An Empirical Comparison of Default Swap Pricing Models[1]

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An Empirical Comparison of Default Swap Pricing Models

In this paper we compare market prices of credit default swaps with model prices. A default swap protects its buyer from losses caused by the occurrence of a default event to a corporate or sovereign debt issuer. In exchange for this default protection, the buyer pays a periodic premium to the protection seller. The no-arbitrage value of the default swap premium can be derived by applying a reduced form credit risk model. We estimate a reduced form model with a constant recovery rate and a polynomial hazard rate function. For comparison, we also implement a method often applied by market practitioners that uses a bond's credit spread as a direct estimate of the default swap premium. We find that the reduced form model outperforms the direct method. Moreover, we shed light on the choice of the default-free term structure of interest rates. We find that swap and repo curves significantly outperform the government curve as proxy for default-free interest rates for investment grade issuers, but that their performance is similar for speculative grade issuers. As such, this is one of the first papers to empirically confirm that financial markets no longer see Treasury bonds as the default-free benchmark. We also pay attention to the choice of the recovery rate. We show that not only bond spreads, but also default swap premiums are relatively insensitive to changes in the recovery rate as long as the integrated hazard function is scaled accordingly.

JEL Codes: C13, G12, G13

1 Introduction

During the last decade, credit derivatives have become important instruments to lay off or take on credit risk. The credit derivatives market has grown exponentially. Until today only very limited empirical research has been devoted to these new instruments, although several reduced form models have been developed to price them. Most empirical papers on credit risk modelling have focussed on defaultable bonds. In this paper, we estimate reduced form models and compare model-implied credit default swap premiums to market data. We show that using a reduced form model outperforms the market practice of directly comparing bonds' credit spreads to default swap premiums. Moreover, we shed light on the choice of the default-free term structure of interest rates. We find that swap and repo curves significantly outperform the government curve as proxy for default-free interest rates for investment grade issuers, but that their performance is similar for speculative grade issuers. As such, this is one of the first papers to empirically confirm that financial markets no longer see Treasury bonds as the default-free benchmark.

A default swap protects its buyer from losses caused by the occurrence of a default event to a corporate or sovereign debt issuer. In exchange for this default protection, the buyer pays a periodic premium to the protection seller. The no-arbitrage value of the default swap premium can be derived by applying a reduced form credit risk model. In these models prices of default-sensitive instruments are jointly determined by the probability of default and the recovered amount at default. Default is often represented by a random stopping time with a stochastic or deterministic arrival intensity (hazard rate), while the recovery rate is oftentimes assumed to be constant. We have a large data set of market quotes on credit default swaps at our disposal, allowing us to conduct empirical testing of reduced form models, which the literature has lacked so far. To the best of our knowledge, the only other paper that analyses credit default swap data is Cossin and Hricko (2001).

They restrained their analyses to regressing default swap premiums on proxies for credit risk, whereas we estimate and apply a reduced form credit risk model.

Market practitioners commonly proxy the default swap premium by the credit spread of a bond with a similar maturity issued by the same borrower. We show analytically that this relationship only holds approximately. Moreover, we show empirically that the approximation results in fairly large deviations between calculated and market default swap premiums. By deriving the risk-neutral pricing formula for a defaultable coupon-bearing bond, we can explicitly express its dependence on risk-neutral processes for default-free interest rates, hazard rates and recovery rates. Since the paper focuses on the estimation and application of credit risk models, we use a priori estimated default-free term structures. The choice for default-free interest rates has received little attention in the literature. Virtually all empirical papers on credit risk modelling used zero-coupon rates extracted from government bonds. However, since 1998 financial markets have moved away from estimating default-free interest rates from government securities, and started using swap and repo contracts instead. We find that using the government curve results in significant overestimation of credit risk for investment grade issuers, that swap curves result in a small but significant bias, and that repo curves estimate credit risk unbiasedly. We also find that the government curve is significantly outperformed by both the swap and repo curves, and that repo curve models performs significantly better than, or not significantly different from swap curve models. For speculative grade issuers the choice for the default-free curve is less important, as the performance differences between the three curves are smaller.

We also pay attention to the choice of the recovery rate. Since it is not possible to extract both the hazard rate and the recovery rate from prices of bonds of a single seniority class, we fix the recovery rate to identify the model. We show that not only bond spreads, but also default swap premiums are relatively insensitive to changes in the recovery rate as long as the integrated hazard function is scaled accordingly. Therefore, there is no need to determine the recovery rate very accurately, as long as it takes a reasonable value.

To model the hazard process, we follow a non-parametric approach that is able to accommodate various shapes. More specifically, we model the hazard function as a constant, linear or quadratic function of time to maturity. The parameters of the hazard function are estimated using non-linear least squares from market prices of bonds of a single issuer. The estimated credit model is subsequently applied to the pricing of credit default swap premiums written on the same issuer. We observe that both the in-sample fit to bonds and the out-of-sample fit to default swaps declines with an issuer's credit quality. We also find that using the various hazard rate functions yield more accurate estimates of default swap premiums than the market practice of directly comparing the premium to the credit spread of a similar bond. An analysis of the deviations between calculated and market premiums reveals that the models overestimate the risk of high grade, short-maturity contracts and underrate low grade, long-term default swaps.

The remainder of this paper is structured as follows. Section 2 discusses the characteristics of credit default swaps. The literature on reduced form credit risk modelling is reviewed in Section 3. In Section 4 we derive reduced form valuation models for bonds and default swaps, and present our estimation framework. The construction of our data set is outlined in Section 5. In Section 6 we present the results of applying the direct comparison methods and the reduced form models. Section 7 concludes the paper.

2 Default Swaps

Credit derivatives were first introduced at the annual meeting of the International Swaps and Derivatives Association (ISDA) of 1992. Since then, the global credit derivatives market has experienced impressive growth. Whereas market size (measured in total outstanding notional) amounted to no more than a few billion US dollars in 1995, participants to the annual "Credit Derivatives Survey by Risk (Patel, 2002) estimated that the market has grown to US$ 1.4 trillion at the end of 2001. To put these figures into perspective, as of June 2001 the total outstanding notional of interest rate swaps amounted to US$ 51 trillion and the market size for interest rate options was estimated at US$ 9.5 trillion (BIS, 2000). Although the market for credit derivatives is still relatively small, it is the fastest growing sector of the global derivatives market (Smith, 2000). According to the biannual survey by the British Bankers' Association (BBA, 2000), the most popular types of credit derivatives are default swaps (accounting for 38% of the global credit derivatives market), total return swaps (11%), credit linked notes (10%) and credit spread options (5%). The first academic papers on credit derivatives were Howard (1995), Smithson (1995), Das (1996), Iacono (1997) and Masters (1998).

The publication of the "Credit Derivatives Definitions" by the International Swaps and Derivatives Association (ISDA, 1999) was a big move to standardizing the terminology in credit derivatives transactions. The ISDA Definitions were amended in 2001 following disagreements in the market on which obligations can be delivered in physically settled contracts in case of a debt restructuring event (ISDA (2001), see also Tolk (2001)). The Definitions established a uniform set of definitions of important terms, such as the range of credit events that could trigger payments or deliveries. In addition to the enhanced enforceability and interpretation of the contracts, the Definitions increased flexibility and reduced the complexity of administration and documentation. The vast majority of all credit derivative transactions are being documented by the ISDA confirms.

A default swap is a contract that protects the holder of an underlying asset from the losses caused by the occurrence of a *credit event* to the asset's issuer, referred to as the *reference entity*. Credit events that trigger a default swap can include one or more of the following: bankruptcy, failure to make a principal or interest payment, obligation acceleration, obligation default, repudiation/moratorium (for sovereign borrowers) and restructuring; these events are jointly referred to as *default*. A default swap only pays out if the reference entity defaults; reductions in value unaccompanied by default do not compensate the buyer in any way. Also, the event of default must be verifiable by publicly available information or an independent auditor. The buyer[1] either pays an up-front amount or makes periodic payments to the seller, typically a specified percentage of the notional amount. In the latter case, the percentage that gives the contract zero value at initiation is called the *spread*, *premium* or *fixed rate*. If default occurs, the default swap can be settled in one of two ways. With a *cash settlement*, the buyer keeps the underlying asset(s), but is compensated by the seller for the loss incurred by the credit event. In a *physical settlement* procedure, the buyer delivers the reference obligation(s) to the seller, and in return, he receives the full notional amount. Either way, the value of the buyer's portfolio is restored to the initial notional amount.[2]

Several features of default swaps are worth mentioning. If the contract specifies periodic payments, and default occurs, the buyer is typically required to pay the part of the premium payment that has accrued since the last payment date; this is called the *accrual payment*. The credit event may apply to a single reference obligation, but more commonly the event refers to any one of a much broader class of debt securities, including bonds and loans. Similarly, the delivery of obligations in case of physical settlement can be restricted to a specific instrument, though more usually the buyer may choose from a list of qualifying obligations, irrespective of currency and maturity as long as they rank *pari passu* with (have the same seniority as) the reference obligation. This latter feature is commonly referred to as the *delivery option*. Theoretically, all deliverable obligations

[1] The party purchasing credit protection is called the *buyer;* similarly, the *seller* refers to the party providing ('selling') protection.

