Chapter 26

Credit Risk

Credit Ratings

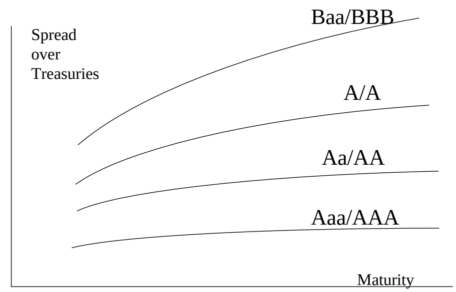
- In the S&P rating system, AAA is the best rating. After that comes AA, A, BBB, BB, B, and CCC
- The corresponding Moody's ratings are Aaa, Aa, A, Baa, Ba, B, and Caa
- Bonds with ratings of BBB (or Baa) and above are considered to be "investment grade"

Information from Bond Prices

- Traders regularly estimate the zero curves for bonds with different credit ratings
- This allows them to estimate probabilities of default in a risk-neutral world

Typical Pattern

(See Figure 26.1, page 611)



The Risk-Free Rate

- Most analysts use the LIBOR rate as the risk-free rate
- The excess of the value of a risk-free bond over a similar corporate bond equals the present value of the cost of defaults

Example (Zero coupon rates; continuously compounded)

Example continued

One-year risk-free bond (principal=$1) sells for

One-year corporate bond (principal=$1) sells for

or at a 0.2497% discount

This indicates that the holder of the corporate bond expects to lose 0.2497% from defaults in the first year

Example continued

- Similarly the holder of the corporate bond expects to lose

  or 0.9950% in the first two years
- Between years one and two the expected loss is 0.7453%

Example continued

- Similarly the bond holder expects to lose 2.0781% in the first three years; 3.3428% in the first four years; 4.6390% in the first five years
- The expected losses per year in successive years are 0.2497%, 0.7453%, 1.0831%, 1.2647%, and 1.2962%

Summary of Results

(Table 26.1, page 612)

Recovery Rates

(Table 26.3, page 614. Source: Moody's Investor's Service, 2000)

Probability of Default

Reason Why This Analysis is Simplistic

- Bonds are assumed to be zero-coupon
- The equation:

  Prob. of Def.×(1-Rec. Rate)=Exp Loss%

  assumes that the claim in the event of default equals the no-default value of the bond

A More Complete Analysis: Definitions

Risk-Neutral Probability of Default

Page 616, equations 26.3 to 26.5

- PV of loss from default
- Reduction in bond price due to default
- Computing *p*'s inductively

Relaxing Assumptions

- This analysis assumes constant interest rates, and known recovery rates and claim amounts
- If default events, risk-free rates, and recovery rates are independent, results hold for stochastic interest rates, and uncertain recovery rates providing the recovery rate is set equal to its expected value in a risk-neutral world,

Extending the Analysis to Allow Defaults at Any Time

- The analysis can be extended to allow defaults at any time
- It is important to distinguish between the default probability density and the hazard rate
- The default probability density, q(t) is defined so that $q(t)\delta t$ as the probability of default between times t and $t+\delta t$ as seen at time zero
- The hazard rate is the probability of default between times t and $t+\delta t$ conditional on no earlier default

What Should We Use as the Claim Amount

The best assumption seems to be that the claim amount for a bond equals the face value plus accrued interest --- not the no-default value

Sample Data (Risk-free Rate=5%; Expected Recovery Rate=30%)

Bond Life	Coupon (%)	Yield (%)
1	7.0	6.6
2	7.0	6.7
3	7.0	6.8
4	7.0	6.9
5	7.0	7.0
10	7.0	7.2

Implied Default Probabilities Assuming That Default Can Happen on Bond Maturity Dates (Table 26.5, page 617)

Time (yrs)	Claim = No-Def Value	Claim=Face Val+Accr Int
1	0.0224	0.0224
2	0.0249	0.0247
3	0.0273	0.0269
4	0.0297	0.0291
5	0.0320	0.0312
10	0.1717	0.1657

Value Additivity

- If claim amount equals no-default value, value of a coupon bond is sum of values of constituent zero-coupon bonds
- The same is not true when claim amount equals face value plus accrued interest

Asset Swaps (page 618)

- An asset swap exchanges the return on a bond for a spread above LIBOR
- Asset swaps are frequently used to extract default probabilities from bond prices. The assumption is that LIBOR is the risk-free rate

Asset Swaps: Example 1

- An investor owns a 5-year corporate bond worth par that pays a coupon of 6%. LIBOR is flat at 4.5%. An asset swap would enable the coupon to be exchanged for LIBOR plus 150bps
- In this case B_j=100 and G_j=106.65 (The value of 150 bps per year for 5 years is 6.65.)

