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# On the harmonious mating strategy through tabu search

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## Abstract

Genetic algorithms (GAs) are well-known heuristic algorithms and have been applied to solve a variety of complicated problems. When adopting GA approaches, two important issues—selection pressure and population diversity—must be considered. This work presents a novel mating strategy, called tabu genetic algorithm (TGA), which harmonizes these two issues by integrating tabu search (TS) into GA's selection. TGA incorporates the tabu list to prevent inbreeding so that population diversity can be maintained, and further utilizes the aspiration criterion to supply moderate selection pressure. An accompanied self-adaptive mutation method is also proposed to overcome the difficulty of determining mutation rate, which is sensitive to computing performance. The classic traveling salesman problem is used as a benchmark to validate the effectiveness of the proposed algorithm. Experimental results indicate that TGA can achieve harmony between population diversity and selection pressure. Comparisons with GA, TS, and hybrids of GA and TS further confirm the superiority of TGA in terms of both solution quality and convergence speed.

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## 1. Introduction

Genetic algorithms (GAs) are well-known heuristic algorithms based on the imitation of natural systems [1]. Their effectiveness in search and optimization problems has received extensive attention [2–5]. However, GA has its costs. Whitley [6] pointed out that there are two major issues in designing GA: selection pressure and population diversity. Selection pressure leads GA to exploit information inside the fitter individuals and results in more superior offspring iteratively. The diversity in GA is attributed to the form of population, which contains a certain number of encoded individuals for exploration. Nevertheless, a tradeoff exists between population diversity and selection pressure. Emphasis on selective pressure accelerates the optimization convergence but potentially causes premature convergence because of hastened loss of diversity. On the contrary, maintaining diversity can yield a better solution quality, but often slows down the convergence speed due to the lack of selection pressure. Therefore, a good GA scheme must simultaneously address these two issues. On one hand, GA must maintain certain diversity to explore the unvisited space, and on the other hand, it must have adequate selective pressure to exploit the relevant solutions. Restated, GA should pursue a good balance between exploration and exploitation in consideration of both convergence speed and optimized solution quality.

In this paper, we present a novel heuristic optimization algorithm, called tabu genetic algorithm (TGA), which incorporates the features of tabu search (TS) into GA. TS is another class of meta-heuristic algorithms, which are based on explicit memory structures [7]. It uses memory to record the search trajectory and guide the search in order to consider both intensification and diversification. Many studies have verified the encouraging capability of TS in combinatorial optimization problems [7–9]. In view of this, several researchers proposed to incorporate TS with GA to enhance the search. Glover and Laguna [9] inceptively introduced the scatter search toward integrating GA and TS. Then, several studies proposed various hybrids of GA and TS. For example, Handa and Kuga [10] considered the difference between the convergence speed of GA and TS in the first half and that in the second half of the search process. They suggested an approach of concatenating GA and TS, which switches the search to TS when GA stops improving solutions. Chin [11] attempted to increase the search intensity of TS with the convergence of GA's population. Another common way of hybridization is to regard TS as a local enhancement for GA. This approach typically selects the best chromosome or random ones from GA's population, and operates on them with TS to explore their neighborhood for better solutions. These results generated by TS are then incorporated into the GA's population and evolution is resumed [12–15,17]. Nara [16] additionally utilized a tabu list to prohibit the mating of chromosomes, whose hamming distances are too close to each other. In general, most of these hybrid approaches are based on running these two algorithms alternately and passing the computation results from one to the other. The original structures of GA and TS are not modified in these approaches.

In contrast to running alternately, TGA implants characteristics of TS into GA's mating strategy. First, the memory structure in TS, the *tabu list*, is used to record the trajectory of evolution and preclude certain mating in consideration of diversification. Second, under the restriction of tabu list, the *aspiration criterion* is applied to supply selection pressure for intensification. By incorporating the tabu list and the aspiration criterion, the selection of GA is expected to achieve harmony in exploring the diverse population and exploiting the superiors. Such a balance of exploration and exploitation has been shown to be able to effectively improve the performance of GA [2]. In addition to selection, a *self-adaptive mutation* operator is proposed so that the mutation is dynamically performed according to the situation of population rather than a fixed probability. This self-adaptation ability can also overcome the difficulty of determining mutation rate, especially when the rate critically affects the performance.

The rest of this paper is organized as follows. In Section 2, related works on mating schemes are reviewed. In Section 3, we propose the TGA algorithm and give detailed descriptions. Section 4 discusses the effectiveness of TGA's schemes and the comparisons of performance with GA, TS, and hybrids of GA and TS. Finally, conclusions are drawn in Section 5.

## 2. Related work

In the conventional GA, no mating strategy is applied to the results of selection; that is, parents are approved without any further examination after they are chosen at random or just by fitness. Indeed, mating in nature is more complicated. For example, with respect to human beings, as well as specific factors such wealth and appearance that objectively contribute to fitness, some implicit factors also potentially guide mating, such as race, bloodline, and others. Inspired by this observation, a number of mating strategies for GA have been proposed to deal with the issues of population diversity or selection pressure in a natural way. According to the adopted metrics used to measure implicit factors, we classify these approaches into two major categories—computation-based and memory-based mating strategies. They are reviewed below.

### *2.1. Computation-based mating*

The most common computation-based approach is to distinguish the degree of similarity among individuals by calculating genotype or phenotype characteristics, and then mate the individuals according to different strategies. Genotype holds the information embedded in individuals; thus it is used to specify mating individuals. Hamming distance is the most popular metric for similarity (or dissimilarity) between two individuals. De Jong [18] introduced the concept of niching and subsequently presented the crowding method to remove the most similar individual when a new one enters a subpopulation. The sharing function [2] takes fitness as a resource and distributes it over all niches, which are delimited by individuals' similarity. Instead of sharing the resources, Petrowski [19] suggested the clearing policy, which encourages the winner to take all resources in a niche. Mauldin [20] proposed the k-bits decreasing uniqueness method in which members of a population must have at least k-bits of differences. The method decreases this uniqueness with evolution. Thus, population diversity is controlled and premature convergence is avoided.

The work on incest prevention [21] introduced a direct way for mating to restrain assimilation from premature convergence. It suggested that individuals with a short hamming distance might be related. Hence mating is only allowed if their hamming distance is above a threshold, which decreases as evolution proceeds. The results show that incest prevention can really avoid premature convergence and improve the solution at the expense of convergence speed. Shimodaira [22] employed the concept of incest prevention in survival selection and increased the diversity by giving individuals with farther hamming distances from the best solution a higher probability of survival. Incorporating with tournament selection, correlative tournament selection (CTS) [23] chooses the mate with higher fitness and hamming distance from a set of candidates. Fernandes and Rosa [24] introduced an analogous concept, called negative assortative mating, into GA with varying population size (GAVaPS). They found that the number of candidates was directly proportional to the performance of the Royal Road function [25]. Bian et al. [26] proposed a strategy of choosing different crossover methods on the basis of the couple's hamming distance in order to prevent premature convergence.

In contrast to incest prevention, De et al. [27] tried to mimic inbreeding in nature and proposed assortative mating, which restricts individuals to mate with one separated by a minimal hamming distance. They stated that this strategy could enhance exploitation and reduce the disruption of schema. Experimental results demonstrated the strategy's effectiveness over both conventional GA and incest prevention in certain problems. The work in [28] used different assortative mating strategies to deal with vector quantization problems, and obtained an opposite outcome: positive assortative mating (inbreeding) did not improve GA performance and was inferior to negative assortative mating (incest prevention). That work revealed that an all-out effort on selection pressure, such as inbreeding, might lead to premature convergence due to the rapid loss of diversity.

