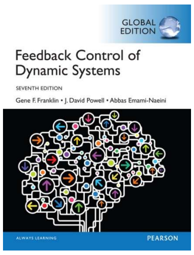

國立清華大學 National Tsing Hua University

PME320702

Feedback Control of Systems

Lecture 4

Instructor : Cheng-Hsien Liu 劉 承 賢

liuch@pme.nthu.edu.tw

http://mx.nthu.edu.tw/~chhsliu

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- Chapter 1: An Overview and Brief History of Feedback Control
- Chapter 2: Dynamic Models
- ⇒ Chapter 3: Dynamic Response
- Chapter 4: A First Analysis of Feedback
- Chapter 5: The Root-locus Design Method
- Chapter 6: The Frequency-response Design Method
- Chapter 7: State-space Design
- Chapter 8: Digital Control
- Chapter 9: Nonlinear Systems
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- Appendix A: Laplace Transforms
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- Appendix C: MATLAB Commands
- Appendix WA: A Review of Complex Variables
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- Appendix WC: Controllability and Observability
- Appendix WD: Ackermann's Formula for Pole Placement
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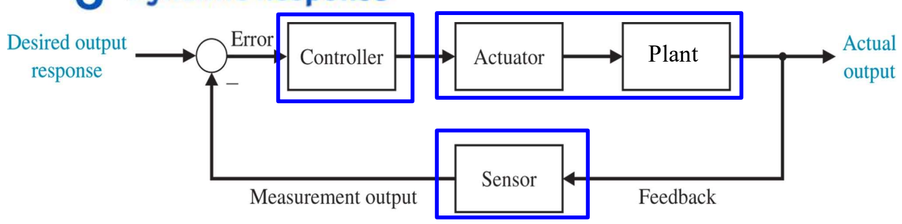
3 Dynamic Response



$$T.F.\quad G(s)=\frac{Y(s)}{R(s)}=\frac{b_m s^m+b_{m-1}s^{m-1}+\cdots+b_1 s+b_0}{s^n+a_{n-1}s^{n-1}+\cdots+a_1 s+a_0}=K\frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}$$

$$\frac{Y}{R}=\frac{N(s)}{D(s)}=\frac{b_1}{s+a_1}+\frac{b_2}{s+a_2}+\cdots+\frac{P_1 s+Q_1}{(s+c_1)^2+d_1}+\frac{P_2 s+Q}{(s+c_2)^2+d_2}+\cdots$$

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Transfer Functions 轉移函數

LTI Dynamic System

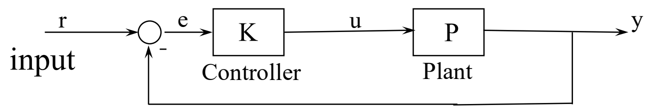
$$\Rightarrow\ T.F.\quad G(s)=\frac{Y(s)}{F(s)}=\frac{b_m s^m+b_{m-1}s^{m-1}+\cdots+b_1 s+b_0}{s^n+a_{n-1}s^{n-1}+\cdots+a_1 s+a_0}$$

$$=K\frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}$$

Def.:

1. G(s) is said to be proper if G(∞) is a finite constant -> $n \ge m$
2. G(s) is said to be strictly proper if G(∞) = 0 -> $n > m$
3. Relative order : n-m
4. The order of system: n
5. Poles: $-p_1, -p_2, \dots, -p_n$ 極點
6. Zeros: $-z_1, -z_2, \dots, -z_n$ 零點

Note: T.F. is to describe the relationship between input and output
T.F. is independent to the input and initial condition



LTI Dynamic System

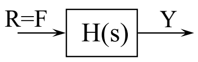
$$T.F.\quad G(s)=\frac{Y(s)}{R(s)}=\frac{b_m s^m+b_{m-1}s^{m-1}+\cdots+b_1 s+b_0}{s^n+a_{n-1}s^{n-1}+\cdots+a_1 s+a_0}$$

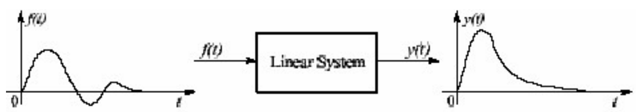
$$=K\frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}$$

$$\Rightarrow \frac{Y}{R}=\frac{N(s)}{D(s)}=\frac{b_1}{S+a_1}+\frac{b_2}{S+a_2}+\cdots$$

$$+\frac{P_1S+Q_1}{(S+C_1)^2+d_1}+\frac{P_2S+Q}{(S+C_2)^2+d_2}+\cdots$$

Dynamic Response





3.3 Effect of Pole Locations (1) First order system

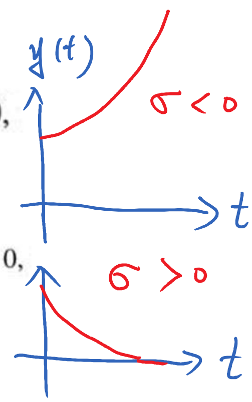
⇒ impulse response

$$\dot{y}+\sigma y=\delta(t) \Rightarrow H(s)=\frac{1}{s+\sigma}$$

$$\Rightarrow y(t)=e^{-\sigma t}u(t) \Rightarrow$$

If $\sigma<0$,

When $\sigma>0$,



(1) First order system

$\dot{y} + \sigma y = \delta(t) \quad H(s) = \frac{1}{s+\sigma} \quad h(t) = e^{-\sigma t} 1(t)$

u(t)

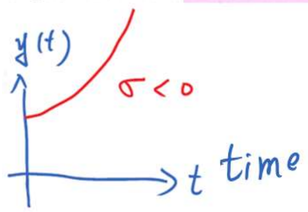
⇨ 看 system 穩定 / 不穩定 ⇒ 只看 pole 的正負號 (實數部)

