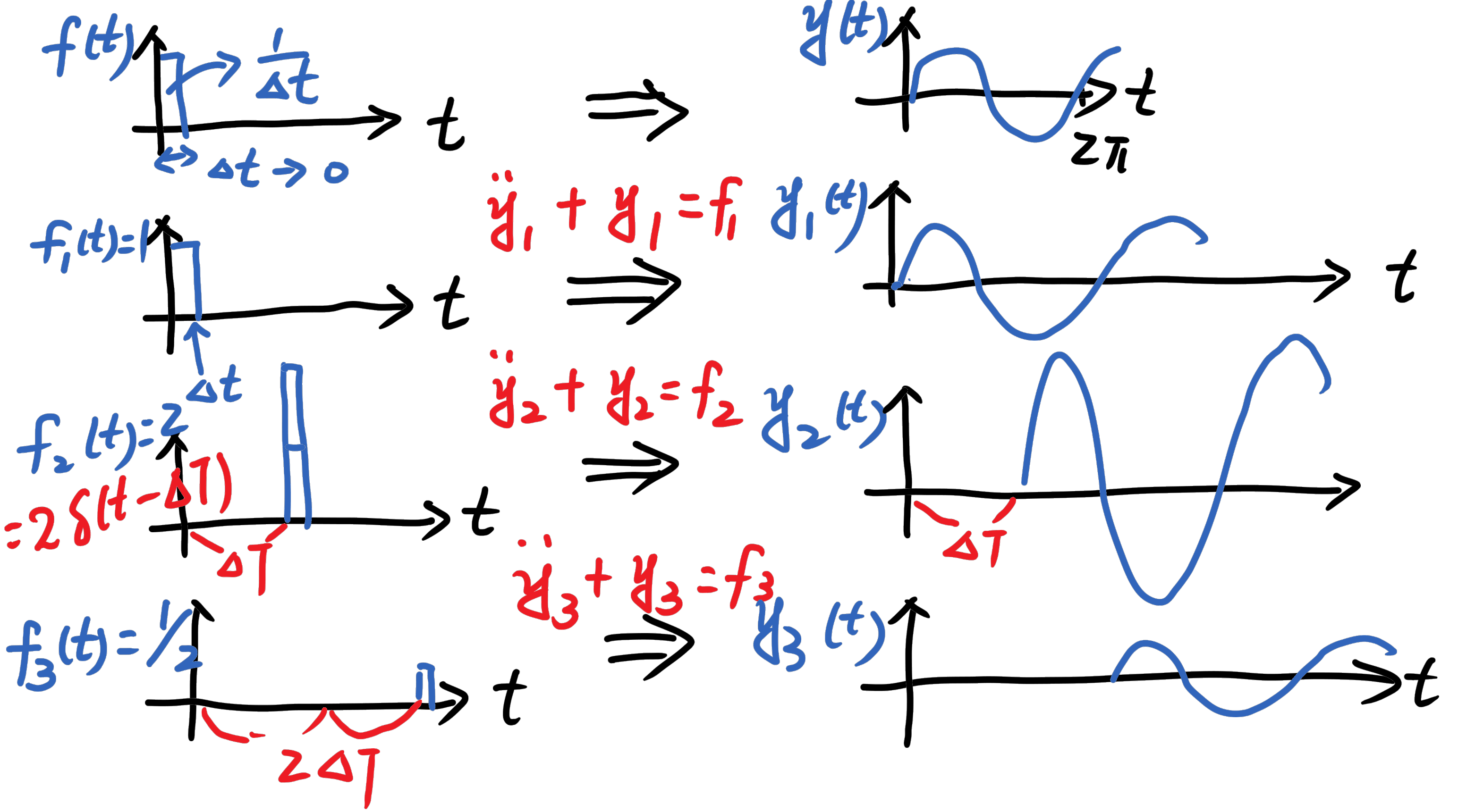
$$= \int_0^\infty e^{-s\tau}\,\delta(\tau - 0)\,d\tau$$

$$= e^{-s0} = 1$$

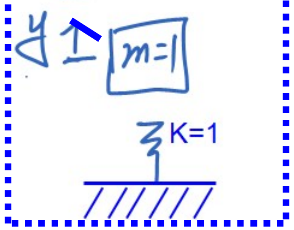
$$\Rightarrow \mathcal{L}[\delta(t)] = 1$$

$$\ddot{y} + y = f(t)$$

$\Rightarrow$ Assume $f(t) = \delta(t) \longleftrightarrow y(t) = g(t) = \sin t$




物理系統




$$\ddot{y} + y = \delta$$

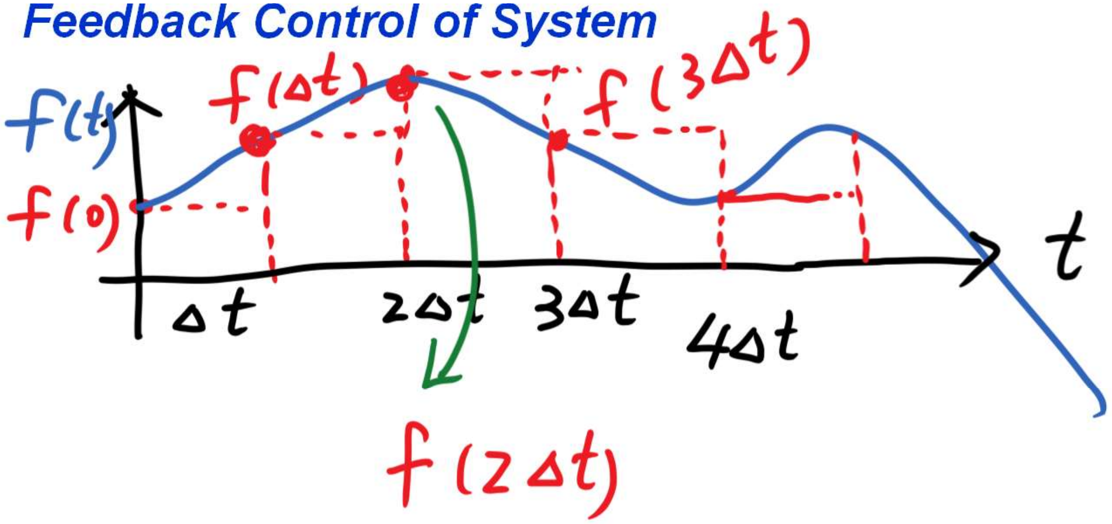
$\downarrow$ Laplace

$$s^2 Y + Y = 1$$

$$Y = \frac{1}{s^2+1}$$

$\downarrow$ $\mathcal{L}^{-1}$aplace

$$y = \sin t \ \#$$




Assume $\Delta t \to 0$

$$\Rightarrow f(t) = \sum_{n=0}^{(t/\Delta t - 1)} f(n\Delta t) \quad //$$

$$\Rightarrow y(t) = f(0)\sin t + f(\Delta t)\sin(t-\Delta t) + f(2\Delta t)\sin(t-2\Delta t) + \cdots\cdots$$

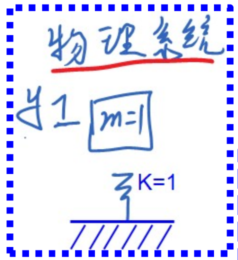
$$= \sum_{n=0}^{N-1} f(n\Delta t)\sin(t-n\Delta t)$$

$$= \sum_{n=0}^{N-1} f(n\Delta t)\cdot g(t-n\Delta t)$$

as $\Delta t \to 0$

$$\Rightarrow y(t) = \int_0^t f(\tau)\, g(t-\tau)\, d\tau$$

$$= f(t) * g(t) \quad \text{convolution}$$




$\ddot{y} + y = \delta$

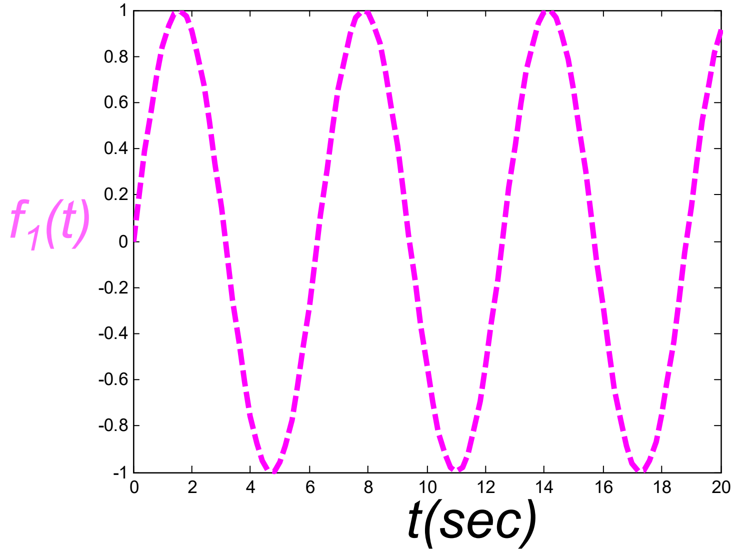
↓ Laplace

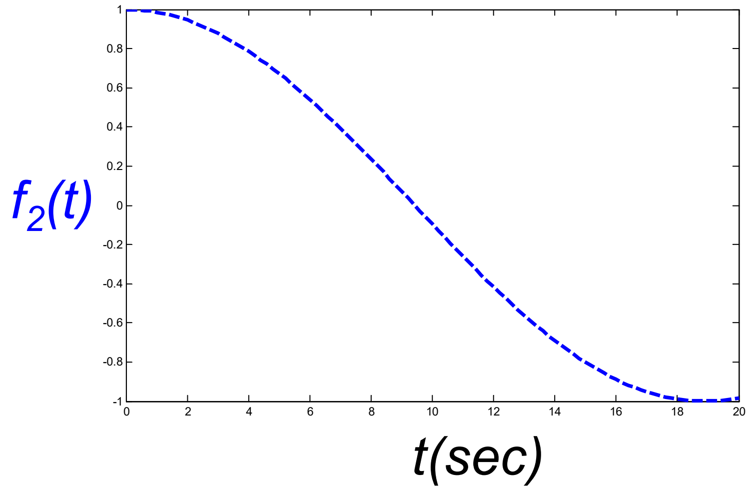
$s^2 Y + Y = 1$

$Y = \dfrac{1}{s^2+1}$

↓ $\text{Laplace}^{-1}$

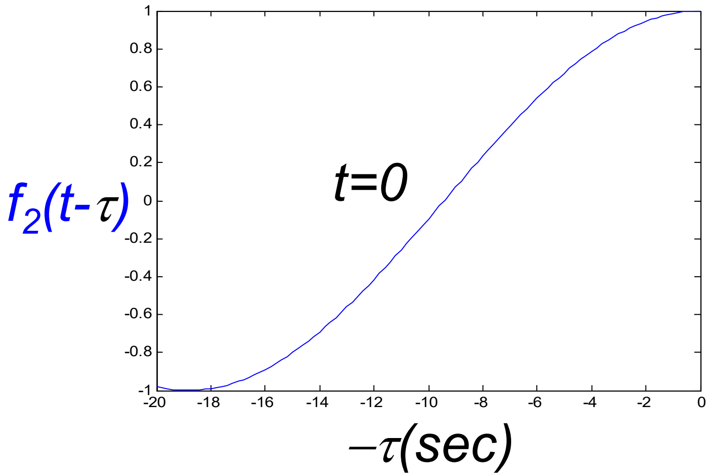
$y = \sin t$ ※

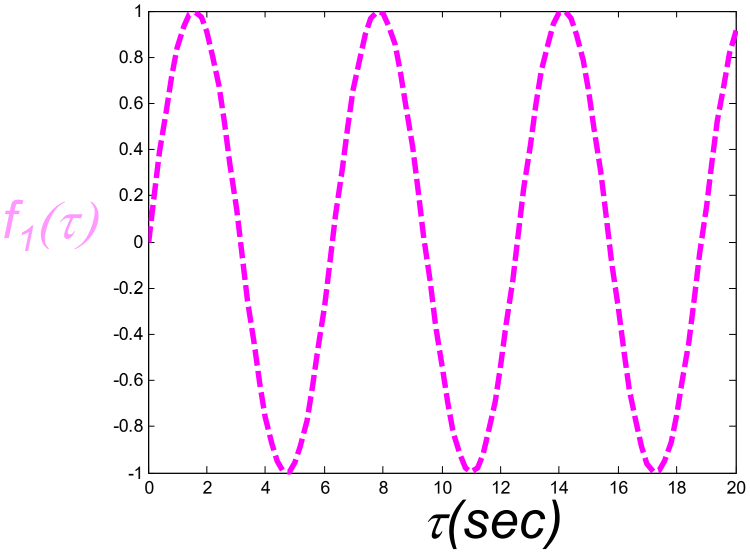


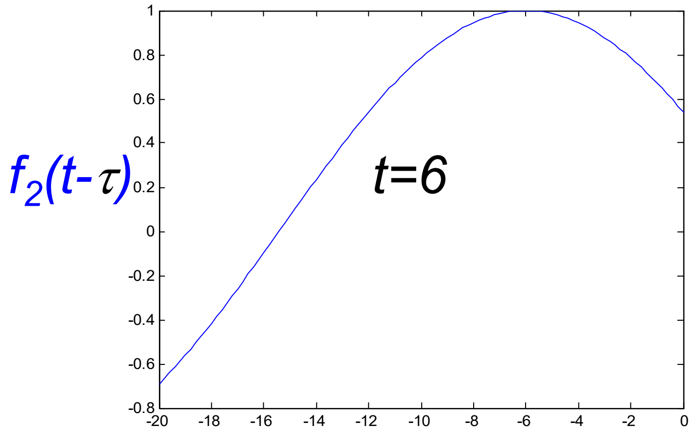




# Convolution

$$f_1(t) * f_2(t) = \int_0^t f_1(\tau) f_2(t-\tau)\, d\tau$$

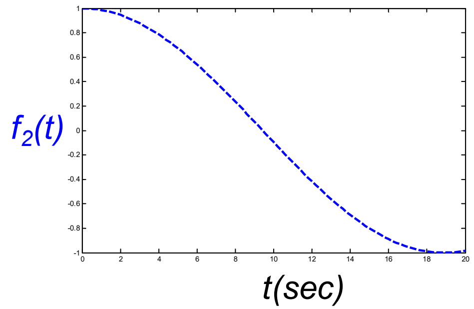


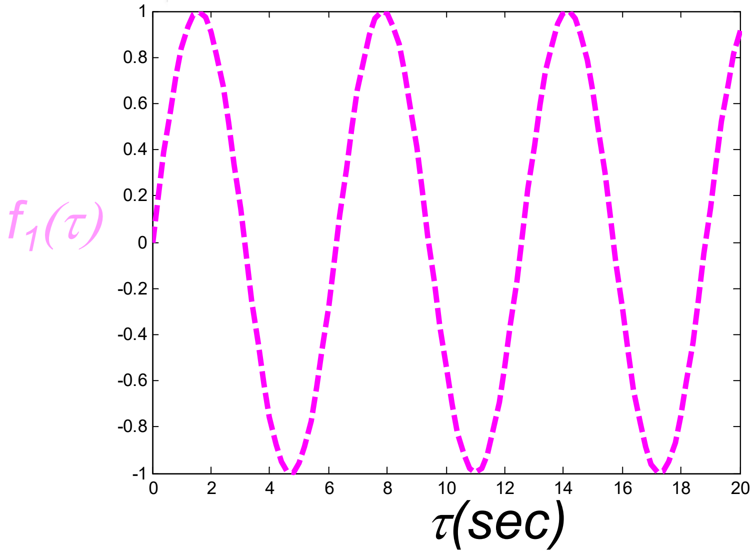



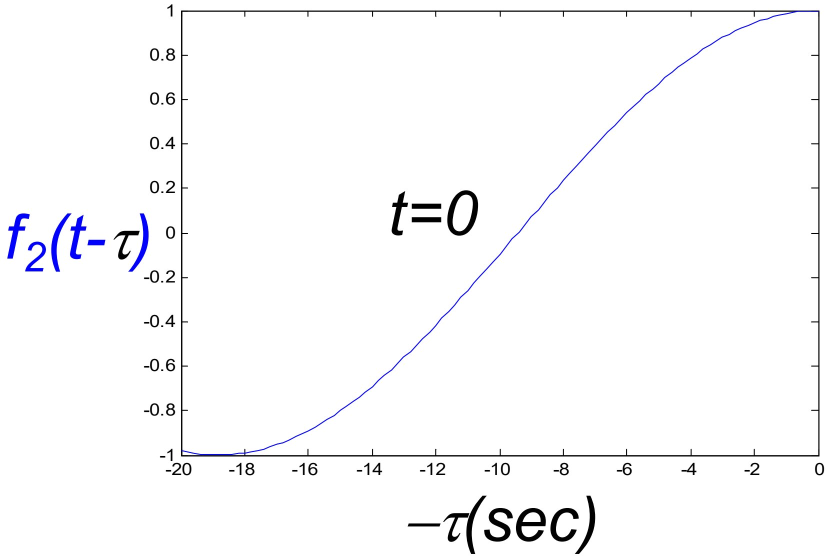



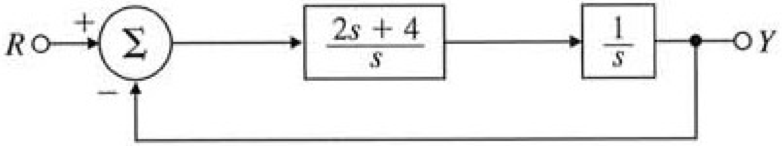
# Convolution

$$f_1(t) * f_2(t) = \int_0^t f_1(\tau) f_2(t-\tau)\, d\tau$$








# Convolution

$$\mathcal{L}\{f_1(t)\} = F_1(s) \text{ and } \mathcal{L}\{f_2(t)\} = F_2(s)$$

$$\Rightarrow \mathcal{L}\{f_1(t) * f_2(t)\} = \int_0^\infty f_1(t) * f_2(t) e^{-st}\, dt = \int_0^\infty \left[\int_0^t f_1(\tau) f_2(t-\tau)\, d\tau\right] e^{-st}\, dt$$

$$= \int_0^\infty \int_\tau^\infty f_1(\tau) f_2(t-\tau) e^{-st}\, dt\, d\tau$$

Multiplying by $e^{-s\tau} e^{s\tau}$

$$= \int_0^\infty f_1(\tau) e^{-s\tau} \left[\int_\tau^\infty f_2(t-\tau) e^{-s(t-\tau)}\, dt\right] d\tau$$

change variables $t' \triangleq t - \tau$

$$= \int_0^\infty f_1(\tau) e^{-st}\, d\tau \int_0^\infty f_2(t') e^{-st'}\, dt' \quad = F_1(s) F_2(s)$$

$$\Rightarrow \mathcal{L}^{-1}\{F_1(s) F_2(s)\} = f_1(t) * f_2(t)$$

## Inverse Laplace Transform by Partial-Fraction Expansion

$m < n$

$$F(s) = \frac{b_1 s^m + b_2 s^{m-1} + \cdots + b_{m+1}}{s^n + a_1 s^{n-1} + \cdots + a_n} = \frac{C_1}{s-p_1} + \frac{C_2}{s-p_2} + \cdots + \frac{C_n}{s-p_n}. \quad (3.43)$$

$$(s-p_1)F(s) = C_1 + \frac{s-p_1}{s-p_2}C_2 + \cdots + \frac{(s-p_1)C_n}{s-p_n}. \quad (3.44)$$

$$C_1 = (s-p_1)F(s)|_{s=p_1} \qquad C_i = (s-p_i)F(s)|_{s=p_i}$$

**EXAMPLE 3.9** $Y(s) = \dfrac{(s+2)(s+4)}{s(s+1)(s+3)}$ Find $y(t)$

$$Y(s) = \frac{C_1}{s} + \frac{C_2}{s+1} + \frac{C_3}{s+3}$$

$$C_1 = \left.\frac{(s+2)(s+4)}{(s+1)(s+3)}\right|_{s=0} = \frac{8}{3}$$

$$C_2 = \left.\frac{(s+2)(s+4)}{s(s+3)}\right|_{s=-1} = -\frac{3}{2}$$

$$C_3 = \left.\frac{(s+2)(s+4)}{s(s+1)}\right|_{s=-3} = -\frac{1}{6}$$

$$\Rightarrow Y = \frac{8/3}{s} + \frac{\left(-\frac{3}{2}\right)}{s+1} + \frac{\left(-\frac{1}{6}\right)}{s+3}$$

$$\Rightarrow y(t) = \frac{8}{3}1(t) - \frac{3}{2}e^{-t}1(t) - \frac{1}{6}e^{-3t}1(t)$$

Ex $F(s)=\dfrac{s^5+1}{(s+1)(s+2)(s+3)^3}$

$$=1+\underline{\left(\frac{A}{s+1}+\frac{B}{s+2}+\frac{C}{(s+3)^3}+\frac{D}{(s+3)^2}+\frac{E}{(s+3)}\right)}$$

⇨ 解 A, B, C, D, E.

