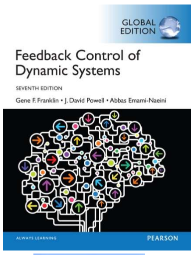

國立清華大學
National Tsing Hua University

PME320702
Feedback Control of Systems

Lecture 5

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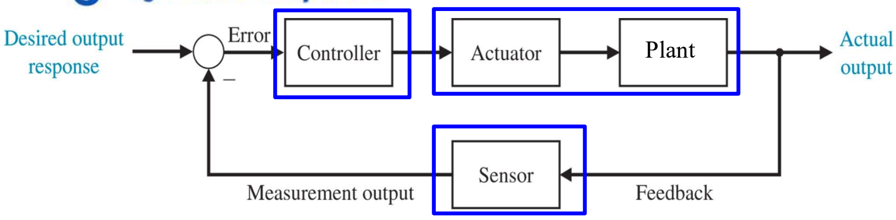
3 Dynamic Response



3.1 Review of Laplace Transforms

3.2.1 The Block Diagram

3.3 Effect of Pole Locations

3.4 Time-Domain Specifications

3.5 Effects of Zeros and Additional Poles

3.6.3 Routh's Stability Criterion

$$\frac{Y(s)}{F(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}$$

$$= K\frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}$$

3.5 Effects of Zeros and Additional Poles

$s = -\alpha\zeta$ zero (零點)

$$\frac{Y(s)}{R(s)} = H(s) = \frac{s/\alpha\zeta + 1}{s^2 + 2\zeta s + 1} = \frac{1}{s^2 + 2\zeta s + 1} + \frac{1}{\alpha\zeta}\frac{s}{s^2 + 2\zeta s + 1}.$$

unit step response

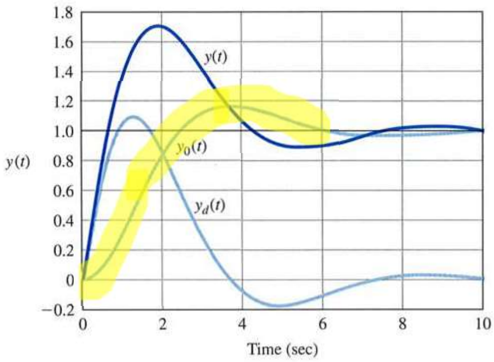
inverse Laplace

$$y(t) = y_0(t) + y_d(t) = y_0(t) + \frac{1}{\alpha\zeta}\dot{y}_0(t)$$

$\alpha \uparrow \Rightarrow \downarrow$

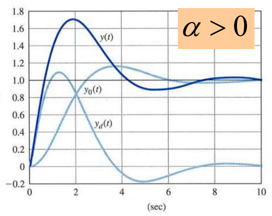
$\alpha > 0$

Figure 3.28
Second-order step responses $y(t)$ of the transfer functions $H(s)$, $H_0(s)$, and $H_d(s)$



$s = -\alpha\zeta$ zero (零點)

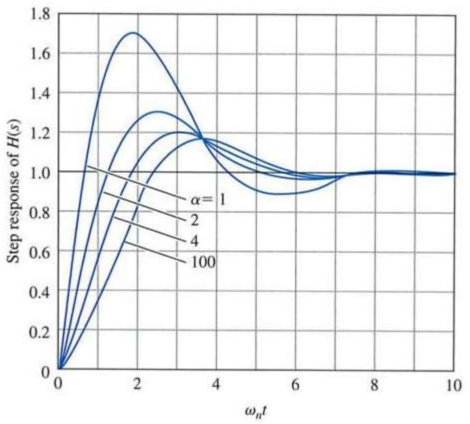
$$\frac{Y(s)}{R(s)} = H(s) = \frac{s/\alpha\zeta + 1}{s^2 + 2\zeta s + 1} = \frac{1}{s^2 + 2\zeta s + 1} + \frac{1}{\alpha\zeta}\frac{s}{s^2 + 2\zeta s + 1}$$



unit step response/*inverse Laplace*

$$y(t) = y_0(t) + y_d(t) = y_0(t) + \frac{1}{\alpha\zeta}\dot{y}_0(t)$$

Figure 3.26
Plots of the step response of a second-order system with a zero ($\zeta = 0.5$)