(stable) (unstable) 極點

⇨ If $\sigma < 0$, the exponential expression here grows with time: the impulse response is unstable

⇨ If $\sigma < 0$,

$e^{-\sigma t}$



$\dot{y} + \sigma y = \delta(t)$

⇨ y(t)= $h(t) = e^{-\sigma t} u(t)$ → 1

⇨ When $\sigma > 0$, the pole is located at $s < 0$, the exponential expression decay the impulse response is stable. 穩定

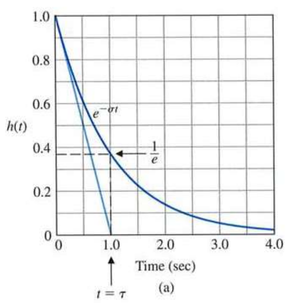


Figure 3.13 First-order system response: (a) impulse response; (b) impulse response and step response using MATLAB®

⇨ time constant $\tau = 1/\sigma$

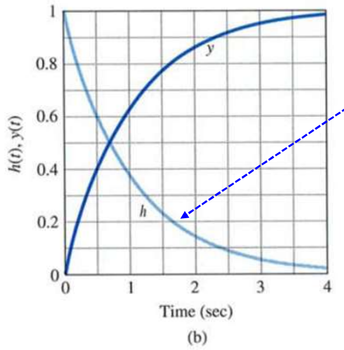
$k=1 \qquad u(t)$

$$\dot{y} + ky = 1(t).$$

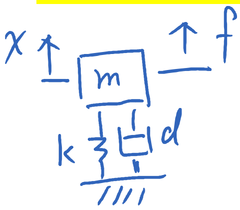
(unit) step response

$$\mathcal{L}\left(\Rightarrow sY + Y = \frac{1}{s} \Rightarrow Y = \frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1} \xrightarrow{\mathcal{L}^{-1}} y = u(t) - e^{-t}u(t)\right.$$

Figure 3.13
First-order system response: (a) impulse response; (b) impulse response and step response using MATLAB®



(2) Standard second order system

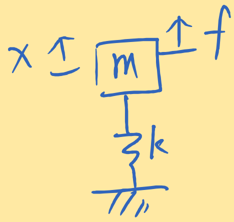


$$m\ddot{x} + d\dot{x} + kx = f \qquad \frac{X}{F} = \frac{1}{ms^2+ds+k}$$

damping coefficient

spring constant

如果 $d=0$



$$m\ddot{x} + kx = f \Rightarrow \ddot{x} + \frac{k}{m}x = \frac{f}{m}$$

$$\left(\frac{1}{2\pi}\right)\sqrt{\frac{k}{m}} \Rightarrow \text{共振頻率}$$

$$\ddot{x} + w_n^2 x = e^t$$

$$x = c_1 \sin(w_n t) + c_2 \cos(w_n t) + Ae^t$$

$$= c\cos(w_n t + \Phi) + Ae^t$$

$$\left\{ \begin{aligned} &\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \\ &A\sin(\alpha) + B\cos(\alpha) = C\cos(\alpha - \beta) \\ &\qquad C = \sqrt{A^2 + B^2} \\ &\qquad \beta = \tan^{-1}\left(\frac{A}{B}\right) \end{aligned} \right\}$$

$j = \sqrt{-1}$

(2) Standard second order system (a good approximation to behavior of many real systems)

$$\frac{Y(s)}{R(s)} = H(s) = \frac{w_n^2}{s^2 + 2\xi w_n s + w_n^2} \quad \underline{= 0} \Rightarrow \quad s = -\sigma \pm j\omega_d$$

$$= -\xi W_n \pm \sqrt{\xi^2 W_n^2 - W_n^2}$$

$$= -\xi W_n \pm W_n \sqrt{1 - \xi^2}\, j$$

$$\left\{ (s + \underbrace{\sigma}_{A} - j\omega_d)(s + \underbrace{\sigma}_{B} + j\omega_d) = (s + \sigma)^2 + \omega_d^2 \right. \quad = A + B$$

$\sigma = \zeta\omega_n$ and $\omega_d = \omega_n\sqrt{1 - \zeta^2}$,

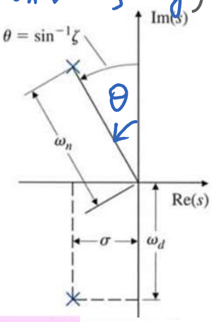
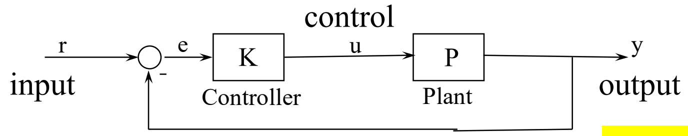


Figure 3.17
s-plane plot for a pair of complex poles

- ζ is the **damping ratio**
- ω_n is the **undamped natural frequency**
- ω_d damped natural frequency

$\zeta = 0$, we have no damping, $\theta = 0$, and the damped natural frequency $\omega_d = \omega_n$



0=<ξ<1

$$\frac{Y(s)}{R(s)} = H(s) = \frac{w_n^2}{s^2 + 2\xi w_n s + w_n^2} = \frac{\omega_n^2}{(s+\zeta\omega_n)^2 + \omega_n^2(1-\zeta^2)}.$$

$\to (w_n\sqrt{1-\xi^2})^2$

$\begin{cases} R=1 \\ r=\delta(t) \end{cases}$ ⇨ $$h(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\sigma t}(\sin\omega_d t)1(t). \quad (3.58)$$

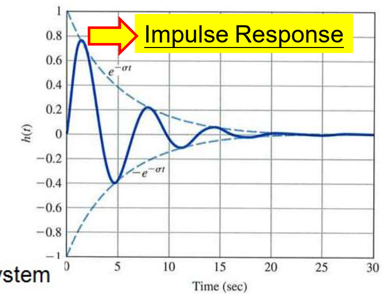


Figure 3.20
Second-order system response with an exponential envelope

⇨ $\left\{ \sigma = \zeta\omega_n \quad \text{and} \quad \omega_d = \omega_n\sqrt{1-\zeta^2} \right\}$

