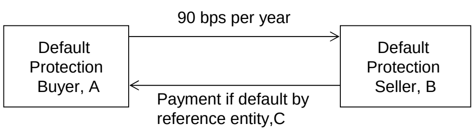
Chapter 27

Credit Derivatives

Credit Default Swap

- Company A buys default protection from B to protect against default on a reference bond issued by the reference entity, C.
- A makes periodic payments to B
- In the event of a default by C
  - A has the right to sell the reference bond to B for its face value, or
  - B pays A the difference between the market value and the face value

CDS Structure



CDS Final Payments

Notation:

L: Face value of bond, notional value of CDS

$A(t)$: Accrued interest on bond per $ of principal at time t

R: Recovery rate, market price as a percent of face value plus accrued interest

s: CDS payment rate per year. Annual payment = sL

τ: Time since last CDS payment

A pays τsL and B pays $L - RL[1 + A(t)]$

Sample Quotes (Jan 2001)

Company	Rating	3yr	5yr	7yr	10yr
Toyota	Aa1/AAA	16/24	20/30	26/37	32/53
Merrill Lynch	Aa3/AA-	21/41	40/55	41/83	56/96
Ford	A+/A	59/80	85/100	95/136	118/159
Enron	Baa1/BBB+	105/125	115/135	117/158	182/233
Nissan	Ba1/BB+	115/145	125/155	200/230	244/274

CDS Valuation

PV of CDS Payments per $1 of Notional

- If default event occurs at $t < T$, PV of payments is
- If no default event, PV of payments is
- Expected PV is

PV of CDS Costs per $1 of Notional Principal

- If default event occurs at $t < T$ cost is
- Expected cost is

Value of CDS to Buyer

- Value is expected PV of payments less expected PV of costs

CDS Rate

CDS rate sets value to zero

CDS Rate continued

When default can happen at any time this becomes

Approximate CDS Spread

- Let
  - y be the yield on bond issued by reference entity with maturity T
  - x be the yield on risk-free bond with maturity T
  - a be average value of $A(t)$
  - a^* be average value for $A(t)$ if reference bond is a par-yield bond with maturity T

Alternative Uses of the Formula

- To calculate CDS spreads from the probabilities of default and expected recovery rate
- To bootstrap the probabilities of default from CDS spreads and expected recovery rates

Sensitivity to Recovery Rate

- Vanilla CDS is not very sensitive to the recovery rate providing the same recovery rate is used to estimate default probabilities and calculate payoffs
- Binary swaps, which provide a fixed payoff in the event of a default, are much more sensitive to recovery rates

First-to-default swaps

- Similar to a regular CDS
- Several reference entities and reference bonds
- First entity to default triggers a payoff
- Settlement is same as ordinary CDS

Valuation

- Must use Monte Carlo simulation
- Each reference entity is simulated to determine when if ever it defaults
- Valuation is sensitive to default correlation
- A conservative (and easy) assumption for the seller is that all correlations are zero
- The easiest way to build in non-zero correlations is with the Gaussian copula model

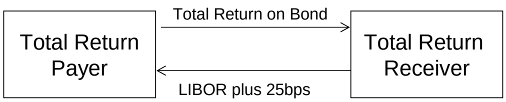
Seller Default Risk

- The impact of seller default risk on a CDS swap can be calculated by jointly simulating the reference entity and the seller
- Suppose Y=PV of payoff and C is PV of payments
- What rules should the simulation have for calculating Y and C?

Total Return Swap

- Company A agrees to pay B the total return earned on a <u>reference bond</u> issued by the <u>reference entity</u>, C, over some period of time.
- Total return includes all coupon payments and any change in the price of the reference bond. (Usually the latter is made at the end)
- B pays A LIBOR plus a spread on a notional equal to the initial value of the reference bond

The Structure



Uses of a TRS

- Total Return Swaps are usually used a financing vehicles
- Receiver wants to invest in bond
- Payer (a financial institution) buys the bond and agrees to the swap
- Payer has less credit exposure than if it had lent Receiver money to buy bond

Valuation of TRS

- If there were no risk of default by receiver, the value of a TRS would be difference between value of reference bond and value of LIBOR bond
- The spread above LIBOR would be zero
- In practice the payer loses money if the receiver defaults at a time when the bond value has declined

Credit Spread Options

- These provide a payoff dependent on movements in a particular credit spread.
- There is usually no payoff in the event of a default on the reference asset
- Payoff may be defined in terms of difference between actual spread and a strike spread or in terms of the difference between the price of an FRN and a strike price

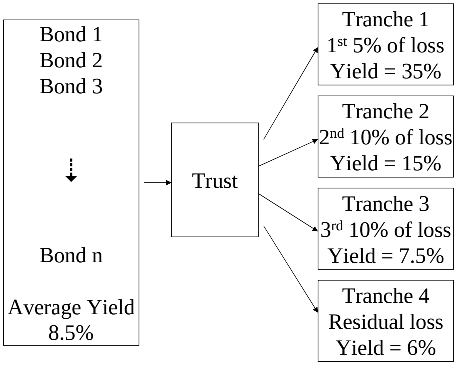
Valuation

- European options can be valued using Black's model
- This assumes that, conditional on no default, spread or FRN price is lognormal
- Need a volatility for forward credit spread or forward FRN price
- Must multiply Black's formula by risk-neutral probability of no default

