Exotic Options

Chapter 19

Types of Exotics

- Package
- Nonstandard American options
- Forward start options
- Compound options
- Chooser options
- Barrier options
- Binary options
- Lookback options
- Shout options
- Asian options
- Options to exchange one asset for another
- Options involving several assets

Packages

- Portfolios of standard options
- Examples from Chapter 9: bull spreads, bear spreads, straddles, etc
- Often structured to have zero cost
- One popular package is a range forward contract

Non-Standard American Options

- Exercisable only on specific dates (Bermudans)
- Early exercise allowed during only part of life (e.g. there may be an initial "lock out" period)
- Strike price changes over the life

Forward Start Options

- Option starts at a future time, T_1
- Most common in employee stock option plans
- Often structured so that strike price equals asset price at time T_1

Compound Option

- Option to buy / sell an option
  - Call on call
  - Put on call
  - Call on put
  - Put on put
- Can be valued analytically
- Price is quite low compared with a regular option

Chooser Option "As You Like It"

- Option starts at time 0, matures at T_2
- At T_1 $(0 < T_1 < T_2)$ buyer chooses whether it is a put or call
- A few lines of algebra shows that this is a package

Chooser Option as a Package

At time T_1 the value is $\max(c, p)$

From put-call parity

$$p = c + e^{-r(T_2 - T_1)}K - S_1 e^{-q(T_2 - T_1)}$$

The value at time T_1 is therefore

$$c + e^{-q(T_2 - T_1)} \max(0, Ke^{-(r-q)(T_2 - T_1)} - S_1)$$

This is a call maturing at time T_2 plus a put maturing at time T_1

Barrier Options

- Option comes into existence only if stock price hits barrier before option maturity
  - 'In' options
- Option dies if stock price hits barrier before option maturity
  - 'Out' options

Barrier Options

- Stock price must hit barrier from below
  - 'Up' options
- Stock price must hit barrier from above
  - 'Down' options
- Option may be a put or a call
- Eight possible combinations

Parity Relations

$$c = c_{\text{ui}} + c_{\text{uo}}$$

$$c = c_{\text{di}} + c_{\text{do}}$$

$$p = p_{\text{ui}} + p_{\text{uo}}$$

$$p = p_{\text{di}} + p_{\text{do}}$$

Binary Options

- Cash-or-nothing: pays Q if $S > K$ at time T, otherwise pays 0. Value = $e^{-rT} Q\, N(d_2)$
- Asset-or-nothing: pays S if $S > K$ at time T, otherwise pays 0. Value = $S_0\, N(d_1)$

Decomposition of a Call Option

Long Asset-or-Nothing option

Short Cash-or-Nothing option (where payoff is K)

Value = $S_0 N(d_1) - e^{-rT} K N(d_2)$

Lookback Options

- Lookback call pays ($S_T - S_{min}$) at time T

  Allows buyer to buy stock at lowest observed price in some interval of time

- Lookback put pays ($S_{max} - S_T$) at time T

  Allows buyer to sell stock at highest observed price in some interval of time

Shout Options

- Buyer can ‘shout’ once during option life
- Final payoff is either
  - Usual option payoff, $\max(S_T - K, 0)$, or
  - Intrinsic value at time of shout, $S_\tau - K$
- Payoff: $\max(S_T - S_\tau, 0) + S_\tau - K$
- Similar to lookback option but cheaper
- How can a binomial tree be used to value a shout option?

Asian Options

- Payoff related to average stock price
- Average Price options pay:
  - $\max(S_{ave} - K, 0)$ (call), or
  - $\max(K - S_{ave}, 0)$ (put)
- Average Strike options pay:
  - $\max(S_T - S_{ave}, 0)$ (call), or
  - $\max(S_{ave} - S_T, 0)$ (put)

Asian Options

- No analytic solution
- Can be valued by assuming (as an approximation) that the average stock price is lognormally distributed

Exchange Options

- Option to exchange one asset for another
- For example:

  an option to exchange U for V
- Payoff is $\max(V_T - U_T, 0)$

Basket Options

- A basket option is an option to buy or sell a portfolio of assets
- This can be valued by calculating the first two moments of the value of the basket and then assuming it is lognormal

How Difficult is it to Hedge Exotic Options ?

- In some cases exotic options are easier to hedge than the corresponding vanilla options. (e.g., Asian options)
- In other cases they are more difficult to hedge. (e.g., barrier options)

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Static Options Replication

- This involves approximately replicating an exotic option with a portfolio of vanilla options
- Underlying principle: If we match the value of an exotic option on some boundary , we have matched it at all interior points of the boundary
- Static options replication can be contrasted with dynamic options replication where we have to trade continuously to match the option

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Example

- A 9-month up-and-out call option an a non-dividend paying stock where $S_0 = 50$, $K = 50$, the barrier is 60, $r = 10\%$, and $\sigma = 30\%$
- Any boundary can be chosen but the natural one is

  $c(S, 0.75) = \text{MAX}(S - 50, 0)$ when $S < 60$

  $c(60, t) = 0$ when $0 \le t \le 0.75$

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Example (continued)

- We might try to match the following points on the boundary

  $c(S, 0.75) = \text{MAX}(S - 50, 0)$ for $S < 60$

  $c(60, 0.50) = 0$

  $c(60, 0.25) = 0$

  $c(60, 0.00) = 0$

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Example continued

(See Table 19.1, page 449)

We can do this as follows:

+1.00 call with maturity 0.75 & strike 50

–2.66 call with maturity 0.75 & strike 60

+0.97 call with maturity 0.50 & strike 60

+0.28 call with maturity 0.25 & strike 60

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Example (continued)

- This portfolio is worth 0.73 at time zero compared with 0.31 for the up-and-out option
- As we use more options the value of the replicating portfolio converges to the value of the exotic option
- For example, with 18 points matched on the horizontal boundary the value of the replicating portfolio reduces to 0.38; with 100 points being matched it reduces to 0.32

Using Static Options Replication

- To hedge an exotic option we short the portfolio that replicates the boundary conditions
- The portfolio must be unwound when any part of the boundary is reached