Asset Swap: Example 2

- Investor owns a 5-year bond is worth $95 per $100 of face value and pays a coupon of 5%. LIBOR is flat at 4.5%.
- The asset swap would be structured so that the investor pays $5 upfront and receives LIBOR plus 162.79 bps. ($5 is equivalent to 112.79 bps per year)
- In this case B_j=95 and G_j=102.22 (162.79 bps per is worth $7.22)

Theory and Practice

- In theory asset swap spreads should be slightly dependent on the bond's coupon
- In practice it is assumed to be the same for all bonds with a particular maturity and the quoted asset swap spread is assume to apply to a bond selling for par
- This means that the spread would be quoted as 162.79 bps in Example 2 and when calculating default probabilities we would assume B_j=100 and G_j=107.22

Historical Data

Historical data provided by rating agencies are also used to estimate the probability of default

Cumulative Average Default Rates (%)

(Table 26.7, page 619; S&P Report, January 2001)

Interpretation

- The table shows the probability of default for companies starting with a particular credit rating
- A company with an initial credit rating of BBB has a probability of 0.24% of defaulting by the end of the first year, 0.55% by the end of the second year, and so on

Do Default Probabilities Increase with Time?

- For a company that starts with a good credit rating default probabilities tend to increase with time
- For a company that starts with a poor credit rating default probabilities tend to decrease with time

Bond Prices vs. Historical Default Experience

- The estimates of the probability of default calculated from bond prices are much higher than those from historical data
- Consider for example a 5 year A-rated zero-coupon bond
- This typically yields at least 50 bps more than the risk-free rate

Bond Prices vs. Historical Default Experience

Possible Reasons for These Results

- The liquidity of corporate bonds is less than that of Treasury bonds
- Bonds traders may be factoring into their pricing depression scenarios much worse than anything seen in the last 20 years

A Key Theoretical Reason

(page 621)

- The default probabilities estimated from bond prices are risk-neutral default probabilities
- The default probabilities estimated from historical data are real-world default probabilities

Risk-Neutral Probabilities

The analysis based on bond prices assumes that

- The expected cash flow from the A-rated bond is 2.47% less than that from the risk-free bond
- The discount rates for the two bonds are the same

This is correct only in a risk-neutral world

The Real-World Probability of Default

- The expected cash flow from the A-rated bond is 0.57% less than that from the risk-free bond
- But we still get the same price if we discount at about 38 bps per year more than the risk-free rate
- If risk-free rate is 5%, it is consistent with the beta of the A-rated bond being 0.076

Which World Should We Use?

- We should use risk-neutral estimates for valuing credit derivatives and estimating the cost of default
- We should use real world estimates for calculating credit VaR and scenario analysis

Merton's Model (page 621 to 623)

- Merton's model regards the equity as an option on the assets of the firm
- In a simple situation the equity value is

$$\max(V_T - D, 0)$$

where V_T is the value of the firm and D is the debt repayment required

Equity vs. Assets

An option pricing model enables the value of the firm's equity today, E_0, to be related to the value of its assets today, V_0, and the volatility of its assets, σ_V

Volatilities

This equation together with the option pricing relationship enables V_0 and σ_V to be determined from E_0 and σ_E

Example

- A company's equity is $3 million and the volatility of the equity is 80%
- The risk-free rate is 5%, the debt is $10 million and time to debt maturity is 1 year
- Solving the two equations yields V_0=12.40 and σ_V=21.23%

Example continued

- The probability of default is $N(-d_2)$ or 12.7%
- The market value of the debt is 9.40
- The present value of the promised payment is 9.51
- The expected loss is about 1.2%
- The recovery rate is 91%

The Implementation of Merton's Model (e.g. KMV Moody's)

- Choose time horizon
- Calculate cumulative obligations to time horizon. This is termed by KMV the "default point". We denote it by D
- Use Merton's model to calculate a theoretical probability of default
- Use historical data or bond data to develop a one-to-one mapping of theoretical probability into either real-world or risk-neutral probability of default.

The Loss Given Default (LGD)

- For derivatives we need to distinguish between a) those that are always assets, b) those that are always liabilities, and c) those that can be assets or liabilities
- What is the loss in each case?

Netting (page 624)

Netting clauses state that is a company defaults on one contract it has with a financial institution it must default on all such contracts

Reducing Credit Exposure (page 625)

- Collateralization
- Downgrade triggers
- Diversification
- Contract design
- Credit derivatives

One-Year Transition Matrix

(S&P, January 2001, p626)

Risk-Neutral Transition Matrix

- A risk-neutral transition matrix is necessary to value derivatives that have payoffs dependent on credit rating changes
- A risk-neutral transition matrix can (in theory) be determined from bond prices

Example

Suppose there are three rating categories and risk-neutral default probabilities extracted from bond prices are:

