清華大學

PME3210

Linear Algebra and Linear Dynamical System Synthesis

線性代數與線性動態系統分析

(Autumn 2008)

Instructor : Cheng-Hsien Liu

劉承賢 動力機械工程學系 副教授

chhsliu@mx.nthu.edu.tw

http://mx.nthu.edu.tw/~chhsliu

http://mx.nthu.edu.tw/~chhsliu/linear/PME3210_Liu.html

課程主旨及目標：

本課程為一新的課程，企圖跨越整合線性代數理論及線性動態系統分析的教學，目的在針對大學部大二及大三學生，介紹矩陣的運算、瞭解矩陣及向量的物理直觀內涵、運用線性代數理論以分析動態系統。本課程所強調的主題介紹將極有助於提供諸如電路分析、訊號處理、通訊系統、控制系統、機器人學、影像處理、機器視覺、系統振動分析等領域的數學基礎及物理掌握與瞭解。

Course Objectives

The goals of the course are to introduce undergraduate to use/ manipulate matrices, understand/get physical intuition of matrices and apply linear algebra to synthesize linear dynamical systems. The emphasizing topics in this course are useful in other disciplines, including circuitry analysis, signal processing, communication, control systems, Robotics, networks, image display, system dynamics and vibration.

Textbook: 1. My class notes
2. *Matrix Analysis and Applied Linear Algebra* by C. D. Meyer
(http://matrixanalysis.com/) (Optional)
(http://www.matrixanalysis.com/DownloadChapters.html)

Reference 1. for Linear Algebra Practice Problem Set- *電機線代(高點) (Required)*
2. Introduction to Linear Algebra 3rd Edition by Gilbert Strang, Wellesley-Cambridge Press (March 2003)

Topics included in this course (tentative syllabus)

Topics	Reading (My Lecture Note pdf)	Lecture	Practice Problems
Introduction to this course and Dynamic Systems	Reader Ch. 1;	09/16, 09/19	
Linear Dynamical Systems	Reader Ch. 2, Ch. 3;	09/23, (No Lecture on 09/26) 09/30, 10/3(Quiz1)	
Matlab/Simulink	Reader Ch. 3;	10/3	
Matlab/Simulink (Competition Project Due: 11/7) Class Room 434	Reader Ch. 3, Matlab notes, Simulink notes,	10/7, 10/14, 10/17(Quiz2),	
The Geometry of Linear Equations	Meyer Ch1.1	10/17, 10/21	
Elimination with Matrices	Meyer Ch1.2; 1.3; 2.1; 2.2; 2.4 Ref1. pp-2-26~2-38	10/21,(No Lecture on 10/24& 10/28) , 10/31, 11/04	

Linear Algebra and Linear Dynamical System Synthesis *Prof. Cheng-Hsien Liu*

Matrix Operations and Inverses	Meyer3.2; 3.3; 3.5; 3.6, 3.7, 6.2 Ref1. pp-2-1~2-25, Ref1. pp2-39~2-66	11/04, 11/7(Quiz3),	
LU and LDU Factorization	Meyer3.9; 3.10 Ref1. pp-2-67~2-79	11/11	
Vector Spaces and Subspaces	Meyer4.1 Ref1. pp5-1~5-39	11/14 (Quiz4)	
Linear Independence	Meyer4.3	11/18	
Basis and Dimension	Meyer4.4 Ref1. pp5-40~5-56	11/18	
The Four Fundamental Subspaces	Meyer4.2; 4.4; 4.5 Ref1. pp5-40~5-56	11/21, 11/25, (No Lecture on 11/28), 12/2 (Quiz5)	
Vector Norm and Orthogonal	Meyer5.1; 5.4	12/5	
Projections and Gram-Schmidt	Meyer5.5; 5.6; 5.13; 5.14 Ref1. pp 1-26	12/9, 12/12	

Determinants	**Meyer Ch 6 (ref)** **Ref1. pp3-1~3-27** **Ref1. pp3-35~3-69**	**12/16 (Quiz6)**	
Eigenvalues and Eigenvectors	**Meyer Ch 7.1; 5.2(ref)** **Ref1. pp7-1~7-77** **Ref1. pp7-87~7-91**	**(Quiz7)**	
Diagonalization	**Meyer Ch 7.2; 7.3** **Ref1. Pp8-1~8-79**	**(Quiz8)**	
Differential Equations	**Meyer Ch 7.4** **Ref1. pp8-80~8-102**		
Singular Value Decomposition	**Meyer Ch 5.12**		
Competition Project Demo (Tentative)		**12/26**	
Final Exam		**01/13 (to be discussed)**	

