Interest Rate Markets

Chapter 5

Types of Rates

- Treasury rates
- LIBOR rates
- Repo rates

Zero Rates

A zero rate (or spot rate), for maturity T is the rate of interest earned on an investment that provides a payoff only at time T

Example **(Table 5.1, page 95)**

Maturity (years)	Zero Rate (% cont comp)
0.5	5.0
1.0	5.8
1.5	6.4
2.0	6.8

Bond Pricing

- To calculate the cash price of a bond we discount each cash flow at the appropriate zero rate
- In our example, the theoretical price of a two-year bond providing a 6% coupon semiannually is

$$3e^{-0.05\times 0.5} + 3e^{-0.058\times 1.0} + 3e^{-0.064\times 1.5} + 103e^{-0.068\times 2.0} = 98.39$$

Bond Yield

- The bond yield is the discount rate that makes the present value of the cash flows on the bond equal to the market price of the bond
- Suppose that the market price of the bond in our example equals its theoretical price of 98.39
- The bond yield is given by solving

$$3e^{-y\times 0.5} + 3e^{-y\times 1.0} + 3e^{-y\times 1.5} + 103e^{-y\times 2.0} = 98.39$$

to get y=0.0676 or 6.76%.

Par Yield

- The par yield for a certain maturity is the coupon rate that causes the bond price to equal its face value.
- In our example we solve

$$\frac{c}{2}e^{-0.05\times0.5}+\frac{c}{2}e^{-0.058\times1.0}+\frac{c}{2}e^{-0.064\times1.5}$$
$$+\left(100+\frac{c}{2}\right)e^{-0.068\times2.0}=100$$

to get c=6.87 (with s.a. compounding)

Par Yield continued

In general if m is the number of coupon payments per year, d is the present value of $1 received at maturity and A is the present value of an annuity of $1 on each coupon date

$$c=\frac{(100-100d)m}{A}$$

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Sample Data for Determining the Zero Curve (Table 5.2, page 97)

Bond Principal (dollars)	Time to Maturity (years)	Annual Coupon (dollars)	Bond Price (dollars)
100	0.25	0	97.5
100	0.50	0	94.9
100	1.00	0	90.0
100	1.50	8	96.0
100	2.00	12	101.6

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The Bootstrapping the Zero Curve

- An amount 2.5 can be earned on 97.5 during 3 months.
- The 3-month rate is 4 times 2.5/97.5 or 10.256% with quarterly compounding
- This is 10.127% with continuous compounding
- Similarly the 6 month and 1 year rates are 10.469% and 10.536% with continuous compounding

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The Bootstrap Method continued

- To calculate the 1.5 year rate we solve

$$4e^{-0.10469\times0.5}+4e^{-0.10536\times1.0}+104e^{-R\times1.5}=96$$

to get $R = 0.10681$ or 10.681%

- Similarly the two-year rate is 10.808%

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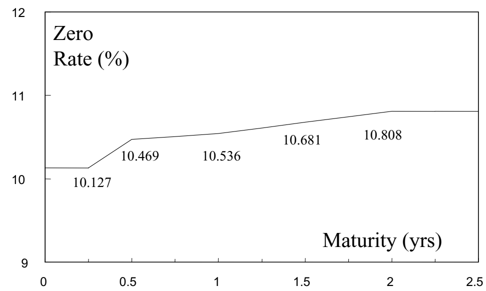
Zero Curve Calculated from the Data (Figure 5.1, page 98)



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Forward Rates

The forward rate is the future zero rate implied by today's term structure of interest rates

Calculation of Forward Rates

Table 5.4, page 98

Year (n)	Zero Rate for an n-year Investment (% per annum)	Forward Rate for n th Year (% per annum)
1	10.0	
2	10.5	11.0
3	10.8	11.4
4	11.0	11.6
5	11.1	11.5

