

## Topic: Bonds and Their Valuation

### I. Definitions:

<b>Bond:</b>	An IOU. - A security that obligates the issuer to make specified payments to the bondholder.
<b>Maturity:</b>	Date when the bond principal is repaid.
<b>Face Value:</b>	Payment at maturity, usually \$1,000 (par value).
<b>Coupon Rate:</b>	Annual interest payment as a percent of the face value

**Example:** 10 year \$1,000 US Treasury Bond paying 7%. New Issue

Maturity:	September 2010
Face Value:	\$1,000
Coupon Rate:	7%
Coupon Payments:	\$70/yr. (Note they actually come semi-annually.)

-1000	70	70	70	70	70	70	70	70	70	70	70+1000
0	1	2	3	4	5	6	7	8	9	10	

#### Who issues Bonds?

US Government - bills(< 1yr.), notes(1-10 yrs) and bonds(10-30 yrs)

Corporations

States

Cities

School Districts

Hospitals

Churches

Note the last 5 are tax free.

**Who buys Bonds?** Anyone can. Explain primary and secondary market.

### II. Bond Pricing (The Discounted Cash Flow Model)

- (1) Price of Bond = Present Value of its future cash flows, discounted at the **appropriate** current interest rate (appropriate based on risk) (not necessarily the coupon rate).

Example: 10-year, 7% (\$1,000) US Treasury bond. Assume comparable investments elsewhere are paying 7%. What is the bond price?

(7%, 10 yr. Annuity)

$$\text{Interest: PVA} = C \left[ \frac{1}{r} - \frac{1}{r(1+r)^t} \right] = \$70 \left[ \frac{1}{.07} - \frac{1}{.07(1.07)^{10}} \right] = \$70 (7.024) = \$491.68$$

$$\text{Principal: } \frac{FV}{(1+r)^t} = \frac{\$1,000}{(1.07)^{10}}$$

$$\begin{aligned} \text{Total: } & \$ 491.68 \\ & + \underline{508.35} \\ & \$ 1,000.03 \quad - \text{ note rounding} \end{aligned}$$

Note 1: Price = face value when coupon rate = discount rate.

Note 2: For a fixed coupon rate bond, the coupon rate stays the same throughout the life of the bond.

Note 3: The coupon rate is the market rate, or the discount rate, at the time the bond is issued.

## (2) Bond Price vs. Interest Rate

**Example:** If you hold the bond for 2 years and the interest rate is still 7%, what is the bond's value?

$$\text{Interest: PVA} = C \left[ \frac{1}{r} - \frac{1}{r(1+r)^t} \right] = \$70 \left[ \frac{1}{.07} - \frac{1}{.07(1.07)^8} \right] = \$70 (5.971) = \$417.97$$

$$\text{Principal: PV} = \frac{FV}{(1+r)^t} = \frac{\$1,000}{(1.07)^8} = \frac{\$1,000}{(1.718)} = \$582.01$$

$$\begin{aligned} \text{Total: } & \$ 417.97 \\ & + \underline{582.01} \\ & \$ 999.98 \quad - \text{ note rounding} \end{aligned}$$

Note: Each year the PV of the interest payments decreases (due to fewer payments) but the PV of the principal increases (because you are closer to the time you get it).

**Example:** Now suppose we hold the bond for 2 years and comparable investments elsewhere are now paying 5%.

$$\text{Interest: PVA} = C \left[ \frac{1}{r} - \frac{1}{r(1+r)^t} \right] = \$70 \left[ \frac{1}{.05} - \frac{1}{.05(1.05)^8} \right] = \$70 (6.463) = \$452.41$$

$$\text{Principal: PV} = \frac{FV}{(1+r)^t} = \frac{\$1,000}{(1.05)^8} = \frac{\$1,000}{(1.48)} = \$676.84$$

$$\begin{aligned} \text{Total: } & \$ 452.41 \\ & + \underline{676.84} \\ & \$ 1,129.25 \quad - \text{ Note that the price went up} \end{aligned}$$

Why did the price of the bond go up?