Grading-

Quiz: 8*5=40%
Competition Project : 3%+7%+18%
Final exam: 35%

Policy for Exam Cheating- Exam cheating will make you fail for this class

Web reference from MIT OpenCourseWare site regarding linear algebra part by using Strang's book :

- **http://ocw.mit.edu/OcwWeb/Mathematics/18-06Spring-2005/CourseHome/index.htm**
- **http://www.twocw.net/mit/Mathematics/18-06Linear-AlgebraFall2002/CourseHome/ (in Chinese)**
- **http://web.mit.edu/18.06/www/ (update course website)**

http://ocw.mit.edu/OcwWeb/Mathematics/18-06Spring-2005/CourseHome/index.htm

1	The Geometry of Linear Equations	1.1-2.1
2	Elimination with Matrices	2.2-2.3
3	Matrix Operations and Inverses	2.4-2.5
4	LU and LDU Factorization	2.6
5	Transposes and Permutations	2.7
6	Vector Spaces and Subspaces	3.1
7	The Nullspace: Solving Ax = 0	3.2
8	Rectangular PA = LU and Ax = b	3.3-3.4
9	Row Reduced Echelon Form	3.3-3.4
10	Basis and Dimension	3.5
11	The Four Fundamental Subspaces	3.6
12	Exam 1: Chapters 1 to 3.5	
13	Graphs and Networks	8.2
14	Orthogonality	4.1
15	Projections and Subspaces	4.2
16	Least Squares Approximations	4.3
17	Gram-Schmidt and A = QR	4.4
18	Properties of Determinants	5.1
19	Formulas for Determinants	5.2
20	Applications of Determinants	5.3
21	Eigenvalues and Eigenvectors	6.1
22	Exam Review	
23	Exam 2: Chapters 1-5	
24	Diagonalization	6.2
25	Markov Matrices	8.3
26	Fourier Series and Complex Matrices	8.5, 10.2
27	Differential Equations	6.3
28	Symmetric Matrices	6.4
29	Positive Definite Matrices	6.5
30	Matrices in Engineering	8.1
31	Singular Value Decomposition	6.7
32	Similar Matrices	6.6
33	Linear Transformations	7.1-7.2
34	Choice of Basis	7.3-7.4
35	Exam Review	
36	Exam 3: Chapters 1-8 (8.1, 2, 3, 5)	
37	Fast Fourier Transform	10.3
38	Linear Programming	8.4
39	Numerical Linear Algebra	9.1-9.3
40	Final Exams	

http://www.stanford.edu/class/ee263/

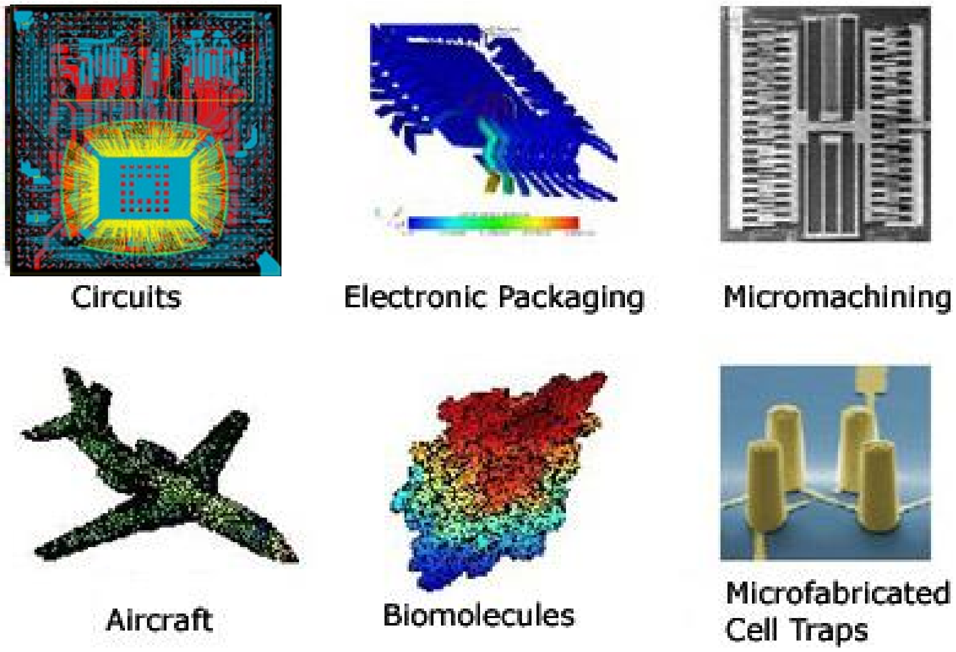
Matrix

$$\begin{aligned} y_1 &= a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ y_2 &= a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ &\vdots \\ y_m &= a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{aligned}$$