$$\begin{cases} A=\lim\limits_{s\to(-1)} F\cdot(s+1) \\ B=\lim\limits_{s\to(-2)} F\cdot(s+2) \\ C=\lim\limits_{s\to(-3)} F\cdot(s+3)^3=\lim\limits_{s\to(-3)}\left[(s+3)^3+A\dfrac{(s+3)^3}{s+1}+B\dfrac{(s+3)^3}{s+2}+C+D(s+3)+E(s+3)^2\right] \end{cases}$$

$$\begin{cases} D = \lim\limits_{s \to (-3)} \dfrac{1}{1!} \dfrac{d}{ds} \left[ F \cdot (s+3)^3 \right] \\ E = \lim\limits_{s \to (-3)} \dfrac{1}{2!} \dfrac{d}{ds^2} \left[ F \cdot (s+3)^3 \right] \end{cases}$$

for E only $\Rightarrow \lim\limits_{s \to \infty} s \cdot \left[ \underline{F(s) - 1} \right]$

$$= \lim_{s \to \infty} s \cdot \frac{s^5 + 1 - (s^5 + 12s^4 + 56s^3 + 126s^2 + 135s + 54)}{(s+1)(s+2)(s+3)^3}$$

$$= -12 = A + B + E$$

## Summary

### The $\mathcal{L}_-$ Laplace Transform

### Inverse Laplace Transform

Laplace transform of $f(t)$, denoted by $\mathcal{L}_-\{f(t)\} = F(s)$

$s = \sigma + j\omega,$

背

$$F(s) \triangleq \int_{0^-}^{\infty} f(t)e^{-st}\,dt. \tag{3.24}$$

$$f(t) = \frac{1}{2\pi j}\int_{\sigma_c - j\infty}^{\sigma_c + j\infty} F(s)e^{st}\,ds, \tag{3.25}$$

背

Basic properties of impulse (δ) function and step (u) function!

$$\begin{cases} \mathcal{L}^{-1}[1] = \delta(t) \\ \mathcal{L}^{-1}\left[\frac{1}{s}\right] = u(t) \end{cases}$$

對

$$\longleftrightarrow \begin{cases} \mathcal{L}^{-1}[s] = \delta'(t) \\ \mathcal{L}^{-1}[s^n] = \delta^{n}(t) \end{cases}$$

對

$$\mathcal{L}[t^a] = \int_0^\infty e^{-st} t^a \, dt$$

令 $st = y \Rightarrow dt = dy/s$

$$= \int_0^\infty \frac{y^a}{s^a} e^{-y} \frac{dy}{s} = \frac{1}{s^{a+1}} \int_0^\infty y^a e^{-y} dy$$

$$= \frac{1}{s^{a+1}} \left[ -y^a e^{-y} \Big|_0^\infty + (a) \int_0^\infty y^{a-1} e^{-y} dy \right]$$

$$= \frac{a}{s^{a+1}} \int_0^\infty y^{a-1} e^{-y} dy$$

$$= \cdots = \frac{a!}{s^{a+1}} \int_0^\infty e^{-y} dy$$

$$= \frac{a!}{s^{a+1}} \left[ -e^{-y} \Big|_0^\infty \right]$$

$$= \frac{a!}{s^{a+1}} [-0 + 1] = \frac{a!}{s^{a+1}}$$

對

$$\Rightarrow \begin{cases} \mathcal{L}[\sin \omega t] = \dfrac{\omega}{s^2+\omega^2} & \text{背} \\ \mathcal{L}[\cos \omega t] = \dfrac{s}{s^2+\omega^2} & \text{背} \end{cases}$$

$$\mathcal{L}[e^{at}] = \int_0^{\infty} e^{at} e^{-st}\, dt = \frac{-e^{-(s-a)t}}{s-a}\Bigg|_0^{\infty}$$

$$= \frac{1}{s-a} \quad \# \quad \text{背}$$

 Textbook pp ~~123~~ (130)~~~125~~(132) 

背 **Superposition**

$$\mathcal{L}\{\alpha f(t)\} = \alpha F(s)$$

$$\mathcal{L}\{\alpha f_1(t) + \beta f_2(t)\} = \alpha F_1(s) + \beta F_2(s). \qquad (3.28)$$

背 **Time Scaling**

$$F_1(s) = \int_0^\infty f(at)e^{-st}\,dt = \frac{1}{|a|}F\left(\frac{s}{a}\right). \qquad (3.31)$$

背 **Time Delay**

$$F_1(s) = \int_0^\infty f(t-\lambda)e^{-st}\,dt = e^{-s\lambda}F(s). \qquad (3.30)$$

背 **Shift in Frequency**

$$F_1(s) = \int_0^\infty e^{-at}f(t)e^{-st}\,dt = F(s+a). \qquad (3.32)$$

背 **Differentiation**

$$\mathcal{L}\left\{\frac{df}{dt}\right\} = \int_{0^-}^\infty \left(\frac{df}{dt}\right)e^{-st}\,dt = -f(0^-) + sF(s). \qquad (3.33)$$

$$\mathcal{L}\{\ddot{f}\} = s^2F(s) - sf(0^-) - \dot{f}(0^-)$$

$$\mathcal{L}\{f^m(t)\} = s^mF(s) - s^{m-1}f(0^-) - s^{m-2}\dot{f}(0^-) - \cdots - f^{(m-1)}(0^-), \qquad (3.35)$$

背 **Integration**

$$F_1(s) = \mathcal{L}\left\{\int_0^t f(\xi)\,d\xi\right\} = \frac{1}{s}F(s), \qquad (3.36)$$

背 **Multiplication by Time**

$$\mathcal{L}\{tf(t)\} = -\frac{d}{ds}F(s). \qquad (3.40)$$

Homework



Proof: Textbook pp ~~827~~(867) (Example A.1)~~~832~~(872) (ExampleA.7)

## Reference Hint

- $$\frac{d}{dt}(e^{-st}) = s e^{-st}$$
- $$\int_0^\infty u' v \, dt = [u v]_0^\infty - \int_0^\infty u v' \, dt$$
- $$F(s) = \int_0^\infty f(t) e^{-st} dt \Rightarrow \alpha F(s) = \int_0^\infty \alpha f(t) e^{-st} dt$$
- $$\alpha F_1(s) + \beta F_2(s) = \int_0^\infty (\alpha f_1(t) + \beta f_2(t)) e^{-st} dt$$
- $$\mathcal{L}[y'(t)] = \int_0^\infty y'(t) e^{-st} dt = [y(t) e^{-st}]_0^\infty - \int_0^\infty y(t) e^{-st} dt$$

$$= 0 - y(0) + s \int_0^\infty y(t) e^{-st} dt$$

$$= s Y(s) - y(0)$$

## The Final Value Theorem 终值定理

背

If all poles of $sY(s)$ are in the left half of the $s$-plane, then

$$\Rightarrow \lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s). \tag{3.46}$$

**EXAMPLE 3.10**

$$Y(s) = \frac{3(s+2)}{s(s^2+2s+10)}$$

$$y(\infty) = sY(s)|_{s=0} = \frac{3\cdot 2}{10} = 0.6.$$

## A.1.3 The Initial Value Theorem 起始值定理

背

For any Laplace transform pair,

$$\Rightarrow \lim_{s\to\infty} sF(s) = f(0^+). \tag{A.27}$$

*Proof: Textbook pp ~~836~~(877) (A.1.3)~~~838~~(879) (A.1.4)*

Homework

Partial Expansion 整理

<1> 先化為真分式

<2> 分解後，一次分母項先解

<3> $\frac{A}{(S+a)^m} + \frac{B}{(S+a)^{m-1}} + \cdots \frac{Z}{(S+a)}$

<i> 消去 $(s+a)^m$ 後代 $(-a)$ 解 A

<ii> 代極限值 $s\underbrace{F(s)}_{\text{真分式}}\Big|_{s\to\infty}$ 解 Z

<iii> $B = \frac{1}{1!}\frac{d}{ds}[\cdots]$

$C = \frac{1}{2!}\frac{d^2}{ds^2}[\quad]$

<4> 分母二次式 $\frac{pS+Q}{(S+a)^2+b^2} = p\frac{(s+a)}{(S+a)^2+b^2} + \frac{m \cdot b}{(s+a)^2+b^2}$

$\mathcal{L}^{-1} \Rightarrow p e^{-at}\cos bt + m e^{-at}\sin bt$ #

# The $\mathcal{L}_-$ Laplace Transform

# Inverse Laplace Transform

**TABLE A.1**

Properties of Laplace Transforms

Mentioned so far

| Number | Laplace Transform | Time Function | Comment | |
|---|---|---|---|---|
| — | $F(s)$ | $f(t)$ | Transform pair | 背 |
| 1 | $\alpha F_1(s) + \beta F_2(s)$ | $\alpha f_1(t) + \beta f_2(t)$ | Superposition | p. 47 背 |
| 2 | $F(s)e^{-s\lambda}$ | $f(t-\lambda)$ | Time delay ($\lambda \geq 0$) | p. 47 背 |
| 3 | $\frac{1}{\lvert a \rvert} F\left(\frac{s}{a}\right)$ | $f(at)$ | Time scaling | p. 47 背 |
| 4 | $F(s+a)$ | $e^{-at}f(t)$ | Shift in frequency | p. 47 背 |
| 5 | $s^m F(s) - s^{m-1}f(0) - s^{m-2}\dot{f}(0) - \cdots - f^{(m-1)}(0)$ | $f^{(m)}(t)$ | Differentiation | p. 47 背 |
| 6 | $\frac{1}{s}F(s)$ | $\int_0^t f(\zeta)\,d\zeta$ | Integration | p. 47 背 |
| 7 | $F_1(s)F_2(s)$ | $f_1(t) * f_2(t)$ | Convolution | p. 33~40 背 |
| 8 | $\lim_{s\to\infty} sF(s)$ | $f(0^+)$ | Initial Value Theorem | p. 49 背 |
| 9 | $\lim_{s\to 0} sF(s)$ | $\lim_{t\to\infty} f(t)$ | Final Value Theorem | p. 49 背 |
| 10 | $\frac{1}{2\pi j}\int_{\sigma_c-j\infty}^{\sigma_c+j\infty} F_1(\zeta)F_2(s-\zeta)\,d\zeta$ | $f_1(t)f_2(t)$ | Time product | |
| 11 | $\frac{1}{2\pi}\int_{-j\infty}^{+j\infty} Y(-j\omega)U(j\omega)\,d\omega$ | $\int_0^\infty y(t)u(t)\,dt$ | Parseval's Theorem | |
| 12 | $-\frac{d}{ds}F(s)$ | $tf(t)$ | Multiplication by time | p. 47 背 |

Mentioned so far

**TABLE A.2** Table of Laplace Transforms

| Number | $F(s)$ | $f(t), t \geq 0$ | |
|---|---|---|---|
| 1 | $1$ | $\delta(t)$ | p. 20 背 |
| 2 | $1/s$ | $1(t)$ | p. 22/ p. 28 背 |
| 3 | $1/s^2$ | $t$ | p. 25-26 |
| 4 | $2!/s^3$ | $t^2$ | p. 25-26 |
| 5 | $3!/s^4$ | $t^3$ | p. 25-26 |
| 6 | $m!/s^{m+1}$ | $t^m$ | p. 25-26 背 |
| 7 | $\frac{1}{s+a}$ | $e^{-at}$ | p. 28 背 |
| 8 | $\frac{1}{(s+a)^2}$ | $te^{-at}$ | p. 28 + p. 25-26 |
| 9 | $\frac{1}{(s+a)^3}$ | $\frac{1}{2!}t^2e^{-at}$ | p. 28 + p. 25-26 |
| 10 | $\frac{1}{(s+a)^m}$ | $\frac{1}{(m-1)!}t^{m-1}e^{-at}$ | p. 28 + p. 25-26 |
| 11 | $\frac{a}{s(s+a)}$ | $1-e^{-at}$ | |
| 12 | $\frac{a}{s^2(s+a)}$ | $\frac{1}{a}(at-1+e^{-at})$ | |
| 13 | $\frac{b-a}{(s+a)(s+b)}$ | $e^{-at}-e^{-bt}$ | |
| 14 | $\frac{s}{(s+a)^2}$ | $(1-at)e^{-at}$ | |
| 15 | $\frac{a^2}{s(s+a)^2}$ | $1-e^{-at}(1+at)$ | |
| 16 | $\frac{(b-a)s}{(s+a)(s+b)}$ | $be^{-bt}-ae^{-at}$ | |
| 17 | $\frac{a}{s^2+a^2}$ | $\sin at$ | p. 27 背 |
| 18 | $\frac{s}{s^2+a^2}$ | $\cos at$ | p. 27 背 |
| 19 | $\frac{s+a}{(s+a)^2+b^2}$ | $e^{-at}\cos bt$ | p. 28 + p.27 |
| 20 | $\frac{b}{(s+a)^2+b^2}$ | $e^{-at}\sin bt$ | p. 28 + p.27 |
| 21 | $\frac{a^2+b^2}{s[(s+a)^2+b^2]}$ | $1-e^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$ | |

## EXAMPLE 3.14 *Forced Differential Equation Solution*

$$\ddot{y}(t)+5\dot{y}(t)+4y(t) = 3, \text{ where } y(0) = \alpha, \quad \dot{y}(0) = \beta$$

u(t)

$$s^2Y(s) - s\alpha - \beta + 5[sY(s) - \alpha] + 4Y(s) = \frac{3}{s} \qquad Y(s) = \frac{s(s\alpha + \beta + 5\alpha) + 3}{s(s+1)(s+4)}$$

$$Y(s) = \frac{\frac{3}{4}}{s} - \frac{\frac{3-\beta-4\alpha}{3}}{s+1} + \frac{\frac{3-4\alpha-4\beta}{12}}{s+4} \Rightarrow y(t) = \left(\frac{3}{4} + \frac{-3+\beta+4\alpha}{3}e^{-t} + \frac{3-4\alpha-4\beta}{12}e^{-4t}\right)1(t).$$

## EXAMPLE 3.15 *Forced Equation Solution with Zero Initial Conditions*

$$\ddot{y}(t)+5\dot{y}(t)+4y(t) = u(t), y(0) = 0, \dot{y}(0) = 0, u(t) = 2e^{-2t}1(t)$$

u(t)

$$s^2Y(s) + 5sY(s) + 4Y(s) = \frac{2}{s+2}. \qquad Y(s) = \frac{2}{(s+2)(s+1)(s+4)}$$

$$Y(s) = -\frac{1}{s+2} + \frac{\frac{2}{3}}{s+1} + \frac{\frac{1}{3}}{s+4} \Rightarrow y(t) = \left(-1e^{-2t} + \frac{2}{3}e^{-t} + \frac{1}{3}e^{-4t}\right)1(t)$$

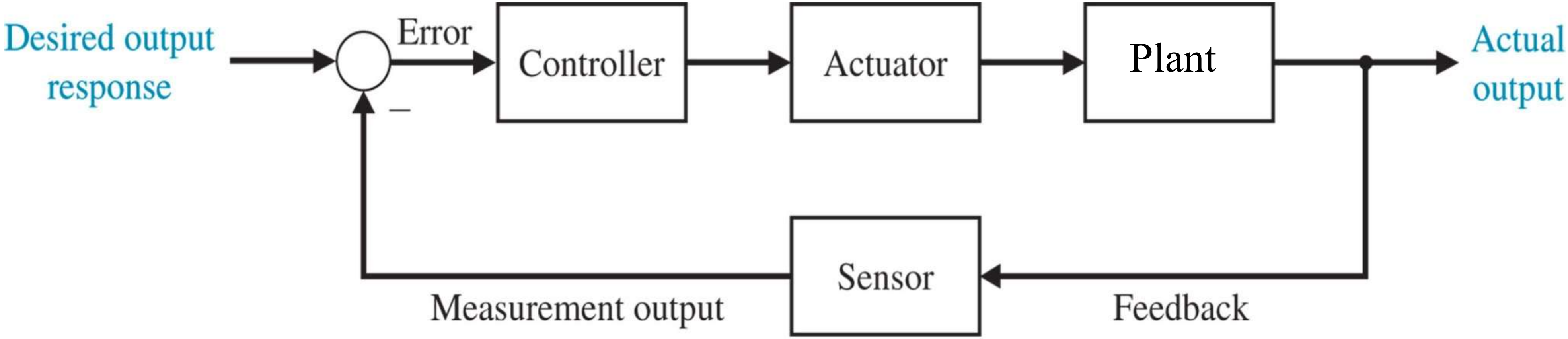
**EXAMPLE 3.5** *Frequency Response*

延後

## 3.1.9 Linear System Analysis Using MATLAB

```
numb=[0 0 100];             % form numerator
denb=[1 10.1 101];          % form denominator
sysb=tf(numb,denb);         % define system by its numerator and denominator
t=0:0.01:5;                 % form time vector
y=step(sysb,t)              % compute step response;
plot(t,y)                   % plot step response
```

# 3 Dynamic Response




3.1 Review of Laplace Transforms

3.2.1 The Block Diagram

3.3 Effect of Pole Locations

3.4 Time-Domain Specifications

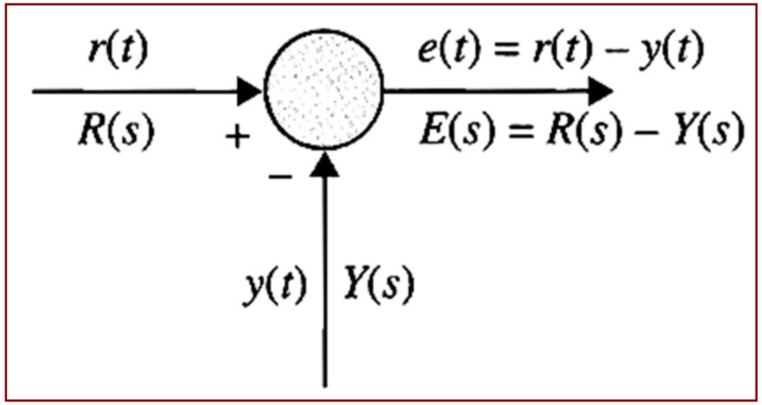
3.5 Effects of Zeros and Additional Poles

3.6.3 Routh's Stability Criterion

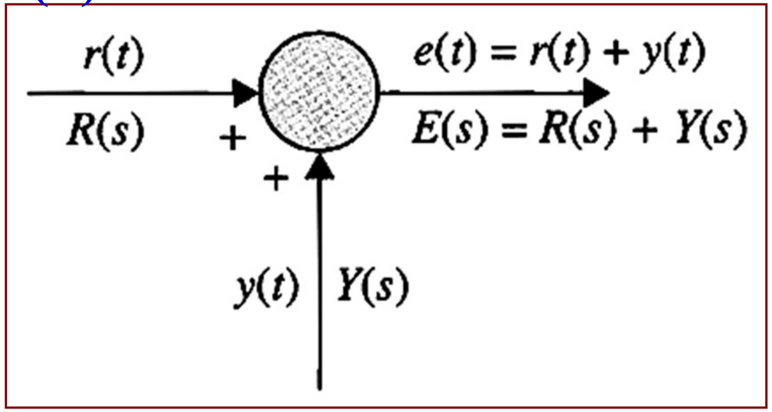
1. To study block diagrams, their components, and their underlying mathematics.
2. To obtain transfer function of systems through block diagram manipulation and reduction.