$s = -\alpha\zeta$ zero (零點)

$$\frac{Y(s)}{R(s)} = H(s) = \frac{s/\alpha\zeta + 1}{s^2 + 2\zeta s + 1} = \frac{1}{s^2 + 2\zeta s + 1} + \frac{1}{\alpha\zeta}\frac{s}{s^2 + 2\zeta s + 1}$$

unit step response/*inverse Laplace*

$$y(t) = y_0(t) + y_d(t) = y_0(t) + \frac{1}{\alpha\zeta}\dot{y}_0(t)$$

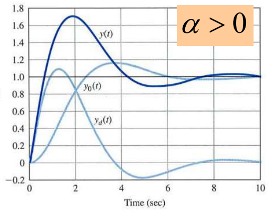
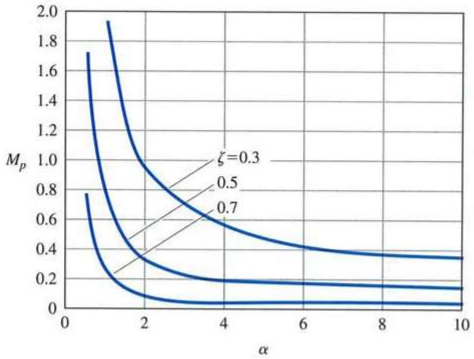


Figure 3.27
Plot of overshoot M_p as a function of normalized zero location α. At $\alpha = 1$, the real part of the zero equals the real part of the poles

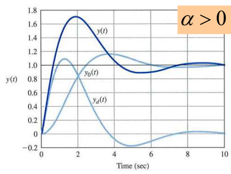


$s = -\alpha\zeta$ zero (零點)

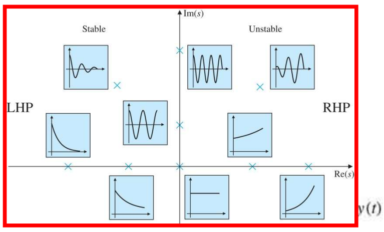
$$\frac{Y(s)}{R(s)} = H(s) = \frac{s/\alpha\zeta + 1}{s^2 + 2\zeta s + 1} = \frac{1}{s^2 + 2\zeta s + 1} + \frac{1}{\alpha\zeta}\frac{s}{s^2 + 2\zeta s + 1}$$

unit step response/*inverse Laplace*

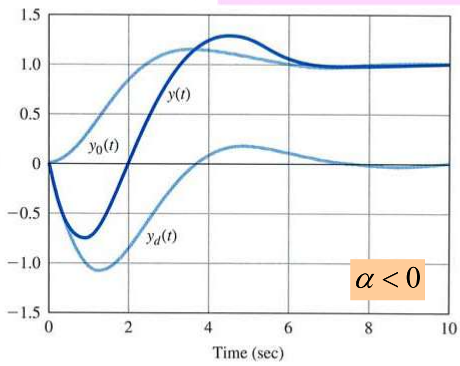
$$y(t) = y_0(t) + y_d(t) = y_0(t) + \frac{1}{\alpha\zeta}\dot{y}_0(t)$$



when $\alpha < 0$ and the zero is in the RHP where $s > 0$



RHP zero ⇒ **nonminimum-phase zero**



RHP ⇒ Right Hand Plane
LHP ⇒ Left Hand Plane

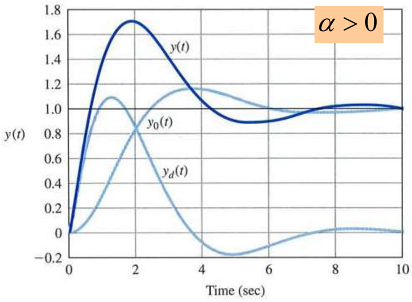
Summary

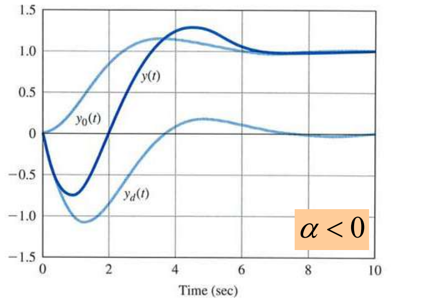
$s=-\alpha\zeta$ zero (零點)