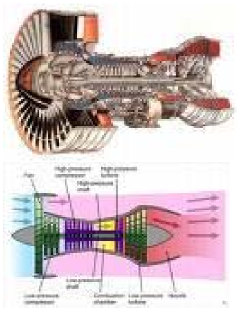
$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} \quad A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

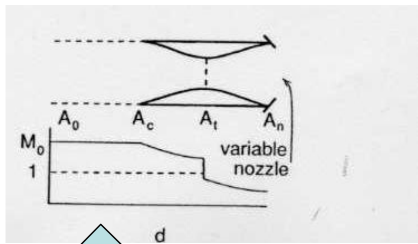
$$y = Ax$$

Numerical Simulation and Synthesis

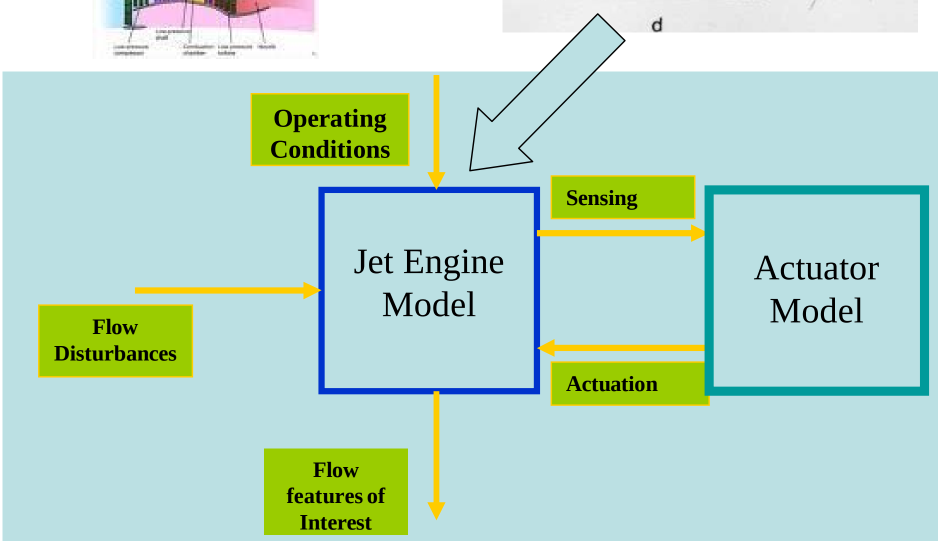


Jet Engine Design and Synthesis

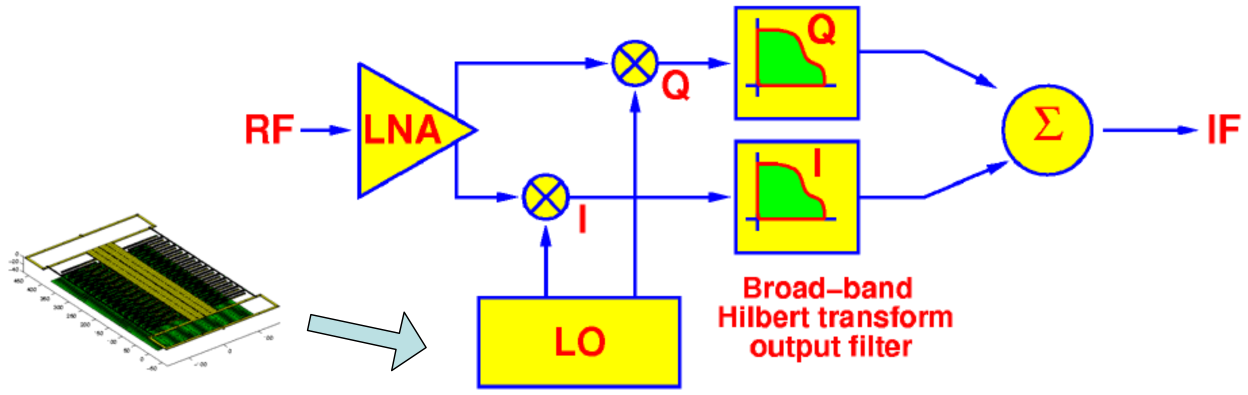




Generate Mathematical models directly from Navier-Stokes Equation based physical performance.