Another significant field in computation-based mating concerns individuals' phenotype information. Phenotype enables individuals to be evaluated by their external or expressed features; mating strategies can be based on such information. The most straightforward metric is fitness. Fitness scaling [2] provides a stable rating and a simple way to prevent super-fit individuals from dominating the population and causing premature convergence. Another way to use fitness is to devise mating schemes according to the differences in individuals' fitness. The phenotypic assortative mating in [27] exactly measures the similarity by fitness. Chakraborty and Chakraborty [29] judged the allowance of mating by a normal function of the normalized fitness. This function gives a higher probability of crossover to the partner with a closer fitness.

In addition to fitness, Ronald [30] proposed the concept of seduction, which can be viewed as a secondary fitness function. It encourages GA to consider more phenotypic features in mating strategies. For example, the first parent can be selected according to fitness, and the second by seduction based on the Royal Road function. Ratford et al. [31] extended the use of seduction for a parallel GA to combine the effects of incest prevention and assortative mating. Mori et al. [32] restated the objective as minimizing free energy. Entropy, defined by the loci of genes, can be regarded as the phenotypic estimate considered in the mating scheme.

### 2.2. Memory-based mating

Incest prevention through genotypic or phenotypic metrics has been shown to maintain population diversity and improve the solution. However, these computation-based mating schemes must distinguish individuals' similarities at the cost of additional computation. Furthermore, the employed metrics significantly influence the results of mating. Motivated by human beings' ancestry, a supplementary memory structure is appended to individuals to enable their ancestry to be recognized. Consequently, the determination of incest is neutral to the genotype or phenotype metrics, and does not require extra computation to measure similarity.

Craighurst and Martin [33] considered the family tree to disallow incest by adhering to the ancestry-based incest law. The levels of the incest law define different numbers of members in a family tree among whom mating is forbidden. A higher level of the incest law corresponds to a more widely encompassing restriction. Experimental results validated its effectiveness on solution quality and diversity maintenance. Although the incest law yielded satisfactory results, it slowed convergence and was sensitive to the mutation rate. Fernandes and coworkers [24,34] further implemented this ancestry table upon GA with variable population size for preserving diversity. The non-incest method obtained better outcomes than the simple GA when applied to the Royal Road function. The improvement increased with the level of incest prevention. The capability of maintaining genetic diversity was also ensured.

Another way of using additional memory tags to handle diversity is to view the subpopulation as a race. Mating is allowed only between individuals with the same racial tag. The coexistence of diverse races (subpopulations) enables individuals to be distributed in a more dispersed fashion and certain diversity to be maintained; meanwhile, the evolution within respective races can also improve the solution quality. These characteristics make it adequate to solve multi-modal problems and reduce the computational cost of measuring distance, as required, for example, by the sharing function. Spears [35] devised three subpopulation schemes to control mating in different ways. The results show that replacing distance metrics with tag bits to estimate similarity is feasible. Ryan [36] proposed the races genetic algorithm (RGA), which applies crossbreeding among different races by a meta-GA in order to improve the possibility of convergence toward global peaks. Bandyopadhyay and Pal [39,40] developed the genetic algorithm with chromosome differentiation (GACD). GACD distinguishes chromosomes into two categories (M and F), labeled by two class bits, and allows crossover only between different categories. GACD's superior performance was verified through many function optimization and pattern classification problems.

Although memory-based mating schemes can prevent premature convergence and relieve computational overhead in determining bloodline, they have some flaws. First, from the viewpoint of maintaining diversity, restricted mating has side effects on the selection pressure; that is, an over-restricted strategy diminishes the selection pressure and then results in slower convergence. Therefore, designing a harmonious way to restrain mating without sacrificing selection pressure is very important. Second, the aforementioned works [24,33] demonstrate that ancestry-based mating schemes are sensitive to mutation rate. Hence, a self-adaptive mutation is suggested to overcome the difficulty of determining the rate and to enhance the solution quality. This paper focuses on developing a novel mating scheme to meet the above requirements.

## 3. Tabu genetic algorithm

The proposed TGA is built upon the evolutionary structure of GA and the restrictive characteristics of TS. Rather than alternately running GA and TS, the mating schemes of GA are combined with a memory structure and the search strategy of TS to augment the salient features of both algorithms simultaneously.

Fig. 1 illustrates the flowchart of the proposed approach in which most procedures of TGA follow the original architecture of GA. If the part of TS, highlighted in gray, is ignored, TGA degenerates into a simple GA. The components of TS supervise the reproduction of GA. After being produced by genetic operators, each pair of offspring undergoes the steps of TS, specifically, the tabu list and aspiration criterion, to check whether their parents are allowed to mate. If the mating does not violate the tabu restriction or is good

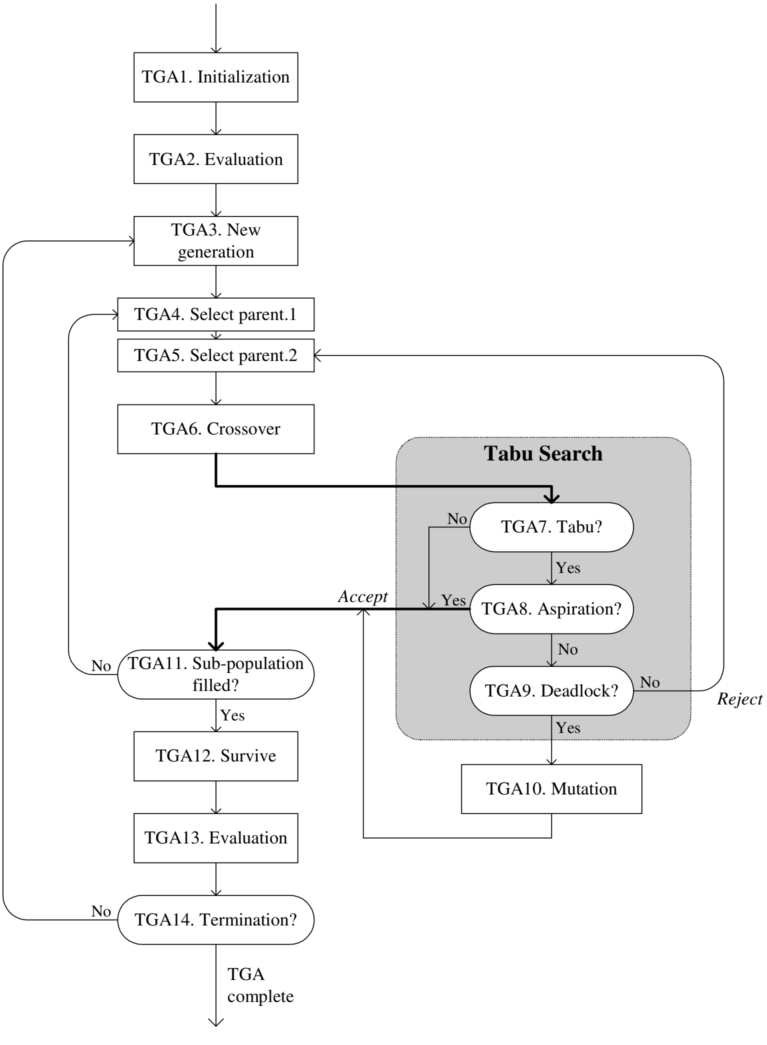

Fig. 1. Flowchart of TGA.

enough to meet the aspiration criterion, it is classified as acceptable and the children are delivered to the subpopulation. Otherwise, the reproduced children are rejected and this mating process returns to the step of selecting a mate. Such a process is repeated until the mating is acceptable. Namely, courtship continues until a potential parent finds its ideal mate. If the number of trials exceeds a predefined threshold, the attempt is classified as a deadlock. In such a case, the selected individuals will be mutated. They are then regarded as acceptable and delivered to the subpopulation as well.

In TGA, besides the partial variation of the original GA process, the components of TS should be redesigned. The following subsections consider the design approaches in great detail.