Cumulative probability of default

	1	2	3	4	5
A	0.67%	1.33%	1.99%	2.64%	3.29%
B	1.66%	3.29%	4.91%	6.50%	8.08%
C	3.29%	6.50%	9.63%	12.69%	15.67%

Matrix Implied Default Probability

- Let M be the annual rating transition matrix and d_i be the vector containing probability of default within i years
- d_1 is the rightmost column of M
- $d_i = M\, d_{i-1} = M^{i-1}\, d_1$
- Number of free parameters in M is number of ratings squared

Transition Matrix Consistent With Default Probabilities

	A	B	C	Default
A	98.4%	0.9%	0.0%	0.7%
B	0.5%	97.1%	0.7%	1.7%
C	0.0%	0.0%	96.7%	3.3%
Default	0.0%	0.0%	0.0%	100%

This is chosen to minimize difference between all elements of $M^{i-1}\, d_1$ and the corresponding cumulative default probabilities implied by bond prices.

Credit Default Correlation

- The credit default correlation between two companies is a measure of their tendency to default at about the same time
- Default correlation is important in risk management when analyzing the benefits of credit risk diversification
- It is also important in the valuation of some credit derivatives

Measure 1

- One commonly used default correlation measure is the correlation between
  1. A variable that equals 1 if company *A* defaults between time 0 and time T and zero otherwise
  2. A variable that equals 1 if company *B* defaults between time 0 and time T and zero otherwise
- The value of this measure depends on T. Usually it increases at T increases.

Measure 1 continued

Denote $Q_A(T)$ as the probability that company *A* will default between time zero and time T, $Q_B(T)$ as the probability that company *B* will default between time zero and time T, and $P_{AB}(T)$ as the probability that both *A* and *B* will default. The default correlation measure is

Measure 2

- Based on a Gaussian copula model for time to default.
- Define t_A and t_B as the times to default of A and B
- The correlation measure, ρ_{AB}, is the correlation between

$$u_A(t_A)=N^{-1}[Q_A(t_A)]$$

and

$$u_B(t_B)=N^{-1}[Q_B(t_B)]$$

where N is the cumulative normal distribution function

Use of Gaussian Copula

- The Gaussian copula measure is often used in practice because it focuses on the things we are most interested in (Whether a default happens and when it happens)
- Suppose that we wish to simulate the defaults for *n* companies . For each company the cumulative probabilities of default during the next 1, 2, 3, 4, and 5 years are 1%, 3%, 6%, 10%, and 15%, respectively

Use of Gaussian Copula continued

- We sample from a multivariate normal distribution for each company incorporating appropriate correlations
- $N^{-1}(0.01) = -2.33$, $N^{-1}(0.03) = -1.88$, $N^{-1}(0.06) = -1.55$, $N^{-1}(0.10) = -1.28$, $N^{-1}(0.15) = -1.04$

Use of Gaussian Copula continued

- When sample for a company is less than -2.33, the company defaults in the first year
- When sample is between -2.33 and -1.88, the company defaults in the second year
- When sample is between -1.88 and -1.55, the company defaults in the third year
- When sample is between -1,55 and -1.28, the company defaults in the fourth year
- When sample is between -1.28 and -1.04, the company defaults during the fifth year
- When sample is greater than -1.04, there is no default during the first five years

Measure 1 vs Measure 2

Modeling Default Correlations

Two alternatives models of default correlation are:

- Structural model approach
- Reduced form approach

Structural Model Approach

- Merton (1974), Black and Cox (1976), Longstaff and Schwartz (1995), Zhou (1997) etc
- Company defaults when the value of its assets falls below some level.
- The default correlation between two companies arises from a correlation between their asset values

Reduced Form Approach

- Lando(1998), Duffie and Singleton (1999), Jarrow and Turnbull (2000), etc
- Model the hazard rate as a stochastic variable
- Default correlation between two companies arises from a correlation between their hazard rates

Pros and Cons

- Reduced form approach can be calibrated to known default probabilities. It leads to low default correlations.
- Structural model approach allows correlations to be as high as desired, but cannot be calibrated to known default probabilities.

Credit VaR (page 630)

Credit VaR asks a question such as:

What credit loss are we 99% certain will not be exceeded in 1 year?

Basing Credit VaR on Defaults Only (CSFP Approach)

- When the expected number of defaults is μ, the probability of n defaults is
- This can be combined with a probability distribution for the size of the losses on a single default to obtain a probability distribution for default losses

Enhancements

- We can assume a probability distribution for μ.
- We can categorize counterparties by industry or geographically and assign a different probability distribution for expected defaults to each category

Model Based on Credit Rating Changes (Creditmetrics)

- A more elaborate model involves simulating the credit rating changes in each counterparty.
- This enables the credit losses arising from both credit rating changes and defaults to be quantified

Correlation between Credit Rating Changes

- The correlation between credit rating changes is assumed to be the same as that between equity prices
- We sample from a multivariate normal distribution and use the result to determine the rating change (if any) for each counterparty