Micromechanical Resonators in a RF Wireless transceiver



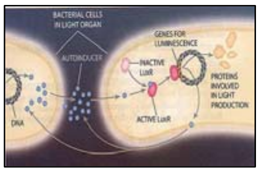
Complicated Geometry, Coupled Electrostatics, Fluids, Elastics

$$\frac{dx_r(t)}{dt} = F(x_r(t)) + b_r u(t)$$

$$y(t) = c_r^T x_r(t)$$

Mathematic Model for
Capturing the input/output response of
Physical System

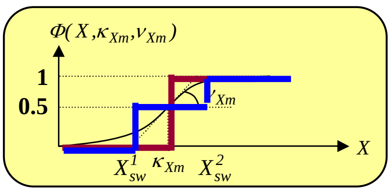
Biological System

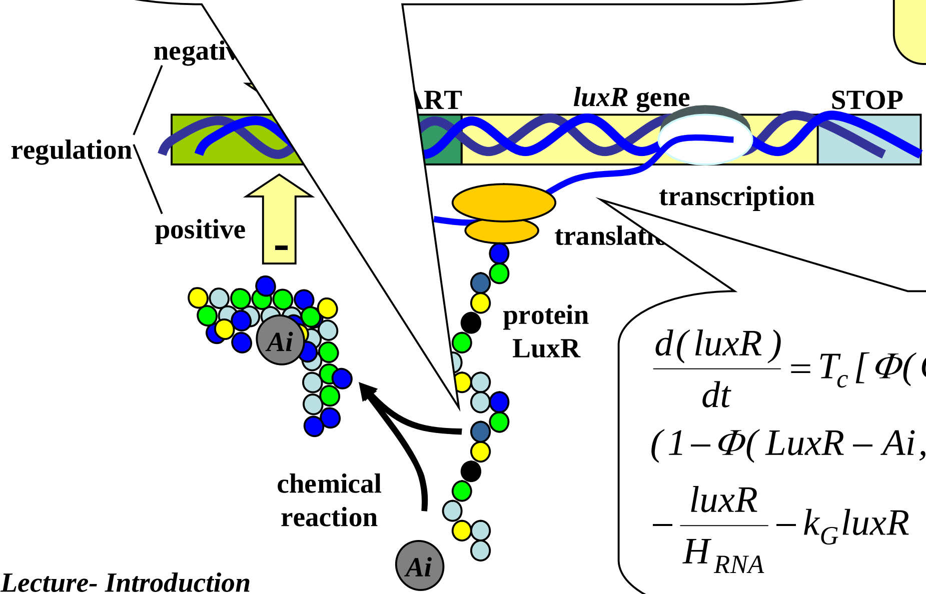


...g smooth functions.

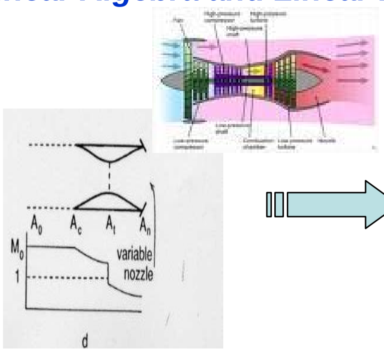
$$\frac{d[x}{dt}$$

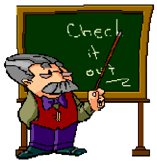
$$\frac{d(LuxR)}{dt} = T_l\ luxR - \frac{LuxR}{H_{sp}} - r^b_{LuxR/Ai}\ Ai\ LuxR + r^d_{LuxR/Ai}\ Co - k_G\ LuxR$$



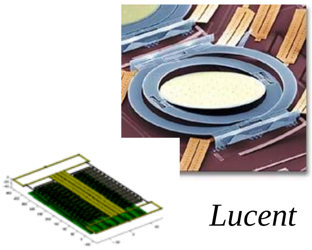


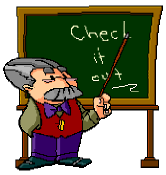
$$\frac{d(luxR)}{dt} = T_c[\Phi(CRP,\kappa_{CRP},\nu_{CRP})(1-\Phi(LuxR-Ai,\kappa_{LuxR-Ai},\nu_{LuxR-Ai}))+b] - \frac{luxR}{H_{RNA}} - k_G luxR$$