[2] In a so-called *binary default swap* the seller pays the buyer a pre-specified amount, independent of the realized loss in case of default.

should have the same price at default and the delivery option would be worthless. However, in some credit events, for instance a debt restructuring, not all obligations become immediately due and payable, so that after such an event bonds with different characteristics will trade at different prices. This is favorable to the buyer, since he can choose the cheapest bonds for delivery to the seller. Counterparties can limit the value of the delivery option by restricting the range of deliverable obligations, e.g., to non-contingent, interest-paying bonds.

Counterparty risk is generally not taken into account in determining deal prices; if a party is unwilling to take on credit risk to its counterparty, it either decides to cancel the trade or to alleviate the exposure, for instance by demanding that a collateral is provided or that the premium is paid up-front instead of periodically (Culp and Neves (1998) and O'Kane and McAdie (2001)).

An important application of credit default swaps is shorting credit risk. The lack of a market for repurchase agreements (*repos*) for most corporates makes shorting bonds unfeasible. So, credit derivatives are the only viable way to go short corporate credit risk. Even if a bond can be shorted on repo, investors can only do so for relative short periods of time (one day to one year), exposing them to changes in the repo rate. On the other hand, default swaps allow investors to go short credit risk at a known cost for long time spans: default swaps with maturities of up to 10 years can be easily contracted, but liquidity rapidly decreases for even longer terms.

3 Literature

In reduced form models, also known as intensity based models, credit risk is jointly determined by the occurrence of default and the recovered amount at default. Default is often represented by a random stopping time with a stochastic or deterministic arrival intensity (hazard rate), while the recovery rate is usually assumed to be constant. The leading frameworks are the Jarrow, Lando, and Turnbull (1997) Markov chain model, which extended the work of Litterman and Iben (1991) and Jarrow and Turnbull (1995) to multiple credit ratings, and the Duffie and Singleton (1999) framework, which allows techniques developed for default-free term structure modelling to be applied to defaultable interest rates. Other important contributions were made by Duffie, Schroder, and Skiadas (1996), Lando (1998), Madan and Unal (1998) and Schönbucher (1998). Bielecki and Rutkowski (2000) provided an overview of both frameworks.

The empirical literature on reduced form models has focused on estimating the parameters of one of three processes: the hazard process, the spread process or the risky short rate process. The first approach seems to be most popular. Cumby and Evans (1997) considered both cross-sectional estimation of a constant hazard rate model and time-series estimation of several stochastic specifications. Madan and Unal (1998) estimated recovery and hazard processes in a two-step procedure using Maximum Likelihood (ML) and Generalized Methods of Methods (GMM). Duffee (1998), Keswani (2000) and Driessen (2001) applied ML with Kalman filtering to obtain parameter estimates of Cox, Ingersoll, and Ross (1985, CIR) processes from time-series data. Janosi, Jarrow, and Yildirim (2000), Bakshi, Madan, and Zhang (2001) and Frühwirth and Sögner (2001) used non-linear squares to estimate the model parameters from cross-sectional data. Janosi *et al.* specified a stochastic hazard rate that depends on the default-free short rate and an equity market index; Bakshi *et al.* estimated a model with correlated interest rates, hazard rates and recovery rates; Frühwirth and Sögner used a constant hazard rate.

The second approach encountered in the empirical literature refrains from modelling the default and/or recovery components of credit risk and directly estimates the spread process instead. Nielsen and Ronn (1998) estimated a log-normal spread model using non-linear least squares on cross-sectional data. Taurén (1999) utilized GMM to estimate the credit spread dynamics as a Chan, Karolyi, Longstaff, and Sanders (1992) process. Dülmann and Windfuhr (2000) and Geyer, Kossmeier, and Pichler (2001) implemented a ML procedure with Kalman filtering to obtain parameter estimates of Vasicek (1977) and/or CIR models for the instantaneous credit spread. Duffie, Pedersen, and Singleton (2000) used an approximate Maximum Likelihood method to estimate a multi-factor model with Vasicek and CIR processes.

The third approach is to consider the sum of the default-free rate and the credit spread and estimate a model for the total risky rate. Duffie and Singleton (1997) utilized this approach to estimate the swap rate as a 2-factor CIR process using Maximum Likelihood.

All discussed papers gauged the quality of the implemented models on their ability to fit spreads and/or bond prices. Since credit derivatives allow credit risk to be traded separately from other sources of risk, they provide a clean way of putting a price on credit risk. Therefore, we may obtain better insights in the performance of credit risk models by applying them to the pricing of credit derivatives.

4 Methodology

In this section, we first present our reduced form credit risk model. Then we discuss the valuation of credit default swaps. Finally, we elaborate on the specification and estimation of the models.

4.1 Valuing Bonds

Following Jarrow and Turnbull (1995), we assume a perfect and arbitrage-free capital market, in which default-free and defaultable zero-coupon bonds, a default-free money-market account and defaultable coupon bonds are traded. The uncertainty is represented by a filtered probability space $(\Omega, \mathcal{F}, \mathbb{Q})$, where Ω denotes the state space, $\mathcal{F}$ is a σ-algebra of measurable events in Ω and $\mathbb{Q}$ is the actual probability measure. The information structure is represented by the filtration $\mathcal{F}(t)$. We take as given some non-negative, bounded and predictable default-free short-rate process $r(t)$, so that the time-t value of the default-free money-market account $B(t)$ is given by

$$B(t) = \exp\left(\int_0^t r(s)\,ds\right).$$

Let $\tilde{\mathbb{Q}}$ denote the equivalent martingale measure that is associated with the numeraire $B(t)$ (see Harrison and Kreps (1979) and Harrison and Pliska (1981)). That is, $\tilde{\mathbb{Q}}$ is the risk-neutral measure. Using risk-neutral valuation, the value $p(t,T)$ at time t of a *default-free zero-coupon bond* with maturity T and notional amount 1, can be calculated as

$$p(t,T) = B(t)\,\tilde{\mathbb{E}}_t\left[\frac{p(T,T)}{B(T)}\right] = \tilde{\mathbb{E}}_t\left[\exp\left(-\int_t^T r(s)\,ds\right)\right],$$

where

$$\tilde{\mathbb{E}}_t[X] = \mathbb{E}^{\tilde{\mathbb{Q}}}[X|\mathcal{F}_t].$$

Let $v(t,T)$ denote the time-t value of a *defaultable zero-coupon bond* with maturity T and face value 1. Default occurs at a random time τ, independent of $r(t)$ under $\tilde{\mathbb{Q}}$.[3] Assuming a constant recovery fraction δ of face value in case of default[4], the time-T value

[3]In our empirical application of the model we use readily available default-free term structures instead of specifying a risk-neutral process for $r(t)$ and estimating its parameters. Therefore, we cannot estimate the correlation between default-free interest rates and the default time, so there is no use in allowing for a correlation parameter in our model.

[4]If we assume a stochastic recovery rate that is risk-neutrally independent from the default-free short-rate process and the default time, all formulas remain valid, except that δ should be interpreted as the *expected* recovery rate under the risk-neutral measure. Moreover, the results of Bakshi, Madan, and Zhang (2001) indicated that a model with a stochastic recovery rate performs equally well as a model with a constant recovery rate.

of the bond equals

$$v(T,T) = \mathbf{1}_{\{\tau>T\}} + \delta \mathbf{1}_{\{\tau\leq T\}},$$

where $\mathbf{1}_{\{A\}}$ is the indicator function of event A. Using the $\tilde{\mathbb{Q}}$-independence of default-free interest rates and the default time, we obtain the following expression for the price of the defaultable zero-coupon bond (cf. Jarrow and Turnbull (1995, formula 49))

$$v(t,T) = B(t)\,\tilde{\mathbb{E}}_t\left[\frac{v(T,T)}{B(T)}\right] = p(t,T)\left[\tilde{\mathbb{P}}(t,T) + \delta\left(1-\tilde{\mathbb{P}}(t,T)\right)\right], \tag{1}$$

where

$$\tilde{\mathbb{P}}(t,T) = \tilde{\mathbb{E}}_t\left[\mathbf{1}_{\{\tau>T\}}\right].$$