### 3.1. Representation

In TGA, a clan number is introduced into chromosomes to identify individuals. Additionally, a tabu list is appended to each chromosome in order to record the trajectory of evolution and carry out the TS strategy. Therefore, a chromosome in TGA is represented as a triple, $(G, \lambda, T)$, where $G$ is a set of original genes; $\lambda$ is a clan number of chromosome, and $T$ is a set of tabu list.

The clan number $\lambda$ is assigned uniquely at the initialization stage. During reproduction offspring inherit the clan number $\lambda$ from one of its parents. Chromosomes with the same clan number indicate that they have the same ancestry; namely, they are genetically identical by descent [41,42].

The tabu list $T$ records several clans that are forbidden to mate. In contrast to a unitary table used in the conventional TS, tabu list $T$ with clan number $\lambda$ is appended to the end of the genes in each chromosome. Such a new chromosome structure thus can accommodate the form of multiple agents in a population and the evolution in GA. Fig. 2 presents an example of representation for the traveling salesman problem (TSP). The genes represent a sequence of cities that the salesman must visit. An additional clan (7) with the tabu list (2, 8) carries the evolutionary information of this chromosome.

### 3.2. Tabu list

The tabu list adopted in TGA contributes to preventing incest in a memory-based manner. Within the mating pool, each successful mating in GA can be

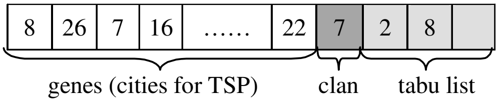

Fig. 2. Example of representation for TSP.

regarded as a move in TS. Just as TS uses the tabu list to prevent some moves from being trapped in local optima, TGA utilizes it to track propagation and forbid incest. Accordingly, assimilation caused by inbreeding can be avoided, and the loss of population diversity can be restrained during evolution.

In practice, if one ignores internal processes of recombination such as crossover and mutation, the external behaviors of selection and reproduction can be viewed as a move. Like the move in TS, the selection is monitored and restricted by the tabu list; restated, mating with a chromosome labeled as tabu is forbidden. Formally, the following function can be defined to determine whether two chromosomes $C_i(G_i, \lambda_i, T_i)$ and $C_j(G_j, \lambda_j, T_j)$ are tabu.

$$\mathrm{Tabu}(C_i, C_j) = \begin{cases} \text{true}, & \text{if } \lambda_i \in T_j \text{ or } \lambda_j \in T_i, \\ \text{false}, & \text{otherwise.} \end{cases} \tag{1}$$

In contrast to the incest law [33], prohibitive context in TGA is updated when the parents mate, instead of when the offspring inherit their genes. In TGA, after a successful mating, each parent updates its tabu list with the mate's clan so that the marriage is traceable. Offspring then inherit the clan and the updated tabu list entirely from one of the parents, rather than from a combination of both parents. Therefore the restriction in TGA is more concerned with the mating relationship than ancestry.

Offspring in TGA inherit the clan identification from one of its parents. This rule differs from that of incest law, which assigns each individual a unique identification. In other words, the tabu list of TGA plays the role of the patrilineal or matrilineal clan pedigree, which tracks of the trajectory of marriage and inheritance, and prevents incest in a broad sense that includes the stepfather or stepmother.

The size of a tabu list is proportional to the size of the population, and can be defined as follows:

$$N_{\mathrm{T}} = \delta \times N, \tag{2}$$

where $N_{\mathrm{T}}$ is the size of the tabu list in TGA, $\delta$ is the parameter of proportionality ($0 \leqslant \delta < 1$), and $N$ is the size of the population. The parameter of proportionality, $\delta$, controls the tendency of the search toward exploration or exploitation. A larger proportion corresponds to a larger tabu list, and thus a longer history for tracking and prohibition makes the selection stricter in preventing inbreeding and encourages mating with new mates. Consequently, the search focuses on exploration. On the contrary, a smaller parameter of proportionality loosens the restriction and supports the search exploitation. As the proportion decreases to zero, TGA degenerates into GA.

The operation of updating the tabu list works as a queue: a newly forbidden clan is inserted in the first position of the tabu list, and the other clans are shifted to the right by one rank. The formerly last clan will be released from the tabu list and regains the qualification to be mated. This process meets the

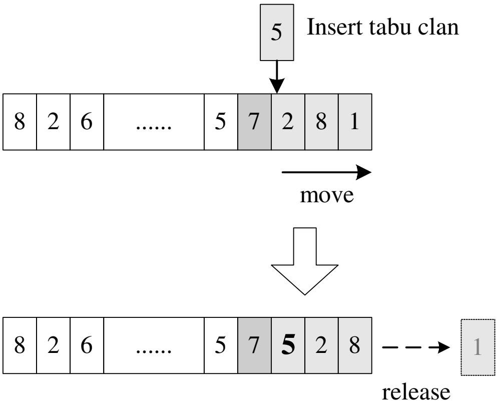

Fig. 3. Updating the tabu list in TGA.

requirement of degeneration of tabu tenure, as defined in TS [24]. Fig. 3 illustrates the operation of updating the tabu list in TGA.

Fig. 4 depicts a more complete process in which TGA applies the tabu list in a permutation problem. The process can be divided into two parts: the left part performs the genetic operators on the genes as in the conventional GA, and the right part handles the clan and the tabu list according to TS. In the right part, TGA first checks whether the mating is acceptable. If the mating does not violate the tabu restriction or satisfies the aspiration criterion, it is classified as acceptable and the reproduced children will be passed to the offspring population. Thereafter, the tabu list is updated in two stages. First, each parent adds the clan number of its mate to its tabu list. This kind of tabu list prevents duplicated mating, which leads to the decreasing of diversity in offspring population. Second, the offspring copies the updated clan number and the tabu list from one of its parents. It inherits the information about ancestry to prevent inbreeding from causing assimilation, which would severely reduce the diversity of the population.

### 3.3. Aspiration criterion

As well as the tabu list, the aspiration criterion is another measure used to judge mating. The criterion is designed to allow superior solutions to override the tabu restriction. Assuming that the fitness of offspring $C'_i$ is $F'_i$ and that the best solution so far obtained is $S$; then the aspiration criterion is defined as,

$$\text{Aspiration}(C'_i) = \begin{cases} \text{true}, & \text{if } F'_i > S, \\ \text{false}, & \text{otherwise.} \end{cases} \tag{3}$$

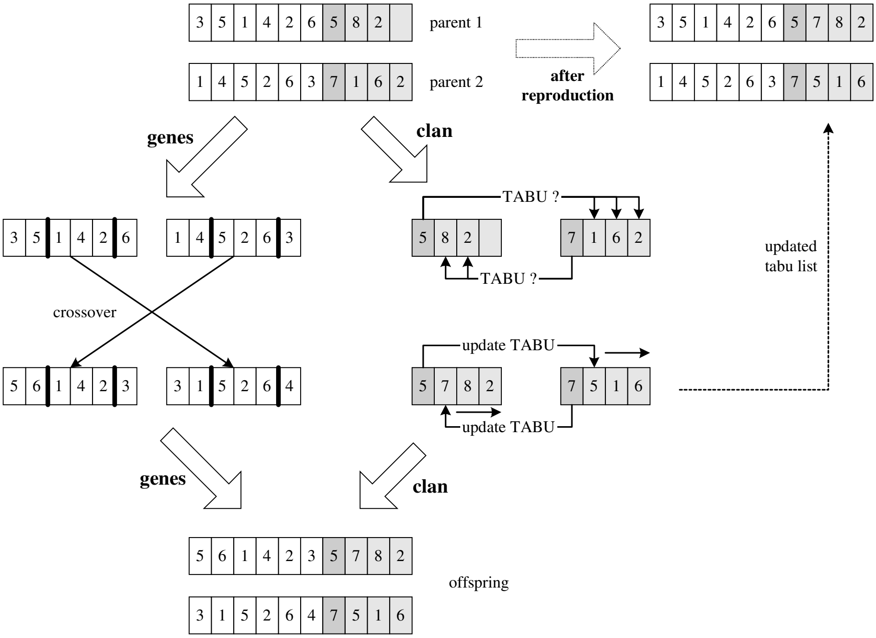

Fig. 4. Operations in TGA.