# Block-Diagram Elements of Comparators

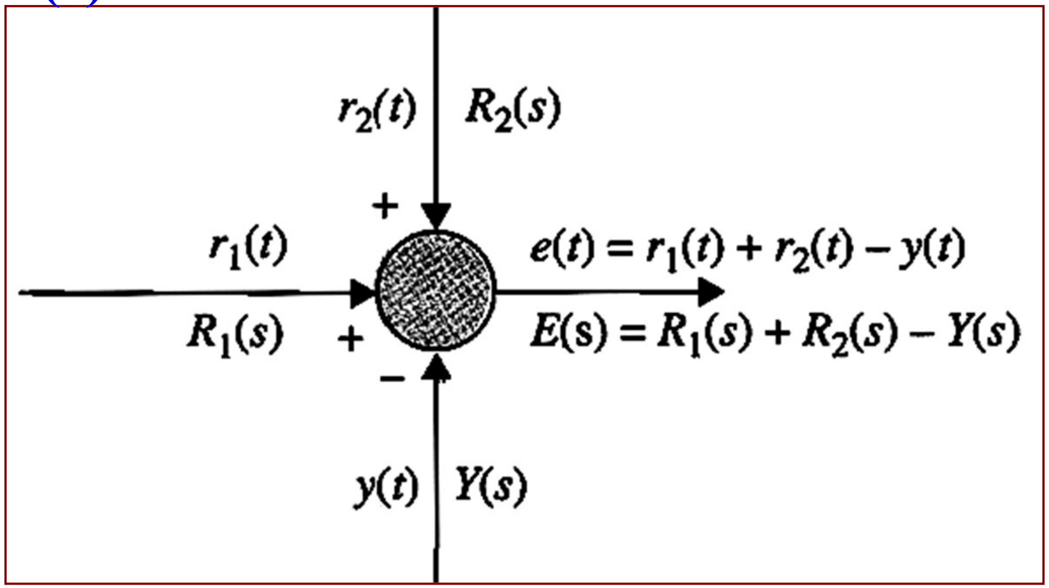
**(a) Subtraction**




**(b) Addition**



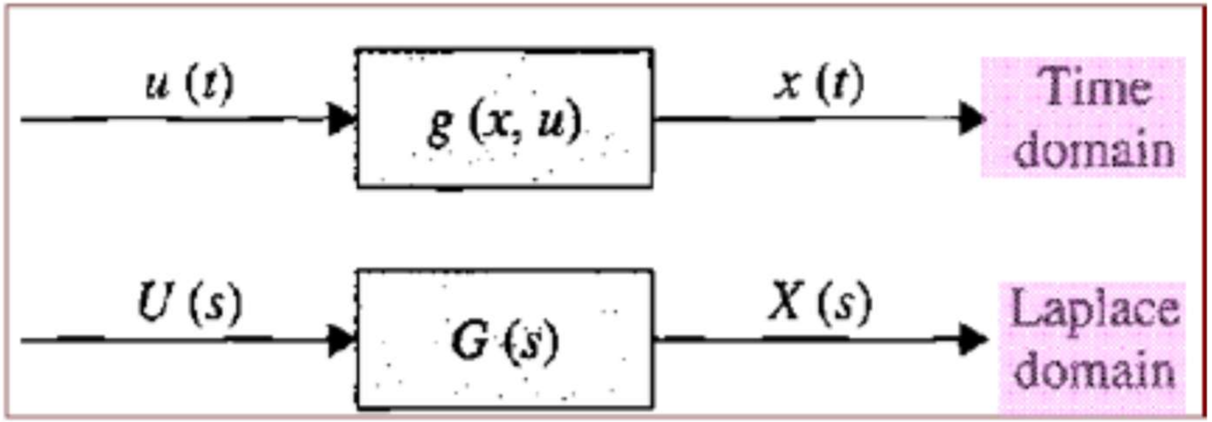

**(c) Addition and subtraction**




**Block diagram elements of control systems.**

# Time & Laplace Domain Block Diagrams

Convolution




$$X(s) = G(s)\,U(s)$$

$$G(s) = \frac{X(s)}{U(s)}$$

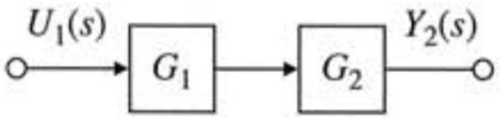
Convolution

$$x(t) = \int_0^{\infty} u(\tau) g(t-\tau) d\tau$$

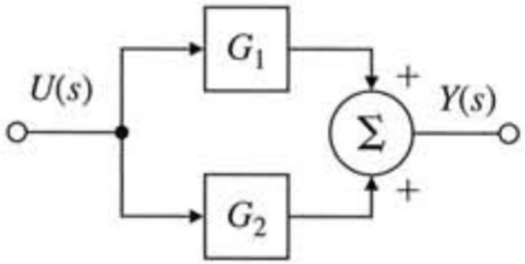
## 3.2 System Modeling Diagrams

G₁ : gain 增益

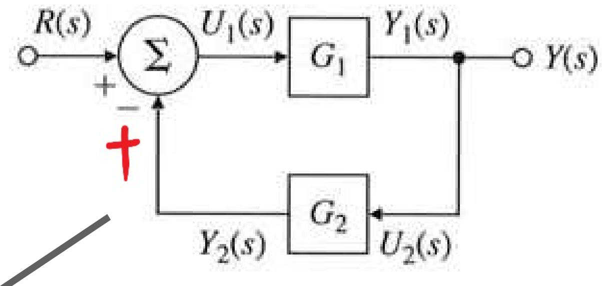
### 3.2.1 The Block Diagram




$$\frac{Y_2(s)}{U_1(s)} = G_1G_2$$




$$\frac{Y(s)}{U(s)} = G_1 + G_2$$




$$\frac{Y(s)}{R(s)} = \frac{G_1}{1 + G_1G_2}$$

**negative feedback**

Feedback Control 回授控制

$$\begin{cases} U_1(s) = R(s) \mp Y_2(s), \\ Y_2(s) = G_2(s)G_1(s)U_1(s) \\ Y_1(s) = G_1(s)U_1(s), \end{cases}$$

$$U_1 = R \mp G_2G_1U_1$$

$$R = (1 \pm G_2G_1)U_1$$

$$U_1 = \frac{1}{1 \pm G_1G_2} R$$

$$\Rightarrow Y_1(s) = \frac{G_1(s)}{1 \pm G_1(s)G_2(s)} R(s).$$

⇒ The gain of a single-loop negative feedback system is given by the forward gain divided by the sum of 1 plus the loop gain.

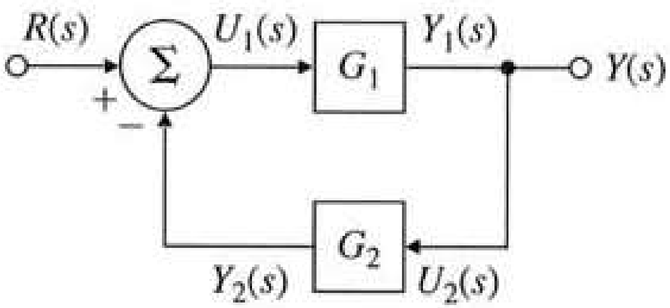
**Negative feedback:** $Y_1(s) = \dfrac{G_1(s)}{1 + G_1(s)G_2(s)} R(s).$

**Positive feedback:** $Y_1(s) = \dfrac{G_1(s)}{1 - G_1(s)G_2(s)} R(s).$

**unity feedback system** $G_2=1$

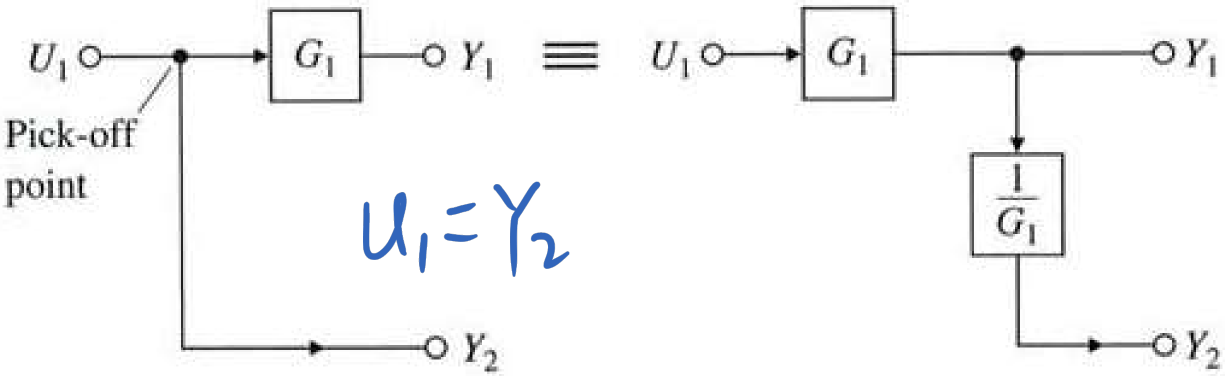
# Rules for Block Diagram Manipulation

**Rule 1:**

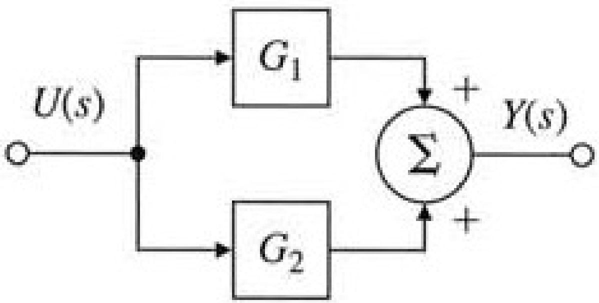



$$\frac{Y(s)}{R(s)} = \frac{G_1}{1 + G_1 G_2}$$

**Rule 2:**

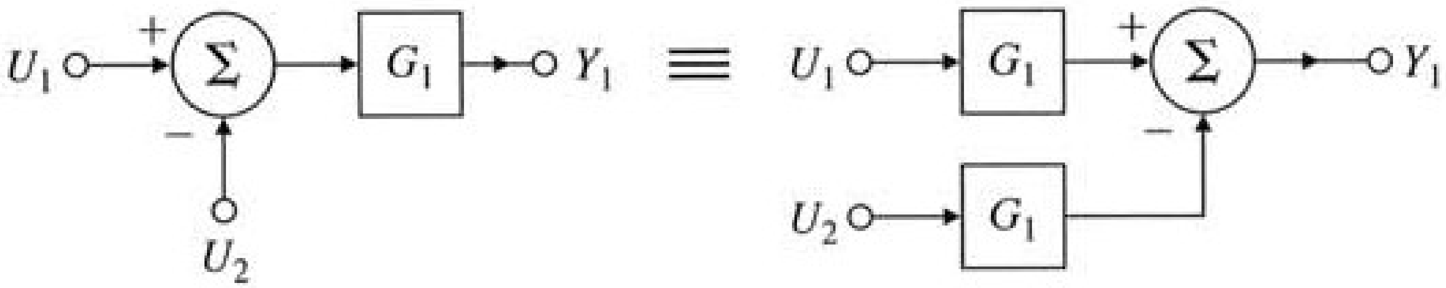



**Rule 3:**

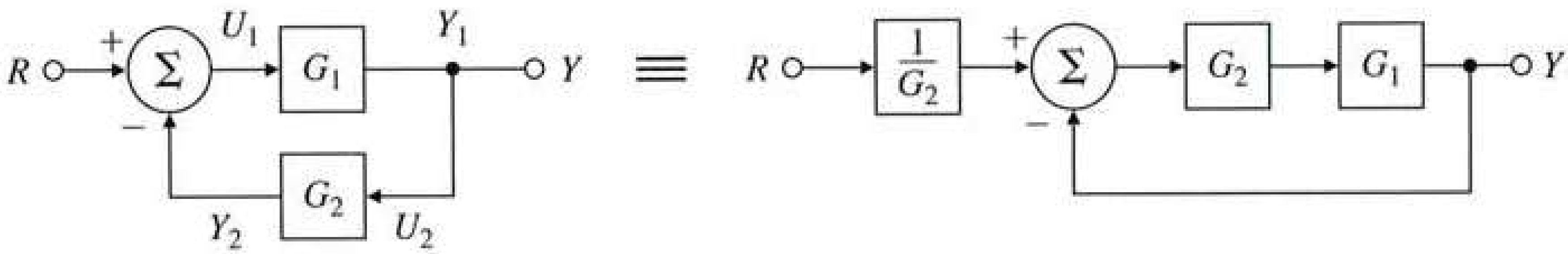



$$\frac{Y(s)}{U(s)} = G_1 + G_2$$

**Rule 4:**




**Rule 5:**

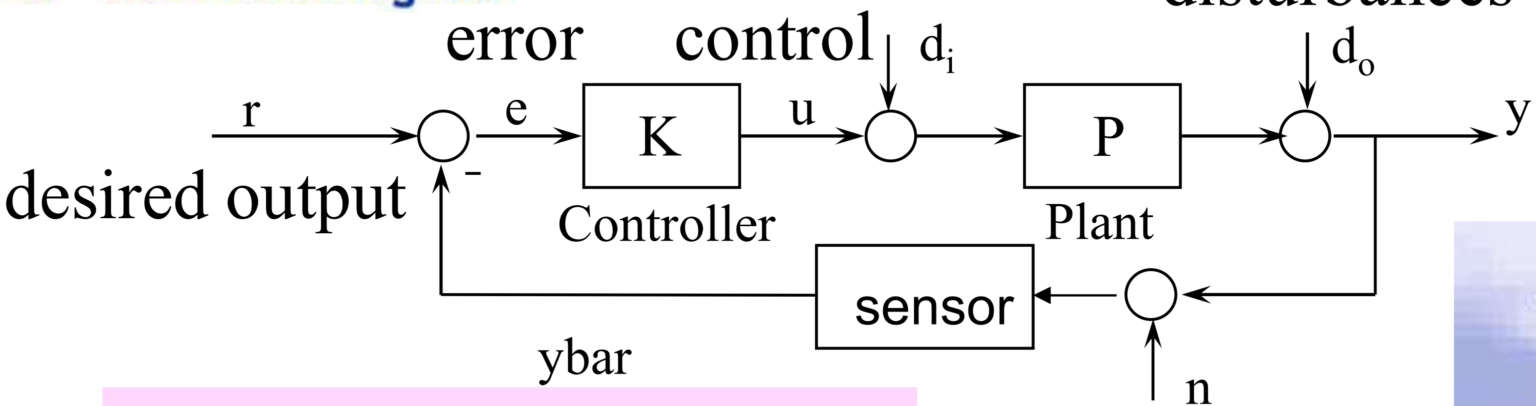



$$\frac{Y}{R} = \frac{G_1}{1 + G_1 G_2}$$

$$\frac{Y}{R\,\frac{1}{G_2}} = \frac{G_1\, G_2}{1 + G_1\, G_2}$$

# Terms

## 3.2.1 The Block Diagram




- r(t): reference input or command signal
- y(t): sensed/actual output
- e(t)= r(t) - ybar(t)
- u(t): control input
- Plant P
- ybar: estimate of y

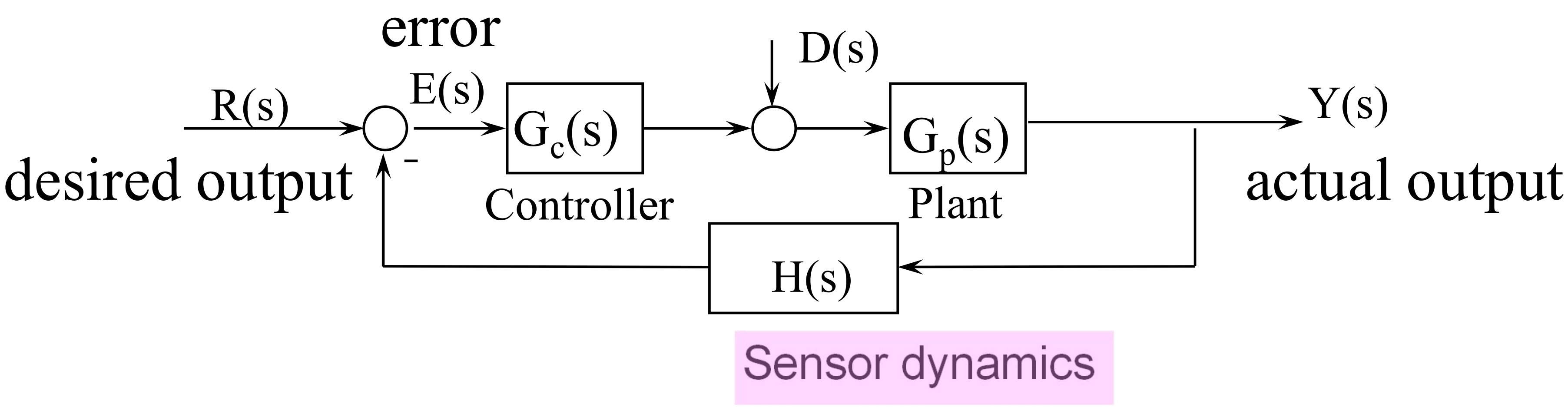
**Purpose**: find compensator/controller
- error is "small"
- stability is "good"

If r(t) is a constant -> "regulator" problem

If r(t) =function of time -> "servo" or "tracking" problem

伺服 追蹤

## 3.2.1 The Block Diagram



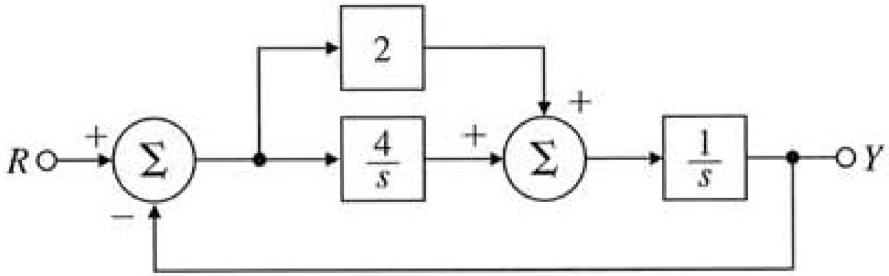

(1) Assume d(t)=0

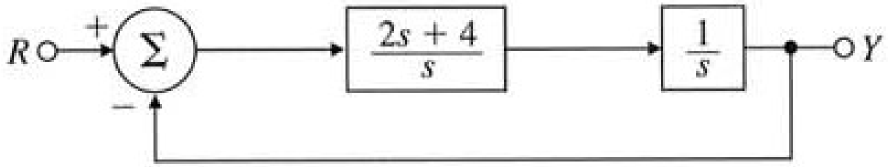
$$\frac{Y}{R} = \frac{G_c \, G_p}{1 + G_c \cdot G_p \cdot H}$$

(2) Assume r(t)=0

$$\frac{Y}{D} = \frac{G_p}{1 + G_c \cdot G_p \cdot H}$$

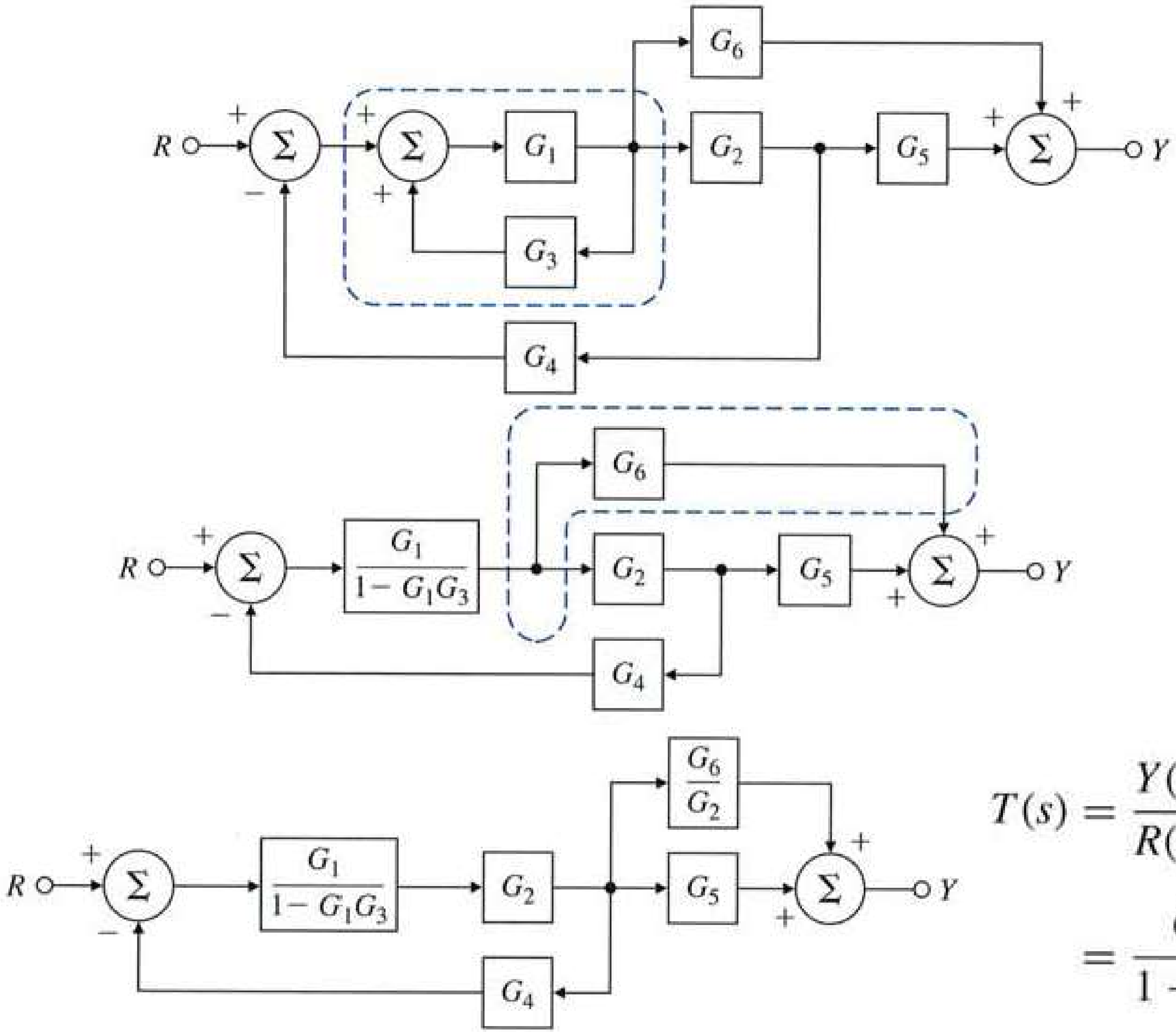
## EXAMPLE 3.20 *Transfer Function from a Simple Block Diagram*




$$T(s) = \frac{Y(s)}{R(s)} = \frac{\frac{2s+4}{s^2}}{1 + \frac{2s+4}{s^2}} = \frac{2s+4}{s^2+2s+4}$$