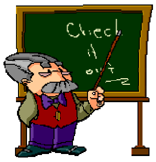


Engine Expert





Microsystem/RF/
Optical Expert



Robot Expert

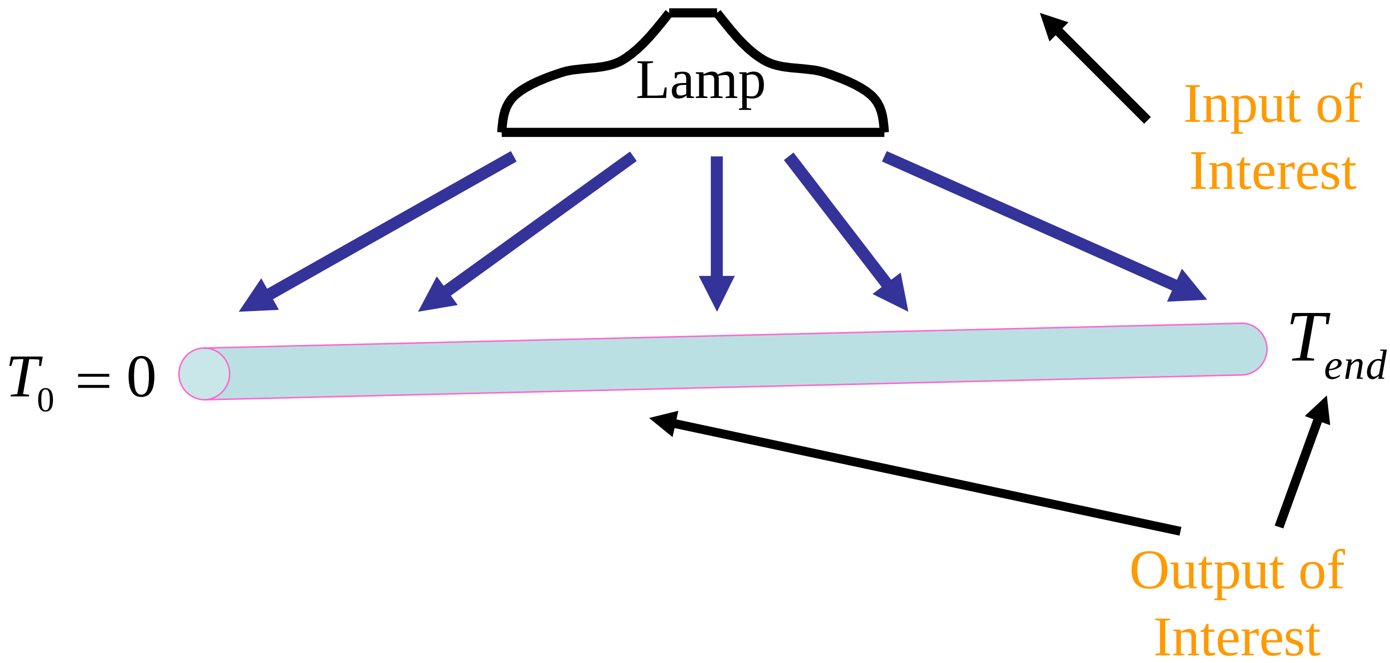
$$\frac{dx_r(t)}{dt} = F(x_r(t)) + b_r u(t)$$

$$y(t) = c_r^{\ T} x_r(t)$$

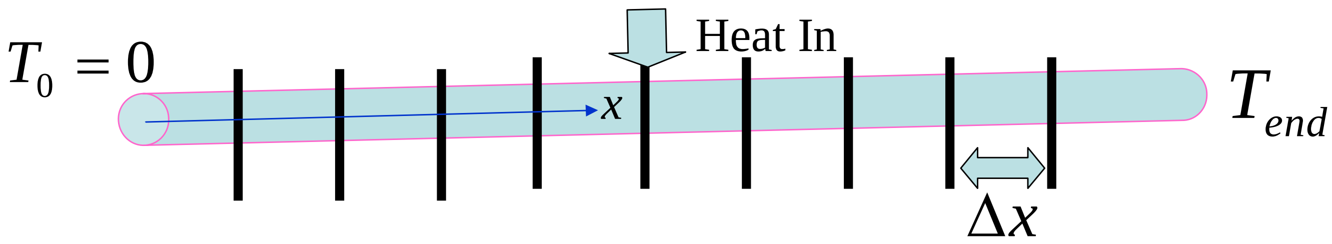
Mathematic Model for Capturing the input/output response of Physical System