$$\frac{Y(s)}{R(s)}=H(s)=\frac{s/\alpha\zeta+1}{s^2+2\zeta s+1}=\frac{1}{s^2+2\zeta s+1}+\frac{1}{\alpha\zeta}\frac{s}{s^2+2\zeta s+1}$$

unit step response/*inverse Laplace*

$$y(t)=y_0(t)+y_d(t)=y_0(t)+\frac{1}{\alpha\zeta}\dot{y}_0(t)$$





LHP zero ⇒ **minimum-phase zero**   **RHP zero** ⇒ **nonminimum-phase zero**

EXAMPLE 3.28 *Effect of the Proximity of the Zero to the Pole Locations on the Transient Response*

$$\frac{Y(s)}{R(s)}=H(s)=\frac{24}{z}\frac{(s+z)}{(s+4)(s+6)}$$

step response $H(s)\frac{1}{s}=\frac{24}{z}\frac{(s+z)}{s(s+4)(s+6)}=\frac{24}{z}\frac{s}{s(s+4)(s+6)}+\frac{24}{s(s+4)(s+6)}$

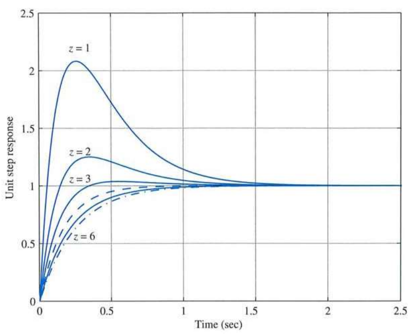
$R(s)=\frac{1}{s}$

$\frac{24}{z(s+4)(s+6)}$

⇒ $y(t)=y_1(t)+y_2(t)$

⇒ $\begin{cases} y_1(t)=\frac{12}{z}e^{-4t}-\frac{12}{z}e^{-6t} \\ y_2(t)=z\int_0^t y_1(\tau)d\tau=-3e^{-4t}+2e^{-6t}+1 \end{cases}$

⇒ $y(t)=1+\left(\frac{12}{z}-3\right)e^{-4t}+\left(2-\frac{12}{z}\right)e^{-6t}$



EXAMPLE 3.28

$$\frac{Y(s)}{R(s)} = H(s) = \frac{24}{z}\frac{(s+z)}{(s+4)(s+6)}$$

$\boxed{\text{step response}}$ $\Longrightarrow$ $y(t) = 1 + \left(\frac{12}{z} - 3\right)e^{-4t} + \left(2 - \frac{12}{z}\right)e^{-6t}$

$R(s) = \frac{1}{s}$

$$\Longrightarrow \lim_{t\to\infty} y(t) = 1$$

$$\|$$

$$s \to 0 \Rightarrow H(s)\big|_{s=0} = \text{DC gain}$$

The Final Value Theorem 终值定理

If all poles of $sY(s)$ are in the left half of the s-plane, then

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s). \tag{3.46}$$

step response $H(s)\frac{1}{s} = \frac{24}{z}\frac{(s+z)}{s(s+4)(s+6)} = \frac{24}{z}\frac{s}{s(s+4)(s+6)} + \frac{24}{s(s+4)(s+6)}$

$$\Longrightarrow \lim_{t\to\infty} y(t) = \lim_{s\to 0} \cancel{s} \cdot H(s) \cdot \cancel{\frac{1}{s}} = \lim_{s\to 0} \frac{24}{z} \cdot \frac{(s+z)}{(s+4)(s+6)}$$