**EXAMPLE 3.21** *Transfer Function from the Block Diagram*




$$T(s) = \frac{Y(s)}{R(s)} = \frac{\frac{G_1G_2}{1-G_1G_3}}{1+\frac{G_1G_2G_4}{1-G_1G_3}}\left(G_5+\frac{G_6}{G_2}\right)$$

$$= \frac{G_1G_2G_5+G_1G_6}{1-G_1G_3+G_1G_2G_4}.$$

# Mason's Rule for Complicated Block Diagram

See Appendix W3.2.3 at www.fpe7e.com

## 3.2.2 Block Diagram Reduction Using MATLAB

**Chapter 1: An Overview and Brief History of Feedback Control**

**Chapter 2: Dynamic Models**

⇨ **Chapter 3: Dynamic Response**

**Chapter 4: A First Analysis of Feedback**

**Chapter 5: The Root-locus Design Method**

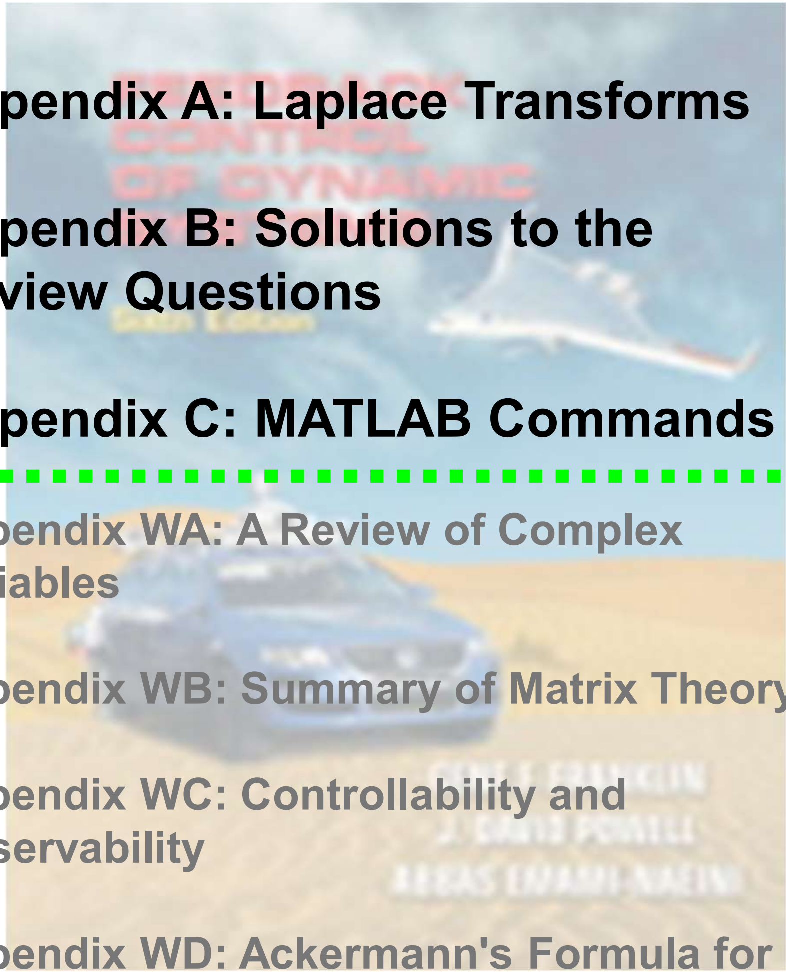
**Chapter 6: The Frequency-response Design Method**

**Chapter 7: State-space Design**

**Chapter 8: Digital Control**

**Chapter 9: Nonlinear Systems**

**Chapter 10: Control Systems Design: Principles and Case Studies**



**Appendix A: Laplace Transforms**

**Appendix B: Solutions to the Review Questions**

**Appendix C: MATLAB Commands**

**Appendix WA: A Review of Complex Variables**

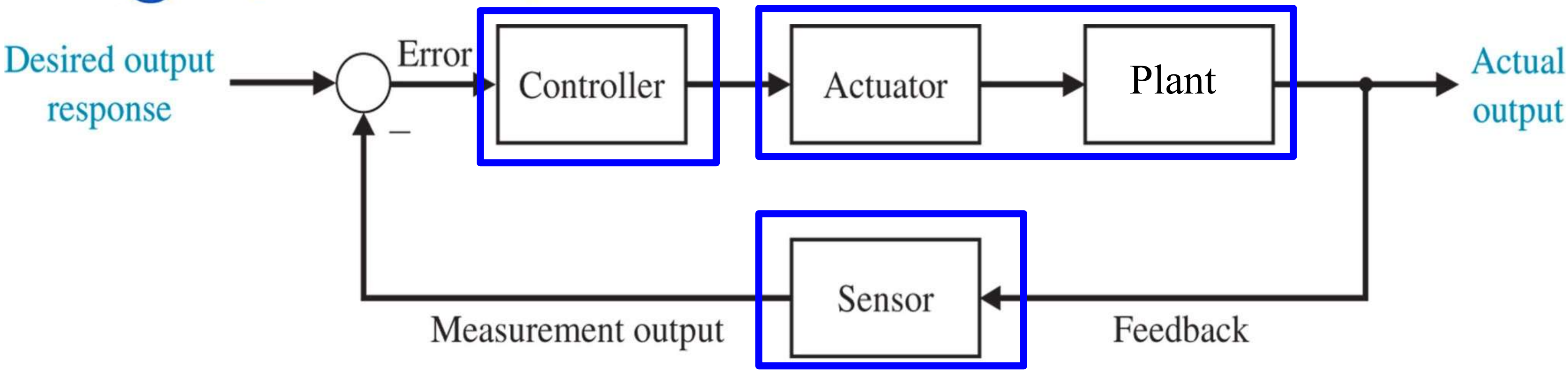
**Appendix WB: Summary of Matrix Theory**

**Appendix WC: Controllability and Observability**

**Appendix WD: Ackermann's Formula for Pole Placement**

**Appendix W2.1.4: Complex Mechanical Systems**

# 3 Dynamic Response




3.1 Review of Laplace Transforms

3.2.1 The Block Diagram

3.3 Effect of Pole Locations

3.4 Time-Domain Specifications

3.5 Effects of Zeros and Additional Poles

3.6.3 Routh's Stability Criterion

$$T.F.\quad G(s)=\frac{Y(s)}{R(s)}=\frac{b_m s^m+b_{m-1}s^{m-1}+\cdots+b_1 s+b_0}{s^n+a_{n-1}s^{n-1}+\cdots+a_1 s+a_0}$$

$$=K\frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}$$

$$\frac{Y}{R}=\frac{N(s)}{D(s)}=\frac{b_1}{s+a_1}+\frac{b_2}{s+a_2}+\cdots$$

$$+\frac{P_1 s+Q_1}{(s+c_1)^2+d_1}+\frac{P_2 s+Q}{(s+c_2)^2+d_2}+\cdots$$

# Transfer Functions

轉移函數

## LTI Dynamic System

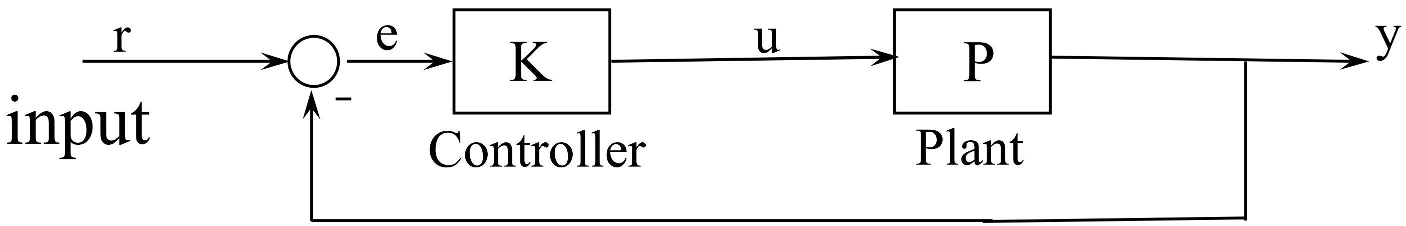
$$\Rightarrow \quad T.F. \quad G(s) = \frac{Y(s)}{F(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

$$= K \frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}$$

Def.:

1. G(s) is said to be proper if G(∞) is a finite constant -> $n \geq m$
2. G(s) is said to be strictly proper if G(∞) = 0 -> $n > m$
3. Relative order : n-m
4. The order of system: n
5. Poles: $-p_1, -p_2, \ldots, -p_n$ 極點
6. Zeros: $-z_1, -z_2, \ldots, -z_n$ 零點

Note:
- T.F. is to describe the relationship between input and output
- T.F. is independent to the input and initial condition




LTI Dynamic System

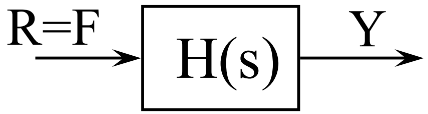
$$T.F. \quad G(s) = \frac{Y(s)}{R(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

$$= K \frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}$$

$$\Rightarrow \frac{Y}{R} = \frac{N(s)}{D(s)} = \frac{b_1}{s+a_1} + \frac{b_2}{s+a_2} + \cdots$$

$$+ \frac{P_1 s + Q_1}{(s+c_1)^2 + d_1} + \frac{P_2 s + Q}{(s+c_2)^2 + d_2} + \cdots$$

# Dynamic Response





## 3.3 Effect of Pole Locations

(1) First order system

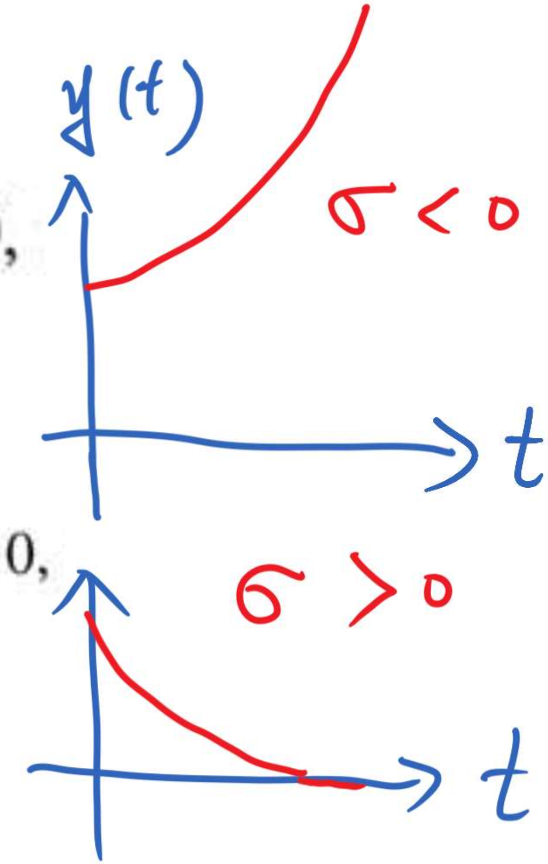
impulse response

$$\dot{y} + \sigma y = \delta(t) \Rightarrow H(s) = \frac{1}{s+\sigma}$$

$$\Rightarrow y(t) = e^{-\sigma t} u(t) \Rightarrow$$

If $\sigma < 0$,

When $\sigma > 0$,



## (1) First order system

u(t)

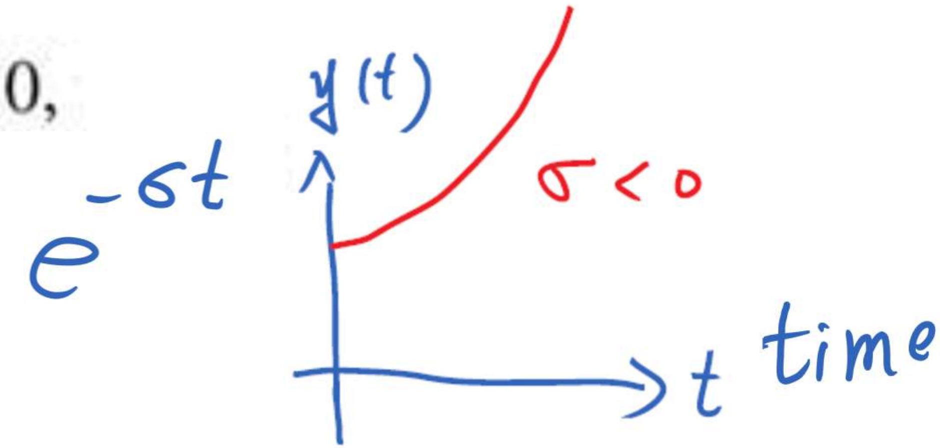
$$\dot{y} + \sigma y = \delta(t) \qquad H(s) = \frac{1}{s+\sigma} \qquad h(t) = e^{-\sigma t} 1(t)$$

⇨ 看 system 穩定/不穩定 ⇒ 只看 pole 的實數部正負號

(stable) (unstable) 極點

⇨ If $\sigma < 0$, the exponential expression here grows with time, the impulse response is **unstable**

⇨ If $\sigma < 0$,



$$\dot{y} + \sigma y = \delta(t)$$

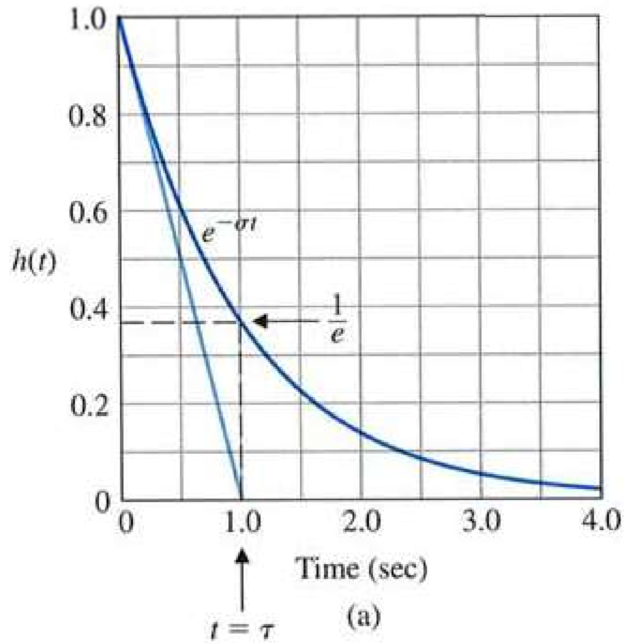
⇨ y(t)= $h(t) = e^{-\sigma t} u(t)$

$\downarrow$

$1$

⇨ When $\sigma > 0$, the pole is located at $s < 0$, the exponential expression decay the impulse response is stable. 穩定

**Figure 3.13**
First-order system response: (a) impulse response; (b) impulse response and step response using MATLAB®




⇨ time constant $\tau = 1/\sigma$

$k=1$ $u(t)$

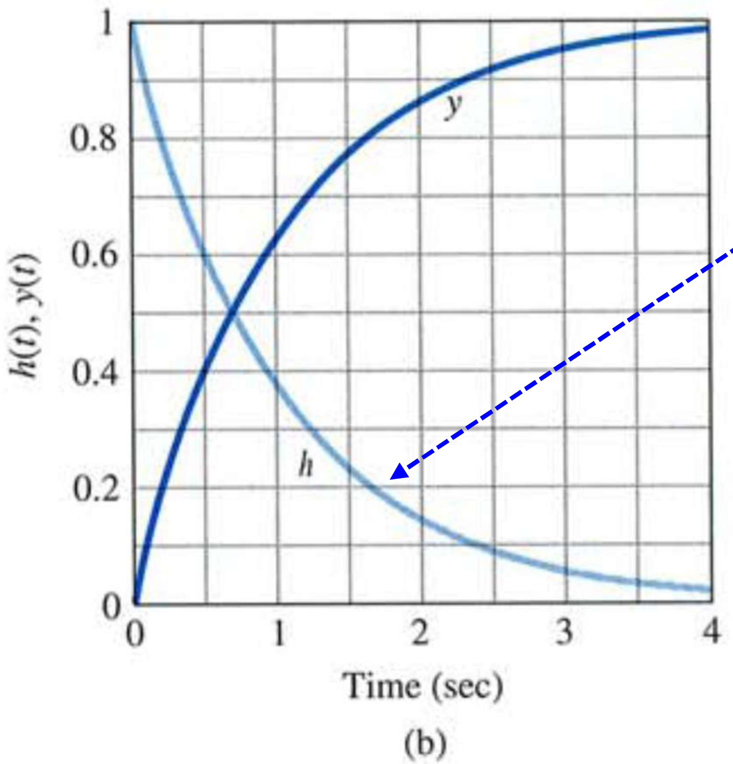
$$\dot{y} + ky = 1(t).$$

(unit) step response

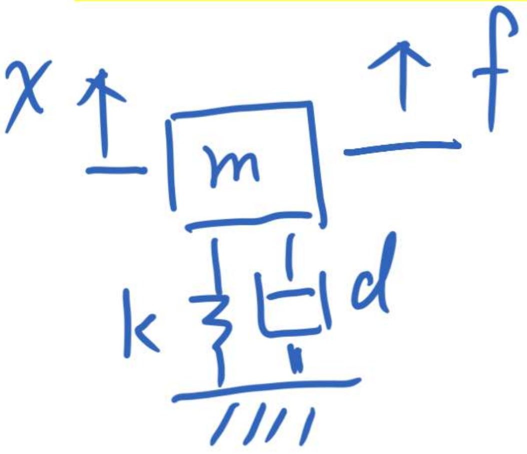
$$\mathcal{L}\{ \quad sY + Y = \frac{1}{s} \Rightarrow Y = \frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1} \xRightarrow{\mathcal{L}^{-1}} y = u(t) - e^{-t}u(t)$$

**Figure 3.13**

First-order system response: (a) impulse response; (b) impulse response and step response using MATLAB®



(2) Standard second order system

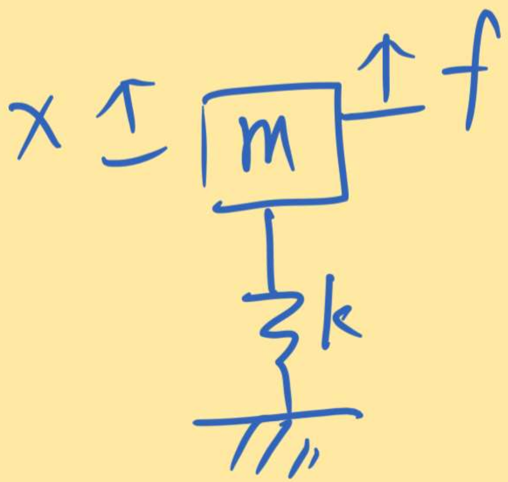



$$m\ddot{x}+d\dot{x}+kx=f \qquad \frac{X}{F}=\frac{1}{ms^2+ds+k}$$

spring constant

damping coefficient

如果 $d=0$




$$m\ddot{x}+kx=f \Rightarrow \ddot{x}+\frac{k}{m}x=f/m$$

$$\left(\frac{1}{2\pi}\right)\sqrt{\frac{k}{m}} \Rightarrow$$ 共振頻率

$$\ddot{x}+w_n^{\,2}x=e^t$$

$$x=c_1\sin(w_n t)+c_2\cos(w_n t)+Ae^t$$

$$=c\cos(w_n t+\Phi)+Ae^t$$

$$\left\{ \begin{aligned} &\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \\ &A\sin(\alpha) + B\cos(\alpha) = C\cos(\alpha - \beta) \\ &\qquad C = \sqrt{A^2 + B^2} \\ &\qquad \beta = \tan^{-1}\left(\frac{A}{B}\right) \end{aligned} \right\}$$

## (2) Standard second order system

(a good approximation to behavior of many real systems)

$$\frac{Y(s)}{R(s)} = H(s) = \frac{w_n^{\ 2}}{s^2 + 2\xi w_n s + w_n^{\ 2}} = 0 \Rightarrow s = -\sigma \pm j\omega_d$$

$$= -\xi w_n \pm \sqrt{\xi^2 w_n^2 - w_n^2}$$

$$= -\xi w_n \pm w_n\sqrt{1-\xi^2}\, j$$

$$(s + \underbrace{\sigma - j\omega_d}_{A})(s + \underbrace{\sigma + j\omega_d}_{B}) = (s+\sigma)^2 + \omega_d^2$$

$= A + B$

$$\sigma = \zeta\omega_n \quad \text{and} \quad \omega_d = \omega_n\sqrt{1-\zeta^2},$$

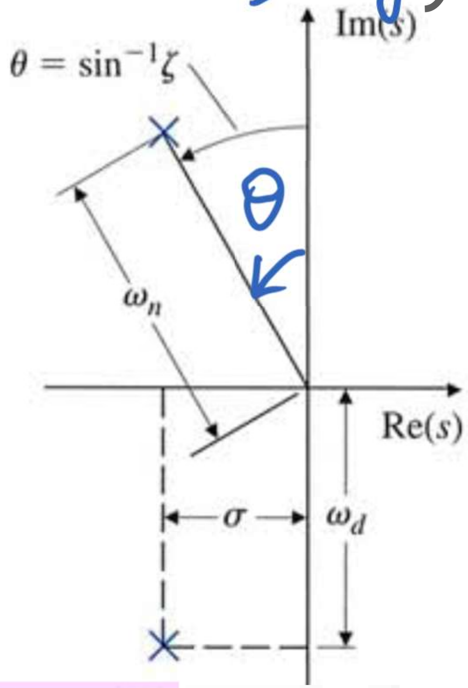
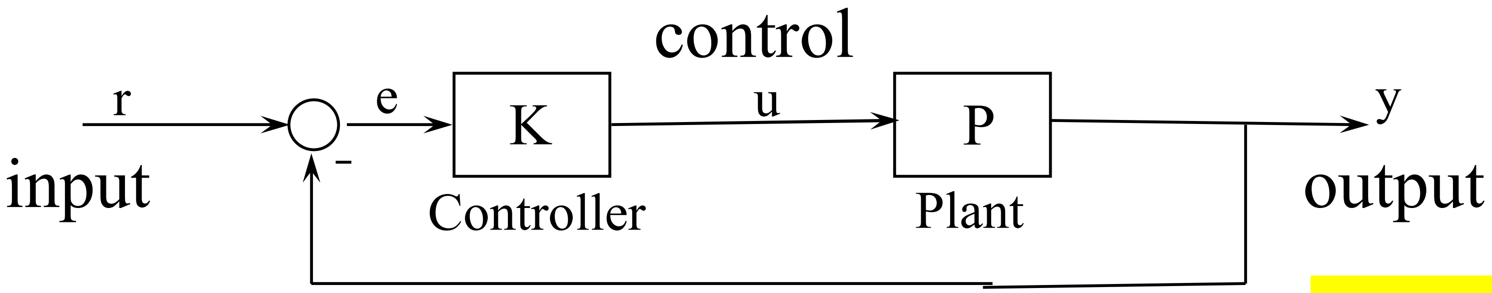



**Figure 3.17** s-plane plot for a pair of complex poles

- $\zeta$ is the **damping ratio**
- $\omega_n$ is the **undamped natural frequency**
- $\omega_d$ damped natural frequency

