Options on Stock Indices, Currencies, and Futures

Chapter 13

European Options on Stocks Providing a Dividend Yield

We get the same probability distribution for the stock price at time T in each of the following cases:

1. The stock starts at price S_0 and provides a dividend yield = q
2. The stock starts at price S_0e^{-qT} and provides no income

European Options on Stocks Providing Dividend Yield continued

We can value European options by reducing the stock price to S_0e^{-qT} and then behaving as though there is no dividend

Extension of Chapter 8 Results

(Equations 13.1 to 13.3)

Lower Bound for calls:

$$c \geq S_0e^{-qT} - Ke^{-rT}$$

Lower Bound for puts

$$p \geq Ke^{-rT} - S_0e^{-qT}$$

Put Call Parity

$$c + Ke^{-rT} = p + S_0e^{-qT}$$

Extension of Chapter 12 Results

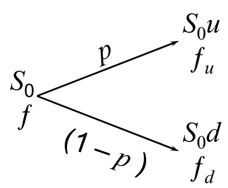
(Equations 13.4 and 14.5)

$$c = S_0e^{-qT}N(d_1) - Ke^{-rT}N(d_2)$$
$$p = Ke^{-rT}N(-d_2) - S_0e^{-qT}N(-d_1)$$

where $$d_1 = \frac{\ln(S_0/K) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0/K) + (r - q - \sigma^2/2)T}{\sigma\sqrt{T}}$$

The Binomial Model



$$f = e^{-rT}[pf_u + (1-p)f_d]$$

The Binomial Model

continued

- In a risk-neutral world the stock price grows at *r-q* rather than at *r* when there is a dividend yield at rate *q*
- The probability, *p*, of an up movement must therefore satisfy

$$pS_0u+(1-p)S_0d=S_0e^{(r-q)T}$$

so that

$$p=\frac{e^{(r-q)T}-d}{u-d}$$

Index Options

- Option contracts are on 100 times the index
- The most popular underlying indices are
  - the Dow Jones Industrial (European) DJX
  - the S&P 100 (American) OEX
  - the S&P 500 (European) SPX
- Contracts are settled in cash

Index Option Example

- Consider a call option on an index with a strike price of 560
- Suppose 1 contract is exercised when the index level is 580
- What is the payoff?

Using Index Options for Portfolio Insurance

- Suppose the value of the index is S_0 and the strike price is K
- If a portfolio has a β of 1.0, the portfolio insurance is obtained by buying 1 put option contract on the index for each $100S_0$ dollars held
- If the β is not 1.0, the portfolio manager buys β put options for each $100S_0$ dollars held
- In both cases, K is chosen to give the appropriate insurance level

Example 1

- Portfolio has a beta of 1.0
- It is currently worth $500,000
- The index currently stands at 1000
- What trade is necessary to provide insurance against the portfolio value falling below $450,000?

Example 2

- Portfolio has a beta of 2.0
- It is currently worth $500,000 and index stands at 1000
- The risk-free rate is 12% per annum
- The dividend yield on both the portfolio and the index is 4%
- How many put option contracts should be purchased for portfolio insurance?

Calculating Relation Between Index Level and Portfolio Value in 3 months

- If index rises to 1040, it provides a 40/1000 or 4% return in 3 months
- Total return (incl dividends)=5%
- Excess return over risk-free rate=2%
- Excess return for portfolio=4%
- Increase in Portfolio Value=4+3-1=6%
- Portfolio value=$530,000

Determining the Strike Price

(Table 13.2, page 274)

Value of Index in 3 months	Expected Portfolio Value in 3 months ($)
1,080	570,000
1,040	530,000
1,000	490,000
960	450,000
920	410,000

An option with a strike price of 960 will provide protection against a 10% decline in the portfolio value

Valuing European Index Options

We can use the formula for an option on a stock paying a dividend yield

Set S_0 = current index level

Set q = average dividend yield expected during the life of the option

Currency Options

- Currency options trade on the Philadelphia Exchange (PHLX)
- There also exists an active over-the-counter (OTC) market
- Currency options are used by corporations to buy insurance when they have an FX exposure

The Foreign Interest Rate

- We denote the foreign interest rate by r_f
- When a U.S. company buys one unit of the foreign currency it has an investment of S_0 dollars
- The return from investing at the foreign rate is $r_f S_0$ dollars
- This shows that the foreign currency provides a "dividend yield" at rate r_f

Valuing European Currency Options

- A foreign currency is an asset that provides a "dividend yield" equal to r_f
- We can use the formula for an option on a stock paying a dividend yield :

  Set S_0 = current exchange rate

  Set $q = r_f$

13.19

Formulas for European Currency Options

(Equations 13.9 and 13.10, page 277)

$$c = S_0 e^{-r_f T} N(d_1) - K e^{-rT} N(d_2)$$

$$p = K e^{-rT} N(-d_2) - S_0 e^{-r_f T} N(-d_1)$$

where $d_1 = \dfrac{\ln(S_0/K) + (r - r_f + \sigma^2/2)T}{\sigma\sqrt{T}}$

$$d_2 = \frac{\ln(S_0/K) + (r - r_f - \sigma^2/2)T}{\sigma\sqrt{T}}$$

13.20

Alternative Formulas

(Equations 13.11 and 13.12, page 278)