If a trial mating could produce the offspring superior to the best solution so far, this mating is allowed despite its being taboo. The criterion thereby encourages intensification under the consideration of diversification. Consequently, the aspiration criterion supplies a moderate selection pressure to overcome the slow convergence, which is the cost of maintaining diversity. Thus it can bring mating into harmony between selection pressure and population diversity. The next section discusses experiments to validate the effectiveness of the aspiration criterion.

### *3.4. Self-adaptive mutation*

In nature, the probability of mutation is typically not fixed. Organisms undergo mutation to adapt to the environment, especially when they suffer a great environmental change. Therefore, the mutation rate should depend on the organism's surroundings. In GA, the mutation rate is generally defined as a fixed probability. This kind of setting does not support proper response to a change in environment and does not reflect the reality of nature. The strategy of adaptive parameters has been shown to be a promising way to enhance evolutionary algorithms [43–46]. Additionally, as stated in previous work

[24,33], the performance of memory-based incest prevention is sensitive to the mutation rate. It is suggested that a self-determined mutation is necessary. Accordingly, we propose a self-adaptive mutation to determine the mutation rate in response to the changing environment.

In TGA, when the population diversity is too low to achieve an acceptable couple, the selection will be repeated indefinitely, causing a deadlock. Technically, according to genetic drift [2,41,42], the number of clans, or the diversity, inevitably shrinks with evolution. The population will eventually converge to a single member or clan. Then, chromosomes can no longer support successful mating, and deadlock occurs. To avoid this, a threshold (TH) is used to limit the number of repeated trial mating ($r$). When an individual courts more often than this threshold allows, but still cannot find an acceptable mate, it is determined that deadlock has occurred.

$$\text{Deadlock} = \begin{cases} \text{true}, & \text{if } r \geqslant \text{TH}, \\ \text{false}, & \text{otherwise}. \end{cases} \tag{4}$$

At this time the mutation operator is performed to vitalize the population to remedy the lack of diversity. Hence, TGA dynamically and adaptively performs the mutation operator according to population diversity. The probability of mutation, triggered by deadlock, varies with evolution, and is therefore not fixed. Moreover, mutation disrupts genetic information and so the mutated individual is viewed as newborn and is assigned a new clan number. The clan identification thus can correspond with the genetic information. The characteristics of dynamic adaptation in response to population circumstances are expected to enhance performance.

### *3.5. TGA algorithm*

Based on the above definitions and discussion, the proposed algorithm TGA is formulated as follows.

**Algorithm TGA.** Assume that the population $P$ consists of $N$ chromosomes $C_1, \dots, C_N$, with fitness $F_1, \dots, F_N$, respectively. The best fitness $S$ is recorded in each generation $t$. The genetic operators are performed to produce offspring $C'_1, \dots, C'_N$, and the deadlock criterion TH is defined to prevent infinite loop. The algorithm terminates at $t_{\max}$ generations, at which point the obtained best fitness $S$ is the optimized result.

**TGA1.** [Initialization.] Set $t \leftarrow 0$, and initialize population $P_t$.

**TGA2.** [Evaluation.] Evaluate population $P_t$ and result in $F_1, \dots, F_N$. Set $S \leftarrow \max(F_1, \dots, F_N)$.

**TGA3.** [New generation.] Set $n \leftarrow 0$ and $r \leftarrow 0$. (Where $n$ is the number of produced offspring, and $r$ is the repeated times of deadlock.)

**TGA4.** [Select parent 1.] Set $i \leftarrow \text{random}(1, \dots, N)$. (Select parents $C_i$.)

**TGA5.** [Select parent 2.] Set $j \leftarrow \text{random}(1, \dots, N)$. (Select parents $C_j$.)

**TGA6.** [Crossover.] Set $p \leftarrow \text{random}(\ )$. If $p > p_{\mathrm{c}}$, $(C_i', C_j') \leftarrow \text{Crossover}(C_i, C_j)$.

**TGA7.** [Tabu?] If $\text{Tabu}(C_i, C_j) = \text{false}$, go to step TGA11. (Check parents $C_i$ and $C_j$ to see if they are forbidden to mate.)

**TGA8.** [Aspiration?] If $\text{Aspiration}(C_i', C_j') = \text{true}$, go to step TGA11. (Check if offspring $C_i'$ or $C_j'$ can override the tabu criterion.)

**TGA9.** [Deadlock?] Set $r \leftarrow r + 1$. If $r < \text{TH}$, go to step TGA5.

**TGA10.** [Mutation.] $(C_i', C_j') \leftarrow \text{Mutation}(C_i', C_j')$. (If deadlock occurs, perform the mutation to activate the subpopulation.)

**TGA11.** [Subpopulation filled?] Insert the offspring $C_i'$ and $C_j'$ into $P_{t+1}$, and set $n \leftarrow n + 2$. If $n < N$, set $r \leftarrow 0$ and return to step TGA4.

**TGA12.** [Survive.] Set $P_{t+1} \leftarrow \text{Survive}(P_t, P_{t+1})$ and $t \leftarrow t + 1$.

**TGA13.** [Evaluation.] Evaluate population $P_t$, and set $S \leftarrow \max(F_1, \ldots F_N)$.

**TGA14.** [Termination?] If $t < t_{\max}$, return to step TGA3. Otherwise the algorithm TGA is complete.

## 4. Performance evaluation

In this paper, several comprehensive experiments are conducted to evaluate the performance of TGA. The impacts of the elements of TGA—tabu list, aspiration criterion, and self-adaptive mutation—are first investigated. The variation in population diversity is also examined to verify the diversification. The superiority of TGA is then confirmed by comparing its performance with that of GA, TS, and hybrids of GA and TS.

The traveling salesman problem (TSP), a classical combinatorial optimization problem, is used as a benchmark to evaluate performance. This problem involves a number of random cities, and the objective is to find the shortest path that passes through every city once. In the experiments, the cities are generated according to the 8th DIMACS Implementation Challenge [37] for generalization and comparability. Chromosomes are encoded by the classical permutation representation [2,3], as illustrated in Fig. 2. A population size of 100 chromosomes is used. The crossover operator adopted here follows the partially matched crossover (PMX), which is commonly used to handle TSP for GA [2]. The mutation operator follows the displacement mutation [3], which randomly displaces an arbitrary section of chromosomes. In the experiments on TGA, the probability of crossover ($P_{\mathrm{c}}$) is set to 1.0, and that of mutation is neglected because an adaptive mechanism is used to determine when the mutation should be performed. Each experiment includes 20 trials. The following subsections provide more details of the evaluation experiments. Section 4.1 examines the effect of the tabu list on GA's convergence and population diversity. Section 4.2 presents the impact of the aspiration criterion upon intensification of TGA. Section 4.3 reveals the role of self-adaptive mutation to improve TGA's performance. Section 4.4

compares the performance of GA, TS, Hybrids of GA and TS, and the proposed TGA.