$\zeta = 0$, we have no damping, $\theta = 0$, and the damped natural frequency $\omega_d = \omega_n$




0=<ξ<1

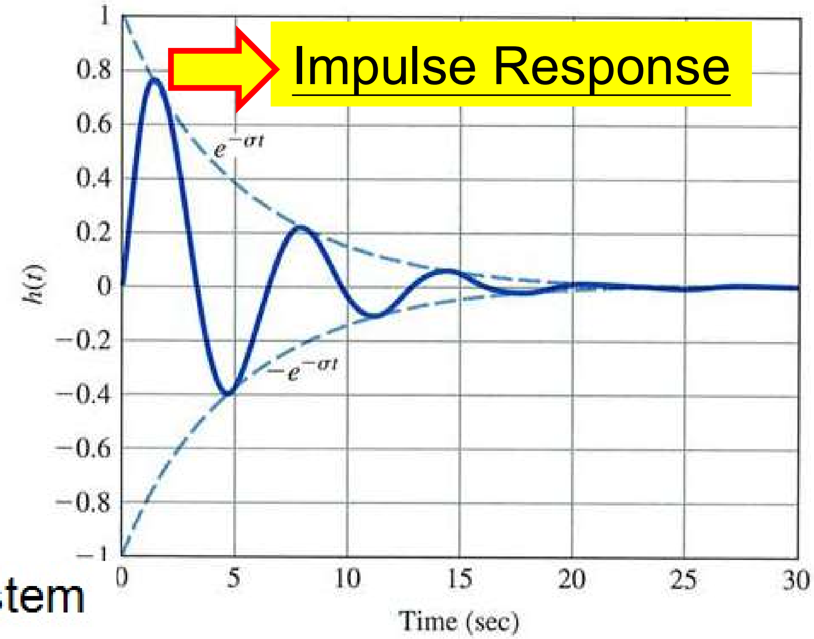
$$\frac{Y(s)}{R(s)} = H(s) = \frac{w_n^2}{s^2 + 2\xi w_n s + w_n^2} = \frac{\omega_n^2}{(s+\zeta\omega_n)^2 + \omega_n^2(1-\zeta^2)}.$$

$\rightarrow (w_n\sqrt{1-\xi^2})^2$

$\begin{cases} R = 1 \\ r = \delta(t) \end{cases}$

$$\Rightarrow h(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\sigma t}(\sin \omega_d t)1(t). \qquad (3.58)$$

**Figure 3.20**
Second-order system response with an exponential envelope




$\Rightarrow \left\{ \sigma = \zeta\omega_n \quad \text{and} \quad \omega_d = \omega_n\sqrt{1-\zeta^2} \right\}$

$\Rightarrow$ $(\sigma, w_d)$ or $(\xi, w_n)$ define the behavior of dynamic system

$$\frac{Y(s)}{R(s)} = H(s) = \frac{w_n^{\ 2}}{s^2 + 2\xi w_n s + w_n^{\ 2}} = \frac{\omega_n^2}{(s+\zeta\omega_n)^2 + \omega_n^2(1-\zeta^2)}.$$

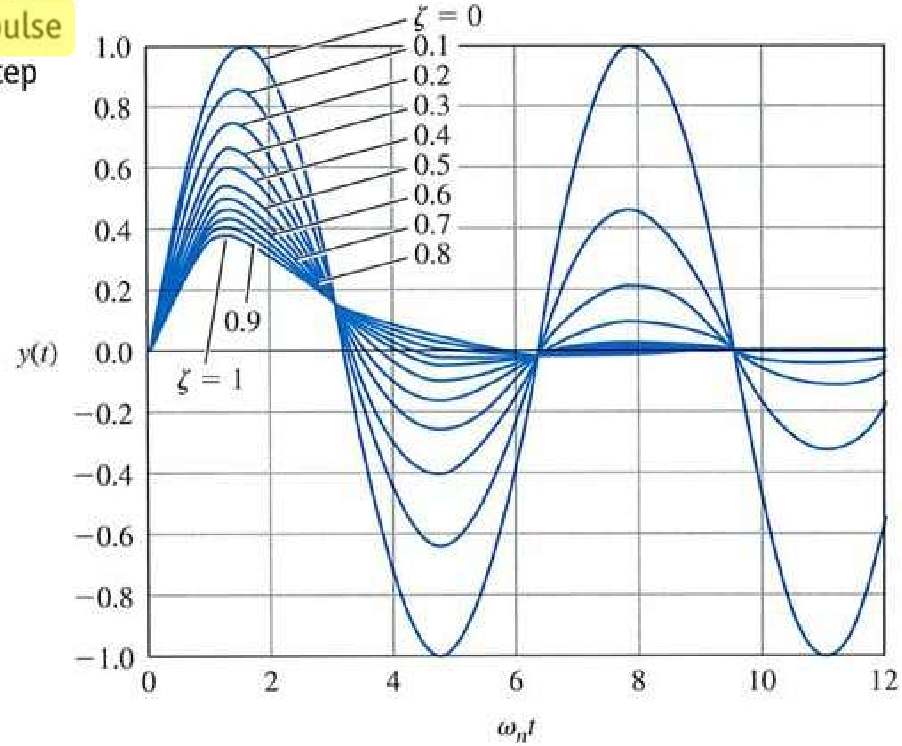
$$\begin{cases} R=1 \\ r=\delta(t) \end{cases} \Rightarrow h(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\sigma t}(\sin \omega_d t)1(t).$$

$0 =< \xi < 1$

$$\{\sigma = \zeta\omega_n \quad \text{and} \quad \omega_d = \omega_n\sqrt{1-\zeta^2}\}$$

$$\begin{cases} y(0) = 0 \\ y'(0) = w_n^2 \end{cases}$$

**Figure 3.18**
Responses of second-order systems versus $\zeta$: (a) impulse responses; (b) step responses




$$\begin{cases} R=1 \to r(t) = \delta(t) \\ \quad \Rightarrow y(t) \text{ impulse response} \\ R = \frac{1}{s} \to r(t) = u(t) \\ \quad \Rightarrow y(t) \text{ step response} \end{cases}$$

$$\begin{cases} y(0) = 0 \\ y'(0) = 0 \end{cases}$$



$$\frac{Y(s)}{R(s)} = H(s) = \frac{w_n^{\ 2}}{s^2 + 2\xi w_n s + w_n^{\ 2}} = \frac{\omega_n^2}{(s+\zeta\omega_n)^2 + \omega_n^2(1-\zeta^2)}.$$

$Y = (H(s))\frac{1}{s}$

0=<ξ<1

(b) step responses $y(t) = 1 - e^{-\sigma t}\left(\cos\omega_d t + \frac{\sigma}{\omega_d}\sin\omega_d t\right),$ $\mathcal{L}^{-1} \Rightarrow$

(1) ξ=0 -> undamped system

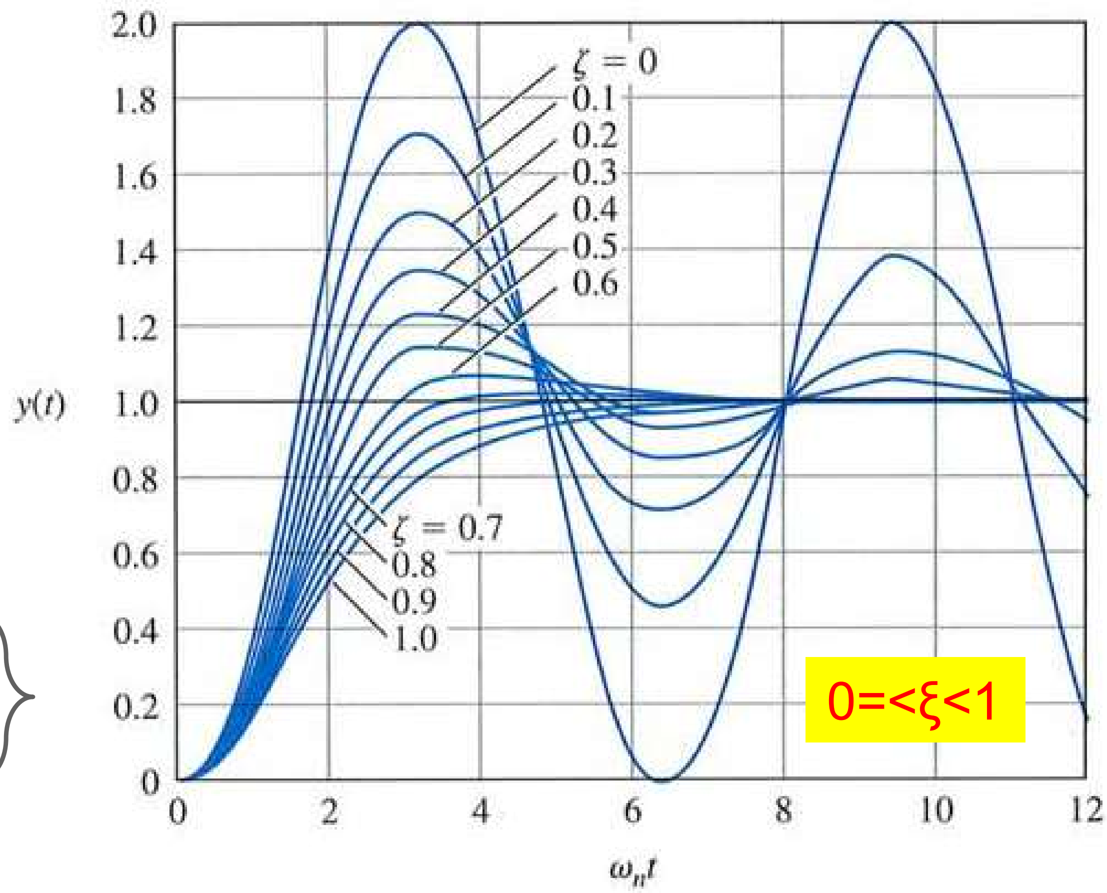
(2) 0<ξ<1 -> under damped system

(3) ξ=1 -> critical damped system

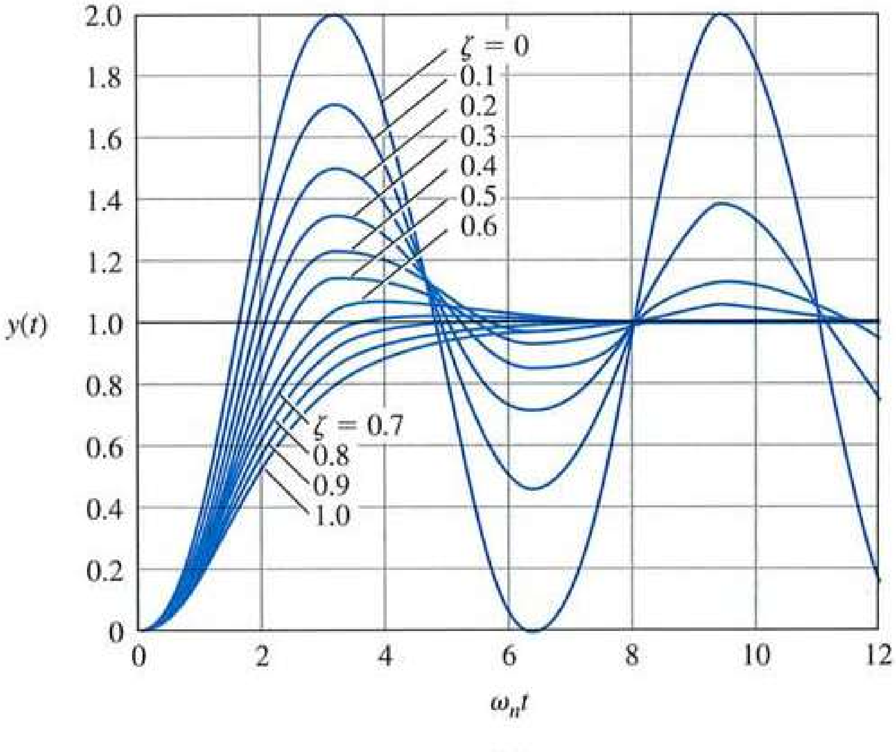
(4) ξ>1 -> over damped system

0=<ξ<1

$\sigma = \zeta\omega_n$ and $\omega_d = \omega_n\sqrt{1-\zeta^2}$




(b)



(b)

$$y(0) = 0 \qquad y'(0) = 0$$

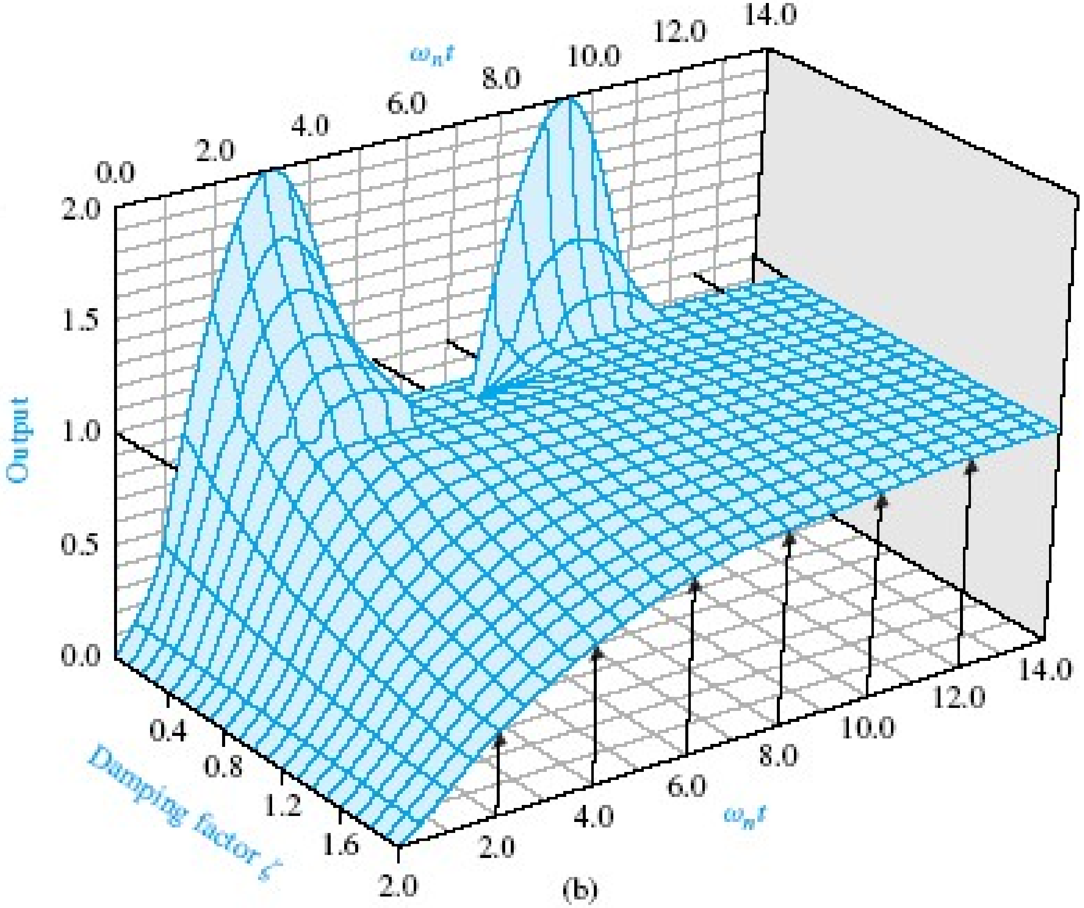
$$\frac{Y(s)}{R(s)} = H(s) = \frac{w_n^{\,2}}{s^2 + 2\xi w_n s + w_n^{\,2}}$$

Step Response

0=<ξ<1

$$y(t) = 1 - e^{-\sigma t}\left(\cos\omega_d t + \frac{\sigma}{\omega_d}\sin\omega_d t\right)$$

$$\left\{\sigma = \zeta\omega_n \quad \text{and} \quad \omega_d = \omega_n\sqrt{1-\zeta^2}\right\}$$



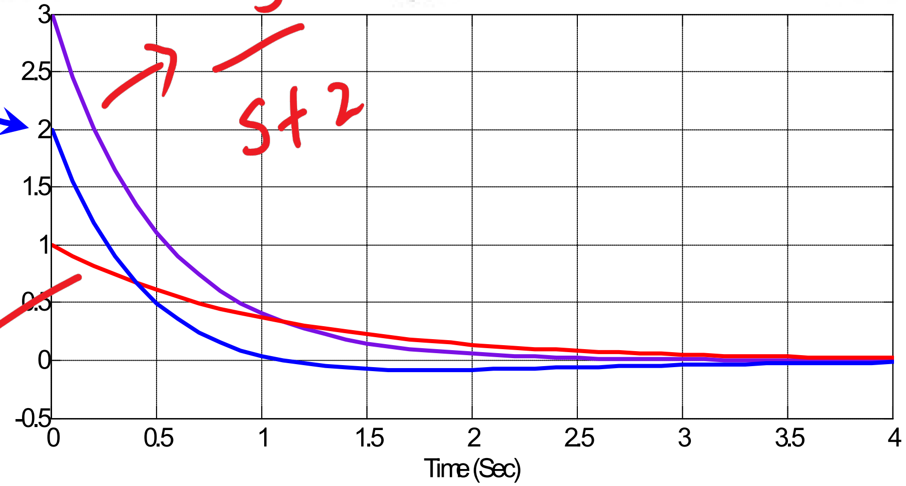
(b)

# EXAMPLE 3.25 *Response versus Pole Locations, Real Roots*

$$H(s) = \frac{2s+1}{s^2+3s+2} = \underbrace{-\frac{1}{s+1} + \frac{3}{s+2}}_{} \Rightarrow h(t) = \begin{cases} -e^{-t} + 3e^{-2t} & t \geq 0, \\ 0 & t < 0. \end{cases}$$

**ξ >1**

$\frac{3}{s+2}$




$\frac{1}{s+1}$

**Figure 3.16**
Impulse response of Example 3.23 [Eq. (3.52)]

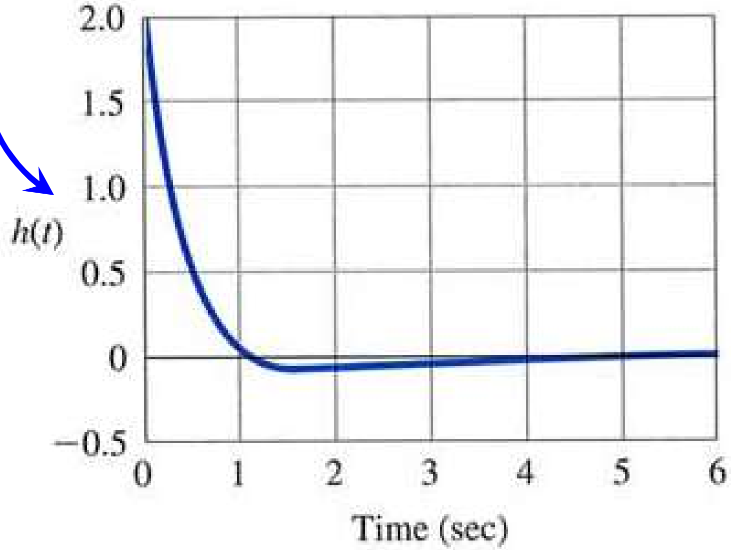
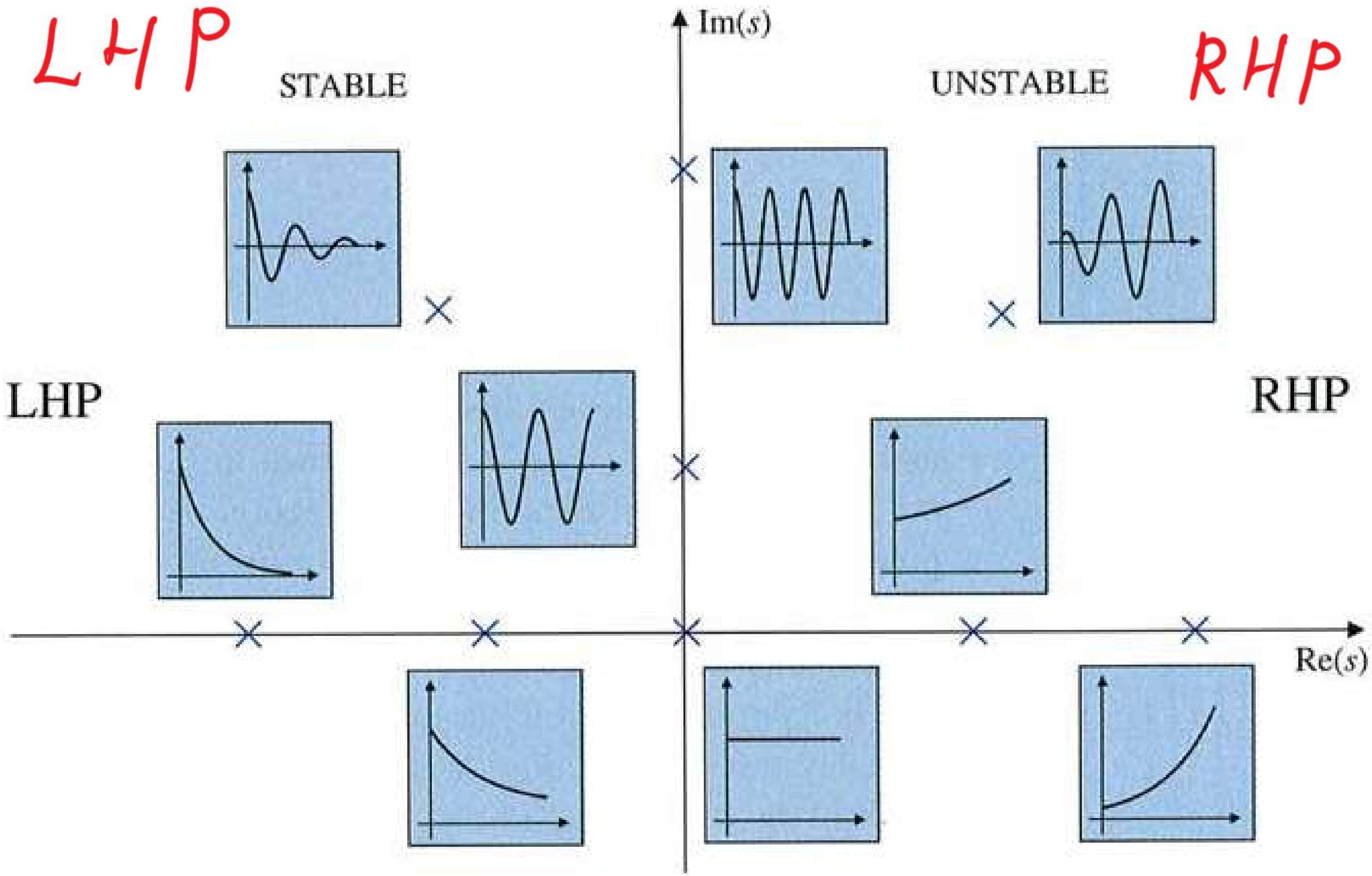