⇨ (σ, w_d) or (ξ, w_n) define the behavior of dynamic system

$$\frac{Y(s)}{R(s)} = H(s) = \frac{w_n^2}{s^2 + 2\xi w_n s + w_n^2} = \frac{\omega_n^2}{(s+\zeta\omega_n)^2 + \omega_n^2(1-\zeta^2)}.$$

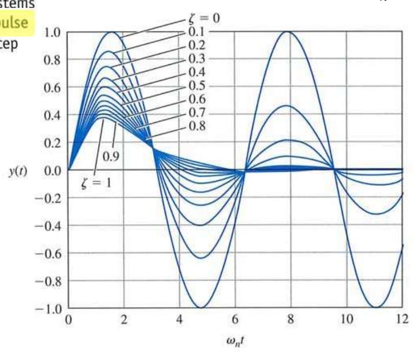
$\begin{cases} R=1 \\ r=\delta(t) \end{cases}$ ⇨ $$h(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\sigma t}(\sin\omega_d t)1(t).$$

0=<ξ<1

$$\left\{ \sigma = \zeta\omega_n \quad \text{and} \quad \omega_d = \omega_n\sqrt{1-\zeta^2} \right\}$$

$$\begin{cases} y(0) = 0 \\ y'(0) = w_n^2 \end{cases}$$

Figure 3.18
Responses of second-order systems versus ζ: (a) impulse responses; (b) step responses



$R = 1 \to r(t) = \delta(t)$
$\Rightarrow y(t)$ impulse response

$R = \frac{1}{s} \to r(t) = u(t)$
$\Rightarrow y(t)$ step response

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$$\begin{cases} y(0)=0 \\ y'(0)=0 \end{cases}$$

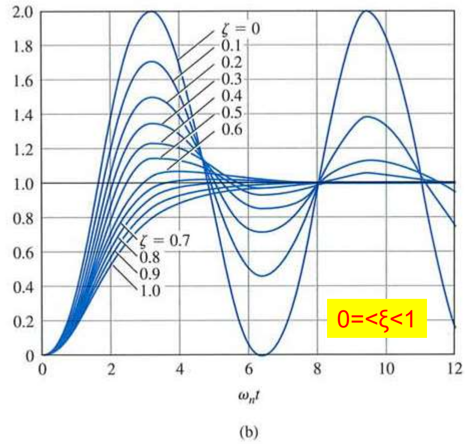
$$\frac{Y(s)}{R(s)} = H(s) = \frac{w_n^2}{s^2+2\xi w_n s + w_n^2} = \frac{\omega_n^2}{(s+\zeta\omega_n)^2+\omega_n^2(1-\zeta^2)}.$$ $Y = H(s)\frac{1}{s}$

0=<ξ<1

(b) step responses $y(t) = 1 - e^{-\sigma t}\left(\cos\omega_d t + \frac{\sigma}{\omega_d}\sin\omega_d t\right),$ $\mathcal{L}^{-1} \Rightarrow$

- (1) ξ=0 -> undamped system
- (2) 0<ξ<1 -> under damped system
- (3) ξ=1 -> critical damped system
- (4) ξ>1 -> over damped system

$$\left\{ \begin{array}{l} 0 =< \xi < 1 \\ \sigma = \zeta\omega_n \quad \text{and} \quad \omega_d = \omega_n\sqrt{1-\zeta^2} \end{array} \right\}$$



(b)

Lecture 4

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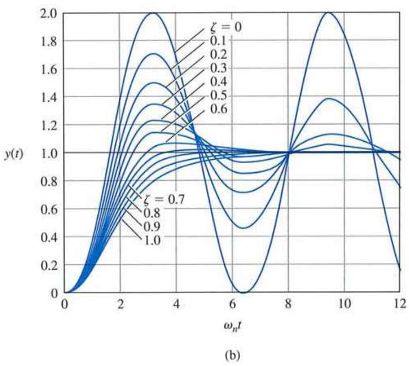


(b)

$$y(0)=0 \qquad y'(0)=0$$

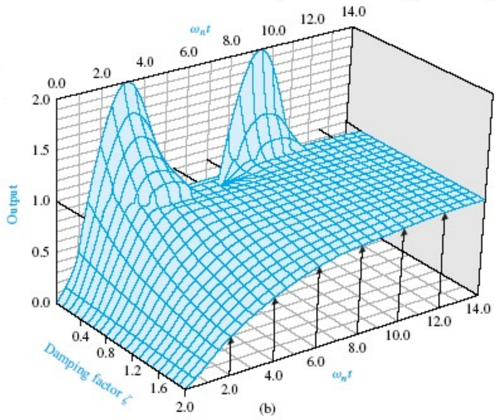
$$\frac{Y(s)}{R(s)} = H(s) = \frac{w_n^2}{s^2+2\xi w_n s + w_n^2}$$

Step Response

0=<ξ<1

$$y(t) = 1 - e^{-\sigma t}\left(\cos\omega_d t + \frac{\sigma}{\omega_d}\sin\omega_d t\right)$$

$$\left\{ \sigma = \zeta\omega_n \quad \text{and} \quad \omega_d = \omega_n\sqrt{1-\zeta^2} \right\}$$



(b)