$$= 1$$

$$\Longrightarrow s \to 0 \Rightarrow H(s)\big|_{s=0} = \text{DC gain}$$

The Final Value Theorem

If all poles of $sY(s)$ are in the left half of the s-plane, then

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s). \tag{3.46}$$

EXAMPLE 3.10

$$Y(s) = \frac{3(s+2)}{s(s^2+2s+10)}$$

$$y(\infty) = sY(s)|_{s=0} = \frac{3\cdot 2}{10} = 0.6.$$

A.1.3 The Initial Value Theorem

For any Laplace transform pair,

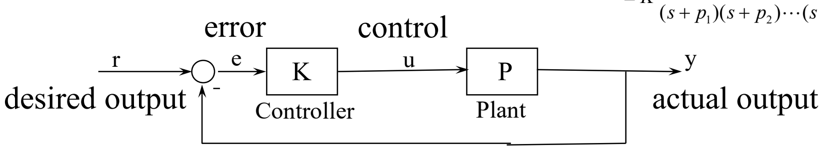
$$\lim_{s\to\infty} sF(s) = f(0^+). \tag{A.27}$$

Proof: Textbook pp 836 (A.1.3)~838 (A.1.4) Homework

3 Dynamic Response

$$T.F.\quad G(s) = \frac{Y(s)}{R(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}$$

$$= K\frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}$$

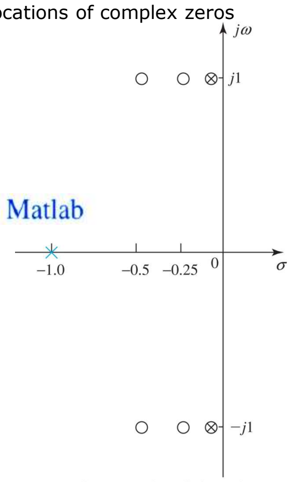


- 3.1 Review of Laplace Transforms
- 3.2.1 The Block Diagram
- 3.3 Effect of Pole Locations
- 3.4 Time-Domain Specifications
- 3.5 Effects of Zeros and Additional Poles
- 3.6.3 Routh's Stability Criterion

Skip page 162 (Example 3.29)~ page 164

$$H(s) = \frac{(s+\alpha)^2 + \beta^2}{(s+1)\left[(s+0.1)^2+1\right]}$$

Figure 3.33 Locations of complex zeros



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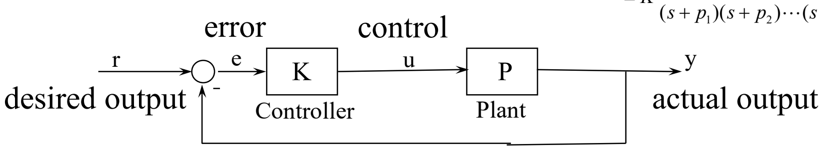
EXAMPLE 3.30 *Aircraft Response Using* Matlab

3 Dynamic Response

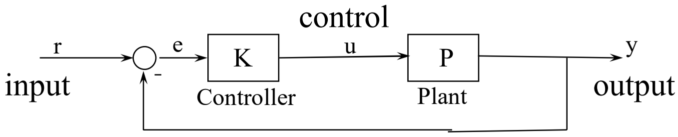
$$T.F.\quad G(s) = \frac{Y(s)}{R(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}$$

$$= K\frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}$$



Skip Session 3.6.1 **3.6.1 Bounded Input–Bounded Output Stability**

3.6 Stability



3.6.2 Stability of LTI Systems

Consider the LTI system $T(s) = \frac{Y(s)}{R(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_m}{s^n + a_1 s^{n-1} + \cdots + a_n}$ If $m < n$

$$= \frac{K \prod_{i=1}^{m}(s - z_i)}{\prod_{i=1}^{n}(s - p_i)}, \quad m \leq n.$$

$= k_1 \frac{1}{s-p_1} + k_2 \frac{1}{s-p_2} + \cdots\cdots$

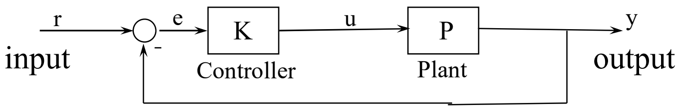
Impulse response

$\Rightarrow$ $y(t) = \sum_{i=1}^{n} K_i e^{p_i t}$, (3.88)