Case A: The derivation of Mathematic Model for Capturing the input/output response of Heat Conducting Bar (Physical System)

$$\text{lamp power } = u(t)$$



Mathematic Model



- Temperature Differential Equation

$$\underbrace{\gamma}_{\text{specific heat}} \frac{\partial T(x,t)}{\partial t} - \underbrace{\kappa}_{\text{thermal conductivity}} \frac{\partial^2 T(x,t)}{\partial x^2} = h(x)\underbrace{u(t)}_{\text{scalar input}}$$

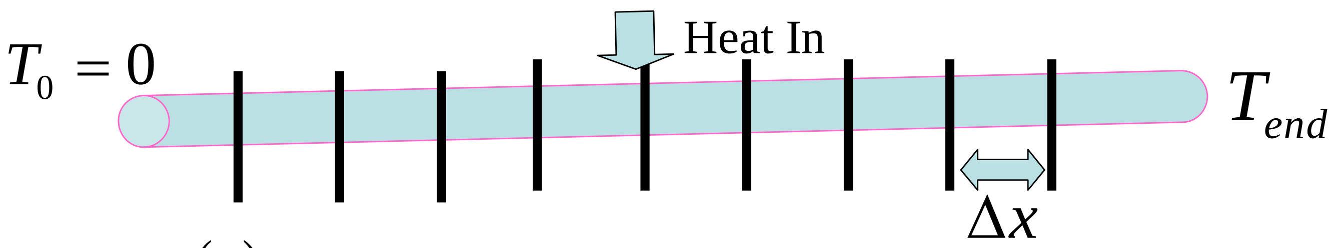
- Spatial Discretization

$$\gamma \frac{d\hat{T}_i}{dt} - \frac{\kappa}{(\Delta x)^2}\left(\hat{T}_{i+1} - 2\hat{T}_i + \hat{T}_{i-1}\right) = h(x_i)u(t) \quad i \in [1,\ldots,N-1]$$

$$T_{end} = \hat{T}_N$$

Linear Algebra and Linear Dynamical System Synthesis Prof. Cheng-Hsien Liu

State-Space Description



$$\frac{dx(t)}{dt} = \underbrace{A}_{NxN} x(t) + \underbrace{b}_{Nx1} \underbrace{u(t)}_{\text{scalar input}} \qquad \underbrace{y(t)}_{\text{scalar output}} = \underbrace{c}_{Nx1}{}^{T} x(t)$$

$$A = \begin{bmatrix} 2 & -1 & 0 & \cdots & 0 \\ -1 & 2 & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 2 & -1 \\ 0 & \cdots & 0 & -1 & 1 \end{bmatrix} \qquad b = \begin{bmatrix} h(x_1) \\ h(x_2) \\ \vdots \\ \vdots \\ h(x_N) \end{bmatrix} \qquad c = \begin{bmatrix} 0 \\ \vdots \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

