The Greek Letters

Chapter 14

Example

• A bank has sold for $300,000 a European call option on 100,000 shares of a nondividend paying stock
• \( S_0 = 49, \quad K = 50, \quad r = 5\%, \quad \sigma = 20\%, \quad T = 20 \text{ weeks}, \quad \mu = 13\% \)
• The Black-Scholes value of the option is $240,000
• How does the bank hedge its risk to lock in a $60,000 profit?

Naked & Covered Positions

Naked position

Take no action

Covered position

Buy 100,000 shares today

Both strategies leave the bank exposed to significant risk

Stop-Loss Strategy

This involves:

• Buying 100,000 shares as soon as price reaches $50
• Selling 100,000 shares as soon as price falls below $50

This deceptively simple hedging strategy does not work well

Delta (See Figure 14.2, page 302)

• Delta (\( \Delta \)) is the rate of change of the option price with respect to the underlying

\[
\Delta = \frac{\text{Option price}}{\text{Slope}}
\]

Stock price

Delta Hedging

• This involves maintaining a delta neutral portfolio

• The delta of a European call on a stock paying dividends at rate \( q \) is \( \Delta = N(d_1)e^{-qT} \)

• The delta of a European put is

\[
e^{-qT}[N(d_1) - 1]
\]
**Delta Hedging continued**

- The hedge position must be frequently rebalanced
- Delta hedging a written option involves a “buy high, sell low” trading rule
- See Tables 14.2 (page 307) and 14.3 (page 308) for examples of delta hedging

**Using Futures for Delta Hedging**

- The delta of a futures contract is $e^{(r-q)T}$ times the delta of a spot contract
- The position required in futures for delta hedging is therefore $e^{(r-q)T}$ times the position required in the corresponding spot contract

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**Theta**

- Theta ($\Theta$) of a derivative (or portfolio of derivatives) is the rate of change of the value with respect to the passage of time
- See Figure 14.5 for the variation of $\Theta$ with respect to the stock price for a European call

**Gamma**

- Gamma ($\Gamma$) is the rate of change of delta ($\Delta$) with respect to the price of the underlying asset
- See Figure 14.9 for the variation of $\Gamma$ with respect to the stock price for a call or put option

**Gamma Addresses Delta Hedging Errors Caused By Curvature**

(Figure 14.7, page 312)

**Interpretation of Gamma**

- For a delta neutral portfolio, $dP \approx Q \, dt + \frac{1}{2} GdS^2$

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Positive Gamma

Negative Gamma
14.13 Relationship Among Delta, Gamma, and Theta

For a portfolio of derivatives on a stock paying a continuous dividend yield at rate $q$

$$\Theta + (r - q)S\Delta + \frac{1}{2} \sigma^2 S^2 \Gamma = r\Pi$$

14.14 Vega

- Vega ($\nu$) is the rate of change of the value of a derivatives portfolio with respect to volatility
- See Figure 14.11 for the variation of $\nu$ with respect to the stock price for a call or put option

14.15 Managing Delta, Gamma, & Vega

- $\Delta$ can be changed by taking a position in the underlying
- To adjust $\Gamma$ & $\nu$ it is necessary to take a position in an option or other derivative

14.16 Rho

- Rho is the rate of change of the value of a derivative with respect to the interest rate
- For currency options there are 2 rhos

14.17 Hedging in Practice

- Traders usually ensure that their portfolios are delta-neutral at least once a day
- Whenever the opportunity arises, they improve gamma and vega
- As portfolio becomes larger hedging becomes less expensive

14.18 Scenario Analysis

A scenario analysis involves testing the effect on the value of a portfolio of different assumptions concerning asset prices and their volatilities
Hedging vs Creation of an Option Synthetically

- When we are hedging we take positions that offset $\Delta, \Gamma, \psi, \text{ etc.}$
- When we create an option synthetically we take positions that match $\Delta, \Gamma, \psi, \text{ and } \nu$

Portfolio Insurance

- In October of 1987 many portfolio managers attempted to create a put option on a portfolio synthetically
- This involves initially selling enough of the portfolio (or of index futures) to match the $\Delta$ of the put option

Portfolio Insurance continued

- As the value of the portfolio increases, the $\Delta$ of the put becomes less negative and some of the original portfolio is repurchased
- As the value of the portfolio decreases, the $\Delta$ of the put becomes more negative and more of the portfolio must be sold

The strategy did not work well on October 19, 1987...