$= k_0 + k_1 \frac{1}{s-p_1} + \cdots$ If $m = n$

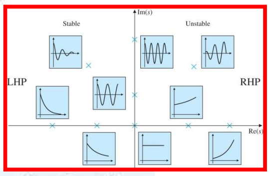
$\Rightarrow$ $\mathrm{Re}\{p_i\} < 0$. $\Rightarrow$ $e^{p_i t} \to 0$

$\Rightarrow$ The system is stable if and only if (necessary and sufficient condition) every term in Eq. (3.88) goes to zero as $t \to \infty$:



Consider the LTI systen $T(s) = \dfrac{Y(s)}{R(s)} = \dfrac{b_0 s^m + b_1 s^{m-1} + \cdots + b_m}{s^n + a_1 s^{n-1} + \cdots + a_n}$

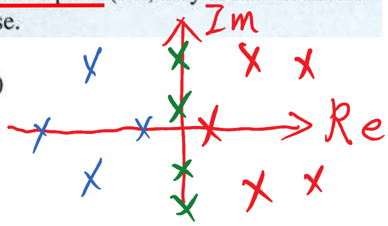
$$= \frac{K \prod_{i=1}^{m}(s - z_i)}{\prod_{i=1}^{n}(s - p_i)}, \quad m \le n.$$

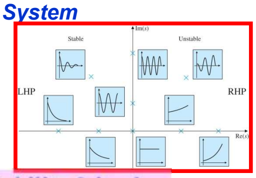


Impulse Response $\Longrightarrow y(t) = \sum_{i=1}^{n} K_i e^{p_i t},$

An LTI system is said to be stable if all the roots of the transfer function denominator polynomial have negative real parts (i.e., they are *all* in the left hand *s*-plane) and is unstable otherwise.

- stable $\mathrm{Re}\{p_i\} < 0. \Longrightarrow e^{p_i t} \to 0$
- unstable
- neutrally stable

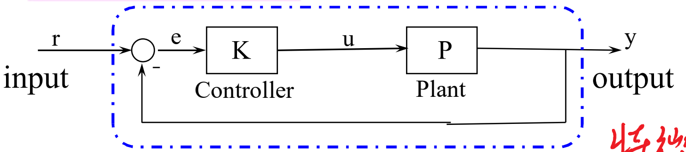




$$T.F. \quad G(s) = \frac{Y(s)}{R(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

$$= K \frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}$$

3.6.3 Routh's Stability Criterion



特徵方程式

$T(s) = \dfrac{Y(s)}{R(s)} = \dfrac{b_0 s^m + b_1 s^{m-1} + \cdots + b_m}{s^n + a_1 s^{n-1} + \cdots + a_n}$ $\Leftarrow=$ characteristic Equation

Consider the LTI system whose transfer function denominator polynomial leads to the characteristic equation

$$\Longrightarrow a(s) = s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_{n-1} s + a_n. \qquad (3.90)$$

A necessary (but not sufficient) condition for stability is that all the coefficients of the characteristic polynomial be positive.

$$a(s) = s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_{n-1} s + a_n. \qquad (3.90)$$

Routh array

$$\begin{array}{lllll}
s^n: & 1 & a_2 & a_4 & \cdots \\
s^{n-1}: & a_1 & a_3 & a_5 & \cdots \\
s^{n-2}: & b_1 & b_2 & b_3 & \cdots \\
s^{n-3}: & c_1 & c_2 & c_3 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \\
s^2: & * & * & & \\
s: & * & & & \\
s^0: & * & & &
\end{array}$$

$$b_1 = -\frac{\det\begin{bmatrix} 1 & a_2 \\ a_1 & a_3 \end{bmatrix}}{a_1} = \frac{a_1 a_2 - a_3}{a_1},$$