Formula for Forward Rates

- Suppose that the zero rates for maturities T_1 and T_2 are R_1 and R_2 with both rates continuously compounded.
- The forward rate for the period between times T_1 and T_2 is

$$\frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$$

Instantaneous Forward Rate

- The instantaneous forward rate for a maturity T is the forward rate that applies for a very short time period starting at T. It is

$$R + T\frac{\partial R}{\partial T}$$

where R is the T-year rate

Upward vs Downward Sloping Yield Curve

- For an upward sloping yield curve:
  Fwd Rate > Zero Rate > Par Yield
- For a downward sloping yield curve
  Par Yield > Zero Rate > Fwd Rate

Forward Rate Agreement

- A forward rate agreement (FRA) is an agreement that a certain rate will apply to a certain principal during a certain future time period

Forward Rate Agreement continued (Page 100)

- An FRA is equivalent to an agreement where interest at a predetermined rate, R_K is exchanged for interest at the market rate
- An FRA can be valued by assuming that the forward interest rate is certain to be realized

Theories of the Term Structure

Pages 102

- Expectations Theory: forward rates equal expected future zero rates
- Market Segmentation: short, medium and long rates determined independently of each other
- Liquidity Preference Theory: forward rates higher than expected future zero rates

Day Count Conventions in the U.S. (Pages 102-103)

Treasury Bonds:	Actual/Actual (in period)
Corporate Bonds:	30/360
Money Market Instruments:	Actual/360

Treasury Bond Price Quotes in the U.S

Cash price = Quoted price + Accrued Interest

Treasury Bill Quote in the U.S.

If Y is the cash price of a Treasury bill that has n days to maturity the quoted price is

$$\frac{360}{n}(100-Y)$$

Treasury Bond Futures

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Cash price received by party with short position =

Quoted futures price × Conversion factor + Accrued interest

Conversion Factor

The conversion factor for a bond is approximately equal to the value of the bond on the assumption that the yield curve is flat at 6% with semiannual compounding

CBOT T-Bonds & T-Notes

Factors that affect the futures price:

- Delivery can be made any time during the delivery month
- Any of a range of eligible bonds can be delivered
- The wild card play

Eurodollar Futures (Page 110)

- If Z is the quoted price of a Eurodollar futures contract, the value of one contract is 10,000[100-0.25(100-Z)]
- A change of one basis point or 0.01 in a Eurodollar futures quote corresponds to a contract price change of $25

Eurodollar Futures continued

- A Eurodollar futures contract is settled in cash
- When it expires (on the third Wednesday of the delivery month) Z is set equal to 100 minus the 90 day Eurodollar interest rate (actual/360) and all contracts are closed out

Forward Rates and Eurodollar Futures (Page 111)

- Eurodollar futures contracts last out to 10 years
- For Eurodollar futures we cannot assume that the forward rate equals the futures rate

Forward Rates and Eurodollar Futures continued

A "convexity adjustment" often made is

$$\text{Forward rate} = \text{Futures rate} - \frac{1}{2}\sigma^2 t_1 t_2$$

where t_1 is the time to maturity of the futures contract, t_2 is the maturity of the rate underlying the futures contract (90 days later than t_1) and σ is the standard deviation of the short rate changes per year (typically σ is about 0.012)

Duration

- Duration of a bond that provides cash flow c_i at time t_i is

$$\sum_{i=1}^{n} t_i \left[\frac{c_i e^{-yt_i}}{B} \right]$$

where B is its price and y is its yield (continuously compounded)

- This leads to $\dfrac{\delta B}{B} = -D\,\delta y$

Duration Continued

- When the yield y is expressed with compounding m times per year

$$\delta B = -\frac{BD\delta y}{1+y/m}$$

- The expression

$$\frac{D}{1+y/m}$$

is referred to as the "modified duration"

Convexity

The convexity of a bond is defined as

$$C = \frac{1}{B}\frac{\partial^2 B}{\partial y^2} = \frac{\sum_{i=1}^{n} c_i t_i^2 e^{-yt_i}}{B}$$

so that

$$\frac{\delta B}{B} = -D\delta y + \frac{1}{2}C(\delta y)^2$$

Duration Matching

- This involves hedging against interest rate risk by matching the durations of assets and liabilities
- It provides protection against small parallel shifts in the zero curve