Hence, $\tilde{\mathbb{P}}(t,T)$ denotes the risk-neutral survival probability. We assume the existence of a non-negative, bounded and predictable process $\lambda(t)$, which represents the intensity for τ under $\tilde{\mathbb{Q}}$. We will refer to $\lambda(t)$ as the *default intensity* or the *hazard rate*. Intuitively, $\lambda(t)\Delta t$ is the risk-neutral probability of default between times t and $t+\Delta t$ as seen at time t, conditional on no earlier defaults. Then,

$$\tilde{\mathbb{P}}(t,T) = \tilde{\mathbb{E}}_t\left[\exp\left(-\int_t^T \lambda(s)\,ds\right)\right] = \tilde{\mathbb{E}}_t\left[\exp\left(-\Lambda(t,T)\right)\right], \tag{2}$$

where $\Lambda(t,T)$ denotes the integrated hazard function

$$\Lambda(t,T) = \int_t^T \lambda(s)\,ds.$$

Finally, consider a *defaultable coupon bond* with coupon payment dates $\mathbf{t} = (t_1,\ldots,t_n)$, coupon payment c, maturity t_n and notional 1. The price $v(t,\mathbf{t},c)$ of this bond at time t

equals the sum of the expected values of its coupons and face value and a potential recovery payment. The i^{th} coupon payment is only made if the bond's issuer has not gone bankrupt yet at time t_i. Similarly, the face value is only paid if the bond is still alive at time t_n. If the bond does default before it matures, a recovery fraction δ of the notional (and *not* of the remaining coupons too, see Jarrow and Turnbull (2000) and Schönbucher (2000)) is made at the random default time τ. Applying the risk-neutral valuation principle to all cash flows, yields

$$v(t,\mathbf{t},c) = \sum_{i=1}^{n} p(t,t_i)\,\tilde{\mathbb{E}}_t\left[c\mathbf{1}_{\{\tau>t_i\}}\right] + p(t,t_n)\,\tilde{\mathbb{E}}_t\left[\mathbf{1}_{\{\tau>t_n\}}\right] + \tilde{\mathbb{E}}_t\left[p(t,\tau)\,\delta\mathbf{1}_{\{\tau\leq t_n\}}\right].$$

Evaluating the expectations, we obtain the following price for a defaultable coupon bond (cf. Duffie and Singleton (1997, formula 26))

$$v(t,\mathbf{t},c) = \sum_{i=1}^{n} p(t,t_i)\,c\tilde{\mathbb{P}}(t,t_i) + p(t,t_n)\,\tilde{\mathbb{P}}(t,t_n) + \int_t^{t_n} p(t,s)\,\delta f(s)\,ds, \tag{3}$$

where $f(t)$ denotes the probability density function associated with the intensity process $\lambda(t)$, i.e.

$$f(t) = \lambda(t)\exp(-\Lambda(0,t)).$$

In our empirical application, we replace the integral expression in (3) by a numerical approximation.[5] To do so, we define a grid of maturities $s_0,\ldots,s_m$, where $s_0 = t$ and $s_m = t_n$ and set

$$\int_t^{t_n} p(t,s)\,\delta f(s)\,ds \approx \sum_{i=1}^{m} p(t,s_i)\,\delta\left(\tilde{\mathbb{P}}(t,s_{i-1}) - \tilde{\mathbb{P}}(t,s_i)\right).$$

[5] This approximation is necessary, because we do not have an analytical expression for $p(t,\cdot)$, but use the market's default-free term structure instead.

In our implementation, we work with a monthly grid.

4.2 Valuing Default Swaps

Similar to a plain vanilla interest rate swap, a default swap contract may be viewed as consisting of two 'legs': a fixed leg and a floating leg. The former contains the payments by the buyer to the seller and is called the fixed leg, because its payments are known at initiation of the contract. The floating leg comprises the potential payment by the seller to the buyer and at the start date it is unknown how much the seller has to pay – if he has to pay at all.

Consider a default swap contract with payment dates $\mathbf{T} = (T_1, \ldots, T_N)$, maturity T_N, premium percentage P and notional 1. Denoting the value of the fixed leg by $\bar{V}(t, \mathbf{T}, P)$ and the value of the floating leg by $\tilde{V}(t)$, the value of the default swap to the buyer equals $\tilde{V}(t) - \bar{V}(t, \mathbf{T}, P)$ and to the seller $\bar{V}(t, \mathbf{T}, P) - \tilde{V}(t)$. At initiation, the premium P is chosen in such a way that the value of the default swap is equal to zero, because only then the value to the buyer equals the value to the seller. Since the value of the fixed leg is homogeneous of degree one P, the premium percentage should be chosen as

$$P = \frac{\tilde{V}(t)}{\bar{V}(t, \mathbf{T}, 1)}.$$

We first determine the value of the *fixed leg*. At each payment date T_i, the buyer has to pay $\alpha(T_{i-1}, T_i) P$ to the seller, where $\alpha(T_{i-1}, T_i)$ is the year fraction between T_{i-1} and T_i (T_0 is equal to t), taking into account the day count convention specified in the contract. If the reference entity does not default during the life of the contract, the buyer makes all payments. However, if default occurs at time $s \leq T_N$, the buyer has made only $I(s)$ payments, where $I(s) = \max(i = 0, \ldots, N : T_i < s)$ and the remaining payments $I(s) + 1, \ldots, N$ are no longer relevant; in addition, he has to make an accrual payment of

$\alpha\left(T_{I(s)}, s\right) P$ at time s.[6] The value of the fixed leg at time t is thus equal to

$$\begin{aligned}\bar{V}(t, \mathbf{T}, P) &= \sum_{i=1}^{N} p(t, T_i)\, \tilde{\mathbb{E}}_t\left[\alpha(T_{i-1}, T_i)\, P \mathbf{1}_{\{\tau > T_i\}}\right] + \tilde{\mathbb{E}}_t\left[p(t, \tau)\, \alpha\left(T_{I(\tau)}, \tau\right) P \mathbf{1}_{\{\tau \leq T_N\}}\right] \\ &= \sum_{i=1}^{N} p(t, T_i)\, \alpha(T_{i-1}, T_i)\, P \tilde{\mathbb{P}}(t, T_i) + \int_t^{T_N} p(t, s)\, \alpha\left(T_{I(s)}, s\right) P f(s)\, ds. \qquad (4)\end{aligned}$$

To calculate the value of the *floating leg*, we have to distinguish between cash settlement and physical settlement. If default occurs, and the contract specifies *cash settlement* the buyer keeps the reference obligation and the seller pays the buyer an amount equal to the difference between the *reference price* and the *final price* of the reference obligation. The reference price is specified in the contract and is typically equal to 100%. The final price is the market value of the reference obligation on the default date as computed by the specified calculation agent by the specified valuation method; commonly, he has to poll one or more dealers for quotes on the reference obligation, disregard the highest and lowest quotes and calculate the arithmetic mean of the remaining quotes. Under our recovery assumption, the final price of the reference obligation is equal to δ, so that the value of the floating leg under cash settlement equals

$$\tilde{V}(t) = \tilde{\mathbb{E}}_t\left[p(t, \tau)(1 - \delta)\, \mathbf{1}_{\{\tau \leq T_N\}}\right] = \int_t^{T_N} p(t, s)(1 - \delta)\, f(s)\, ds. \qquad (5)$$

If the default swap contract specifies *physical settlement* things work somewhat differently. At the default time, the buyer delivers one or more of the deliverable obligations with a total notional of 1 to the seller and the seller pays 1 to the buyer. Assuming one deliverable, the net value of these transfers is equal to $1 - \delta$, so that the value of the floating leg under physical settlement is equal to its value under cash settlement. How-

[6] We assume that if the default time exactly coincides with a payment date T_i, the buyer does *not* make the regular payment, but makes an accrual payment, i.e. $I(T_i) = i - 1$. Since the regular payment and the accrual payment are equal on a payment date, this assumption does not affect the value of the default swap.

ever, a default swap contract generally allows the buyer to choose from a list of qualifying obligations, irrespective of currency and maturity as long as they have the same seniority as the reference obligation. This feature is called the *delivery option*, see also Section 2. We refrain from valuing the delivery option, and use the value of the floating leg under cash settlement. To numerically approximate the integrals in (4) and (5) , we use the same method as for the defaultable coupon bond price. Now a grid $S_0, \ldots, S_M$ is chosen with $S_0 = t$ and $S_M = T_N$. Again, in our implementation we use a monthly grid.

Our default swap pricing formula is very similar to other models encountered in the literature. The models by Aonuma and Nakagawa (1998), Brooks and Yan (1998), Scott (1998), Jarrow and Turnbull (1998) and Duffie (1999) are equal to our model, except that they only allow defaults on premium payment dates. Consequently, they lack the accrual payment in the fixed leg valuation. Cheng (1999) formally showed that the last four models, as well as a special version of Das and Sundaram (1999), are all mathematically equivalent. Nakagawa (1999) and Hull and White (2000), like us, also allowed defaults to occur on other dates than payment dates. However, Nakagawa (1999) did not incorporate the accrual payment and Hull and White (2000) assumed that the protection buyer makes a continuous stream of premium payments, rather than a set of discrete payments.

4.3 Specification

Since the focus of the paper is the estimation and application of credit risk models, we refrain from estimating a model for the default-free short-rate. Instead we use a priori estimated zero-coupon curves and use them to calculate the prices of default-free zero-coupon bonds. To completely specify the model, we have to (1) select a model for the risk-neutral hazard process, (2) pick a recovery rate and (3) choose a proxy for the default-free term structure.

4.3.1 Hazard Process

In the existing empirical literature, as discussed in Section 3, all studies that use time series estimation model the hazard rate stochastically, typically as a Vasicek or CIR process. Papers that apply cross-sectional estimation techniques consider either constant or stochastic hazard rates; in the latter case, the stochastic process is chosen in such a way that the survival probability curve (2) is known analytically.