No Dynamics (Steady-State) Case

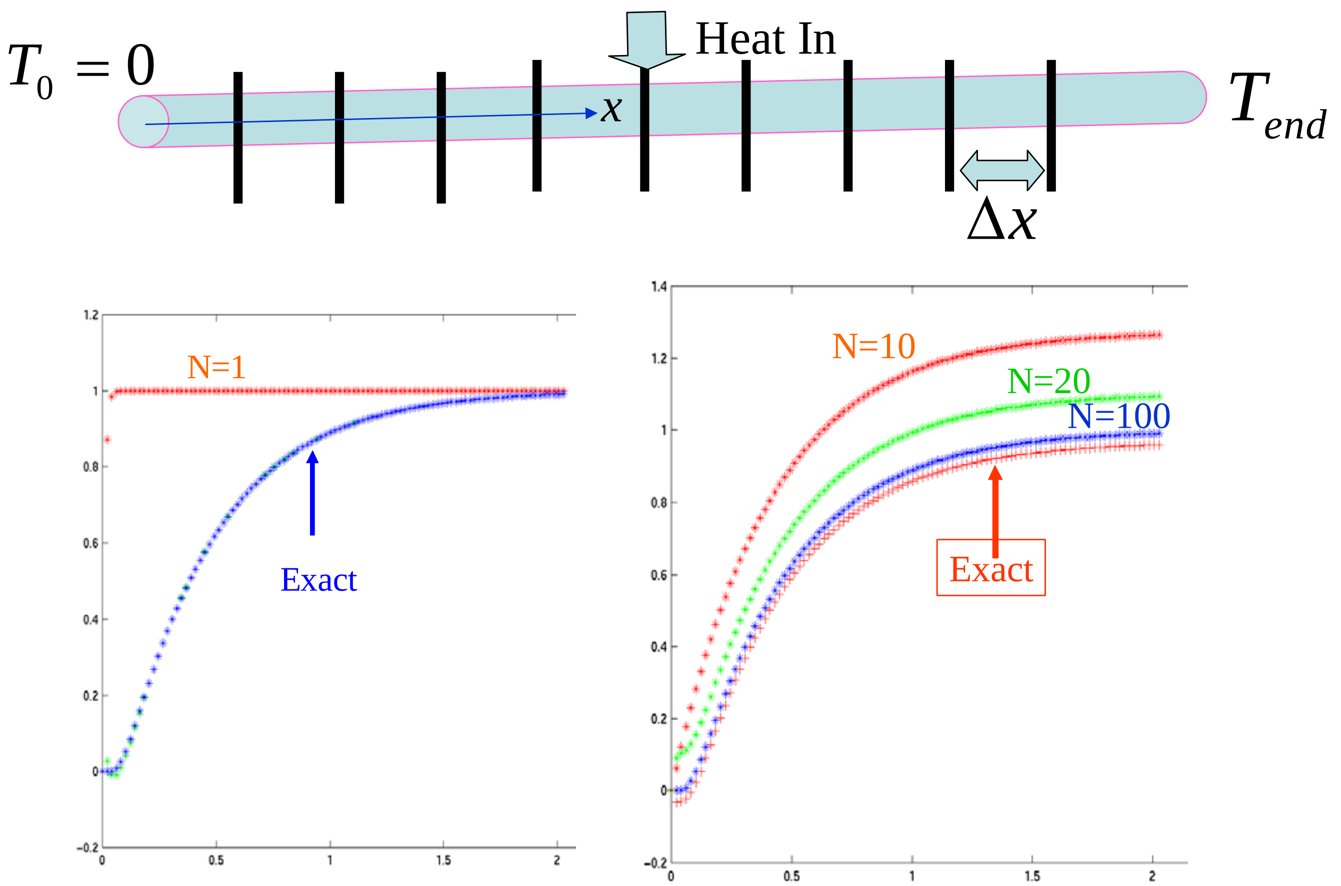
- State Space (Matrix) Representation

$$0 = \underbrace{A}_{NxN} x + \underbrace{b}_{Nx1} \underbrace{u}_{\substack{scalar\\input}} \qquad \underbrace{y}_{\substack{scalar\\output}} = \underbrace{c}_{Nx1}{}^{T} x$$

- Original System - Single Input/Output

$$y = -\underbrace{c^{T} A^{-1} b}_{1x1} u$$

  - Exactly reproduces Steady-state I/O Behavior



Dynamic Linear case

- State Space (Matrix) Representation

$$\frac{dx(t)}{dt} = \underbrace{A}_{NxN} x(t) + \underbrace{b}_{Nx1} \underbrace{u(t)}_{\substack{scalar\\input}} \qquad \underbrace{y(t)}_{\substack{scalar\\output}} = \underbrace{c}_{Nx1}{}^{T} x(t)$$

Laplace Transform $\mathcal{L}\left[\dot{f}(t)\right] = sF(s) - f(0)$

- Original System - Single Input/Output

$$sX(s) = AX(s) + bU(s) \qquad Y(s) = c^{T}X(s)$$

$$\Rightarrow Y(s) = \underbrace{c^{T}(sI - A)^{-1}b}_{H(s)}U(s)$$

← Transfer Function

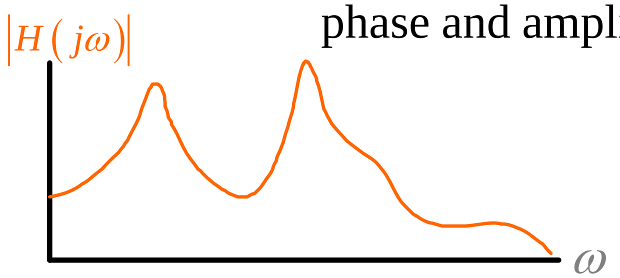
$$H(s) = \tilde{c}^{T}\begin{bmatrix} \frac{1}{s-\lambda_1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{1}{s-\lambda_N} \end{bmatrix}\tilde{b}$$

