Interest Rate Derivatives: More Advanced Models

Chapter 24

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The Two-Factor Hull-White Model (Equation 24.1, page 571)

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Analytic Results

- Bond prices and European options on zero-coupon bonds can be calculated analytically when $f(r) = r$

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Options on Coupon-Bearing Bonds

- We cannot use the same procedure for options on coupon-bearing bonds as we do in the case of one-factor models
- If we make the approximate assumption that the coupon-bearing bond price is lognormal, we can use Black's model
- The appropriate volatility is calculated from the volatilities of and correlations between the underlying zero-coupon bond prices

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Volatility Structures

- In the one-factor Ho-Lee or Hull-White model the forward rate S.D.s are either constant or decline exponentially. All forward rates are instantaneously perfectly correlated
- In the two-factor model many different forward rate S.D. patterns and correlation structures can be obtained

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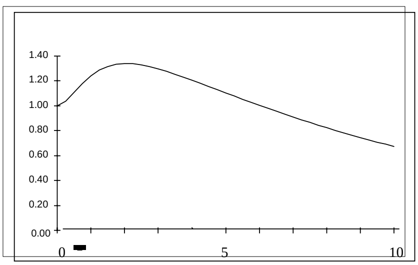
Example Giving Humped Volatility Structure (Figure 24.1, page 572)

a=1, b=0.1, σ_1=0.01, σ_2=0.0165, ρ=0.6



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Transformation of the General Model

Transformation of the General Model continued

Attractive Features of the Model

- It is Markov so that a recombining 3-dimensional tree can be constructed
- The volatility structure is stationary
- Volatility and correlation patterns similar to those in the real world can be incorporated into the model

HJM Model: Notation

$P(t,T)$: price at time t of a discount bond with principal of $1 maturing at T

Ω_t : vector of past and present values of interest rates and bond prices at time t that are relevant for determining bond price volatilities at that time

$v(t,T,\Omega_t)$: volatility of $P(t,T)$

Notation continued

$f(t,T_1,T_2)$: forward rate as seen at t for the period between T_1 and T_2

$F(t,T)$: instantaneous forward rate as seen at t for a contract maturing at T

$r(t)$: short-term risk-free interest rate at t

$dz(t)$: Wiener process driving term structure movements

Modeling Bond Prices

Modeling Forward Rates

Equation 24.7, page 575)

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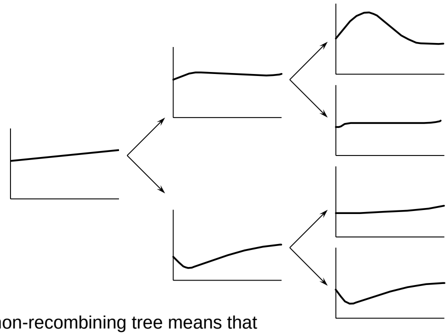
Tree For a General Model



A non-recombining tree means that the process for *r* is non-Markov

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The LIBOR Market Model

The LIBOR market model is a model constructed in terms of the forward rates underlying caplet prices

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Notation

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Volatility Structure

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In Theory the Λ's can be determined from Cap Prices

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Example 24.1 (Page 579)

- If Black volatilities for the first three caplets are 24%, 22%, and 20%, then

Λ_0=24.00%

Λ_1=19.80%

Λ_2=15.23%

Example 24.2 (Page 579)

The Process for F_k in a One-Factor LIBOR Market Model

Rolling Forward Risk-Neutrality (Equation 24.16, page 579)

It is often convenient to choose a world that is always FRN wrt a bond maturing at the next reset date. In this case, we can discount from t_{i+1} to t_i at the δ_i rate observed at time t_i. The process for F_k is

The LIBOR Market Model and HJM

In the limit as the time between resets tends to zero, the LIBOR market model with rolling forward risk neutrality becomes the HJM model in the traditional risk-neutral world

Monte Carlo Implementation of BGM Cap Model

(Equation 24.18, page 580)

Multifactor Versions of BGM

- BGM can be extended so that there are several components to the volatility
- A factor analysis can be used to determine how the volatility of F_k is split into components

Ratchet Caps, Sticky Caps, and Flexi Caps

- A plain vanilla cap depends only on one forward rate. Its price is not dependent on the number of factors.
- Ratchet caps, sticky caps, and flexi caps depend on the joint distribution of two or more forward rates. Their prices tend to increase with the number of factors

Valuing European Options in the LIBOR Market Model

There is a good analytic approximation that can be used to value European swap options in the LIBOR market model. See pages 582 to 584.

Calibrating the LIBOR Market Model

- In theory the LMM can be exactly calibrated to cap prices as described earlier
- In practice we proceed as for the one-factor models in Chapter 23 and minimize a function of the form
- where U_i is the market price of the ith calibrating instrument, V_i is the model price of the ith calibrating instrument and P is a function that penalizes big changes or curvature in a and σ

Types of Mortgage-Backed Securities (MBSs)

- Pass-Through
- Collateralized Mortgage Obligation (CMO)
- Interest Only (IO)
- Principal Only (PO)

Option-Adjusted Spread (OAS)

- To calculate the OAS for an interest rate derivative we value it assuming that the initial yield curve is the Treasury curve + a spread
- We use an iterative procedure to calculate the spread that makes the derivative's model price = market price. This is the OAS.