We follow an intermediate approach by using a deterministic function of time to maturity. This specification facilitates parameter estimation, while still allowing for time-dependency. We model the integrated hazard function non-parametrically as a polynomial function of time to maturity

$$\Lambda(t,T) = \sum_{i=1}^{d} \lambda_i (T-t)^i ,$$

where d is the degree of the polynomial and $\lambda_1, \ldots, \lambda_d$ are unknown parameters. Note that we have imposed the required restriction $\Lambda(t,t) = 0$ by omitting the constant term. This specification implies that the hazard rate itself is polynomial of degree $d-1$. The survival probabilities follow directly from (2) as $\exp(-\Lambda(t,T))$. To the extent that the survival probability curve from a stochastic hazard specification can be approximated by our exponential-polynomial function, deterministic and stochastic models will yield similar results.

4.3.2 Recovery Rate

There are two approaches for the estimation of the recovery rate. The first is to consider it as just another parameter, and estimate it from the data along with the other parameters. The second method is to a priori fix a value. Although the first method seems preferable, it turns out that it is hard to identify the recovery rate from the data.

[insert Figure 1 around here]

Figure 1.a illustrates this for a constant hazard rate model estimated on a data set of Deutsche Bank bonds on May 4th, 1999 (the first day in our sample) using the swap curve as proxy for the default-free curve. We vary the recovery rate from 10% to 90% in steps of 10% and for each value we estimate the hazard rate. It is clear from the figure that the fitted zero-coupon curves are virtually identical, except for the one estimated with a recovery rate of 90%. To get some intuition for this outcome, consider the price of defaultable zero-coupon bond in equation (1). This equation may be rewritten as

$$v(t,T) = p(t,T)\left[1 - (1-\delta)\left(1 - \tilde{\mathbb{P}}(t,T)\right)\right].$$

So, given a default-free curve $p(t,T)$, the price only depends on the product of $1-\delta$ and $1 - \tilde{\mathbb{P}}(t,T)$. Using formula (2) and a first order Taylor expansion, the bond price can be approximated as

$$v(t,T) \approx p(t,T)\left[1 - (1-\delta)\Lambda(t,T)\right].$$

For the constant hazard rate model, $\Lambda(t,T) = \lambda_1(T-t)$, so that the zero-coupon spread $s(t,T)$ with respect to the default-free rate is approximately equal to $s(t,T) \approx (1-\delta)\lambda_1$ (see also Duffie and Singleton (1999, below equation 5)). Decreasing $1-\delta$ and simultaneously increasing λ_1 by the same ratio will result in approximately the same spread. Figure 1.b shows that this indeed happens when we estimate the hazard rate for different values of recovery rate. As long as the recovery rate is chosen between roughly 10% and 80%, the product of $1-\delta$ and λ_1 is approximately constant.

It is clear that it is hard to identify both the hazard and recovery processes from bond data, see also Duffee (1998, p. 203), Duffie (1999, p. 80) and Duffie and Singleton (1999,

p. 705). This may pose a problem for some applications, but for our purpose of pricing default swaps it fortunately does not. It turns out that the default swap premium is also relatively insensitive to the assumed recovery rate. Figure 1.c shows the premiums for a 5 year default swap written on Deutsche Bank for varying recovery rates (and thus varying hazard rates). As long as the recovery rate is chosen between roughly 10% and 80%, the estimated default swap premium is approximately between 13 and 15 basis points (bps). A smaller range of 14 to 15 bps is obtained, if the recovery is chosen between 10% and 60%.

In conclusion, both the bond spread and the default swap premium only depend on the product of one minus the recovery rate and the integrated hazard function. Therefore, it is not very important to choose the recovery rate very accurately, as long as it takes a reasonable value. In our implementation, we set $\delta = 50\%$.

4.3.3 Default-Free Interest Rates

Our bond and default swap valuation models require a term structure of default-free interest rates as input data. Since a few years, fixed income investors have moved away from using government securities to extract default-free interest rates and started using plain vanilla interest rate swap rates instead. Golub and Tilman (2000) and Kocić, Quintos, and Yared (2000) mentioned the diminishing amounts of US and European government debts, the credit and liquidity crises of 1998, and the introduction of the euro in 1999 as primary catalyzing factors for this development. Nowadays, government securities are considered to be unsuitable for pricing and hedging other fixed income securities, because in addition to interest rate risk they have become sensitive to liquidity risk. Swaps, on the other hand, being synthetic instruments, are available in unlimited quantities, allowing investors to go long or short any desired amount. A disadvantage of swap rates is that they contain a credit risk premium due to two sources. First, being a bilateral agreement between two parties, an investor is exposed to the potential default of its counterparty. Duffie and Huang (1996) showed that this premium is quite small however: only one or two basis points for typical differences in counterparties' credit qualities. Second, the swap's floating leg payments are indexed on a short-term[7] LIBOR rate, which is a default-risky rate. Therefore, the swap *rate* will be higher than the default-free rate even though the swap *contract* is virtually default-free, see Collin-Dufresne and Solnik (2001).

An instrument that is less sensitive to the risk of counterparty default and is not linked to a risky rate is a repurchase agreement (*repo* for short, see, e.g., Duffie (1996)). A repo is basically a collateralized loan, typically between two banks for a relatively short time period (1 day to, at most, 1 year). Each instrument has its own repo rate, and the highest repo rate is referred to as the *general collateral* (GC) *rate*.[8] GC rates have historically been close to swap rates, but they were typically several basis points lower. The usage of repo rates as default-free interest rates was recommended by Duffie (1999, p. 75).

Even though the above seems to be well-known to practitioners, the academic literature has paid little attention to the choice of the default-free curve. This is demonstrated by the fact that almost all empirical papers that estimate reduced form credit risk models used the government curve as the default-free curve; Duffie, Pedersen, and Singleton (2000) are the only exception by using the swap curve. We estimate our models for all three proxies – government, swap and repo curves – and see which curve gives the best fit to bond prices and default swap premiums.

[7]For US dollar-denominated swap contracts the floating payments are bound to 3-month LIBOR, whereas euro-denominated swaps are indexed on the 6-month LIBOR rate.

[8]Instruments whose repo rates are at or near the GC rate, are called *general collateral*. Instruments whose repo rates are significantly below the GC rate are referred to as *special*. Since data on repo specialness is hard to find, we assume that all considered bonds are general collateral.

4.4 Estimation

We use cross-sectional estimation to estimate the parameters of our model. So, if we are given a default-free zero-coupon curve at time t, and the market prices $P_1(t), \ldots, P_{b(t)}(t)$ of $b(t)$ defaultable bonds issued by a single entity, where the i^{th} bond has payment dates $\mathbf{t}_i$ and coupon percentages c_i, we may estimate the parameters of that entity's integrated hazard function. We minimize the criterion function,

$$\sum_{i=1}^{b(t)} \left(P_i(t) - v(t, \mathbf{t}_i, c_i)\right)^2,$$

using the Gauss-Newton algorithm, see e.g. Greene (2000, Chapter 10). We repeatedly estimate the model until all residuals are smaller than 2.5 standard deviations, removing the bond with the largest residual (in absolute sense) each time this condition is not met. This procedure prevents strongly mispriced bonds from unreasonably affecting the estimated curves, see also Perraudin and Taylor (1999, Section 2.2). To estimate a curve on a particular trading day, we also consistently exclude all bonds with a remaining maturity of less than 3 months. In our data set, such bonds showed constant prices or they were not quoted at all for several consecutive days. Moreover, we require that on each day, quotes should be available for at least 5 bonds. This ensures some degree of statistical reliability of the estimated parameters.

Strictly speaking, our approach of cross-sectional estimation, where we re-estimate the model parameters on a daily basis, is inconsistent with our assumption of a deterministic hazard function. Nevertheless, it is used in order to fit the model to observed bond prices as closely as possible. This approach could be compared to the practice of estimating the implied volatility of the Black-Scholes model using one or more observed option prices, rather than estimating the (assumed constant) volatility using a time-series of returns of the underlying asset.

5 Data

For our purposes we require market data on defaultable bonds, default swaps and default-free interest rates.

The *bond data set* consists of corporate and sovereign bonds and is obtained from two sources. From Bloomberg, we obtain bond characteristics, like maturity dates, coupon percentages and seniorities; a time series of credit ratings for each issuer is also downloaded from Bloomberg. Clean bid and ask price quotes are retrieved daily at 4.00pm from Reuters' TREASURY and EUROBOND pages. These Reuters pages are connected to bank and broker pages, and each time a party updates a quote for a bond, that quote is also refreshed on the Reuters page. Therefore, the TREASURY and EUROBOND pages provide a good representation of the market for euro-denominated bonds. The data covers the period from January 1, 1999 to January 10, 2001 and contains prices of almost 10800 bonds issued by over 1600 different entities. The total number of price quotes is close to 2.5 million. To estimate the credit risk models, we construct a sample of fixed-coupon, bullet, senior unsecured bonds that are denominated in euros or in one of the currencies of the participating countries. This reduces the number of bonds to 3920, the number of unique issuers to 704 and the number of quotes to approximately 1.1 million.