For Stable Systems, H(j*w*) is the frequency response

If $u(t) = e^{j\omega t}$ ← Sinusoid

then $y(t) = H(j\omega)e^{j\omega t}$

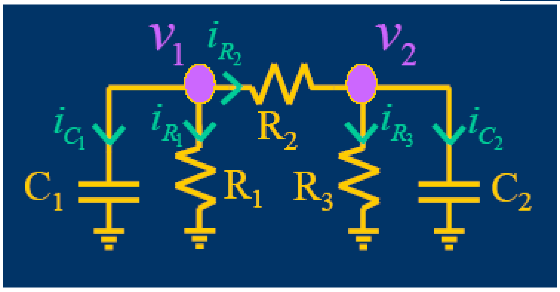
Sinusoid with shifted phase and amplitude



Note: Important Topics for Modern Control

Case B: The derivation of Mathematic Model for Capturing the Behavior of Integrated Circuit (Physical System)

$$\frac{dx(t)}{dt} = Ax(t)$$



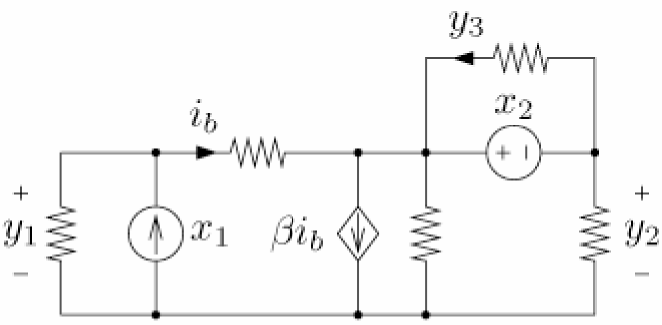
Constitutive Equations	Conservation Laws
$i_c = C\frac{dv_c}{dt}$	$i_{C_1} + i_{R_1} + i_{R_2} = 0$
$i_R = \frac{1}{R}v_R$	$i_{C_2} + i_{R_3} - i_{R_2} = 0$

Nodal Equations Yields 2x2 System

$$\begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \begin{bmatrix} \frac{dv_1}{dt} \\ \frac{dv_2}{dt} \end{bmatrix} = - \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_3} + \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Linear static circuit

interconnection of resistors, linear dependent (controlled) sources, and independent sources

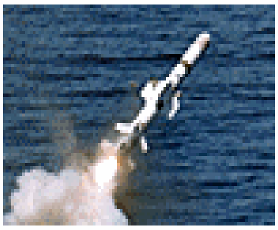


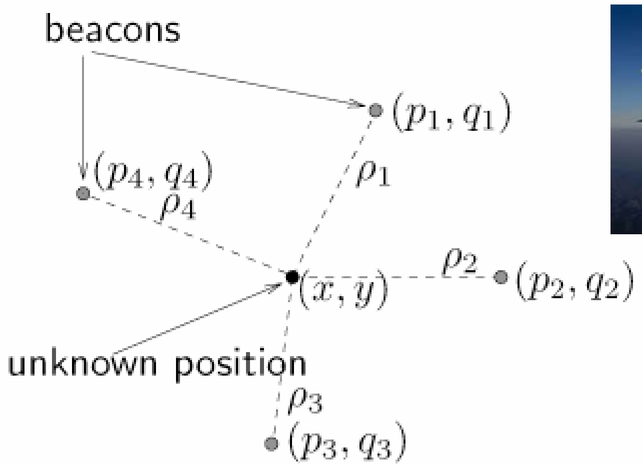
- x_j is value of independent source j
- y_i is some circuit variable (voltage, current)
- we have $y = Ax$
- if x_j are currents and y_i are voltages, A is called the *impedance* or *resistance* matrix

Navigation by range measurement

Also Space Travel (Star Track)

- (x, y) unknown coordinates in plane
- (p_i, q_i) known coordinates of beacons for $i = 1, 2, 3, 4$
- ρ_i measured (known) distance or range from beacon i





Control or design

$$y = Ax$$

- x is vector of design parameters or inputs (which we can choose)
- y is vector of results, or outcomes
- A describes how input choices affect results

sample problems:

- find x so that $y = y_{\text{des}}$
- find all x's that result in $y = y_{\text{des}}$ (*i.e.*, find all designs that meet specifications)
- among x's that satisfy $y = y_{\text{des}}$, find a small one (*i.e.*, find a small or efficient x that meets specifications)

$$\dot{x} = Ax + Bu, \quad y = Cx, \quad x(0) = 0$$