Using $F_0 = S_0 e^{(r - r_f)T}$

$$c = e^{-rT}[F_0 N(d_1) - K N(d_2)]$$

$$p = e^{-rT}[K N(-d_2) - F_0 N(-d_1)]$$

$$d_1 = \frac{\ln(F_0/K) + \sigma^2 T/2}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

13.21

Mechanics of Call Futures Options

When a call futures option is exercised the holder acquires

1. A long position in the futures
2. A cash amount equal to the excess of the futures price over the strike price

13.22

Mechanics of Put Futures Option

When a put futures option is exercised the holder acquires

1. A short position in the futures
2. A cash amount equal to the excess of the strike price over the futures price

13.23

The Payoffs

If the futures position is closed out immediately:

Payoff from call = $F_0 - K$

Payoff from put = $K - F_0$

where F_0 is futures price at time of exercise

13.24

Put-Call Parity for Futures Options

(Equation 13.13, page 284)

Consider the following two portfolios:

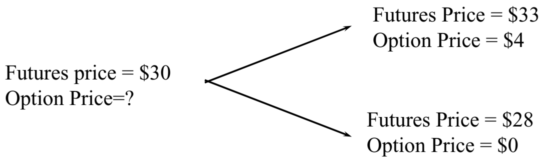
1. European call plus Ke^{-rT} of cash
2. European put plus long futures plus cash equal to F_0e^{-rT}

They must be worth the same at time T so that

$$c + Ke^{-rT} = p + F_0 e^{-rT}$$

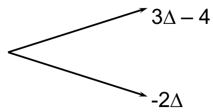
Binomial Tree Example

A 1-month call option on futures has a strike price of 29.



Setting Up a Riskless Portfolio

- Consider the Portfolio: long Δ futures
  short 1 call option



- Portfolio is riskless when $3\Delta - 4 = -2\Delta$ or $\Delta = 0.8$

Valuing the Portfolio
(Risk-Free Rate is 6%)

- The riskless portfolio is:

  long 0.8 futures
  short 1 call option

- The value of the portfolio in 1 month is -1.6
- The value of the portfolio today is $-1.6e^{-0.06/12} = -1.592$

Valuing the Option

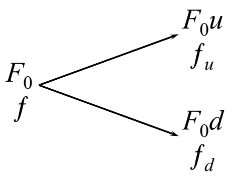
- The portfolio that is

  long 0.8 futures
  short 1 option

  is worth -1.592
- The value of the futures is zero
- The value of the option must therefore be 1.592

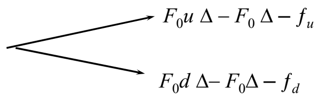
Generalization of Binomial Tree Example (Figure 13.3, page 285)

- A derivative lasts for time T and is dependent on a futures price



Generalization
(continued)

- Consider the portfolio that is long Δ futures and short 1 derivative



- The portfolio is riskless when

$$\Delta = \frac{f_u - f_d}{F_0 u - F_0 d}$$

Generalization
(continued)

- Value of the portfolio at time T is
  $F_0 u \Delta - F_0 \Delta - f_u$
- Value of portfolio *today* is $-f$
- Hence
  $f = -[F_0 u \Delta - F_0 \Delta - f_u]e^{-rT}$

Generalization
(continued)

- Substituting for Δ we obtain

$f = [\, p\, f_u + (1 - p\,) f_d\ ]e^{-rT}$

where

$$p = \frac{1 - d}{u - d}$$

Valuing European Futures Options

- We can use the formula for an option on a stock paying a dividend yield

  Set S_0 = current futures price (F_0)

  Set q = domestic risk-free rate (r)
- Setting $q = r$ ensures that the expected growth of F in a risk-neutral world is zero

Growth Rates For Futures Prices

- A futures contract requires no initial investment
- In a risk-neutral world the expected return should be zero
- The expected growth rate of the futures price is therefore zero
- The futures price can therefore be treated like a stock paying a dividend yield of r

Black's Formula
(Equations 13.17 and 13.18, page 287)

- The formulas for European options on futures are known as Black's formulas

$$c = e^{-rT}\left[F_0\, N(d_1) - K\, N(d_2)\right]$$
$$p = e^{-rT}\left[K\, N(-d_2) - F_0\, N(-d_1)\right]$$

where $d_1 = \dfrac{\ln(F_0 / K) + \sigma^2 T / 2}{\sigma\sqrt{T}}$

$$d_2 = \frac{\ln(F_0 / K) - \sigma^2 T / 2}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

Futures Option Prices vs Spot Option Prices

- If futures prices are higher than spot prices (normal market), an American call on futures is worth more than a similar American call on spot. An American put on futures is worth less than a similar American put on spot
- When futures prices are lower than spot prices (inverted market) the reverse is true

Summary of Key Results

- We can treat stock indices, currencies, and futures like a stock paying a dividend yield of q
  - For stock indices, q = average dividend yield on the index over the option life
  - For currencies, $q = r_f$
  - For futures, $q = r$