The *default swap data set* is constructed by combining quotes from two sources. Firstly, it contains indicative bid and ask quotes from daily sheets posted by commercial and investment banks, such as J.P. Morgan Chase, Salomon Brothers, Deutsche Bank and Credit Suisse, and by brokers, such as Prebon, Tradition and ICAP. Secondly, it comprises bid and ask quotes from internet trading services *creditex* and *CreditTrade*, whose participants, in addition to banks and brokers, also include other financial institutions and corporates. The data period ranges from May 1, 1999 to January 10, 2001. In this period, we observed 48098 quotes on default swaps on 837 distinct reference entities. Contracts denominated in US dollars make up 82% of the quotes, euro-denominated contracts account for 17% and the remaining 1% is comprised of British pounds, Japanese yens and Australian dollars. Quotes on dollar contracts are observed in the entire data period, whereas quotes on euro-denominated default swaps are only observed from March 2000 to January 2001. All contracts specify quarterly payments by the protection buyer. Virtually all quotes (99.7%) are for contracts with a notional amount of 10 million (denominated in one of the above mentioned currencies); the remaining contracts have face values of 1, 5, 15, 20, 25 or 50 million. The maturity of the default swaps ranges from 1 month to 20 years, with multiples of 6 months up to 10 years being most common; 5-year deals are most popular, making up 53% of the observations, followed by 3-year (10%), 10-year (7%) and 1-year (4%) contracts. For our subsequent analyses, we constrain ourselves to default swaps that are euro- or dollar-denominated, have a maturity of at most 10 years and a notional amount of 10 million. Imposing these constraints reduces the number of observations by 2.7%, but creates a more uniform data set by removing the least liquid contracts. Whenever we refer to 'the default swap data set' we mean this restricted version of the original data set. For our research, we need reference entities for which both bond and default swap prices are available. Restricting the data sets to this subset of entities, leaves us with 225 reference entities, 1131 bonds, about 258000 bond prices and about 23000 default swap prices.

As proxy for *default-free interest rates*, we consider three alternatives: government rates, swap rates, and general collateral (GC) repo rates. The zero-coupon 'euro' government curve is estimated on a daily basis from a subset of the bond data set, consisting of liquid German government bonds. We model the discount function as a linear combination of third degree B-splines basis functions with knots at 2, 5 and 10 years (see, e.g., Houweling, Hoek, and Kleibergen (2001) for more details). Euro swap rates are downloaded from Bloomberg. We apply a standard bootstrapping procedure to extract zero-coupon rates and interpolate linearly between the available maturities to get a curve for all required maturities. Finally, we download euro repo benchmark rates from the website of the British Bankers' Association (BBA, 2001). Unfortunately, the longest maturity for which GC rates are available (1 year) is too short for our purposes. Therefore, we use the following method to calculate approximate GC rates for all required maturities: on each day, we determine the 1-year swap-GC spread and assume that this spread may be subtracted from the swap rates of all other maturities to get the GC rates. Analogously to the swap curve, we use bootstrapping and linear interpolation to obtain a zero-coupon curve.

[insert Figure 2 around here]

Figure 2 depicts the average zero-coupon swap, repo and government curves over the sample period. By construction, the swap and repo curves have identical shapes and levels. The government curve, however, is on average much lower than the other curves and also less concave.

6 Results

In this section we first discuss the properties of the default swap data set and implement an approximate default swap pricing method often applied by financial market participants. Then, we present the results of applying our reduced form credit risk model to our data set. We conclude by analyzing the pricing errors of the model.

6.1 Analyzing Default Swap Premiums

[insert Table 1 around here]

[insert Figure 3 around here]

Since the empirical literature on credit default swaps is restricted to just one other paper (Cossin and Hricko, 2001), it is interesting to look at the properties of the data first. Table 1 summarizes the default swap quotes on several characteristics. Panel I subdivides the 46820 observations by the reference entity's *credit rating* at the quote date. This information is also depicted in Figure 3, which shows histograms of the default swap premiums per rating category. As may be expected, the rating is a very important determinant of default swap premiums as average premiums decrease monotonously with credit quality. Cossin and Hricko (2001) found a similar pattern of premiums with credit rating; their average premiums per rating differ from ours, but this may be due to the different sample period. Within the class of investment grade ratings, a linear relation seems to exist: average premiums roughly double each time the credit quality decreases by one rating. For speculative grade issuers, average default swap quotes rapidly increase. In addition, the histograms in Figure 3 show that not only the average premium but also the variation of premiums increases with the issuer's credit rating.

In panel II of Table 1 the sample is further subdivided by *deal type.* The number of bid quotes is roughly equal to the number of ask quotes. As usual in financial markets, a spread exists between them. The average bid-ask spread is 8 bps, but an increasing pattern with ratings may be observed. For AAA-rated entities the average bid-ask spread amounts 4.4 bps, rising gradually within high grade ratings to 12.6 bps for rating BBB. A much larger spread exists between bid and ask quotes for speculative grade issuers: about 75 bps for BB and 110 for B. The bid-ask spread for CCC-rated default swaps is negative, but this is most likely caused by the small number of observations. Note that the bid-ask spreads are relatively large compared to the quote size. For instance, for AA the average bid-ask spread of 6.8 bps amounts to about 28% of the average quote of 43.4 bps.

Contracts in our database are denominated in one of two *currencies:* either US dollars or euros. Panel III shows that dollar-denominated default swaps prevail, but recall that euro-denominated default swaps are only observed during the second half of the data period. For all ratings, dollar quotes are on average larger than euro quotes, except for rating B where the number of euro observations is rather small. This finding still holds, if we also control for quote date, though to a lesser extent (not shown here).

The relation between premium level and contract *maturity* is assessed in panel IV. Notice that more than 50% of the observations resides in the 4- to 5- year maturity range; Cossin and Hricko (2001) also reported this asymmetric distribution of the quotes over the maturity range. Aggregated over all ratings, there does not seem to be clear relation between the average default swap premium and maturity. Only for A- and B-rated reference entities an increasing pattern may be detected, but for the other ratings premiums are approximately constant as a function of time to maturity. Our findings are in line with Cossin and Hricko's (2001), who tested several specifications for the maturity effect, but none of them appeared to be significant.

Finally, we look at the behavior of average premiums over *time*. To that end we group the default swaps by quote date into three-month periods, see panel V. Except for AA and BB, the quotes for all ratings roughly follow a U-shape pattern over time: in the middle of the sample period, the average premium is higher than both at the start and at the end.

6.2 Comparing Bond Spreads and Default Swap Premiums

To admit a direct comparison between bonds and default swaps, we make the following intuitive argument. Suppose an investor in a coupon-bearing defaultable bond buys protection by entering into a credit default swap. The package consisting of the bond and the default swap is free of default risk, so "defaultable bond + default swap = default-free bond". Therefore, we also have "default swap = default-free bond − defaultable bond". Consequently, the default swap premium should be equal to the spread between the defaultable and the default-free bond.

To formalize our argument, consider a defaultable bond with coupon payment dates $\mathbf{t} = (t_1, \ldots, t_n)$, coupon c, maturity t_n and notional 1. Further, consider a default swap with the same maturity, premium percentage P and notional 1. For simplicity, assume that the default swap's and bond's payment dates coincide.

The value $V(t)$ of the package to the investor is given by $v(t, \mathbf{t}, c) - \bar{V}(t, \mathbf{t}, P) + \tilde{V}(t)$, whose formulas are given by $(3) - (5)$. We replace the integrals in the pricing formulas by the approximations from Section 4; the grids for both the bond and default swap are chosen to be equal to the payment dates $\mathbf{t}$. Under these assumptions, the loss of $1 - \delta$ on the bond on a potential default date is exactly offset by the reception of $1 - \delta$ from the default swap. Moreover, the accrual payments of the default swap's fixed leg vanish, because for the current grid choice default can only occur on a payment date. Therefore, the value of the package can be written as

$$V(t) = \sum_{i=1}^{n} p(t, t_i) (c - P\alpha(t_{i-1}, t_i)) \tilde{\mathbb{P}}(t, t_i) + \tag{6}$$

$$\sum_{i=1}^{n} p(t, t_i) \left(\tilde{\mathbb{P}}(t, t_{i-1}) - \tilde{\mathbb{P}}(t, t_i) \right) + p(t, t_n) \tilde{\mathbb{P}}(t, t_n). \tag{7}$$

The first line indicates that the coupon payments c on the bond are reduced by the 'insurance premium' $P\alpha(t_{i-1}, t_i)$ on the default swap. The second line shows that the notional of 1 will be paid eventually, but that the timing depends on the occurrence of the credit event. The investor is thus protected against default risk, but is now exposed to the risk of premature redemption of the notional and missing some of the promised coupons (*prepayment risk*).

If we additionally assume that both the default-free and defaultable bond are priced at par, then their yields are equal to their coupons rates (ignoring the prepayment risk). Let y and Y denote the yield of the defaultable and the default-free bond, respectively, then $y = c$ and $Y = c - P$, so that $y - Y = P$. This confirms our intuition that the bond spread should equal the default swap premium. Note however that we had to make several assumptions to get this result, so that it is only approximately valid. Obviously, its validity is further impaired by the implicit assumptions that the bond and default swap pricing models are correct, and that bonds and default swaps are priced off the same survival curve by financial market participants. Nevertheless, bond spreads and default swap premiums should be comparable. This relation was also presented by Cossin and Hricko (2001), though without proof. Duffie (1999) showed that this relation holds *exactly* for par floating rate notes instead of coupon-bearing bonds. O'Kane and McAdie (2001) discussed a large number of factors that may affect the difference between bond spreads and default swap premiums.