Lecture 4

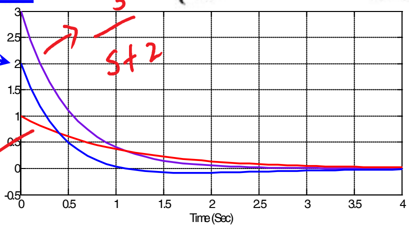
EXAMPLE 3.25 *Response versus Pole Locations, Real Roots*

$$H(s) = \frac{2s+1}{s^2+3s+2} = \underbrace{-\frac{1}{s+1} + \frac{3}{s+2}}_{} \Rightarrow h(t) = \begin{cases} -e^{-t}+3e^{-2t} & t \geq 0, \\ 0 & t < 0. \end{cases}$$

$\xi > 1$

$\frac{3}{s+2}$

$\frac{1}{s+1}$



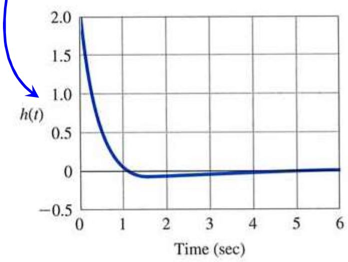
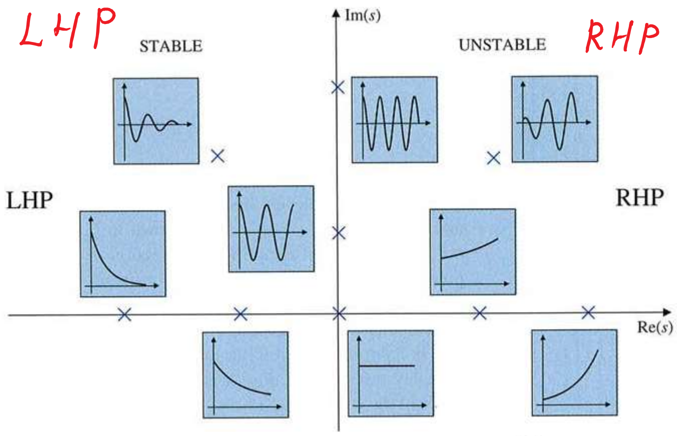


Figure 3.16
Impulse response of Example 3.23 [Eq. (3.52)]

Figure 3.15
Time functions associated with points in the s-plane (LHP, left half-plane; RHP, right half-plane)



$0 =< \xi < 1$

$$\frac{Y(s)}{R(s)} = H(s) = \frac{w_n^2}{s^2+2\xi w_n s + w_n^2} = \frac{\omega_n^2}{(s+\zeta\omega_n)^2+\omega_n^2(1-\zeta^2)}, \begin{cases} s = -\sigma \pm j\omega_d \\ = -\xi w_n \pm \sqrt{\xi^2 w_n^2 - w_n^2} \\ = -\xi w_n \pm w_n \sqrt{1-\xi^2}\, j \end{cases}$$

$$\Rightarrow h(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\sigma t}(\sin \omega_d t)1(t).$$

$$\left\{ \sigma = \zeta\omega_n \quad \text{and} \quad \omega_d = \omega_n\sqrt{1-\zeta^2} \right\}$$

EXAMPLE 3.26 *Oscillatory Time Response*

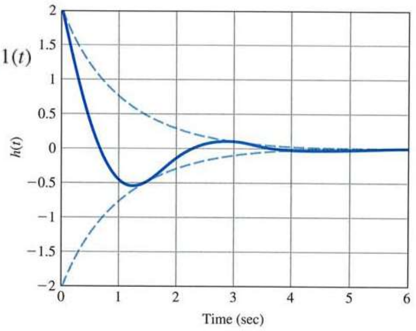
$$H(s) = \frac{2s+1}{s^2+2s+5}$$ find the exact impulse response. $= \frac{?}{s^2+2\zeta\omega_n s+\omega_n^2}$

$\Rightarrow$ $\omega_n^2 = 5 \Rightarrow \omega_n = \sqrt{5} = 2.24$ rad/sec $2\zeta\omega_n = 2 \Rightarrow \zeta = \frac{1}{\sqrt{5}} = 0.447.$

$\Rightarrow$ $$H(s) = \frac{2s+1}{(s+1)^2+2^2} = 2\frac{s+1}{(s+1)^2+2^2} - \frac{1}{2}\frac{2}{(s+1)^2+2^2}$$

the impulse response is

$\Rightarrow$ $$h(t) = \left(2e^{-t}\cos 2t - \frac{1}{2}e^{-t}\sin 2t\right)1(t)$$



$$F(s) = \frac{b_1 s^m + b_2 s^{m-1} + \cdots + b_{m+1}}{s^n + a_1 s^{n-1} + \cdots + a_n} = \frac{C_1}{s-p_1} + \frac{C_2}{s-p_2} + \cdots + \frac{C_n}{s-p_n}.$$

$\Rightarrow$ $$(s-p_1)F(s) = C_1 + \frac{s-p_1}{s-p_2}C_2 + \cdots + \frac{(s-p_1)C_n}{s-p_n}.$$

EXAMPLE

$$Y(s) = \frac{(s+2)(s+4)}{s(s+1)(s+3)}$$ Find $y(t)$ Homework

$$Y(s) = \frac{C_1}{s} + \frac{C_2}{s+1} + \frac{C_3}{s+3}$$

$\Rightarrow$ $$y(t) = \frac{8}{3}1(t) - \frac{3}{2}e^{-t}1(t) - \frac{1}{6}e^{-3t}1(t)$$

EXAMPLE 3.16 *Forced Differential Equation Solution*

$\ddot{y}(t)+5\dot{y}(t)+4y(t)=3$, where $y(0)=\alpha$, $\dot{y}(0)=\beta$

$$s^2Y(s)-s\alpha-\beta+5[sY(s)-\alpha]+4Y(s)=\frac{3}{s} \qquad Y(s)=\frac{s(s\alpha+\beta+5\alpha)+3}{s(s+1)(s+4)}$$

$$Y(s)=\frac{\frac{3}{4}}{s}-\frac{\frac{3-\beta-4\alpha}{3}}{s+1}+\frac{\frac{3-4\alpha-4\beta}{12}}{s+4} \qquad y(t)=\left(\frac{3}{4}+\frac{-3+\beta+4\alpha}{3}e^{-t}+\frac{3-4\alpha-4\beta}{12}e^{-4t}\right)1(t).$$