### *4.1. Tabu list*

In TGA, the tabu list supports diversification. Its restrictive characteristics protect diversity through strategic mating and further prevent premature convergence. Several experiments are performed on the tabu list to confirm the effectiveness. In these experiments, TGA** denotes that the aspiration criterion and self-adaptive mutation in TGA are both disabled in order to remove their influences on selection pressure and mutation. At first, the probability of mutation ($P_M$) is set to zero in order to identify the effect of the tabu list on convergence. As illustrated in Fig. 5, though both GA and TGA** are trapped into premature convergence due to the lack of mutation, TGA** yields better solutions than GA. This improvement is attributed to the tabu list that can maintain diversity during evolution. Second, the performance is compared in a general situation, in which the mutation rate is set to a fixed probability, $P_M = 0.005$, for both GA and TGA. Fig. 5 shows that TGA** converges more slowly than GA. Such a defect is expected because the tabu list attempts to

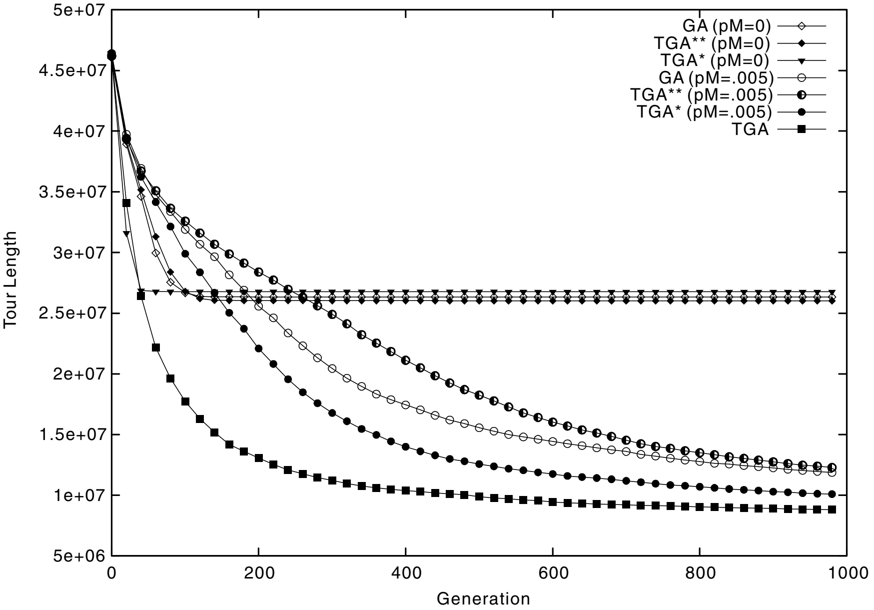

Fig. 5. Comparison of convergence of GA and TGA, both without mutation ($P_M = 0$) and with mutation ($P_M = 0.005$), at the city number $c = 100$. *: TGA without self-adaptive mutation; **: TGA without aspiration criterion and self-adaptive mutation.

maintain diversity at the expense of the selection pressure. The next subsection discusses an aspiration criterion, accompanied with the tabu list, to complement selection pressure and thereby overcome this defect.

A measurement of the variation in the population diversity during evolution is required to quantify the capacity of maintaining diversity. With respect to order-based encoding, the metrics must account for cyclic and inverse characteristics. Accordingly, Ronald [38] proposed a distance function to measure such a cyclic-TSP as,

$$d_{i,j} = l - e_{i,j}, \tag{5}$$

where $d_{i,j}$ is Ronald's proposed distance between two chromosomes $C_i$ and $C_j$; $l$ is the number of genes, which equals the number of cities, $c$; $e_{i,j}$ is the number of common edges between the corresponding tours constituted by two chromosomes. Furthermore, the degree of diversity in the whole population, $D$, can be defined as,

$$D = \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j=i+1}^{N} d_{i,j}. \tag{6}$$

Fig. 6 plots the variation of population diversity with and without mutation for GA and TGA. A higher degree corresponds to a more diverse population. The degree of population diversity for TGA declines more slowly than that for GA, independently of whether mutation is performed. This result confirms the fact that selection with the tabu list restrains inbreeding from leading to assimilation and a monotonous population. This outcome demonstrates that TGA with the tabu list can retard the lost of diversity.

Next, the impact of the size of tabu list on the performance of TGA is considered. The parameter of proportionality, $\delta$, determining the size of tabu list, is experimentally changed from 0.1 to 0.6 in increments of 0.1. Two population sizes ($N = 20$ and $N = 100$) are tested. The other parameters are crossover rate ($P_c$) $= 1.0$, aspiration criterion enabled, and self-adaptive mutation enabled. Table 1 lists the number of deadlocks per generation and the percentage of deadlock occurrences in each selection. The results demonstrate that the probability of deadlocks increases with the parameter of proportionality till $\delta = 0.4$. When the parameter exceeds 0.4, the probability is nearly fixed. Fig. 7 shows an outcome consistent with the direct proportionality between the probability of deadlock and the size of tabu list in Table 1. Moreover, this figure shows that the probability of deadlock increases as evolution proceeds, rather than being uniform. This phenomenon occurs in response to the decreasing diversity. Section 4.3 further examines the interaction between the probability of deadlock and self-adaptive mutation.

The effect of different sizes of tabu list on convergence is also examined. Fig. 8 depicts the convergence of TGA for different parameters of proportionality

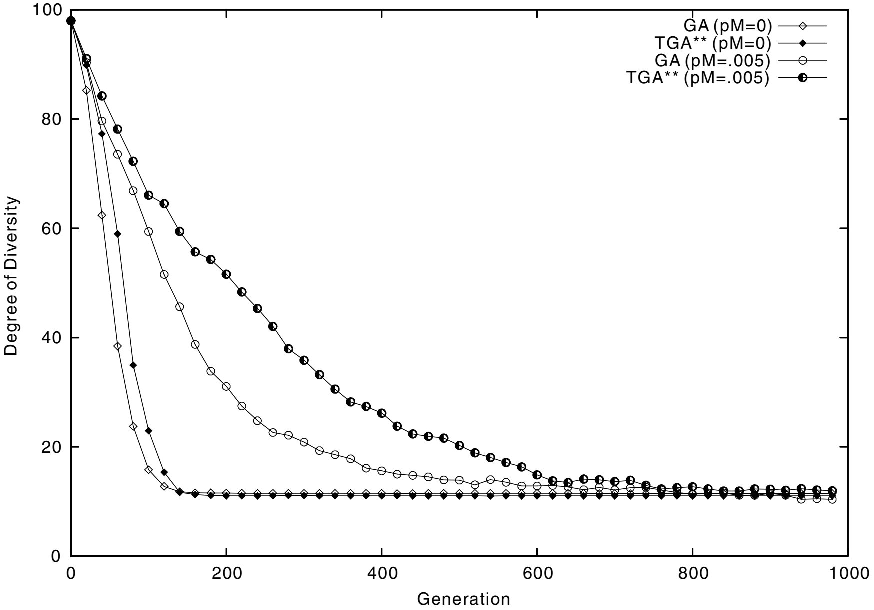

Fig. 6. Comparison of population diversity between GA and TGA both without mutation ($P_{\mathrm{M}} = 0$) and with mutation ($P_{\mathrm{M}} = 0.005$), at $c = 100$. **: TGA without aspiration criterion and self-adaptive mutation.

Table 1
Average number of deadlocks per generation for various $\delta$ at $c = 100$

| $\delta$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 |
|---|---|---|---|---|---|---|
| Number of deadlocks at $N = 20$ | 1.72 | 5.86 | 7.61 | 8.19 | 8.46 | 8.46 |
| Number of deadlocks at $N = 100$ | 15.68 | 30.32 | 35.20 | 36.80 | 37.24 | 36.97 |
| *Percentage of deadlocks at $N = 20$ | 17.2% | 58.6% | 76.1% | 81.9% | 84.6% | 84.6% |
| *Percentage of deadlocks at $N = 100$ | 31.3% | 60.6% | 70.4% | 73.6% | 74.5% | 73.9% |

*Percentage = (number of deadlocks)/(number of selections).

of the tabu list with a population size of 100. Although the algorithm with a proportionality parameter exceeds 0.4 performs as well as one with $\delta = 0.4$, it takes more time to check the tabu condition, thus being less efficient. For this reason, the parameter of proportionality, $\delta = 0.4$, is suggested and adopted in following experiments.