Example: Look at the same thing, but comparable investments are now 9%.

$$\text{Interest: PVA} = C \left[ \frac{1}{r} - \frac{1}{r(1+r)^t} \right] = \$70 \left[ \frac{1}{.09} - \frac{1}{.09(1.09)^8} \right] = \$70 (5.535) = \$387.45$$

$$\text{Principal: PV} = \frac{\text{FV}}{(1+r)^t} = \frac{\$1,000}{(1.09)^8} = \frac{\$1,000}{1.99} = \$501.87$$

Total:           \$     387.45  
                  +     501.87  
                  \$     889.32     Note that the price went down

Why did the price of the bond go down?

Note 1: Bond price and interest rate move in opposite directions

Note 2: When coupon rate    > discount rate,    price > face value    (premium)  
                                      =                                =                                (par)  
                                      <                                <                                (discount)

Bond prices are usually expressed as a percentage of their face value.

**Example:**    #1:    100    Selling at par  
                  #2:    112    Selling at a premium  
                  #3:    88    Selling at a discount

**Premium** - Above the face value

**Discount** - Below the face value

Is it better to buy a bond at a discount or a premium?

Either way, you'll get the same effective interest rate.

Bonds pay interest semi-annually. Our 7% bond actually pays \$35 every 6 months.

This actually increases the effective annual rate. 7% is the APR and EAR > APR

Coupon Rate is always APR. We'll assume annual payments to simplify.

(3) **Current Yield** - Annual coupon payments divided by the market value

Example of Current Yield: 7% 10 year bond

After 2 yrs, the interest rates are now 5%

Market value of bond = \$1,129.25 (we just did)

$$\text{Current Yield} = \frac{70}{1,129.25} = 6.20\%$$

Note: This 6.20% is the return I get immediately, but it is not my yield to maturity since I don't get back my \$1,129.25. I get back \$1,000 at maturity. Actually the YTM is 5%.

(4) Quotations: Bond prices are quoted as % of their face value.

{ **Bid:** Price investors receive if they sell an asset. Sell at the Bid.  
 { **Ask:** Price investors pay if they buy an asset. Buy at the Ask.

Ask > Bid

Note: Think of the broker who holds the bonds bidding for new bonds or asking you for money for his current bonds.

(a) **Government Bonds: Prices** are quoted in 32<sup>nds</sup> after the decimal

{	<b>Rate:</b>	Coupon rate
	<b>Current Yld:</b>	coupon / closing price
	<b>Maturity:</b>	Month and year
	<b>Bid/Ask:</b>	The difference is the spread – The profit to the broke. Spread is usually 2/32 of 1%. Note that if the Ask Yld is above the Rate, the price is below 100 and vice versa.
	<b>Change:</b>	Change in price (in 32 <sup>nds</sup> ) from the previous day's close (+ or -). Note that if the prices went up, it probably because interest rates (expectations) dropped.
	<b>Ask Yld:</b>	Yield to maturity based on the ask price. Note that the longer maturities on the government bonds show higher yields.

**Example:**      "Ask"      115:28      [=115 28/32 % of face value (= \$1,158.75)]  
                   "Chg"      -3            [declined by 3/32 % (or 0.094%)]

If you had bought the bond yesterday, you would have paid \$1,159.69 (94 cents loss).

(b) **Corporate Bonds:** Prices are quoted in 8<sup>ths</sup> after the decimal

### III. Yield to Maturity (YTM)

- Also known as IRR (Internal Rate of return): The interest rate for which the PV of the bond's payments equals the bond price.
- The rate of return you get if you hold the bond to maturity.
- The true return on all cash flows.
- It should equal the rate for comparable bonds.

- Similar bonds (in terms of risk and maturity) should have the same YTM.  
For similar bonds, market will adjust the prices to give you the same YTM. – If a bond has a higher yield, people buy it which bids up the price, and the yield falls.

**Example:** \$10,000 10-year bond with 4 years remaining. Coupon Rate=7%  
Price = \$10,346.52 What is the yield to maturity?

$$\$10,346.52 = \frac{700}{(1+r)} + \frac{700}{(1+r)^2} + \frac{700}{(1+r)^3} + \frac{700}{(1+r)^4} + \frac{10,000}{(1+r)^4}$$

Trail and Error: If the price is > \$10,000, the interest rates went down from 7%. In this case, the answer is 6%