$u(t)$

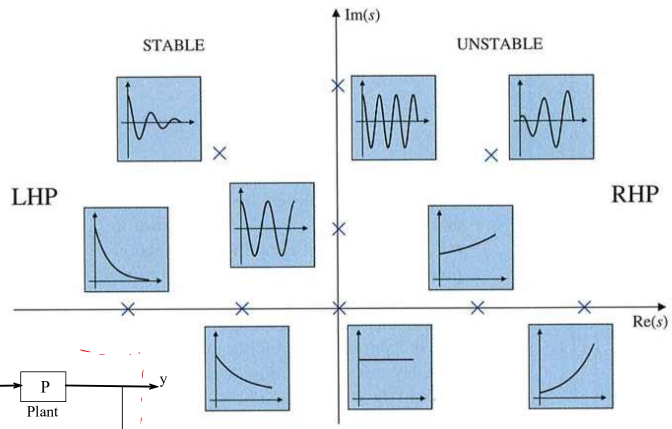
EXAMPLE 3.17 *Forced Equation Solution with Zero Initial Conditions*

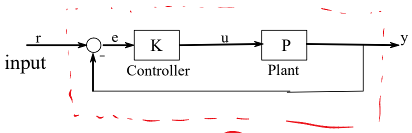
$\ddot{y}(t)+5\dot{y}(t)+4y(t)=u(t),\ y(0)=0,\ \dot{y}(0)=0,\ u(t)=2e^{-2t}1(t)$

$$s^2Y(s)+5sY(s)+4Y(s)=\frac{2}{s+2} \qquad Y(s)=\frac{2}{(s+2)(s+1)(s+4)}$$

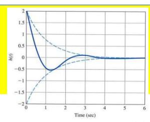
$$Y(s)=-\frac{1}{s+2}+\frac{\frac{2}{3}}{s+1}+\frac{\frac{1}{3}}{s+4} \qquad y(t)=\left(-1e^{-2t}+\frac{2}{3}e^{-t}+\frac{1}{3}e^{-4t}\right)1(t)$$

Figure 3.15
Time functions associated with points in the *s*-plane (LHP, left half-plane; RHP, right half-plane)





$$H(s)=\frac{2s+1}{s^2+2s+5}$$

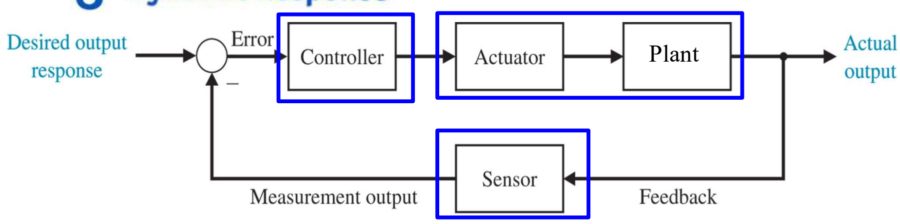


the impulse response is

$$h(t)=\left(2e^{-t}\cos 2t-\frac{1}{2}e^{-t}\sin 2t\right)1(t)$$

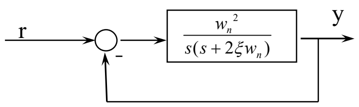
$$\frac{Y}{R}=\frac{N(s)}{D(s)}=\frac{b_1}{s+a_1}+\frac{b_2}{s+a_2}+\cdots+\frac{P_1 s+Q_1}{(s+c_1)^2+d_1}+\frac{P_2 s+Q}{(s+c_2)^2+d_2}+\cdots$$

3 Dynamic Response



3.4 Time-Domain Specifications

Unity Feedback for a second order system

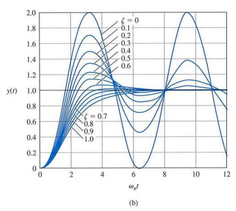


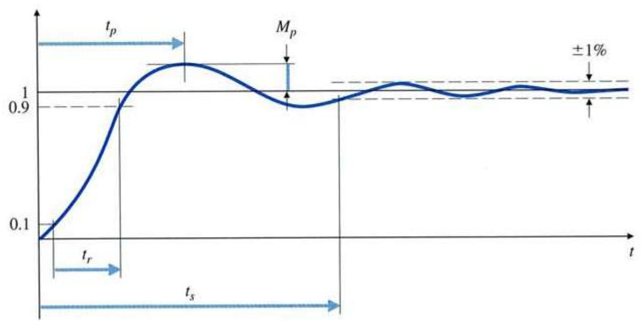
$$y(0)=0 \qquad y'(0)=0$$

$$\frac{Y(s)}{R(s)} = H(s) = \frac{w_n^2}{s^2+2\xi w_n s + w_n^2} = \frac{\omega_n^2}{(s+\zeta\omega_n)^2+\omega_n^2(1-\zeta^2)}.$$

(b) step responses $\Rightarrow$ $y(t) = 1 - e^{-\sigma t}\left(\cos\omega_d t + \frac{\sigma}{\omega_d}\sin\omega_d t\right)$