### 4.2. Aspiration criterion

The experimental results in Section 4.1 have shown the capacity of the tabu list to maintain population diversity. However, the tabu list achieves this goal

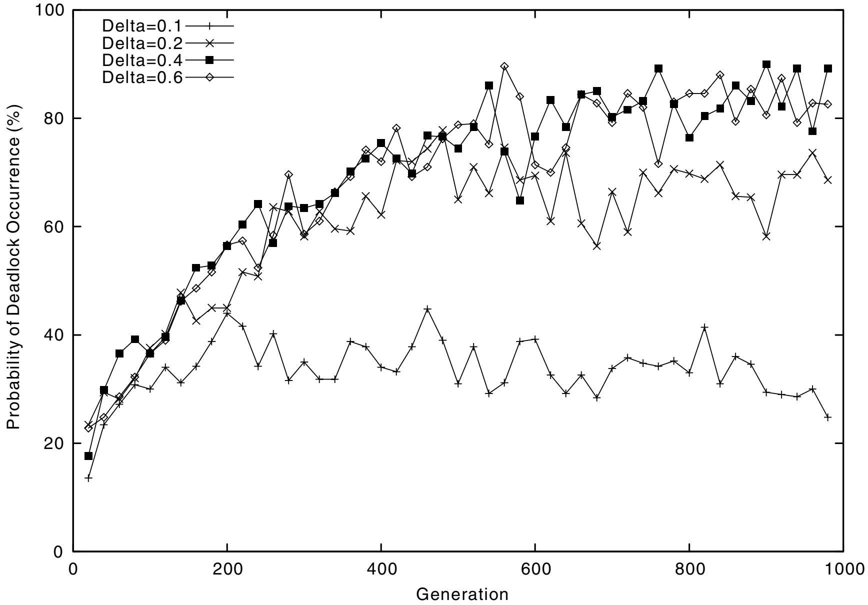

Fig. 7. Variation in probability of deadlock occurrence for different proportionality parameters ($\delta$).

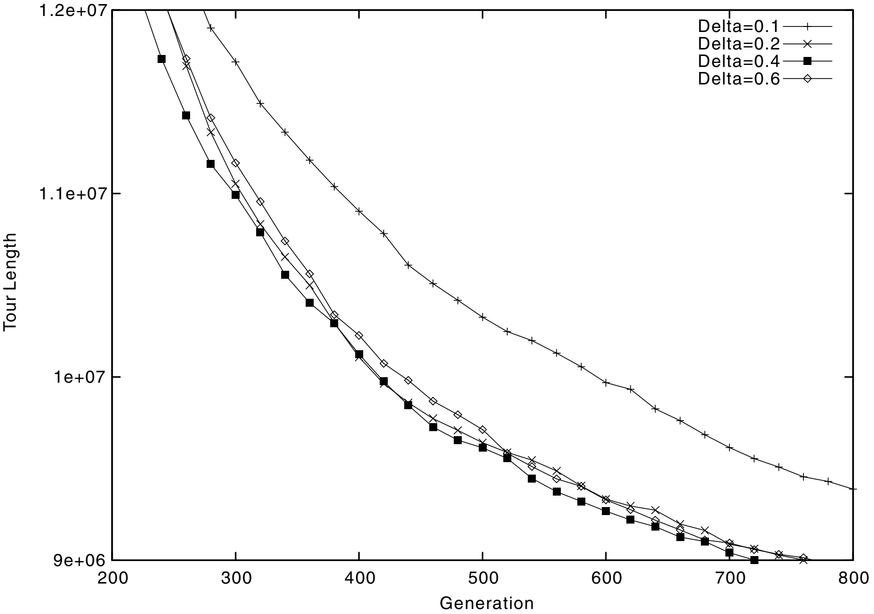

Fig. 8. Convergence with different parameters of proportionality of the tabu list at $c = 100$, $N = 100$.

through restrictive mating at the expense of selection pressure. It causes the search to focus on exploration and mitigate exploitation, and then leads to slower convergence. The aspiration criterion is proposed to supply moderate selection pressure to yield a good balance between exploration and exploitation.

Experiments are conducted on TGA with and without the aspiration criterion to verify the algorithm's effectiveness. At first the offspring has higher probability to meet aspiration criterion. As evolution proceeds, the offspring is harder and harder to be superior to the best solution, and the aspiration thus occurs less often. In terms of the influence on convergence speed, Fig. 5 shows that TGA with the aspiration criterion, TGA$^*$, converges faster in spite of the application of mutation. Without mutation, TGA with the aspiration criterion converges faster at the cost of solution quality. The reason is that the lack of mutation reduces diversification although the tabu list provides a mechanism to maintain diversity. This makes TGA with the aspiration criterion tend to over-exploitation and causes premature convergence. The other results with mutation ($P_{\mathrm{M}} = 0.005$) validate this inference. TGA with the aspiration criterion (TGA$^*$) converges faster than GA and TGA without the aspiration criterion (TGA$^{**}$). Moreover, it does not suffer from premature convergence because diversification is maintained by mutation.

### 4.3. Self-adaptive mutation

In TGA, self-adaptive mutation dynamically performs the mutation operator in response to population diversity rather than with a fixed probability. Mutation is performed to activate the population and continue the evolution when the diversity is so low that the individual cannot find an acceptable mate. A threshold number of trial mating determines whether the diversity is too low and when mutation should be performed. A higher threshold gives individuals more opportunities to find an acceptable mate, but too many trials may increase the time taken in mating. As well as the deadlock threshold, the size of the tabu list also affects the occurrence of deadlock. The descriptions in Section 3.2 and the results in Section 4.1 indicate that a larger tabu list restricts mating more. Thus, the number of trails is prone to exceed the deadlock threshold, and the chances of deadlock are correspondingly increased. The size of tabu list is set as 0.4 of the population size according to the suggestion in Section 4.1 for a better performance.

The sensitivity of the deadlock threshold (TH) to the search performance is investigated as follows. Fig. 9 plots the convergence and the consumed time is given in parentheses. These results show that the performance increases with the level of threshold, as does the consumed time. Thus the threshold can be determined as follows: a lower threshold is suited to faster convergence at the expense of solution quality, whereas a higher threshold obtains better solution

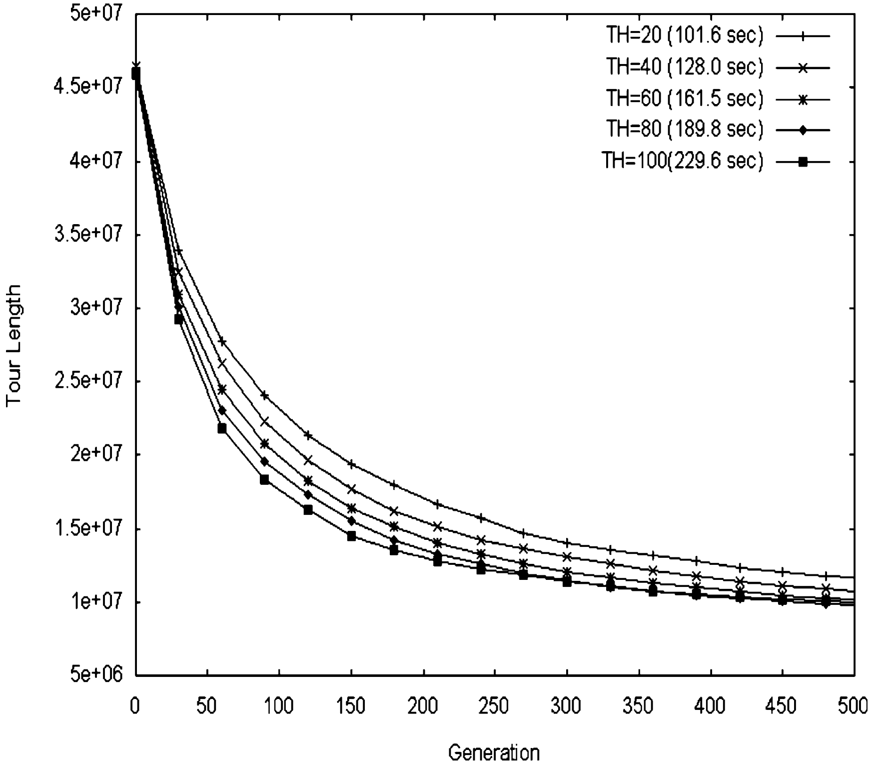

Fig. 9. Comparison of different levels of deadlock thresholds (TH) at $c = 100$, $N = 100$.

but slower convergence. In the following experiments, the highest threshold, equaling the population size, $N$, is adopted for a better solution quality. Furthermore, the worthiness of taking this extra computation effort is verified in Section 4.4.