$$b_2 = -\frac{\det\begin{bmatrix} 1 & a_4 \\ a_1 & a_5 \end{bmatrix}}{a_1} = \frac{a_1 a_4 - a_5}{a_1},$$

$$b_3 = -\frac{\det\begin{bmatrix} 1 & a_6 \\ a_1 & a_7 \end{bmatrix}}{a_1} = \frac{a_1 a_6 - a_7}{a_1},$$

$$c_1 = -\frac{\det\begin{bmatrix} a_1 & a_3 \\ b_1 & b_2 \end{bmatrix}}{b_1} = \frac{b_1 a_3 - a_1 b_2}{b_1},$$

$$c_2 = -\frac{\det\begin{bmatrix} a_1 & a_5 \\ b_1 & b_3 \end{bmatrix}}{b_1} = \frac{b_1 a_5 - a_1 b_3}{b_1},$$

A system is stable if and only if *all* the elements in the first column of the Routh array are positive.

$$a(s) = s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_{n-1} s + a_n. \qquad (3.90)$$

Routh array

$$\begin{array}{lllll}
s^n: & 1 & a_2 & a_4 & \cdots \\
s^{n-1}: & a_1 & a_3 & a_5 & \cdots \\
s^{n-2}: & b_1 & b_2 & b_3 & \cdots \\
s^{n-3}: & c_1 & c_2 & c_3 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \\
s^2: & * & * & & \\
s: & * & & & \\
s^0: & * & & &
\end{array}$$

若 $a_i > 0 \Rightarrow$ 不知系统 stable/unstable

若 stable system $\Rightarrow a_i > 0$

- ✓ A necessary (but not sufficient) condition for stability is that all the coefficients of the characteristic polynomial be positive.
- A system is stable if and only if *all* the elements in the first column of the Routh array are positive.
- if the elements of the first column are not all positive,

  then the number of roots in the RHP equals the number of sign changes in the column.

EXAMPLE 3.32 *Routh's Test*

The polynomial $a(s) = s^6 + 4s^5 + 3s^4 + 2s^3 + s^2 + 4s + 4$

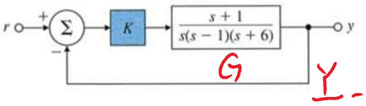
$$
\begin{array}{llll}
s^6: & 1 & 3 & 1 & 4 \\
s^5: & 4 & 2 & 4 & 0 \\
s^4: & \frac{5}{2} = \frac{4\cdot 3 - 1\cdot 2}{4} & 0 = \frac{4\cdot 1 - 4\cdot 1}{4} & 4 = \frac{4\cdot 4 - 1\cdot 0}{4} & \\
s^3: & 2 = \frac{\frac{5}{2}\cdot 2 - 4\cdot 0}{\frac{5}{2}} & -\frac{12}{5} = \frac{\frac{5}{2}\cdot 4 - 4\cdot 4}{\frac{5}{2}} & 0 & \\
s^2: & 3 = \frac{2\cdot 0 - \frac{5}{2}\left(-\frac{12}{5}\right)}{2} & 4 = \frac{2\cdot 4 - \left(\frac{5}{2}\cdot 0\right)}{2} & & \\
s: & -\frac{76}{15} = \frac{3\left(-\frac{12}{5}\right) - 8}{3} & 0 & & \\
s^0: & 4 = \frac{-\frac{76}{15}\cdot 4 - 0}{-\frac{76}{15}}. & & &
\end{array}
$$

Right Hand Plane

there are two poles in the RHP

EXAMPLE 3.33 *Stability versus Parameter Range*

$$\frac{Y}{R} = \frac{K\frac{s+1}{s(s-1)(s+6)}}{1+K\frac{s+1}{s(s-1)(s+6)}}$$



$$\frac{Y}{R} = \frac{KG}{1+KG}$$

The characteristic equation for the system is given by

$$\Rightarrow \quad 1 + K\frac{s+1}{s(s-1)(s+6)} = 0, \quad \Rightarrow \quad s^3 + 5s^2 + (K-6)s + K = 0.$$