Figure 3.22
Definition of rise time t_r, settling time t_s, and overshoot M_p



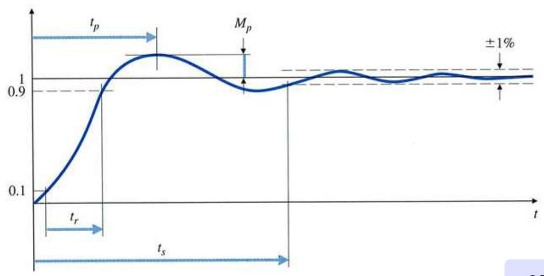


0=<ξ<1

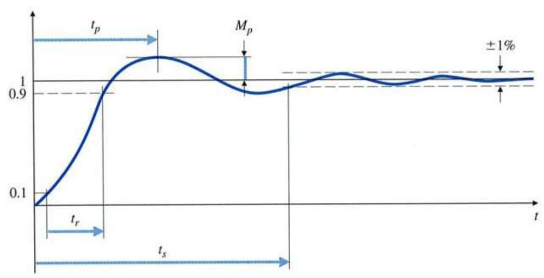
$$y(t) = 1 - e^{-\sigma t}\left(\cos\omega_d t + \frac{\sigma}{\omega_d}\sin\omega_d t\right), \qquad \omega_d = \omega_n\sqrt{1-\zeta^2} \text{ and } \sigma = \zeta\omega_n$$

$$\left\{\begin{array}{ll} \multicolumn{2}{c}{\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta} \\ A\sin(\alpha) + B\cos(\alpha) = C\cos(\alpha-\beta) & A = \frac{\sigma}{\omega_d},\ B = 1, \text{ and } \alpha = \omega_d t \\ C = \sqrt{A^2+B^2} = \dfrac{1}{\sqrt{1-\zeta^2}} & \beta = \tan^{-1}\left(\dfrac{A}{B}\right) = \tan^{-1}\left(\dfrac{\zeta}{\sqrt{1-\zeta^2}}\right) \end{array}\right\}$$

$$\Rightarrow y(t) = 1 - \frac{e^{-\sigma t}}{\sqrt{1-\zeta^2}}\cos(\omega_d t - \beta)$$



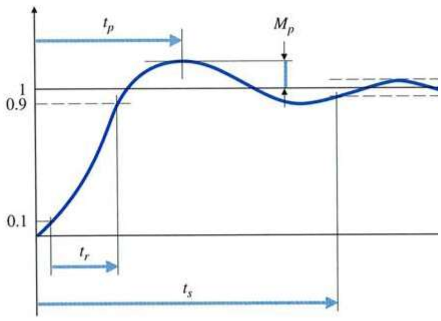
$$\Rightarrow y(t) = 1 - \frac{e^{-\sigma t}}{\sqrt{1-\zeta^2}}\cos(\omega_d t - \beta).$$

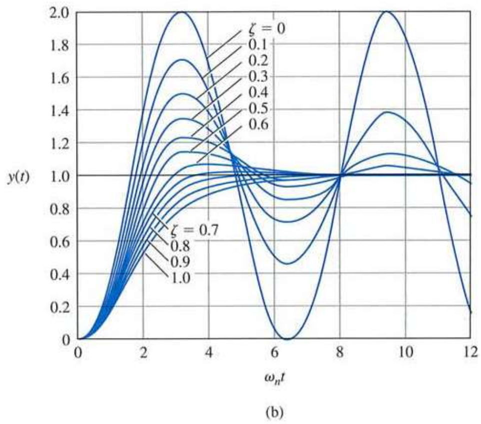


- **rise time** t_r — for $\zeta = 0.5$ $\quad t_r \cong \dfrac{1.8}{\omega_n}$.
- **settling time** t_s — $t_s = \dfrac{4.6}{\zeta\omega_n} = \dfrac{4.6}{\sigma}$,
- **overshoot** M_p — $M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}, \quad 0 \le \zeta < 1$,
- **peak time** t_p — $t_p = \dfrac{\pi}{\omega_d}$.

$$y(t) = 1 - \frac{e^{-\sigma t}}{\sqrt{1-\zeta^2}}\cos(\omega_d t - \beta). \qquad \omega_d = \omega_n\sqrt{1-\zeta^2} \text{ and } \sigma = \zeta\omega_n$$

⇨ **rise time t_r** for $\zeta = 0.5$ $\quad t_r \cong \frac{1.8}{\omega_n}$.





(b)

3.4.2 Overshoot and Peak Time

0=<ξ<1

$$y(t) = 1 - e^{-\sigma t}\left(\cos\omega_d t + \frac{\sigma}{\omega_d}\sin\omega_d t\right), \qquad \omega_d = \omega_n\sqrt{1-\zeta^2} \text{ and } \sigma = \zeta\omega_n$$

⇨ $$y(t) = 1 - \frac{e^{-\sigma t}}{\sqrt{1-\zeta^2}}\cos(\omega_d t - \beta).$$

$$\dot{y}(t) = \sigma e^{-\sigma t}\left(\cos\omega_d t + \frac{\sigma}{\omega_d}\sin\omega_d t\right) - e^{-\sigma t}(-\omega_d\sin\omega_d t + \sigma\cos\omega_d t) = 0$$

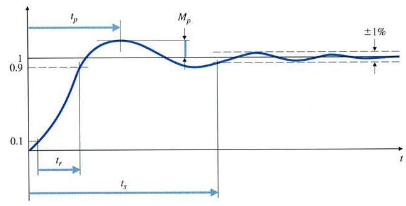
$$= e^{-\sigma t}\left(\frac{\sigma^2}{\omega_d} + \omega_d\right)\sin\omega_d t = 0.$$

⇨ $$t_p = \frac{\pi}{\omega_d}.$$

$$y(t_p) \triangleq 1 + M_p = 1 - e^{-\sigma\pi/\omega_d}\left(\cos\pi + \frac{\sigma}{\omega_d}\sin\pi\right)$$