**Figure 3.15**
Time functions associated with points in the s-plane (LHP, left half-plane; RHP, right half-plane)




$0 =< \xi < 1$

$$\frac{Y(s)}{R(s)} = H(s) = \frac{w_n^{\ 2}}{s^2 + 2\xi w_n s + w_n^{\ 2}} = \frac{\omega_n^2}{(s+\zeta\omega_n)^2 + \omega_n^2(1-\zeta^2)}.$$

$$\begin{cases} s = -\sigma \pm j\omega_d \\ = -\xi w_n \pm \sqrt{\xi^2 w_n^2 - w_n^2} \\ = -\xi w_n \pm w_n \sqrt{1-\xi^2}\, j \end{cases}$$

$$\Rightarrow h(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\sigma t}(\sin\omega_d t)1(t).$$

$$\left\{ \sigma = \zeta\omega_n \quad \text{and} \quad \omega_d = \omega_n\sqrt{1-\zeta^2} \right\}$$

## EXAMPLE 3.26 *Oscillatory Time Response*

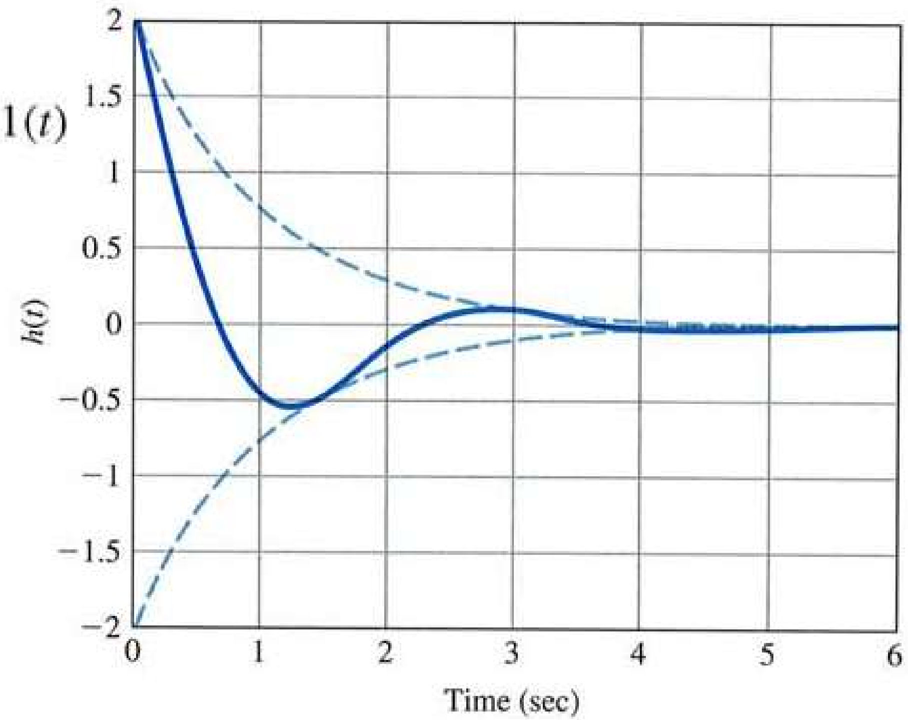
$$H(s) = \frac{2s+1}{s^2+2s+5} \quad \text{find the exact impulse response.} \quad = \frac{?}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$\Rightarrow \omega_n^2 = 5 \Rightarrow \omega_n = \sqrt{5} = 2.24 \text{ rad/sec} \qquad 2\zeta\omega_n = 2 \Rightarrow \zeta = \frac{1}{\sqrt{5}} = 0.447.$$

$$\Rightarrow H(s) = \frac{2s+1}{(s+1)^2+2^2} = 2\frac{s+1}{(s+1)^2+2^2} - \frac{1}{2}\frac{2}{(s+1)^2+2^2}$$

the impulse response is

$$\Rightarrow h(t) = \left(2e^{-t}\cos 2t - \frac{1}{2}e^{-t}\sin 2t\right)1(t)$$



$$F(s) = \frac{b_1 s^m + b_2 s^{m-1} + \cdots + b_{m+1}}{s^n + a_1 s^{n-1} + \cdots + a_n} = \frac{C_1}{s - p_1} + \frac{C_2}{s - p_2} + \cdots + \frac{C_n}{s - p_n}.$$

$$\Rightarrow (s - p_1)F(s) = C_1 + \frac{s - p_1}{s - p_2}C_2 + \cdots + \frac{(s - p_1)C_n}{s - p_n}.$$

**EXAMPLE** $Y(s) = \frac{(s+2)(s+4)}{s(s+1)(s+3)}$ Find $y(t)$ Homework

$$Y(s) = \frac{C_1}{s} + \frac{C_2}{s+1} + \frac{C_3}{s+3}$$

$$\Rightarrow y(t) = \frac{8}{3}1(t) - \frac{3}{2}e^{-t}1(t) - \frac{1}{6}e^{-3t}1(t)$$

## EXAMPLE 3.16 *Forced Differential Equation Solution*

$$\ddot{y}(t)+5\dot{y}(t)+4y(t)=3, \text{ where } y(0)=\alpha, \quad \dot{y}(0)=\beta$$

u(t)

$$s^2Y(s)-s\alpha-\beta+5[sY(s)-\alpha]+4Y(s)=\frac{3}{s} \qquad Y(s)=\frac{s(s\alpha+\beta+5\alpha)+3}{s(s+1)(s+4)}$$

$$Y(s)=\frac{\frac{3}{4}}{s}-\frac{\frac{3-\beta-4\alpha}{3}}{s+1}+\frac{\frac{3-4\alpha-4\beta}{12}}{s+4} \qquad y(t)=\left(\frac{3}{4}+\frac{-3+\beta+4\alpha}{3}e^{-t}+\frac{3-4\alpha-4\beta}{12}e^{-4t}\right)1(t).$$

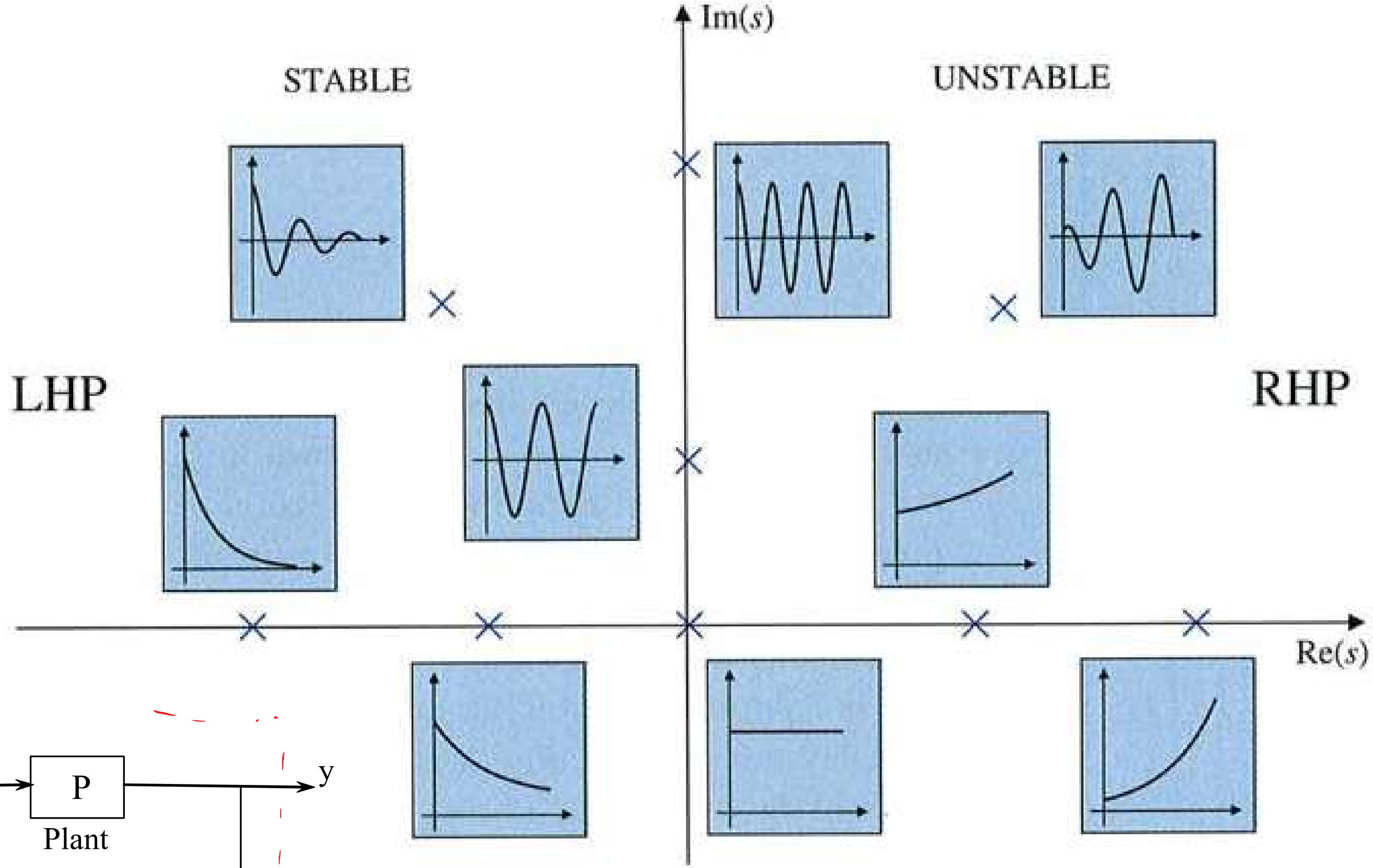
## EXAMPLE 3.17 *Forced Equation Solution with Zero Initial Conditions*

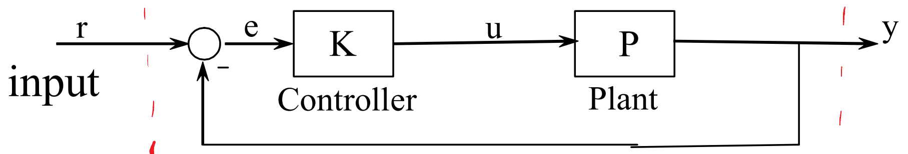
$$\ddot{y}(t)+5\dot{y}(t)+4y(t)=u(t), y(0)=0, \dot{y}(0)=0, u(t)=2e^{-2t}1(t)$$

$$s^2Y(s)+5sY(s)+4Y(s)=\frac{2}{s+2}. \qquad Y(s)=\frac{2}{(s+2)(s+1)(s+4)}$$

$$Y(s)=-\frac{1}{s+2}+\frac{\frac{2}{3}}{s+1}+\frac{\frac{1}{3}}{s+4} \qquad y(t)=\left(-1e^{-2t}+\frac{2}{3}e^{-t}+\frac{1}{3}e^{-4t}\right)1(t)$$

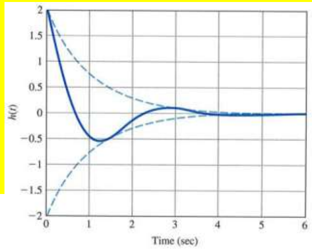
**Figure 3.15**
Time functions associated with points in the $s$-plane (LHP, left half-plane; RHP, right half-plane)




$$\frac{Y}{R} = \frac{N(s)}{D(s)} = \frac{b_1}{s+a_1} + \frac{b_2}{s+a_2} + \cdots + \frac{P_1 s + Q_1}{(s+c_1)^2 + d_1} + \frac{P_2 s + Q}{(s+c_2)^2 + d_2} + \cdots$$

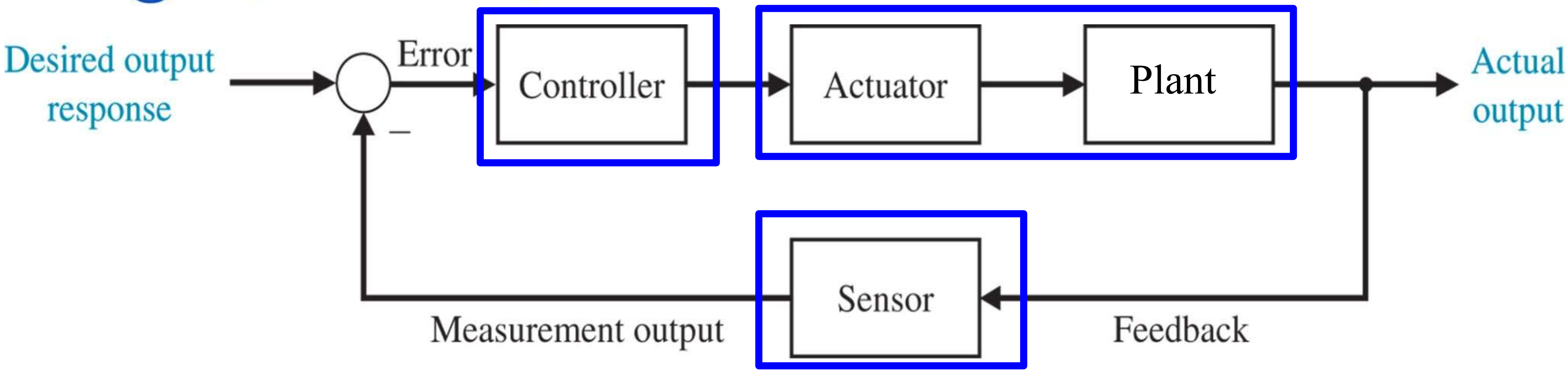
$$H(s) = \frac{2s+1}{s^2+2s+5}$$




the impulse response is

$$h(t) = \left(2e^{-t}\cos 2t - \frac{1}{2}e^{-t}\sin 2t\right)1(t)$$

# 3 Dynamic Response




3.1 Review of Laplace Transforms

3.2.1 The Block Diagram

3.3 Effect of Pole Locations

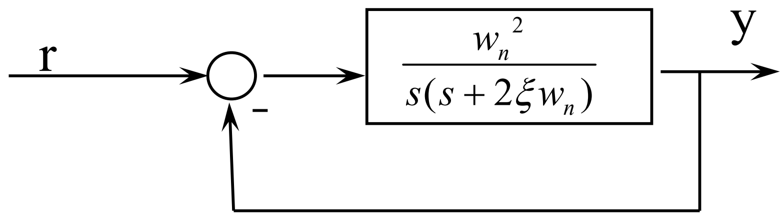
3.4 Time-Domain Specifications

3.5 Effects of Zeros and Additional Poles

3.6.3 Routh's Stability Criterion

# 3.4 Time-Domain Specifications

Unity Feedback for a second order system

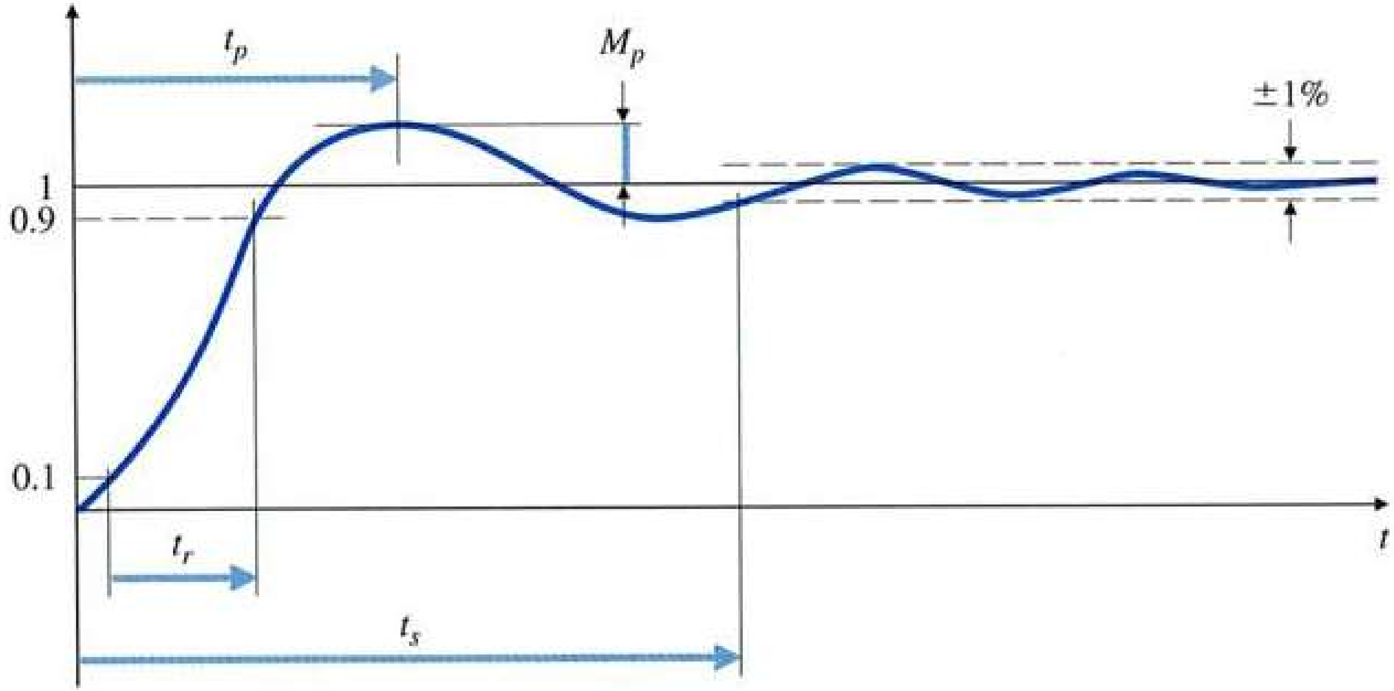


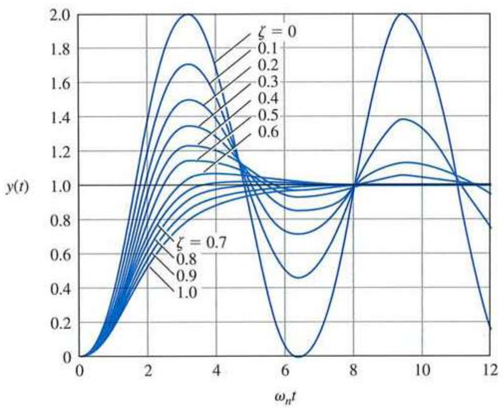

$$y(0)=0 \qquad y'(0)=0$$

$$\frac{Y(s)}{R(s)}=H(s)=\frac{w_n^{\ 2}}{s^2+2\xi w_n s+w_n^{\ 2}}=\frac{\omega_n^2}{(s+\zeta\omega_n)^2+\omega_n^2(1-\zeta^2)}.$$

(b) step responses $\Rightarrow$ $y(t)=1-e^{-\sigma t}\left(\cos\omega_d t+\frac{\sigma}{\omega_d}\sin\omega_d t\right)$

**Figure 3.22**
Definition of rise time $t_r$, settling time $t_s$, and overshoot $M_p$



(b)

