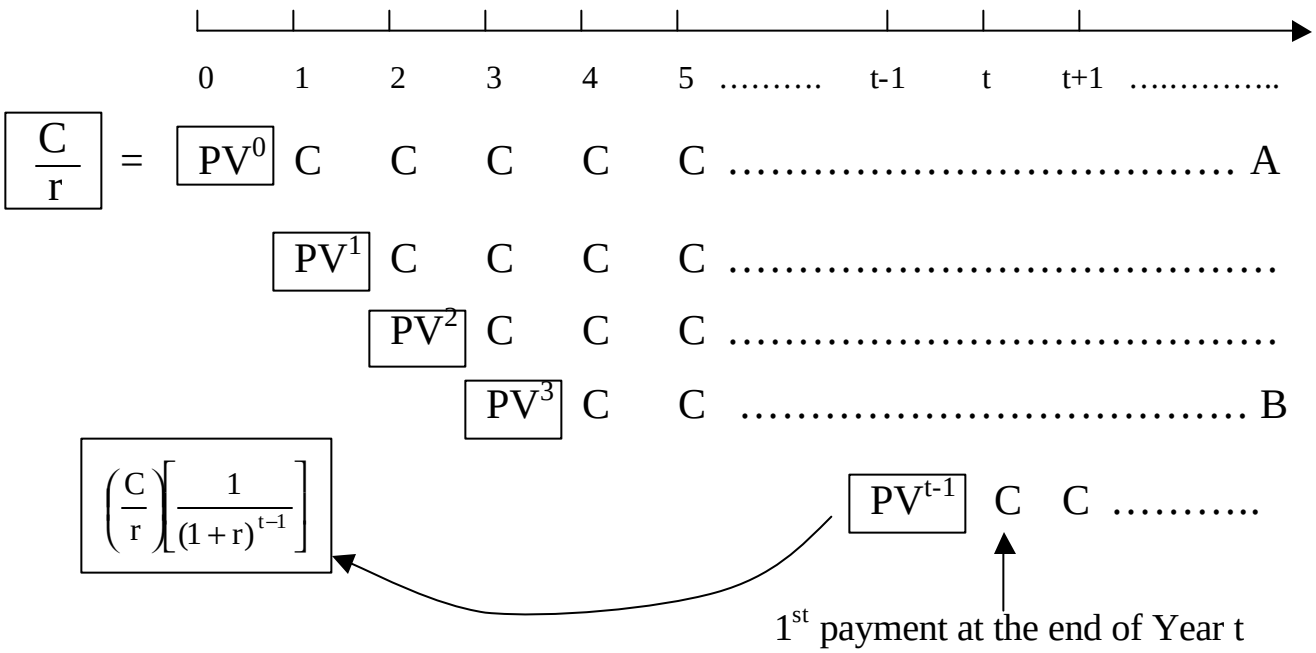
Topic: Time Value of Money

I. $FV = PV(1+r)^t$ $(1+r)^t$: growth factor

II. $PV = FV/(1+r)^t$ $1/(1+r)^t$: discount factor

III. Multiple CF's

(1) Perpetuity: A stream of level cash payments that never end.



Ex: C = \$1, r=5%

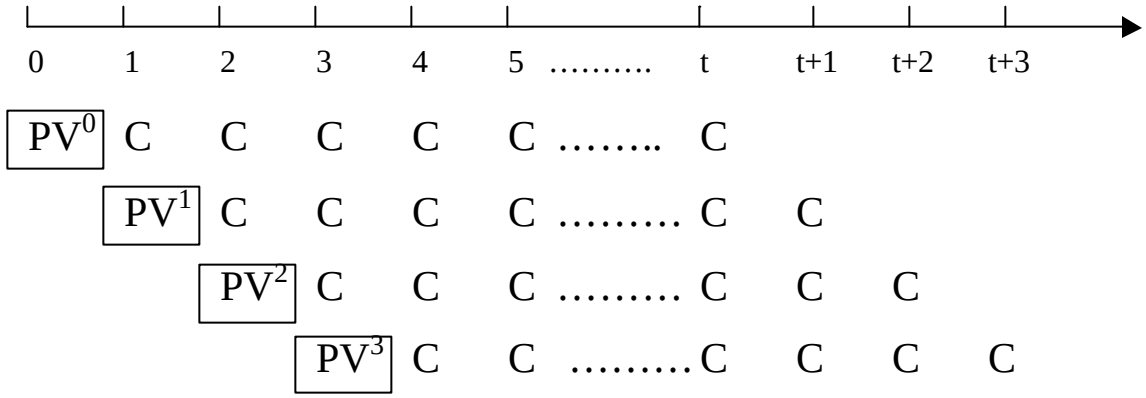
PV(A)=PV^0 = 1/.05 = 20 PV(B)=$PV^3/(1.05)^3$ = 17.28