We will now determine to what extent this relation holds for our data set. For each quoted default swap written on an entity, we have to find a quoted bond issued by that same entity with the same maturity. Unfortunately, a bond with exactly the same maturity as the default swap is rarely available. Therefore, we examine two alternative methods that are often used by practitioners:[9]

1. Find a quoted bond whose maturity differs at most 10% from the default swap's maturity;

2. Find two quoted bonds, one whose maturity is smaller than, but at most twice as small as, the default swap's maturity, and one whose maturity is larger than, but at most twice as large as, the default swap's maturity, and linearly interpolate their spreads.

We will refer to method 1 as the *matching method,* and to method 2 as the *interpolation method*. The performance of each method is evaluated for all three proxies for the default-free term structure. That is, a bond's spread is calculated by subtracting either the swap, repo or government rate from its yield-to-maturity. This gives a total of six method-proxy combinations.

[9] Only the methods are used in practice, the percentages are our own implementation of them.

Each time a pair can be formed of a default swap premium and a (matched or interpolated) bond spread, we calculate two *pricing errors.* One by subtracting bond bid spreads from default swap ask quotes, and the other by subtracting bond ask spreads from default swap bid quotes.[10] The pricing errors are summarized in two ways. First, as the average, denoted by the Mean Pricing Error (MPE), and second as the average of the absolute values, called the Mean Absolute Pricing Error (MAPE). A negative (positive) sign of the MPE statistic indicates that bond spreads are, on average, too large (small) and thus that the bond market's estimate of the issuer's credit risk is larger (smaller) than the default swap's market estimate. To test if this under- or overestimation is significant, we create a time series $\text{MPE}_{i1},\ldots,\text{MPE}_{iS}$, where MPE_{it} denotes the mean pricing error for method i on date t. We then apply a *one-sample Z-test* (see, e.g., Arnold (1990, Chapter 11))

$$Z_i = \sqrt{S}\frac{\text{MPE}_i}{s_i},$$

where MPE_i and s_i are the sample mean and sample standard deviation of the MPE_{it} series, respectively, and S is the sample size. Asymptotically, Z_i has a standard normal distribution. Similarly, to determine if significant performance differences exist between our methods, we create a time series $\text{MAPE}_{i1},\ldots,\text{MAPE}_{iS}$ of mean absolute pricing errors for each method i. Then a *paired Z-test* (Arnold, 1990, Chapter 11) can be used to determine if method i's pricing performance is significantly different from method j's. The test lets us find out whether two time series have the same mean, while allowing for non-zero correlation and unequal variances. The test statistic is defined as

$$Z_{ij} = \sqrt{S}\frac{\bar{d}_{ij}}{s_{ij}},$$

[10] In this way, we are contrasting similar sides of the market. For instance, an investor can create an exposure to an issuer's default by buying a bond – for which he pays the ask price – or by writing default swap protection – for which he receives the bid premium.

where $\bar{d}_{ij}$ and s_{ij} are the sample mean and sample standard deviation, respectively, of $d_{ijt} = \text{MAPE}_{it} - \text{MAPE}_{jt}$, $t = 1, \ldots, S$. Asymptotically, Z_{ij} has a standard normal distribution.

[insert Table 2 around here]

[insert Table 3 around here]

[insert Figure 4 around here]

Figure 4 depicts scatter plots of pricing errors versus default swap premiums per rating for the interpolation method that uses the swap curve as default-free curve; the plots for the other methods are similar. If interpolated bond spreads over the swap curve are good estimates of default swap premiums, all (default swap premium, bond spread) pairs should lie on the horizontal axis. For ratings AAA, AA and A, this seems indeed to be the case, so that on average the method does a good job. This is confirmed by the MPE values in Table 2, which are approximately zero, though only for AAA the MPE is insignificant. For rating BBB, the scatter plot and MPE statistic indicate that bond spreads are on average smaller than default swap premiums; for ratings BB and B this is almost always the case. Moreover, the Z-test indicates that the MPEs are significantly different from zero, so that for BBB, BB and B bond spreads are biased estimates of default swap premiums. For all ratings, the dispersion around the horizontal axis is fairly large. In fact, the MAPE statistics in Table 2, together with the average default swap premiums in Table 1, imply that the calculated premiums deviate on average 19% (for BBB) to 68% (for AAA) in absolute value from the market values.

Tables 2 and 3 also shed light on the performance of the other methods. A striking feature of these results is the abominable performance of the government-based methods for AAA- to A-graded issuers. Their MAPE values are up to four times higher than for methods that proxy the default-free curve with the swap or repo curve. As the paired Z-

tests in Table 3 point out, the underperformance of the government curve for investment grade issuers (so including BBB too) is highly significant. Moreover, since the MPEs are negative, almost identical in size to the MAPEs and statistically significant, bond spreads relative to the government curve are virtually always larger than default swap premiums for high grade issuers. The MPE values for the swap and repo curve methods are much closer to – but still significantly different from – zero. Looking at the MAPE statistics as well, the swap curve methods perform significantly better than the repo curve methods for investment grade, but significantly worse for speculative grade entities. For speculative grade issuers, the government curve methods significantly outperform their swap- and repo-based counterparts. For all methods the MPE statistics take large, significantly positive values though, indicating that they all result in bond spreads being smaller than default swap premiums.

Our overall impression is that bond and default swap markets deviate considerably, with absolute deviations increasing for lower credit ratings. For AAA- to A-rated issuers, bond spreads over swap rates give reasonable estimates of default swap premiums. For ratings BBB to B, default swap premiums are substantially larger than bond spreads for all considered methods. Recall that the applied direct comparison method is only an approximation to the actual relationship between bonds and default swaps; specifically, a part of the difference between bond spreads and default swap premiums is due to the presence of prepayment risk (see the discussion below equation (6)). An application of our reduced form credit risk model, which specifies the exact dependence of both instruments on interest rates, hazard rates and recovery rates, may yield better results. This is the topic of the remainder of the paper.

6.3 Estimating Hazard Functions

[insert Table 4 around here]

We now turn to the estimation of credit risk models as described in Section 4.4. We consider three proxies for the default-free curve – government, swap and repo curves – and three degrees for the integrated hazard function – linear, quadratic and cubic. This yields a total of nine models. For each issuer, we estimate all models for each day that we have at least one default swap quote and at least five bond quotes. The first row of Table 4 shows the number of 'issuer-days' on which this was the case. Most observations are for AAA- and AA-rated issuers even though rating classes A and BBB contain the largest number of default swap quotes (see Table 1). This is caused by the composition of the bond database, which contains (a) a small number of BBB-rated bonds and (b) a small number of bonds per A-rated issuer. The remainder of Table 4 shows the average fit of the model to the bond prices. A model's quality at time t may be assessed by looking at the root mean squared error (RMSE) of the deviations between the market prices and the model prices

$$RMSE\,(t) = \sqrt{\frac{1}{b(t)} \sum_{i=1}^{b(t)} e_i\,(t)^2}$$

where $e_i\,(t) = P_i\,(t) - v\,(t, \mathbf{t}_i, c_i)$. A model's RMSE is calculated as the average of its $RMSE(t)$ values over all days in the sample.

Looking at the differences between ratings first, we observe that the goodness of fit deteriorates as the rating declines. AAA- and AA-rated bond can be fitted quite accurately with RMSE values of 15 to 20 bps. The RMSEs for other investment grade bonds are well below 1% and usually below 50 bps. For speculative grade bonds, RMSE statistics range from 1% to 2%. Next we try to determine which model performs best. We find that using more parameters – obviously – yields a better in-sample fit. This holds for all three proxies for the default-free curve, but especially for the government curve if we move from a linear to a quadratic model. Table 1 also indicates that using a cubic model has little advantage over a quadratic model for high grade bonds, but does improve the fit for low grade bonds. As to the choice of the default-free curve, we see that for the linear model, the government curve clearly underperforms the swap and repo curves. For quadratic and cubic models, on the other hand, choosing a proxy for the default-free curve is less important. Overall it seems sufficient for AAA and AA to use a linear model with a swap or repo curve, for A and BBB a quadratic model with a swap or repo curve, and for BB and B a cubic model with any proxy for the default-free curve.

[insert Table 5 around here]

Now we turn to the discussion of the estimated coefficients of the integrated hazard function, see Table 5. Let us first look at the case where the integrated hazard function is modelled as a first degree polynomial. Since the hazard function is a constant in this model, the λ_1 parameter may be interpreted as the issuers' average risk-neutral default intensity. The estimation results for all three default-free curve proxies indicate that the average default intensity increases with the issuers' credit rating, except that B's is somewhat below BB's. Therefore, on average, credit ratings do a good job in ranking firms by credit worthiness. However, the level of the default intensity differs considerably between the models: the default rate for the government curve model is about 50 bps to 60 higher than the default rate in the repo curve model, which in turn is about 10 bps higher than in the swap curve model. Obviously, these differences reflect the average spreads between these three curves, but especially for investment grade bonds they lead to substantially different default rates. For instance, if we use the swap curve we would conclude that AAA's default rate is only 7 bps, but if we use the government curve that number would be ten times as large.