0=<ξ<1

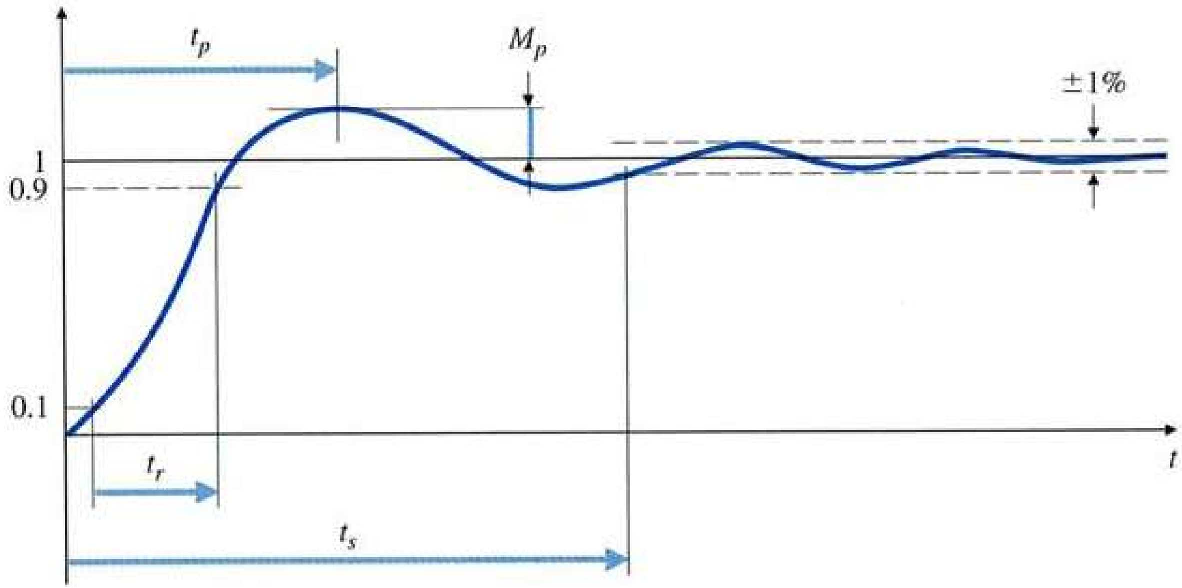
$$y(t) = 1 - e^{-\sigma t}\left(\cos\omega_d t + \frac{\sigma}{\omega_d}\sin\omega_d t\right),\qquad \omega_d = \omega_n\sqrt{1-\zeta^2} \text{ and } \sigma = \zeta\omega_n$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

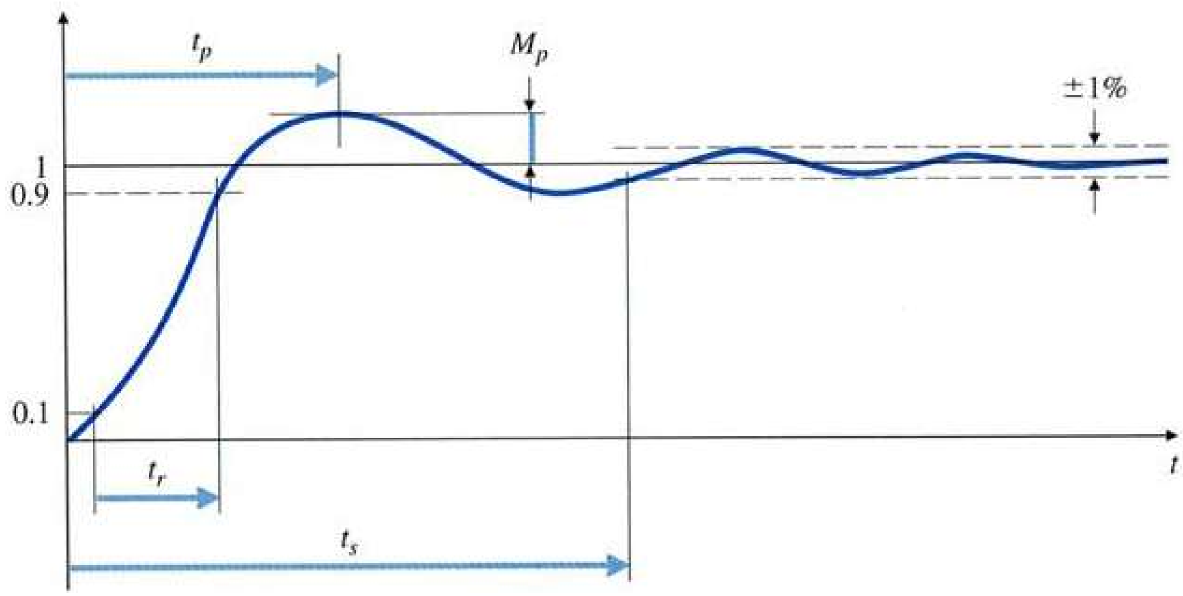
$$A\sin(\alpha) + B\cos(\alpha) = C\cos(\alpha - \beta) \qquad A = \frac{\sigma}{\omega_d},\ B = 1, \text{ and } \alpha = \omega_d t$$

$$C = \sqrt{A^2+B^2} = \frac{1}{\sqrt{1-\zeta^2}} \qquad \beta = \tan^{-1}\left(\frac{A}{B}\right) = \tan^{-1}\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right)$$

$$\Rightarrow y(t) = 1 - \frac{e^{-\sigma t}}{\sqrt{1-\zeta^2}}\cos(\omega_d t - \beta)$$



$$y(t) = 1 - \frac{e^{-\sigma t}}{\sqrt{1-\zeta^2}}\cos(\omega_d t - \beta).$$

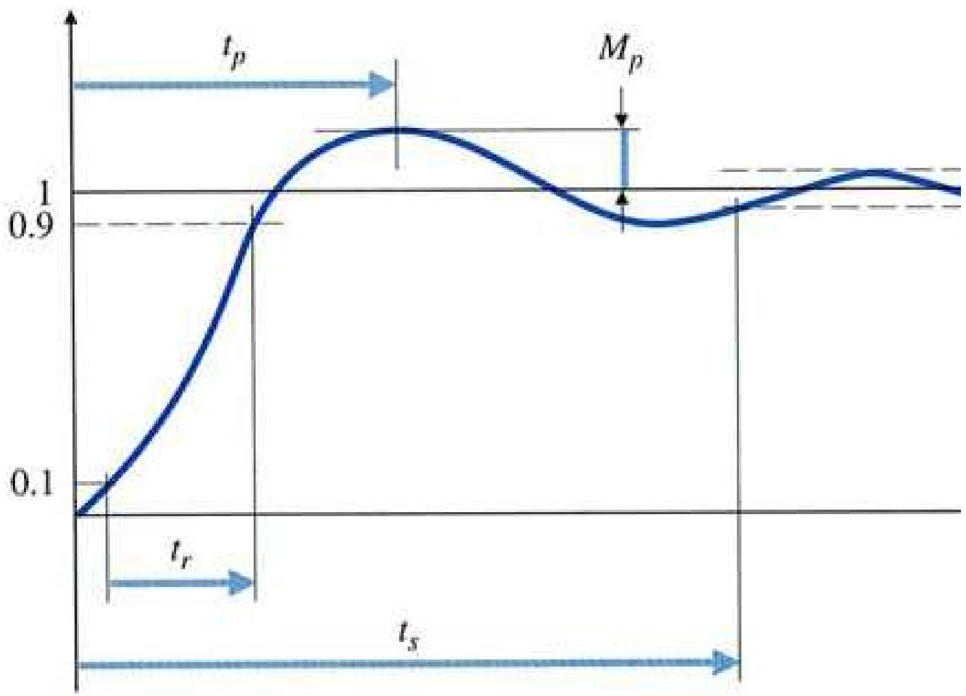


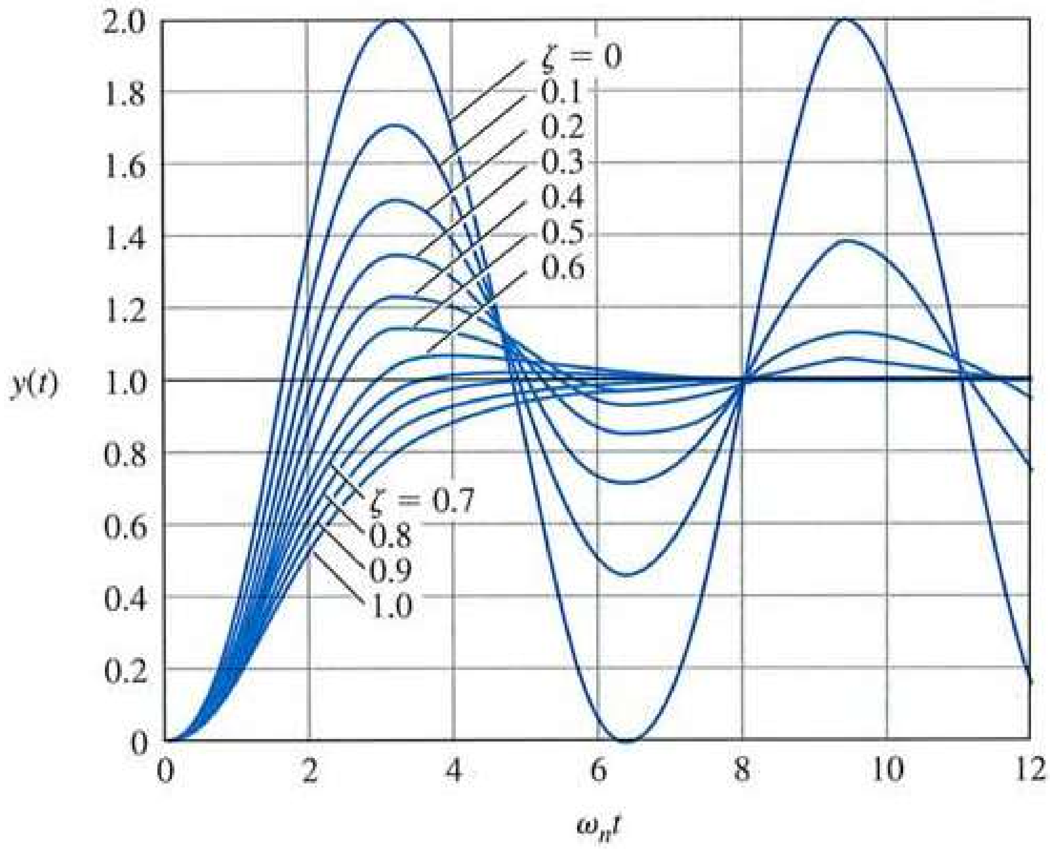

| | |
|---|---|
| rise time $t_r$ | for $\zeta = 0.5$ $\quad t_r \cong \frac{1.8}{\omega_n}.$ |
| settling time $t_s$ | $t_s = \frac{4.6}{\zeta\omega_n} = \frac{4.6}{\sigma},$ |
| overshoot $M_p$ | $M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}, \quad 0 \le \zeta < 1,$ |
| peak time $t_p$ | $t_p = \frac{\pi}{\omega_d}.$ |

$$y(t) = 1 - \frac{e^{-\sigma t}}{\sqrt{1-\zeta^2}}\cos(\omega_d t - \beta).$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \text{ and } \sigma = \zeta\omega_n$$

⇨ **rise time $t_r$**　　for $\zeta = 0.5$　　$t_r \cong \frac{1.8}{\omega_n}.$




(b)

## 3.4.2 Overshoot and Peak Time

0=<ξ<1

$$y(t) = 1 - e^{-\sigma t}\left(\cos\omega_d t + \frac{\sigma}{\omega_d}\sin\omega_d t\right), \qquad \omega_d = \omega_n\sqrt{1-\zeta^2} \text{ and } \sigma = \zeta\omega_n$$

$$\Rightarrow \quad y(t) = 1 - \frac{e^{-\sigma t}}{\sqrt{1-\zeta^2}}\cos(\omega_d t - \beta).$$

$$\dot{y}(t) = \sigma e^{-\sigma t}\left(\cos\omega_d t + \frac{\sigma}{\omega_d}\sin\omega_d t\right) - e^{-\sigma t}(-\omega_d\sin\omega_d t + \sigma\cos\omega_d t) = 0$$

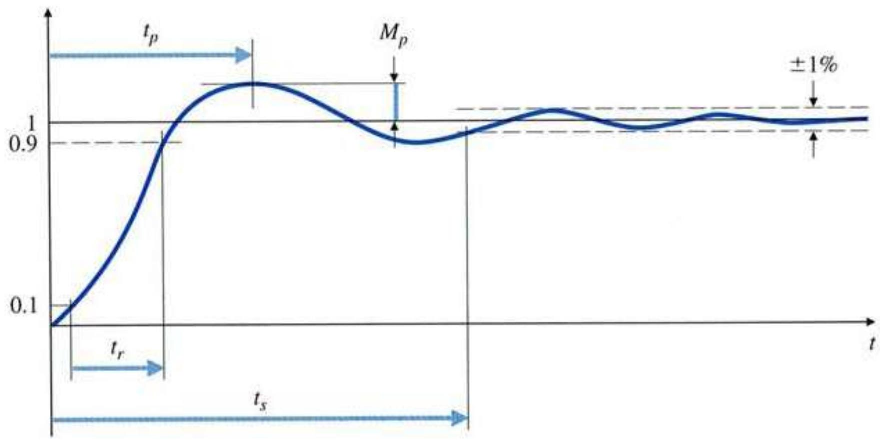
$$= e^{-\sigma t}\left(\frac{\sigma^2}{\omega_d} + \omega_d\right)\sin\omega_d t = 0.$$

$$\Rightarrow \quad t_p = \frac{\pi}{\omega_d}.$$

$$y(t_p) \triangleq 1 + M_p = 1 - e^{-\sigma\pi/\omega_d}\left(\cos\pi + \frac{\sigma}{\omega_d}\sin\pi\right)$$

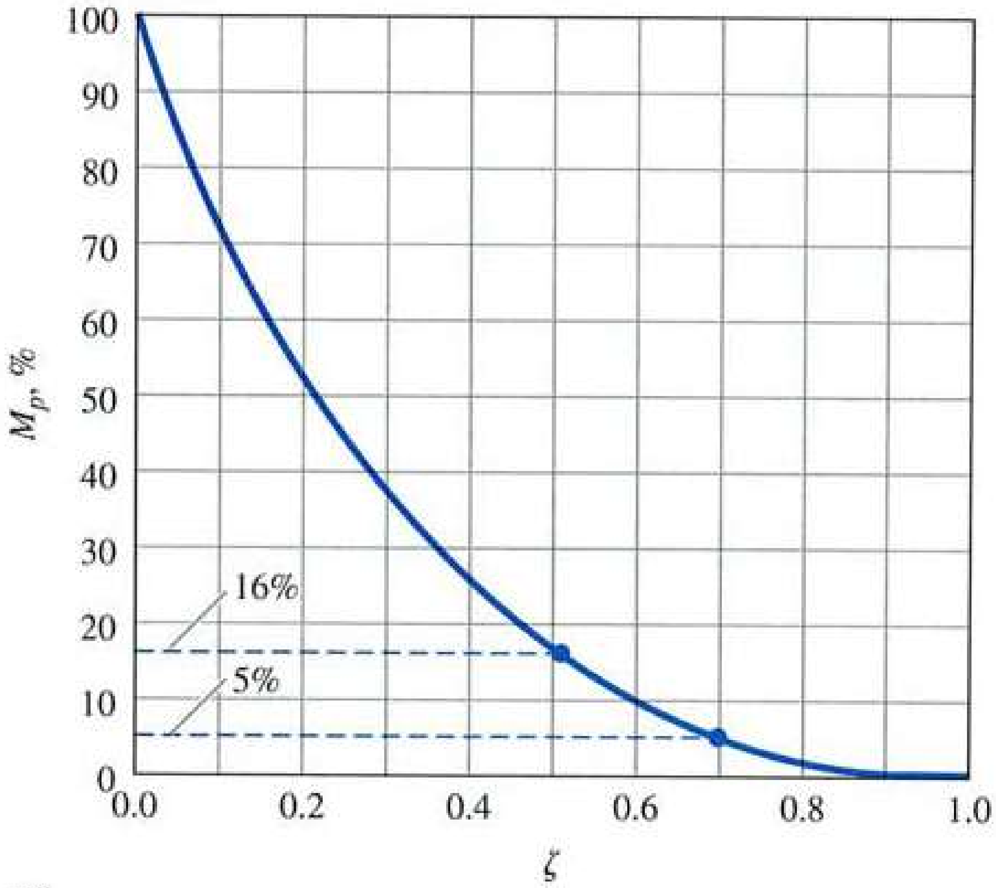
$$= 1 + e^{-\sigma\pi/\omega_d}.$$

$$\Rightarrow \quad M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}, \quad 0 \le \zeta < 1,$$

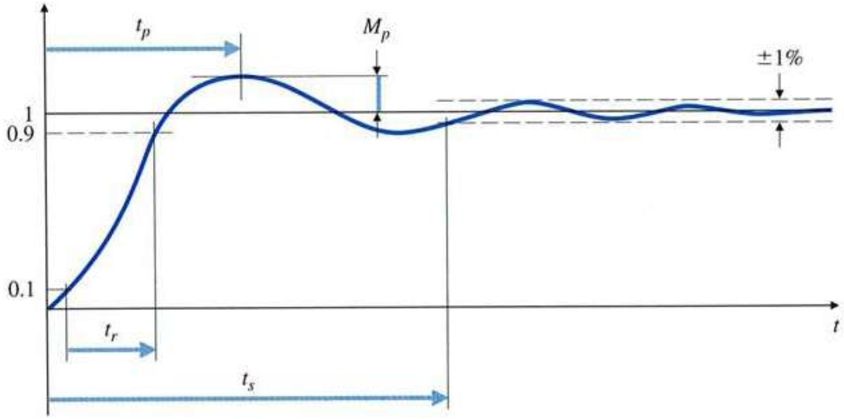


$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}, \quad 0 \le \zeta < 1,$$

**Figure 3.23**
Overshoot $M_p$ versus damping ratio $\zeta$ for the second-order system




$$\frac{Y(s)}{R(s)} = H(s) = \frac{w_n^{\ 2}}{s^2 + 2\xi w_n s + w_n^{\ 2}}$$



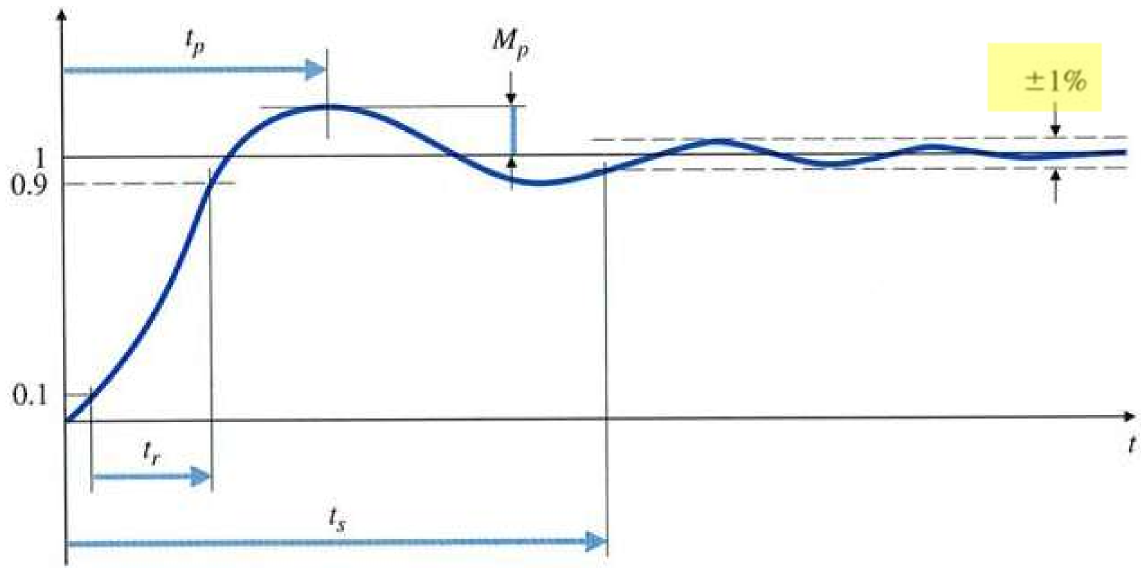
## 3.4.3 Settling Time

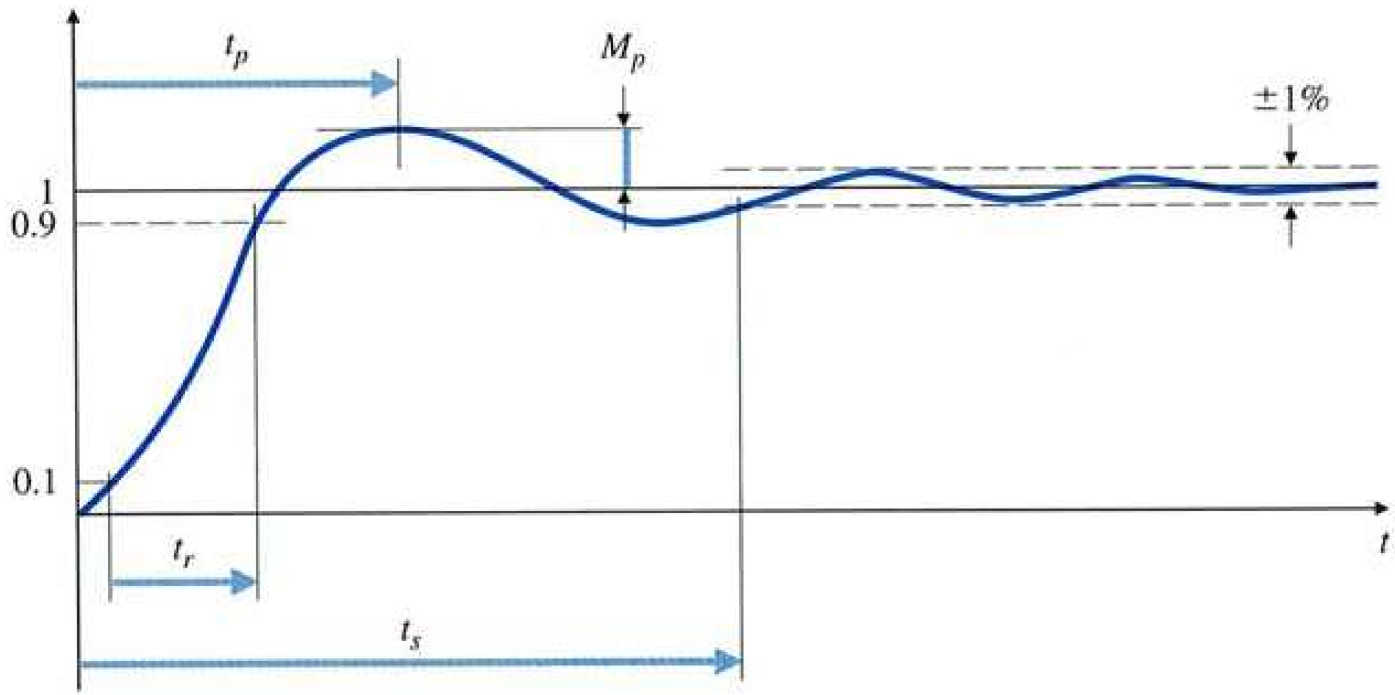
$$\omega_d = \omega_n\sqrt{1-\zeta^2} \text{ and } \sigma = \zeta\omega_n$$

$$y(t) = 1 - e^{-\sigma t}\left(\cos\omega_d t + \frac{\sigma}{\omega_d}\sin\omega_d t\right),$$

$$\Rightarrow e^{-\zeta\omega_n t_s} = 0.01 \Rightarrow \zeta\omega_n t_s = 4.6$$

$$\Rightarrow t_s = \frac{4.6}{\zeta\omega_n} = \frac{4.6}{\sigma},$$






| | | |
|---|---|---|
| **rise time $t_r$** | for $\zeta = 0.5$ | $t_r \cong \dfrac{1.8}{\omega_n}$. |
| ✓ **settling time $t_s$** | $t_s = \dfrac{4.6}{\zeta\omega_n} = \dfrac{4.6}{\sigma}$, | |
| ✓ **overshoot $M_p$** | $M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$, | $0 \le \zeta < 1$, |
| ✓ **peak time $t_p$** | $t_p = \dfrac{\pi}{\omega_d}$. | |