Fig. 10 compares different mutation rates with self-adaptive mutation. In the case of fixed probabilities of mutation, the performance deteriorates with the mutation rate as the rate exceeds 0.002. Eventually the search approaches a random search when the mutation rate increases beyond 0.01. However, the results in this figure clearly demonstrate that TGA with self-adaptive mutation outperforms TGA with any fixed mutation rate. In terms of solution quality and convergence speed, the self-adaptation not only overcomes the difficulty of determining an optimal mutation rate, but its dynamically adaptive nature also enhances the performance.

The variation in the mutation occurrence probability for self-adaptive mutation is further examined. As Fig. 11 shows, self-adaptation involves dynamic mutation rather than mutation with a fixed probability. Initially, the

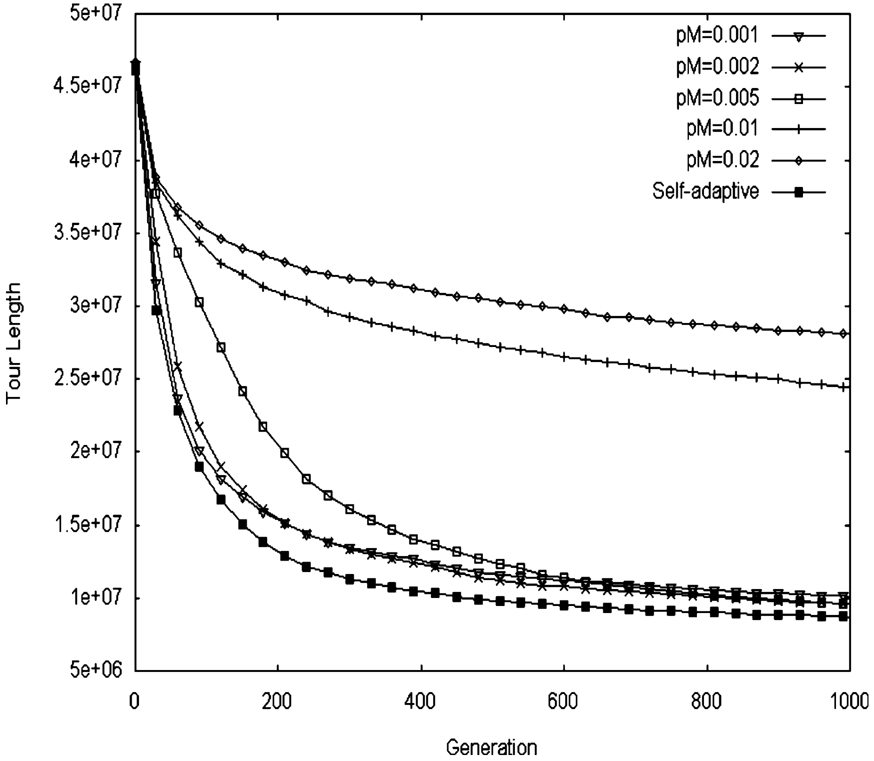

Fig. 10. Comparison of different mutation rates ($P_M$) and self-adaptive mutation at $c = 100$, $N = 100$.

probability is low. As the information contained inside the population is exploited stepwise, diversity is gradually lost. The population eventually becomes uniform and stops improving. Hence, the self-adaptive mutation in TGA is performed with increasing frequency as a gradually strengthening force to activate the population. The outcome presented in Fig. 11 is consistent with this suggestion. The improved performance further confirms the effectiveness of this self-adaptive mutation strategy.

### *4.4. Performance comparison*

TGA is compared with GA, TS, and GA + TS, which alternately run GA and TS, to further establish the better performance of TGA. The 100-city TSP is used as benchmark [37]. The population size ($N$) is set to 100 chromosomes. For GA, GA + TS, and TGA, the crossover operator adopted here is PMX; the mutation operator is displacement mutation. TS sequentially

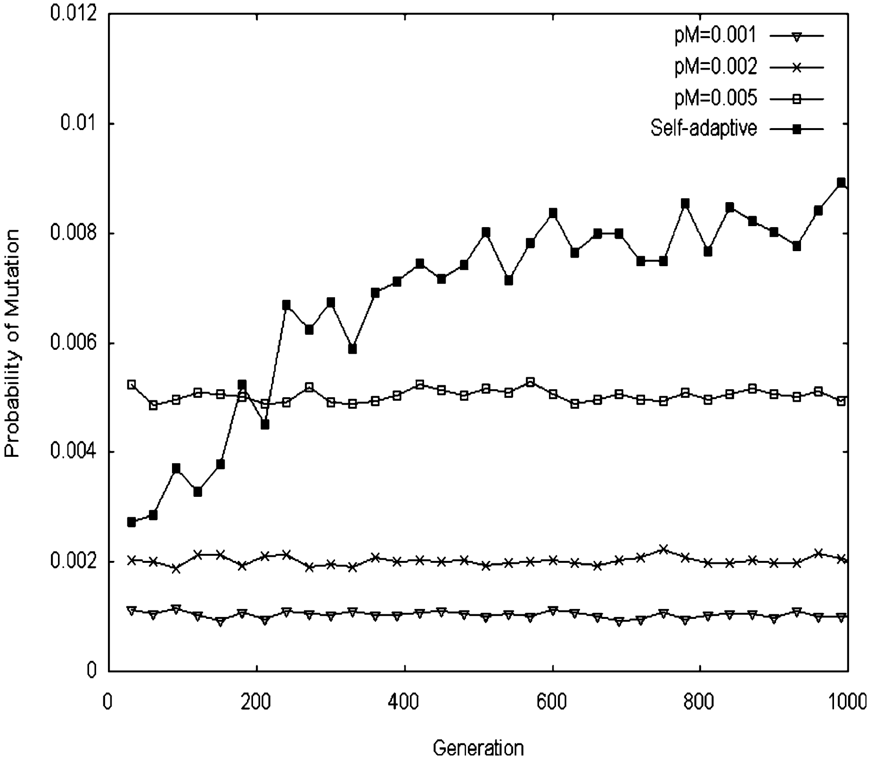

Fig. 11. Variation in the probability of mutation occurrence during evolution at $c = 100$, $N = 100$.

performs the move operator on all adjacent solutions to generate the neighborhood, and the best solution is selected as the current solution. The optimal parameters, obtained by our many experiments, are set as crossover rate $(P_c) = 1.0$ and mutation rate $(P_M) = 0.002$ for GA, and the tabu list has size 12 for TS. The hybrid approach, $(GAn_1 + TSn_2)$, alternately runs $n_1$ generations of GA and $n_2$ generations of TS. Three settings are proposed to simulate the typical hybrid of GA and TS: GA10 + TS10, GA100 + TS100, and GA100 + TS10. Their parameters are set as those of GA and TS. In regard to TGA, it uses the same population size and crossover rate as GA, but dynamically performs the proposed self-adaptive mutation. The size of the tabu list for TGA is set to 0.4 of the population size, as suggested in Section 4.1. Experiments run 1000 generations and 20 trials for each set of parameters.

Table 2 compares solution quality. TGA achieved the best solution on average. $t$-test is further used to test for statistical significance of the difference. The null and alternative hypotheses are as follows.