In the quadratic integrated hazard model the hazard function is linear, so that λ_1 and

λ_2 are the level and slope coefficient of the hazard function, respectively. The estimates for both coefficients rise as the rating declines (with two exceptions: the level of B for all default-free curve proxies is smaller than that of BB and BBB's slope in the government curve model is slightly smaller than A's). These findings imply that if we compare the estimated spread curves of two ratings, the worst rating's spread curve both starts at a higher level and is steeper. Again, noticeable differences exist between the three default-free curve types. Using the government curve results in higher and steeper spread curves than using the swap curve. Comparing swap and repo curves, we find that the levels differ by about 10 bps just like in the linear model, but the slopes are exactly equal. This simply reflects the construction of our repo curve as a parallel shift of the swap curve.

Finally, we look at the results of the cubic model for the integrated hazard function. Unlike the nicely ordered level and slope coefficients, there does not seem to be relation between λ_3 and the credit rating. Both the size and sign of this parameter appear to be uncorrelated with credit worthiness. Considering the results for the government curve, we observe that, compared to the quadratic model, the level coefficient has decreased and the slope coefficient has increased for all ratings (except BB) . Interestingly, the λ_3 parameters for the government curve models are almost equal to those in the swap and repo curve models. Further, in the latter models, the level coefficient is virtually unchanged for AAA, AA and A, has increased for BBB and BB and has decreased for B. Similarly, the level coefficients is virtually unchanged for AAA, AA and A, has decreased for BBB and BB and has increased for B.

In conclusion, the choice of a proxy for the default-free curve has a significant impact on the estimated credit risk model. Both the level and shape of the hazard function are substantially effected by this decision. Moreover, the fit of the model to investment grade bonds is better if we use the swap or repo curve instead of the government curve. For speculative grade bonds, choosing a proxy is less important. Which combination of a default-free curve proxy and polynomial degree is the 'best', can be judged by applying the models to the pricing of credit default swaps.

6.4 Comparing Model and Market Premiums

Having estimated a credit risk model for a specific issuer allows us to calculate model premiums of credit default swaps written on that issuer. Like above, we define the pricing error as the market premium minus the model premium and summarize it by the Mean Pricing Error (MPE), and Mean Absolute Pricing Error (MAPE). A negative (positive) sign of the MPE statistic implies that the model premiums are, on average, too large (small) and thus that the model overestimates (underestimates) the issuer's credit risk.

[insert Table 6 around here]

[insert Table 7 around here]

Table 6 contains the MPE and MAPE figures for all nine models subdivided by rating, as well as one-sample Z-tests for the MPE statistics. Table 7 shows the outcomes of the paired Z-tests for the investment grade and speculative grade subsamples. If we compare these figures to the ones in Tables 2 and 3, we see very similar patterns. First, mean absolute pricing errors increase with credit ratings for all models, except for high grade entities in the models that use the government curve. Second, government-based models perform very badly for investment grade issuers: their MAPE statistics take significantly larger values than for swap- and repo-curve models, and their significantly positive MPE statistics indicate a large overestimation of default swap premiums. Third, the MPE values for the swap and repo curve models are close to zero, and for first degree swap and second and third degree repo models mostly statistically insignificant. Fourth, for speculative grade bonds, the government curve models outperform the swap- and repo-based models by two to 20 basis points; for quadratic and cubic models this outperformance is significant.

Finally, for all models the MPE statistics take significantly positive values, indicating that they all underestimate the credit risk of low grade entities.

Estimating a hazard rate model gives a clear improvement over the direct methods of Section 6.2. Comparing the best direct method to the best model for each default-free curve proxy, we find that MAPE statistics are reduced by 35% to 55% for investment grade issuers. For A-rated entities the reduction is less spectacular with a decrease of about 15%. Also, for government-based approaches the best method and best model perform similarly poor. For speculative grade issuers hazard rate models outperform the direct methods by 15% to 20%. Even though using a model works better than directly comparing bond spreads and default swap premiums, the model premiums of the best-performing model still deviate on average 20% to 50% in absolute value from the market values. So the models fit rather well to bonds, but their out-of-sample performance on default swaps is somewhat poor.

We now try to identify the best model. The results show that models that use the swap or the repo curve on average do a reasonable job for investment grade issuers. The swap-based models somewhat underestimate the true default swap premiums, and except for the linear model, this bias is significant; the repo-based model, on the other hand, slightly overestimates the market premiums, but these differences are mostly insignificant (except for the linear model). Looking at the MAPE statistics as well, the linear and quadratic swap curve models significantly outperform their repo- and government-based counterparts for investment grade entities; the differences between swap and repo models are economically limited though. For speculative grade issuers, the quadratic and cubic government curve models significantly work better than swap and repo models with equal degrees. As to the choice of the optimal degree of the integrated hazard function, the paired Z-tests indicate that the quadratic model works significantly better than, or not significantly different from, the linear and cubic models. This result holds for both investment grade and speculative grade issuers.

All in all, a cubic model that uses the repo curve seems to be the best choice for investment grade issuers: it gives unbiased estimates, and has the second lowest MAPE values. For speculative grade entities, none of the considered models can be recommended, since they all significantly underestimate credit risk.

6.5 Analyzing Pricing Errors

In the previous section, we showed that the mean absolute pricing error increases if the credit rating deteriorates. Moreover, speculative grade default swaps are grossly underpriced, but investment grade contracts can be priced more or less unbiasedly. It is also interesting to see if differences between market and model premiums can be related to other characteristics than the issuer's credit rating. We try to accomplish this by regressing absolute pricing errors on dummy variables for the following characteristics: deal type (bid or ask), currency (euro or dollar), rating (AAA, AA, A, BBB, BB), maturity (1-year segments up to 5 years, and a segment from 5 to 10 years) and quote date (6-month periods). Since each set of dummies contains mutually exclusive categories, their values sum to 1 for each observation. Usually, the coefficient of one of the dummies is set to 0 as identifying restriction. We take a different approach here, and set a linear combination of the coefficients to 0, where the weight of a coefficient equals the sample mean of the corresponding dummy variable. For instance, if β and α denote the coefficients of the bid and ask dummies, we could have set one of them to 0. Instead, we impose $b\beta + a\alpha = 0$, where b is set to the percentage of bid observations and a to the percentage of ask observations. The advantage of imposing these restrictions is that the constant of the regression equals the sample mean of the dependent variable. Furthermore, each coefficient can be interpreted as the change in the absolute pricing error for that dummy for an otherwise representative observation.

[insert Table 8 around here]

Table 8 shows the regression results for the quadratic specification of the integrated hazard function for all three proxies for the default-free curve; the results for the linear and cubic model are similar. We observe that most parameters are statistically different from zero, the signs of the parameters are largely consistent between the models and the R^2 values are about 50%. Mispricings strongly differ between *deal types* as errors on bid quotes are larger than on ask quotes. Since we use hazard functions estimated from bond bid quotes to calculate default swap ask premiums, and vice versa, this implies that the bid-side of the bond market is somewhat more in line with the ask-side of the default swap market than vice versa. The *maturity* of the default swap contract is also predictive of the pricing error, since the coefficients of the maturity dummies are significant (except for the interval from 3 to 4 years) and monotonously increasing. All models tend to overprice short-term contracts and underprice long-term contracts. As noted before, pricing errors for investment grade *ratings* are usually negative, and for speculative grade ratings strongly positive. The *currency* dummies are only significant for the government model, indicating that the swap and repo models can price both dollar- and euro-denominated default swaps. Finally, the parameter estimates for the contract's *quote date* dummies show that for most models pricing errors in 2000 were smaller than in 1999 or 2001, although not all coefficients are statistically different from zero.

7 Summary

In this paper we have empirically investigated the market prices of credit default swaps. We have shown that a simple reduced form model better prices credit default swaps than the market practice of directly comparing bonds' credit spreads to default swap premiums. The model works reasonably well for investment grade issuers, but quite poor in the high yield environment; so, there is definitely room for more empirical research and further model development. Further, we found evidence that government bonds are no longer considered by the markets to be the reference default-free instrument. Swap and/or repo rates have taken over this position. We also showed that the value of the assumed constant recovery rate has only marginal influence on model prices, since a reduction in the recovery rate comes along with a reduction in the probability of default.