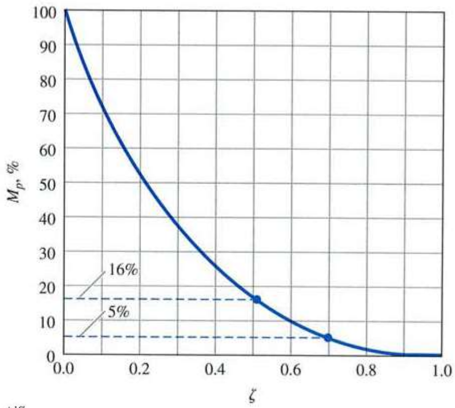
$$= 1 + e^{-\sigma\pi/\omega_d}.$$

⇨ $$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}, \quad 0 \le \zeta < 1,$$

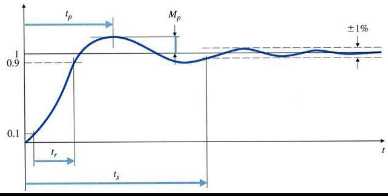


$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}, \quad 0 \le \zeta < 1,$$

Figure 3.23
Overshoot M_p versus damping ratio ζ for the second-order system



$$\frac{Y(s)}{R(s)} = H(s) = \frac{w_n^2}{s^2 + 2\xi w_n s + w_n^2}$$



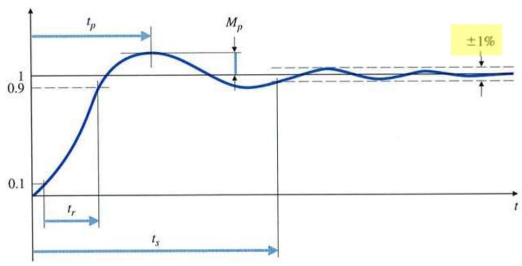
3.4.3 Settling Time

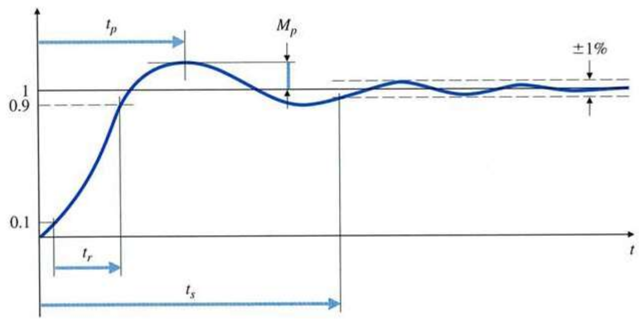
$$\omega_d = \omega_n\sqrt{1-\zeta^2} \text{ and } \sigma = \zeta\omega_n$$

$$y(t) = 1 - e^{-\sigma t}\left(\cos\omega_d t + \frac{\sigma}{\omega_d}\sin\omega_d t\right),$$

$$\Rightarrow e^{-\zeta\omega_n t_s} = 0.01 \quad \Rightarrow \quad \zeta\omega_n t_s = 4.6$$

$$\Rightarrow t_s = \frac{4.6}{\zeta\omega_n} = \frac{4.6}{\sigma},$$





rise time t_r for $\zeta = 0.5$ $t_r \cong \frac{1.8}{\omega_n}$.

✓ settling time t_s $t_s = \frac{4.6}{\zeta\omega_n} = \frac{4.6}{\sigma}$,

✓ overshoot M_p $M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$, $0 \le \zeta < 1$,

✓ peak time t_p $t_p = \frac{\pi}{\omega_d}$.

3.4 Time-Domain Specifications Time-Domain Specs

Design synthesis

$$\omega_n \ge \frac{1.8}{t_r},$$
$$\zeta \ge \zeta(M_p)$$
$$\sigma \ge \frac{4.6}{t_s}.$$

$$T.F.\quad G(s) = \frac{Y(s)}{F(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0} = K\frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}$$

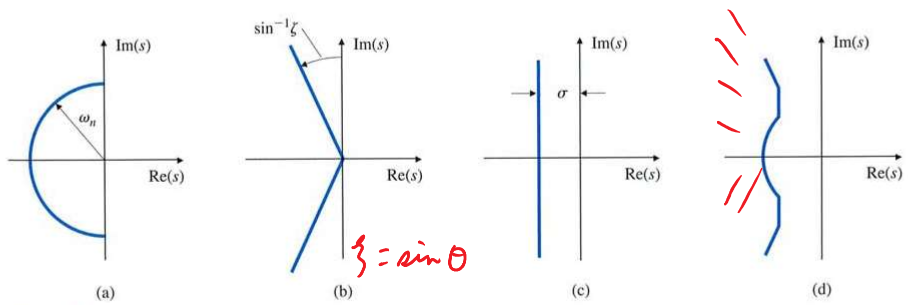


Figure 3.25

Graphs of regions in the s-plane delineated by certain transient requirements: (a) rise time; (b) overshoot; (c) settling time; (d) composite of all three requirements

(2) Standard second order system

(a good approximation to behavior of many real systems)

$$\frac{Y(s)}{R(s)} = H(s) = \frac{w_n^{\,2}}{s^2 + 2\xi w_n s + w_n^{\,2}} = 0 \Rightarrow s = -\sigma \pm j\omega_d$$

$$= -\xi w_n \pm \sqrt{\xi^2 w_n^2 - w_n^2}$$

$$(s + \underbrace{\sigma}_{A} - \underbrace{j\omega_d}_{})(s + \underbrace{\sigma}_{B} + j\omega_d) = (s+\sigma)^2 + \omega_d^2 = A + B$$

$$= -\xi w_n \pm w_n \sqrt{1-\xi^2}\, j$$

$$\sigma = \zeta\omega_n \quad \text{and} \quad \omega_d = \omega_n\sqrt{1-\zeta^2},$$

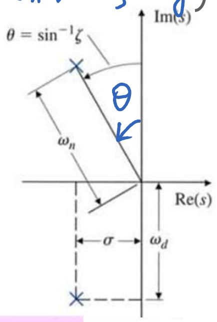


Figure 3.17
s-plane plot for a pair of complex poles

- ζ is the **damping ratio**
- ω_n is the **undamped natural frequency**
- ω_d damped natural frequency