Table 2
Average ($\overline{X}$) and standard deviation ($S$) of best solutions

| | GA | TS | GA10 + TS10 | GA100 + TS100 | GA100 + TS10 | TGA |
|---|---|---|---|---|---|---|
| $\overline{X}$ | 11935491 | 12795811 | 10533845 | 10283457 | 10051794 | 8724187 |
| $S$ | 499557 | 1068378 | 433165 | 653625 | 446210 | 356133 |

Table 3
Results of statistical test for significance ($\alpha = 0.05$)

| $X$ | $Y$ | df | $t$ | $t_{\mathrm{df},0.05}$ | $H_0$ |
|---|---|---|---|---|---|
| TGA | GA | 34.35 (35) | −23.41 | −1.645 | Reject |
| | TS | 23.17 (24) | −16.17 | −1.711 | Reject |
| | GA10 + TS10 | 36.63 (37) | −14.43 | −1.645 | Reject |
| | GA100 + TS100 | 29.37 (30) | −9.37 | −1.645 | Reject |
| | GA100 + TS10 | 36.23 (37) | −10.40 | −1.645 | Reject |

df: degrees of freedom.

$$\begin{cases} H_0 : X - Y \geqslant 0, \\ H_1 : X - Y < 0. \end{cases}$$

TGA is tested as $X$ with other algorithms as $Y$ to confirm the superiority of TGA. Table 3 displays the statistical results of $t$-test, based on the data in Table 2. Given a significance level ($\alpha$) of 0.05, each $t$ value is less than the critical value $t_{\mathrm{df},0.05}$; the null hypothesis is thus rejected. These results establish that the superiority of TGA over other algorithms (GA, TS, and hybrids) in terms of solution quality is statistically significant.

Fig. 12 compares the convergence of TGA with that of GA, TS, and GA + TS. The plot shows that TGA outperforms the three hybrid approaches, which nevertheless yielded superior solutions to those obtained by running GA or TS individually. Moreover, TGA and GA + TS converge faster than GA, but slower than TS in the first half of convergence.

The comparative results in Fig. 12 concern convergence speed in number of iterations. However, an algorithm may spend more time on computation, while nevertheless converging faster with respect to iterations. Each algorithm conducts a different number of computations in each iteration, and so comparing speeds only by iterations is unfair. Thus, the time taken in each generation, instead of the number of iterations, is recorded and represented on the $x$-axis. Each algorithm takes 20 trials to obtain the average trajectory of the convergence.

Fig. 13 compares convergence speed in terms of running time. Clearly, the interval between two TGA samples is longer than that between two GA samples, indicating that TGA takes more time in each generation. However, TGA still converges faster than GA in spite of requiring more computation. This improvement suggests that the TGA strategy does make evolution more

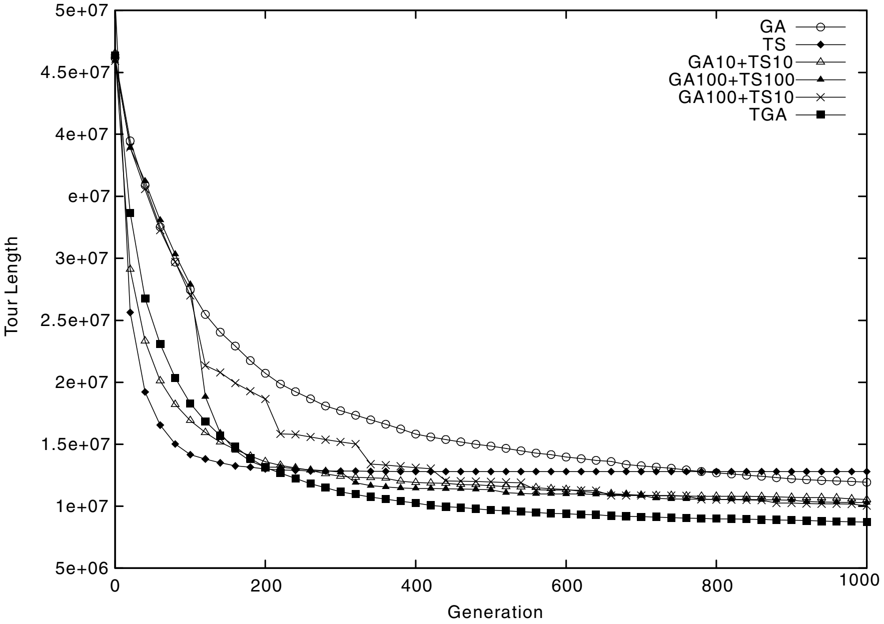

Fig. 12. Comparison of GA, TS, GA + TS, and TGA in terms of generations at $c = 100$, $N = 100$.

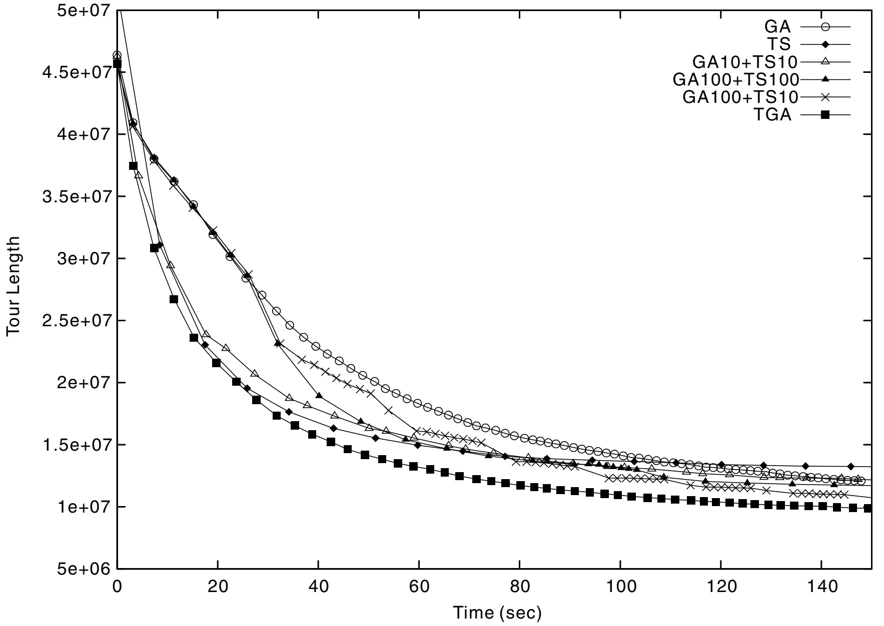

Fig. 13. Comparison of GA, TS, and TGA in terms of running time at $c = 100$, $N = 100$.

efficient; in other words, it enhances intensification. Also, a solution of higher quality reflects the effectiveness of the diversification in TGA. Consequently, the extra computation cost of TGA is worthwhile in terms of solution quality and convergence speed.

## 5. Conclusions

This work presents a novel heuristic optimization algorithm, called TGA, which incorporates the characteristics of TS into the mating strategy of GA. First, an additional memory structure, which consists of a clan number and a tabu list, is appended to each chromosome to record the trajectory of evolution as a basis for the mating strategy. The tabu list contributes TGA to maintain diversity through broad-sense incest prevention. Second, the aspiration criterion for overriding the restriction of tabu list is used to supply a moderate selection pressure and to overcome the retardation caused by restrictive mating. As well as promoting intensification and diversification in the mating strategy, TGA applies a self-adaptive mutation approach to dynamically perform the mutation operator according to the situation of the population. The traveling salesman problem is used as a benchmark to evaluate TGA. The comprehensive experimental results confirm the effectiveness of the elements of TGA: tabu list, aspiration criterion, and self-adaptive mutation. The comparisons demonstrate that TGA outperforms GA, TS, and conventional hybrids of GA and TS, in terms of solution quality and convergence speed. These favorable results show that the tabu list and the aspiration criterion constitute a harmonious mating strategy for TGA in regard to population diversity and selection pressure. They also show that the self-adaptive mutation in TGA effectively enhances the performance. Altogether, these results demonstrate the superiority of TGA due to the integration of tabu search and genetic algorithm in the mating strategy.

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