## 3.4 Time-Domain Specifications Time-Domain Specs

Design synthesis

$$\begin{cases} \omega_n \geq \dfrac{1.8}{t_r}, \\ \zeta \geq \zeta(M_p) \\ \sigma \geq \dfrac{4.6}{t_s}. \end{cases}$$

$$T.F. \quad G(s) = \frac{Y(s)}{F(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$
$$= K \frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}$$

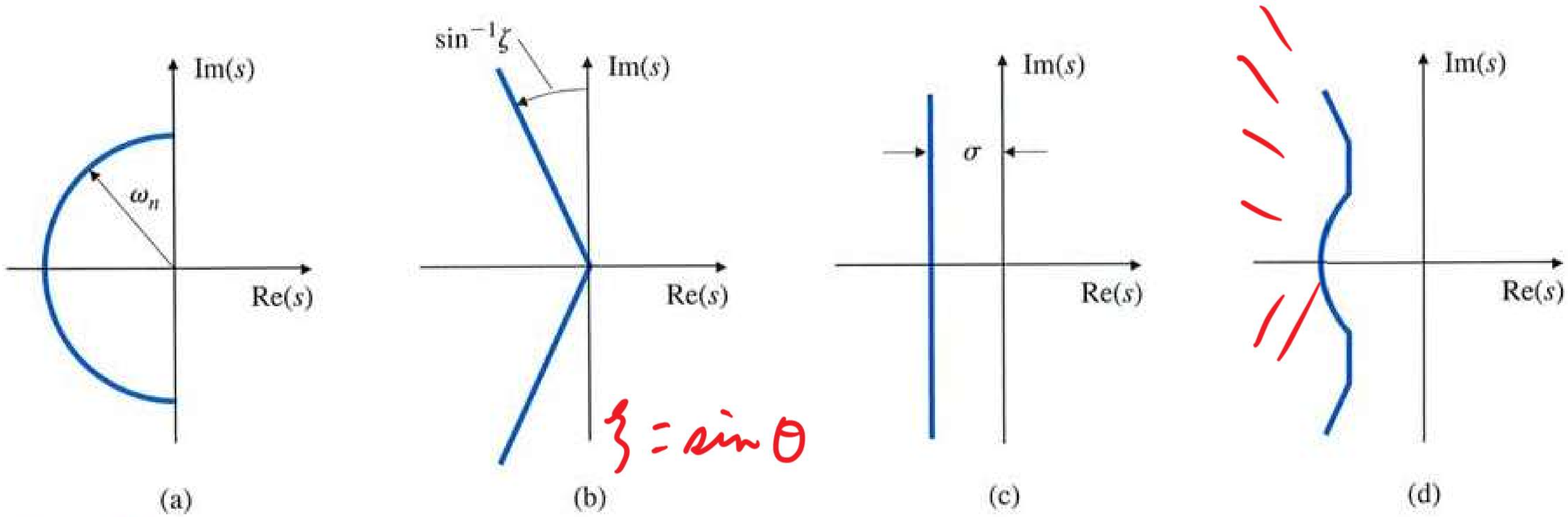



**Figure 3.25**

Graphs of regions in the $s$-plane delineated by certain transient requirements: (a) rise time; (b) overshoot; (c) settling time; (d) composite of all three requirements

## (2) Standard second order system

(a good approximation to behavior of many real systems)

$$\frac{Y(s)}{R(s)} = H(s) = \frac{w_n^{\ 2}}{s^2 + 2\xi w_n s + w_n^{\ 2}}$$

$= 0 \Rightarrow$ $s = -\sigma \pm j\omega_d$

$= -\xi w_n \pm \sqrt{\xi^2 w_n^2 - w_n^2}$

$= -\xi w_n \pm w_n \sqrt{1-\xi^2}\, j$

$$(s + \sigma - j\omega_d)(s + \sigma + j\omega_d) = (s + \sigma)^2 + \omega_d^2$$

A  B  $= A + B$

$$\sigma = \zeta\omega_n \quad \text{and} \quad \omega_d = \omega_n\sqrt{1 - \zeta^2},$$

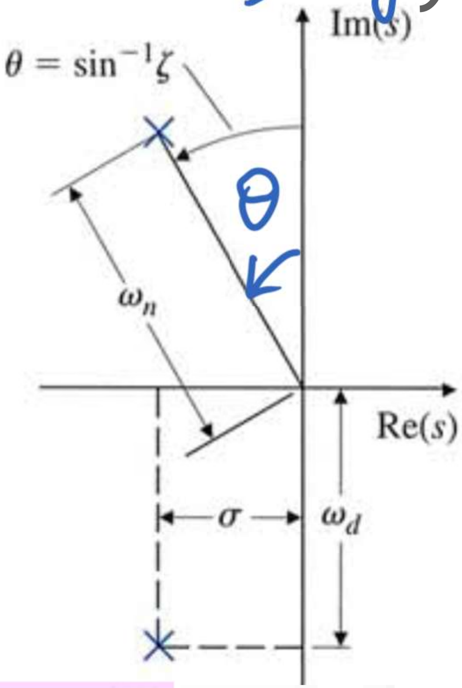



**Figure 3.17**
s-plane plot for a pair of complex poles

$\zeta$ is the **damping ratio**

$\omega_n$ is the **undamped natural frequency**

$\omega_d$ damped natural frequency

$\zeta = 0$, we have no damping, $\theta = 0$, and the damped natural frequency $\omega_d = \omega_n$

# 3.4 Time-Domain Specifications Time-Domain Specs

**Design synthesis**

$$\begin{cases} \omega_n \geq \dfrac{1.8}{t_r}, \\ \zeta \geq \zeta(M_p) \\ \sigma \geq \dfrac{4.6}{t_s}. \end{cases}$$

$$T.F. \quad G(s) = \frac{Y(s)}{F(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0} = K\frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}$$

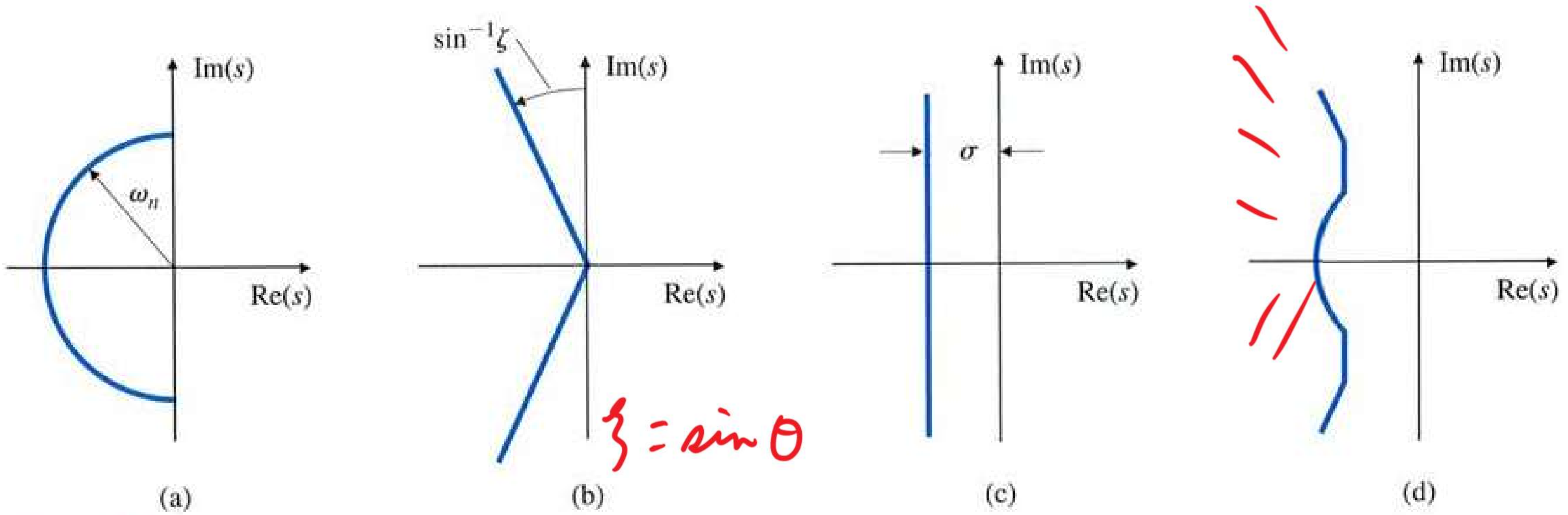
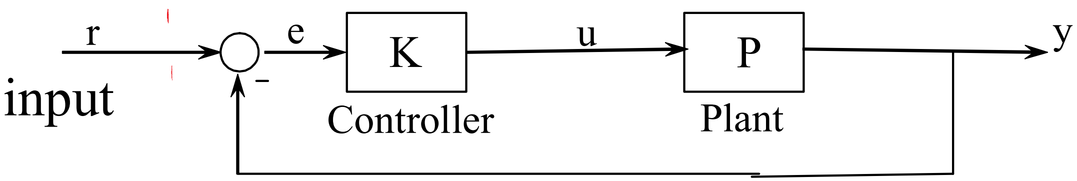
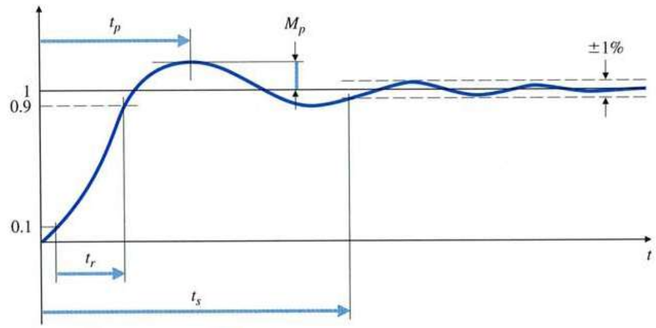



**Figure 3.25**
Graphs of regions in the s-plane delineated by certain transient requirements: (a) rise time; (b) overshoot; (c) settling time; (d) composite of all three requirements




## EXAMPLE 3.27 *Transformation of the Specifications to the s-Plane*

the system response requirements are $t_r \leq 0.6$ sec, $M_p \leq 10\%$, and $t_s \leq 3$ sec.

$$\begin{cases} \omega_n \geq \dfrac{1.8}{t_r}, \\ \zeta \geq \zeta(M_p) \\ \sigma \geq \dfrac{4.6}{t_s}. \end{cases}$$

$$\omega_n \geq \frac{1.8}{t_r} = 3.0 \text{ rad/sec}$$

$$\zeta \geq 0.6 \qquad M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}, \quad 0 \leq \zeta < 1$$

$$\sigma \geq \frac{4.6}{3} = 1.5 \text{ sec}$$

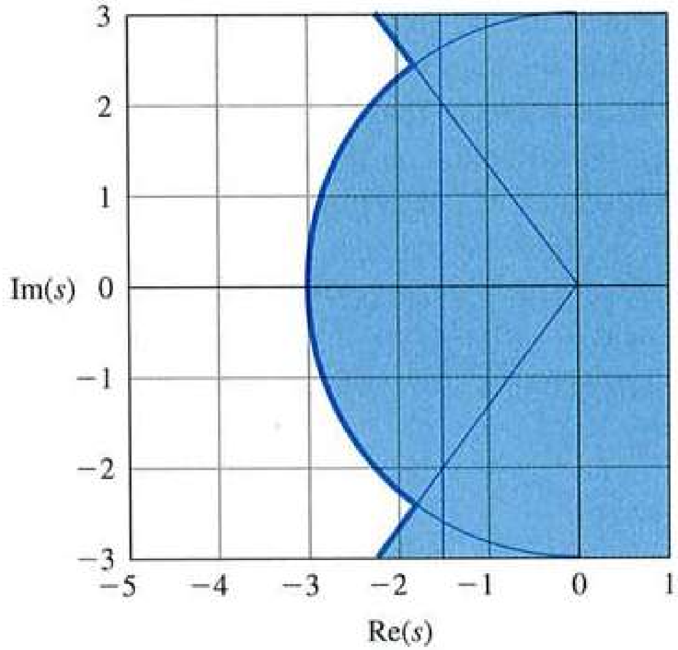
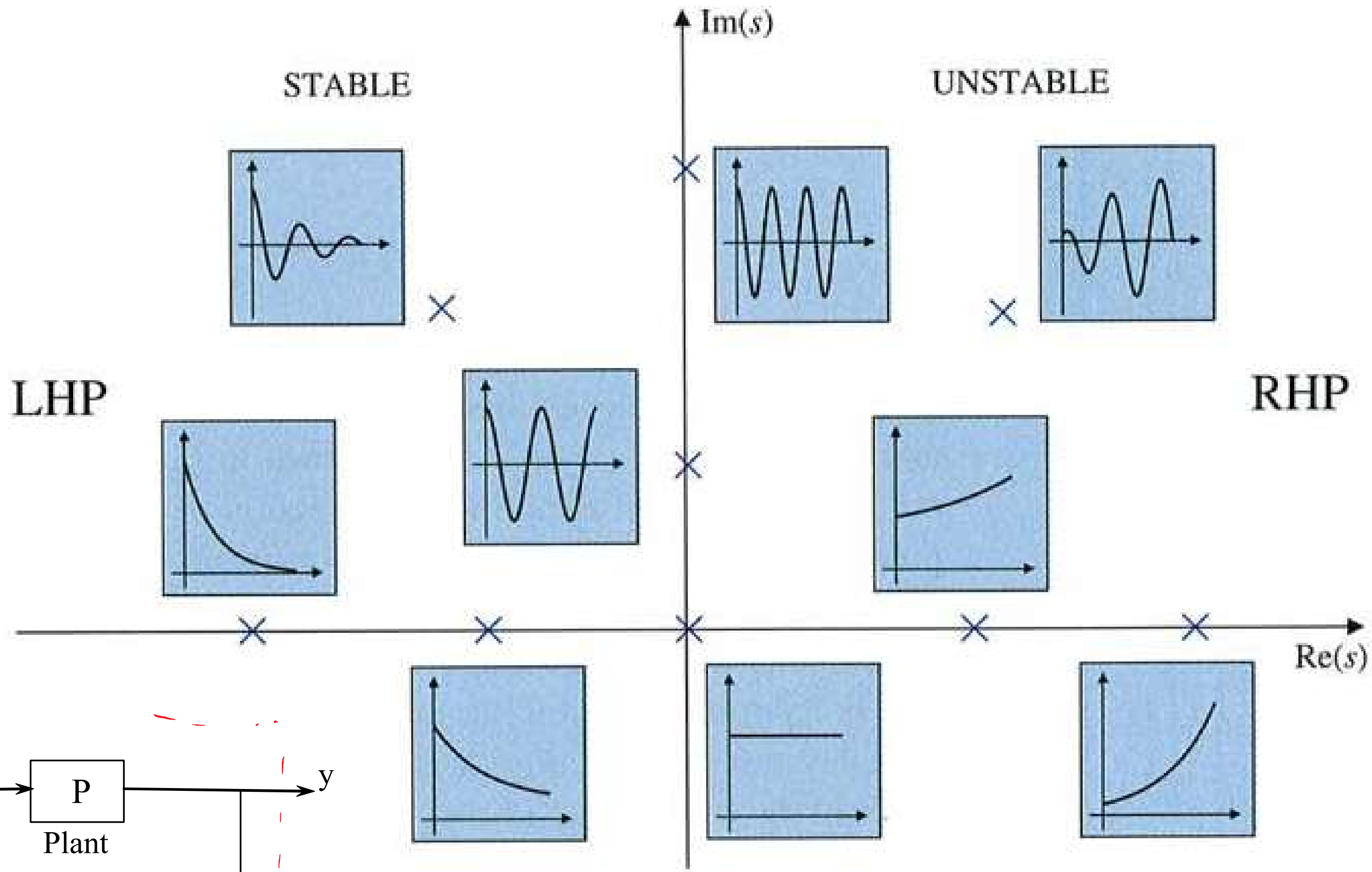
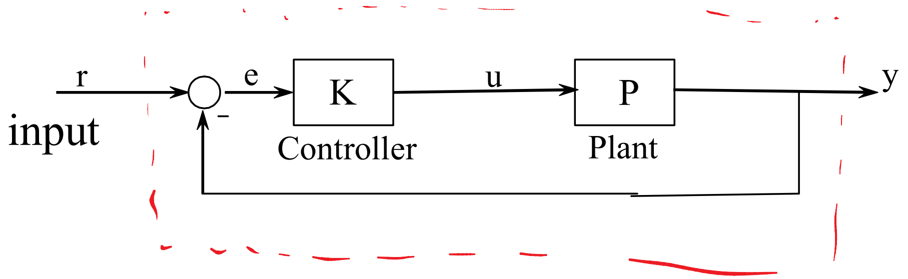


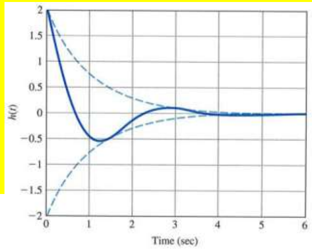
**Figure 3.15**
Time functions associated with points in the $s$-plane (LHP, left half-plane; RHP, right half-plane)




$$\frac{Y}{R} = \frac{N(s)}{D(s)} = \frac{b_1}{s+a_1} + \frac{b_2}{s+a_2} + \cdots + \frac{P_1 s + Q_1}{(s+c_1)^2 + d_1} + \frac{P_2 s + Q}{(s+c_2)^2 + d_2} + \cdots$$

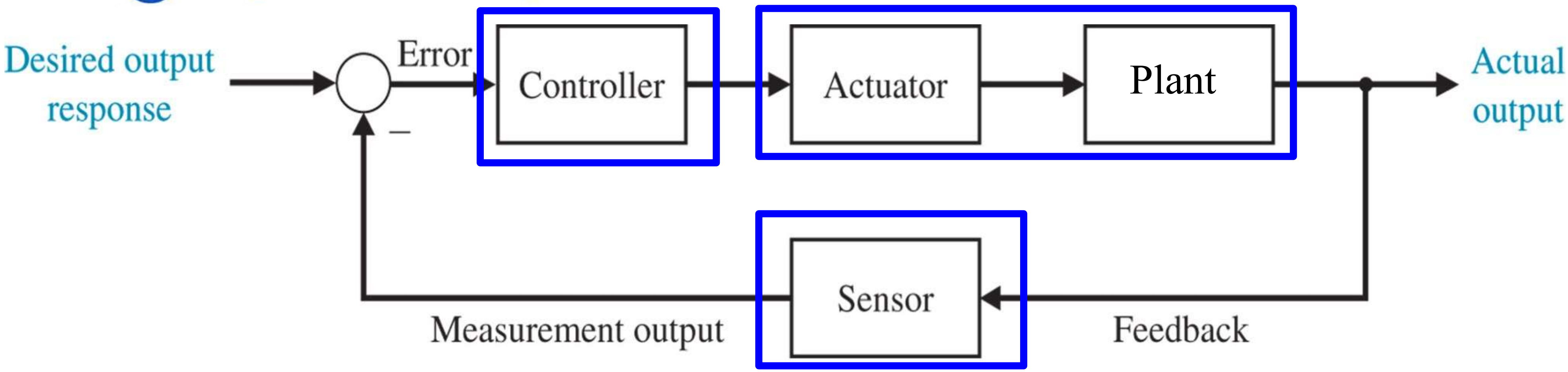
$$H(s) = \frac{2s+1}{s^2+2s+5}$$




the impulse response is

$$h(t) = \left(2e^{-t}\cos 2t - \frac{1}{2}e^{-t}\sin 2t\right)1(t)$$

# 3 Dynamic Response




3.1 Review of Laplace Transforms

3.2.1 The Block Diagram

3.3 Effect of Pole Locations

3.4 Time-Domain Specifications

3.5 Effects of Zeros and Additional Poles

3.6.3 Routh's Stability Criterion

$$\frac{Y(s)}{F(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}$$

$$= K\frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}$$