$\zeta = 0$, we have no damping, $\theta = 0$, and the damped natural frequency $\omega_d = \omega_n$

3.4 Time-Domain Specifications Time-Domain Specs

Design synthesis

$$\begin{cases} \omega_n \geq \dfrac{1.8}{t_r}, \\ \zeta \geq \zeta(M_p) \\ \sigma \geq \dfrac{4.6}{t_s}. \end{cases}$$

$$T.F.\quad G(s) = \frac{Y(s)}{F(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}$$
$$= K\frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}$$

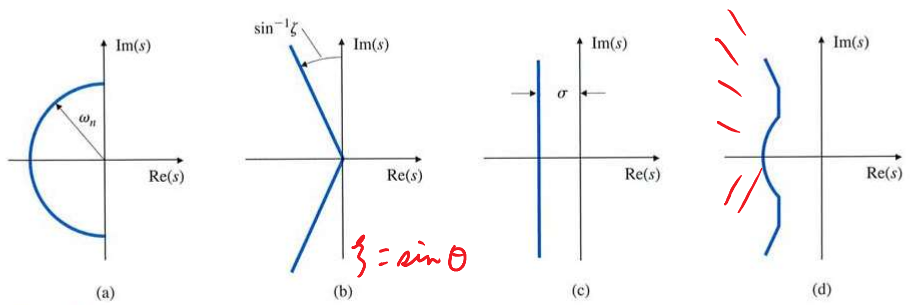
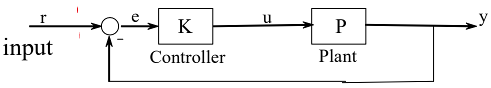
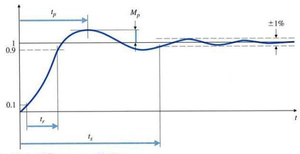


Figure 3.25
Graphs of regions in the s-plane delineated by certain transient requirements: (a) rise time; (b) overshoot; (c) settling time; (d) composite of all three requirements





EXAMPLE 3.27 *Transformation of the Specifications to the s-Plane*

the system response requirements are $t_r \le 0.6$ sec, $M_p \le 10\%$, and $t_s \le 3$ sec.

$$\begin{cases} \omega_n \ge \dfrac{1.8}{t_r}, \\ \zeta \ge \zeta(M_p) \\ \sigma \ge \dfrac{4.6}{t_s}. \end{cases}$$

$$\omega_n \ge \frac{1.8}{t_r} = 3.0 \text{ rad/sec}$$

$$\zeta \ge 0.6 \Leftarrow M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}, \quad 0 \le \zeta < 1$$

$$\sigma \ge \frac{4.6}{3} = 1.5 \text{ sec}$$

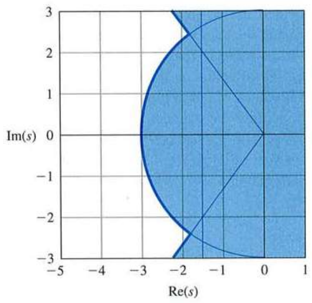
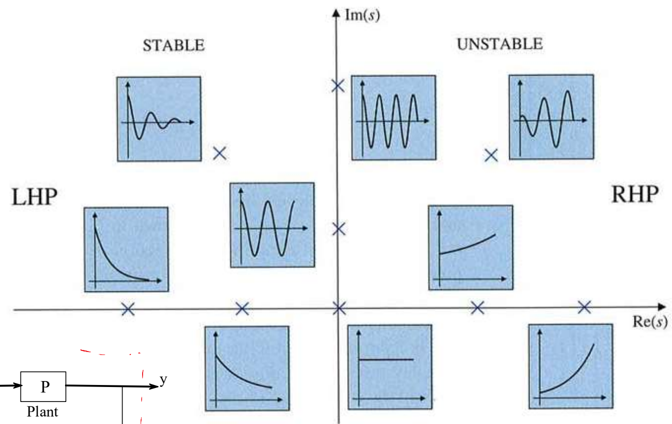
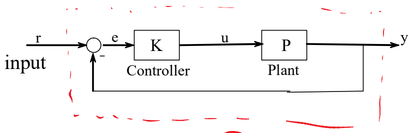


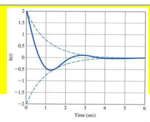
Figure 3.15
Time functions associated with points in the s-plane (LHP, left half-plane; RHP, right half-plane)





$$\frac{Y}{R} = \frac{N(s)}{D(s)} = \frac{b_1}{s+a_1} + \frac{b_2}{s+a_2} + \cdots + \frac{P_1 s + Q_1}{(s+c_1)^2 + d_1} + \frac{P_2 s + Q}{(s+c_2)^2 + d_2} + \cdots$$

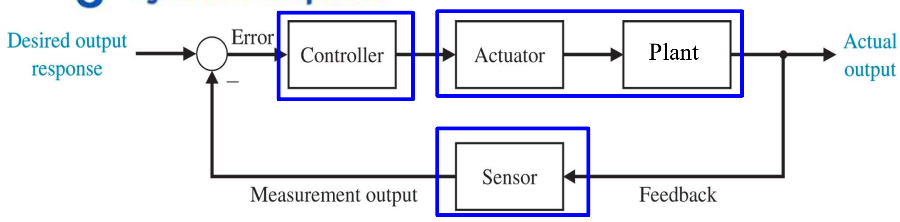
$$H(s) = \frac{2s+1}{s^2+2s+5}$$



the impulse response is

$$h(t) = \left(2e^{-t}\cos 2t - \frac{1}{2}e^{-t}\sin 2t\right) 1(t)$$

3 Dynamic Response



$$\frac{Y(s)}{F(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}$$

$$= K\frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}$$