Routh array is

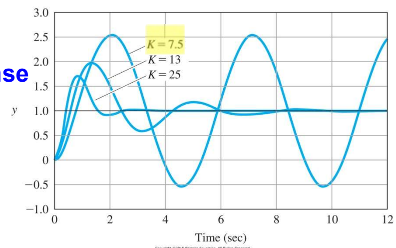
$$
\begin{array}{lll}
s^3: & 1 & K-6 \\
s^2: & 5 & K \\
s: & (4K-30)/5 & \\
s^0: & K. &
\end{array}
$$

For the system to be stable, it is necessary that

$$\Rightarrow \quad \frac{4K-30}{5} > 0 \quad \text{and} \quad K > 0.$$

$$\Rightarrow \quad K > 7.5 \quad \text{and} \quad K > 0.$$

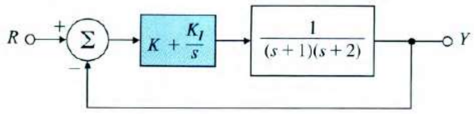
Unit Step Response



$$T(s) = \frac{Y(s)}{R(s)} = \frac{K(s+1)}{s^3 + 5s^2 + (K-6)s + K}$$

EXAMPLE 3.34 *Stability versus Two Parameter Ranges*

Figure 3.41
System with proportional-integral (PI) control



The characteristic equation of the closed-loop system is

$$\Rightarrow \quad 1 + \left(K + \frac{K_I}{s}\right)\frac{1}{(s+1)(s+2)} = 0, \qquad \Rightarrow \quad s^3 + 3s^2 + (2+K)s + K_I = 0$$

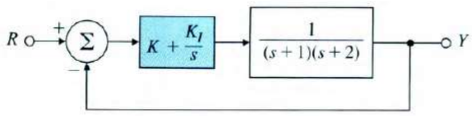
Routh array is

$$\begin{array}{lcc} s^3: & 1 & 2+K \\ s^2: & 3 & K_I \\ s\ : & (6+3K-K_I)/3 & \\ s^0: & K_I. & \end{array}$$

stability we must have

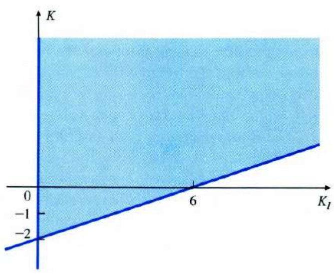
$$\Rightarrow \quad K_I > 0 \quad \text{and} \quad K > \frac{1}{3}K_I - 2.$$

Figure 3.41
System with proportional-integral (PI) control



stability we must have

$$\Rightarrow \quad K_I > 0 \quad \text{and} \quad K > \frac{1}{3}K_I - 2.$$



Routh's Test for Special Case

EXAMPLE

For the polynomial

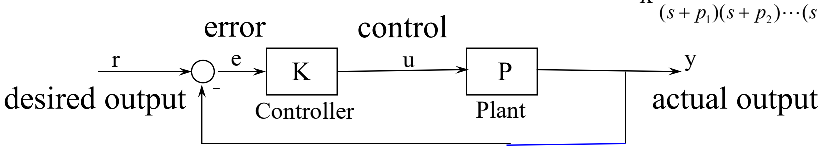
$$a(s) = s^5 + 5s^4 + 11s^3 + 23s^2 + 28s + 12,$$

s^5:	1	11	28	
s^4:	5	23	12	
s^3:	6.4	25.6	0	
s^2:	3	12		
s:	0	0		$\leftarrow a_1(s) = 3s^2 + 12$
New s:	6	0		$\leftarrow \dfrac{da_1(s)}{ds} = 6s$
s^0:	12,			

3 Dynamic Response

$$T.F. \quad G(s) = \frac{Y(s)}{R(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}$$

$$= K\frac{(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}$$



3.1 Review of Laplace Transforms

3.2.1 The Block Diagram

3.3 Effect of Pole Locations

3.4 Time-Domain Specifications

3.5 Effects of Zeros and Additional Poles

3.6.3 Routh's Stability Criterion