➔ PV(\$1 payment for 3 years) = 20-17.28 = 2.72

Ex: In 1997, Bill Gates donated \$20 mill to Harvard University. In order to provide \$800,000 scholarship in perpetuity, what interest rate must he get? [Answer: 4%]

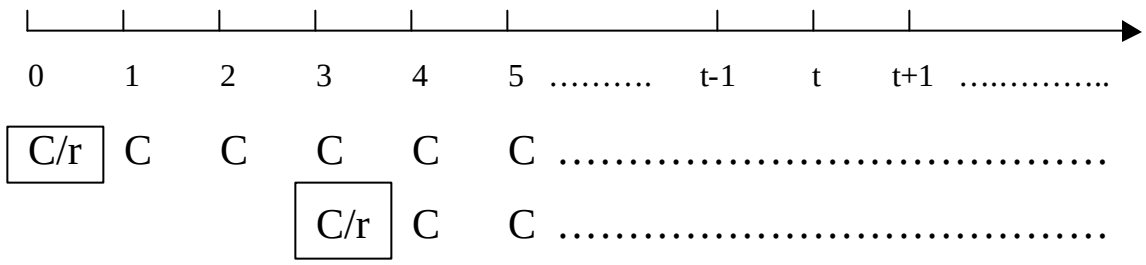
Ex: Donate \$500,000 to charity to provide food for the homeless. In order to provide \$35,000 of food per year, in perpetuity, what interest rate must you get? [Answer: 7%]

(2) Annuity



$$PV = C\left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right] \longrightarrow \text{for a t-year annuity}$$

Example:



$$PV(\text{3-year \$C annuity}) = \left(\frac{C}{r}\right) - \left(\frac{C}{r}\frac{1}{(1+r)^3}\right)$$

$$= C\left[\frac{1}{r} - \frac{1}{r(1+r)^3}\right]$$

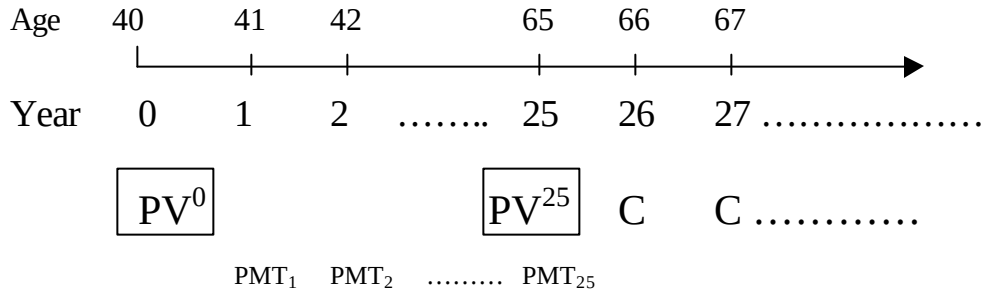
By deduction,

$$PV(\text{t-year \$C annuity}) = C * \underbrace{\left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right]}_{\text{Annuity Factor}} = C^* \text{ AF(@r, t years)}$$

Ex: Find the PV of 5-year $1,000 annuity @ 5%. [Ans: $4,329]

Ex: Joan Murray is a 40-year-old executive at BestLife Insurance, Inc. Her annual income is $72,000. If Joan wants to live happily *forever*, after retiring in 25 years, she needs to spend $56,000 annually *ever after*. How much does she have to set aside at the beginning of each year for the next 25 years to accomplish her goal? (Assume the discount rate is 7%.)

Sol.



$PV^{25} = C/r = 800{,}000$

$PV^{0} = PV^{25}/(1 + r)^{25} = 147{,}399.34$

PV^{0} = PMT * A.F.(@7%, 25 years)

So PMT = $12,643.30/year