Collateralized Debt Obligation

- A pool of debt issues are put into a special purpose trust
- Trust issues claims against the debt in a number of tranches
  - First tranche covers x% of notional and absorbs first x% of default losses
  - Second tranche covers y% of notional and absorbs next y% of default losses
  - …

Collateralized Debt Obligation



CDOs continued

- Note that average yield on tranches equals average yield on bonds less fee taken by trust manager
- Often trust manager holds first tranche

CDO Applications

- Can provide a range of credit quality debt objects
- Can create high quality debt from low quality debt
- Can create high yield debt from average risk debt
- Can create artificial short by selling tranches before buying bonds

Valuing CDO Tranches

- Depends on default correlation of bonds in portfolio
- Must use Monte Carlo simulation
- It is easiest to handle the default correlation with the Gaussian copula model

Quantifying the Cost of Default on a stand-alone derivatives contract

Two Categories of Derivatives:

- Those that are always assets to one party and liabilities to the other (e.g., options)
- Those that can become assets or liabilities (e.g., swaps, forward contracts)

Independence Assumption

- The independence assumption states that the variables affecting the price of a derivative are independent of the variables determining defaults
- This assumption (although not perfect) makes pricing for default risk possible

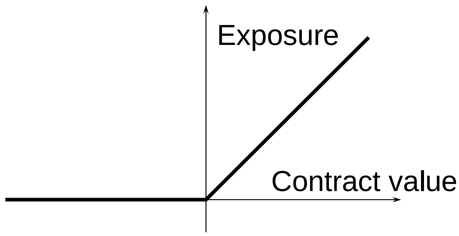
Notation

Contracts that are Assets

A Simple Interpretation

- Use the "risky" discount rate rather than the risk-free discount rate when discounting cash flows in a risk-neutral world
- Note that this does not mean we simply increase the interest rate in option pricing formulas

Credit Exposure for Contracts That Can be Assets or Liabilities



The Impact of Defaults

Example

- 5 year fixed-for-fixed annual-pay currency swap where interest at 10% in £ is exchanged for interest at 5% in $
- Principals are exchanged at the end of the life of the swap
  - Initial exchange rate = 2.000
  - Volatility of exchange rate = 15%
  - £ principal = 50 $ principal = 100
  - £ yield curve flat at 10% pa (ann comp) $ yield curve flat at 5% pa (ann comp)
- 1-, 2-, 3-, 4-, & 5-year zero-coupon bonds issued by the counterparty would have yields that are spreads of 25, 50, 70, 85, & 95 basis points above the risk-free rate
- Defaults can occur only at the end of years 1, 2, 3, 4, & 5

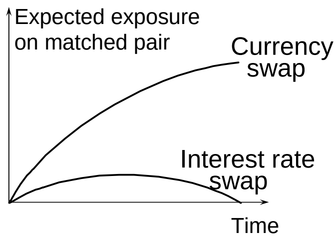
Evaluating the Cost of Defaults

Maturity t_i	u_i	when we receive $s: v_i	when we receive $s: $u_i v_i$	when we pay $s: v_i	when we pay $s: $u_i v_i$
1	0.00250	5.9785	0.0149	5.9785	0.0149
2	0.00745	10.2140	0.0761	5.8850	0.0439
3	0.01083	13.5522	0.1468	5.4939	0.0595
4	0.01265	16.2692	0.2058	5.0169	0.0634
5	0.01296	18.4967	0.2398	4.5278	0.0587
Total			0.6834		0.2404

Example continued

- The total cost of defaults on a matched pair of swaps with similar counterparties is 0.6834+0.2404=0.9236% of principal.
- This means that a bid-offer spread of 20 to 21 basis points is required to compensate for credit risk
- Why do we have more credit risk when we are receiving dollars in this example?
- From a credit perspective, is it better to receive fixed or floating in an interest rate swap when yield curve is upward sloping?

Why Cost of Defaults for Currency Swaps > Interest Rate Swaps



Convertibles

A convertible bond is a corporate bond that can be exchanged for equity at certain times in the future at a predetermined exchange ratio (shares per bond)

Convertibles continued

- One of the problems in valuing convertibles is that, in order to value the corporate bond correctly, it is necessary to take account of the chance of default in some way
- Otherwise we are implicitly assuming it is a no-default Treasury bond

Valuing Convertible Bonds

The value at a node is

MAX[MIN(Q_1, Q_2), Q_3]

where

Q_1 is the value given by the rollback

Q_2 is the call price, &

Q_3 is the value if conversion takes place.

Valuing Convertible Bonds
(continued)

- We divide the value of the bond at each node into two components
  - a component that arises from situations where the bond ultimately ends up as equity
  - a component that arises from situations where the bond ultimately ends up as debt

Example 27.6

- 9-month zero-coupon bond with face value of $100
- Convertible into 2 shares
- Callable for $115 at any time
- Initial share price = $50, volatility = 30%, no dividends
- Risk-free rates all 10%
- Yields on issuer's non-convertible bonds = 15%

The Tree

(Numbers at each node in descending order are the stock price, equity component, debt component & total value)