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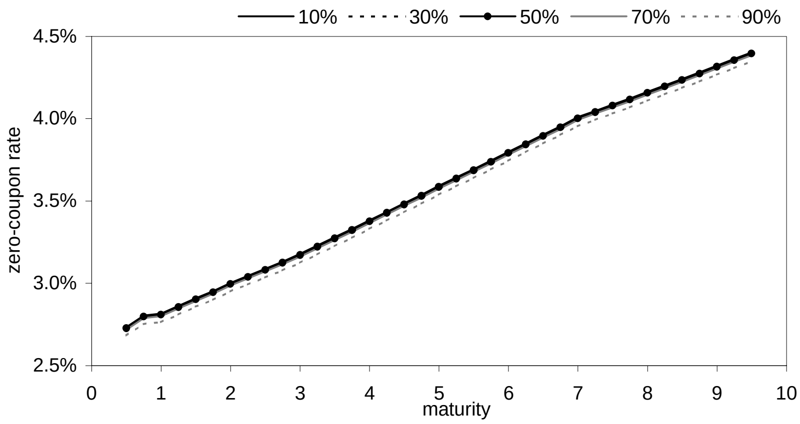
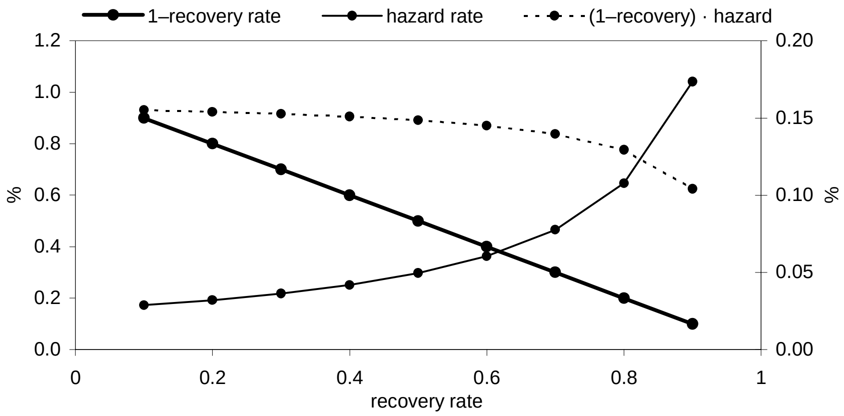


(a)



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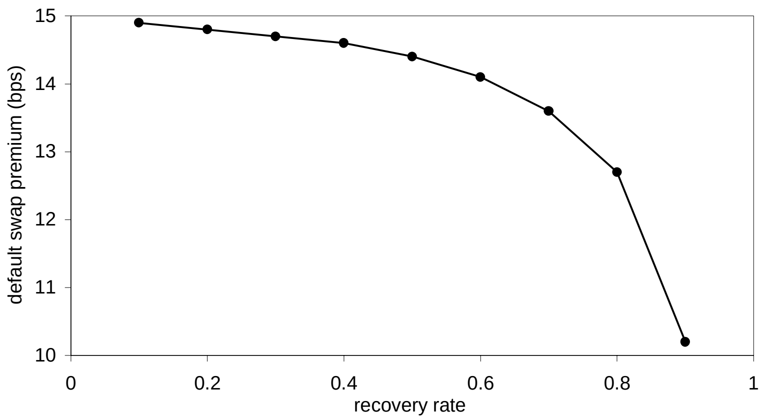

(c)

Figure 1. Illustration of the insensitivity of spreads and default swap premiums to the assumed recovery rate.

A reduced form credit risk model is fitted to market bid quotes of Deutsche Bank bonds on 4 May 1999 using non-linear least squares estimation. The integrated hazard function is modelled as a linear function of time to maturity, so that the hazard rate is a constant. The default-free term structure is approximated by the swap curve. The recovery rate is varied between 10% and 90%. Panel **(a)** shows the fitted zero-coupon curves for varying recovery rates; **(b)** portrays 1 minus the recovery rate and the hazard rate on the left axis, and their product on the right axis; **(c)** depicts the calculated premiums for a 5 year default swap.

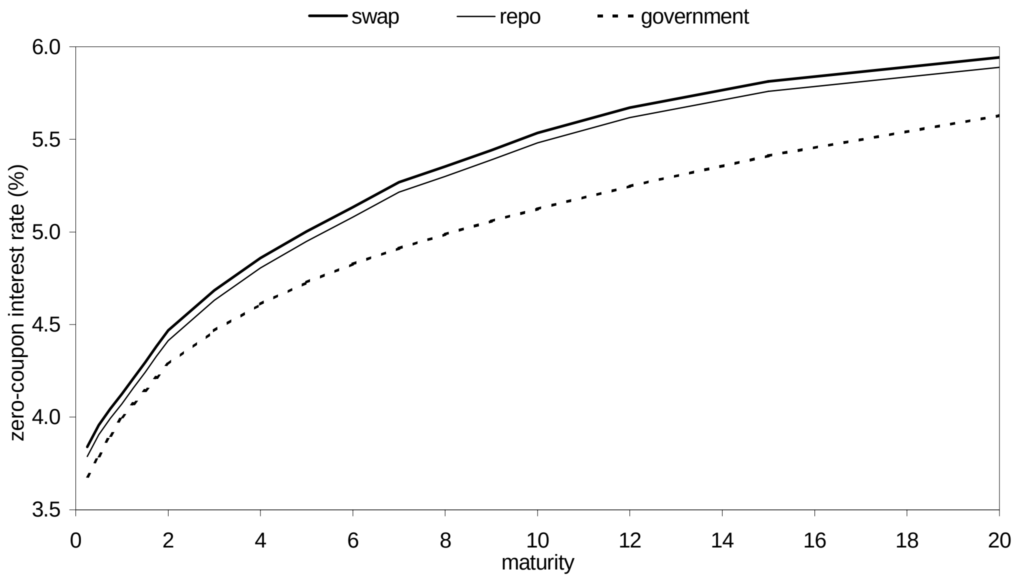

Figure 2. Swap, repo and government curves.
Average zero-coupon swap, repo and government curves over the sample period from May 1999 to January 2001.

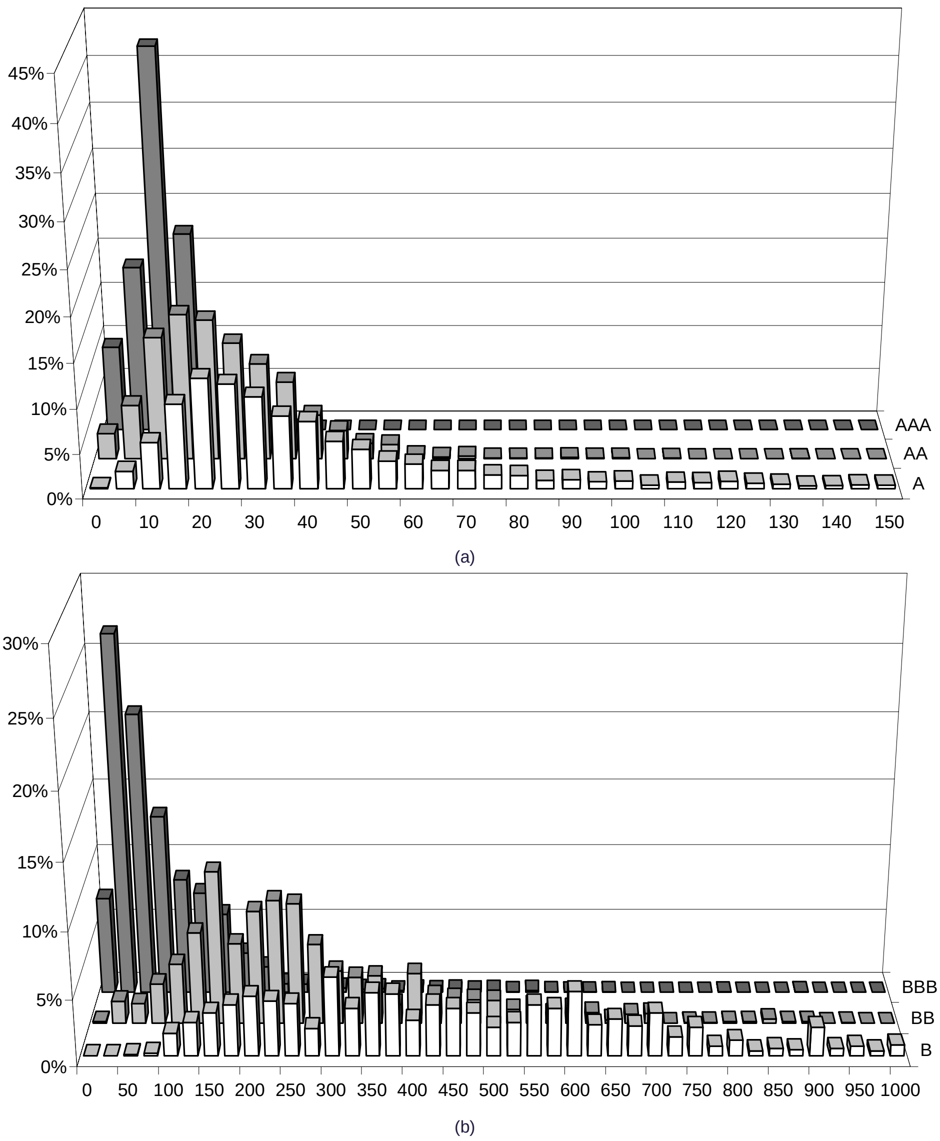

Figure 3. Histograms of default swap premiums.
The data set consists of indicative bid and ask quotes on default swaps obtained from internet trading services *creditex* and *CreditTrade* and from daily price sheets posted by banks and brokers. The quotes are observed from May 1, 1999 to January 10, 2001. All contracts have a notional of 10 million (euros or US dollars) and a maturity of at most 10 years. per credit rating.
Figure **(a)** shows histograms of default swap quotes for ratings AAA, AA and A, and figure **(b)** for BBB, BB and B. For rating class CCC, we have only 10 observations, so it is not shown here.

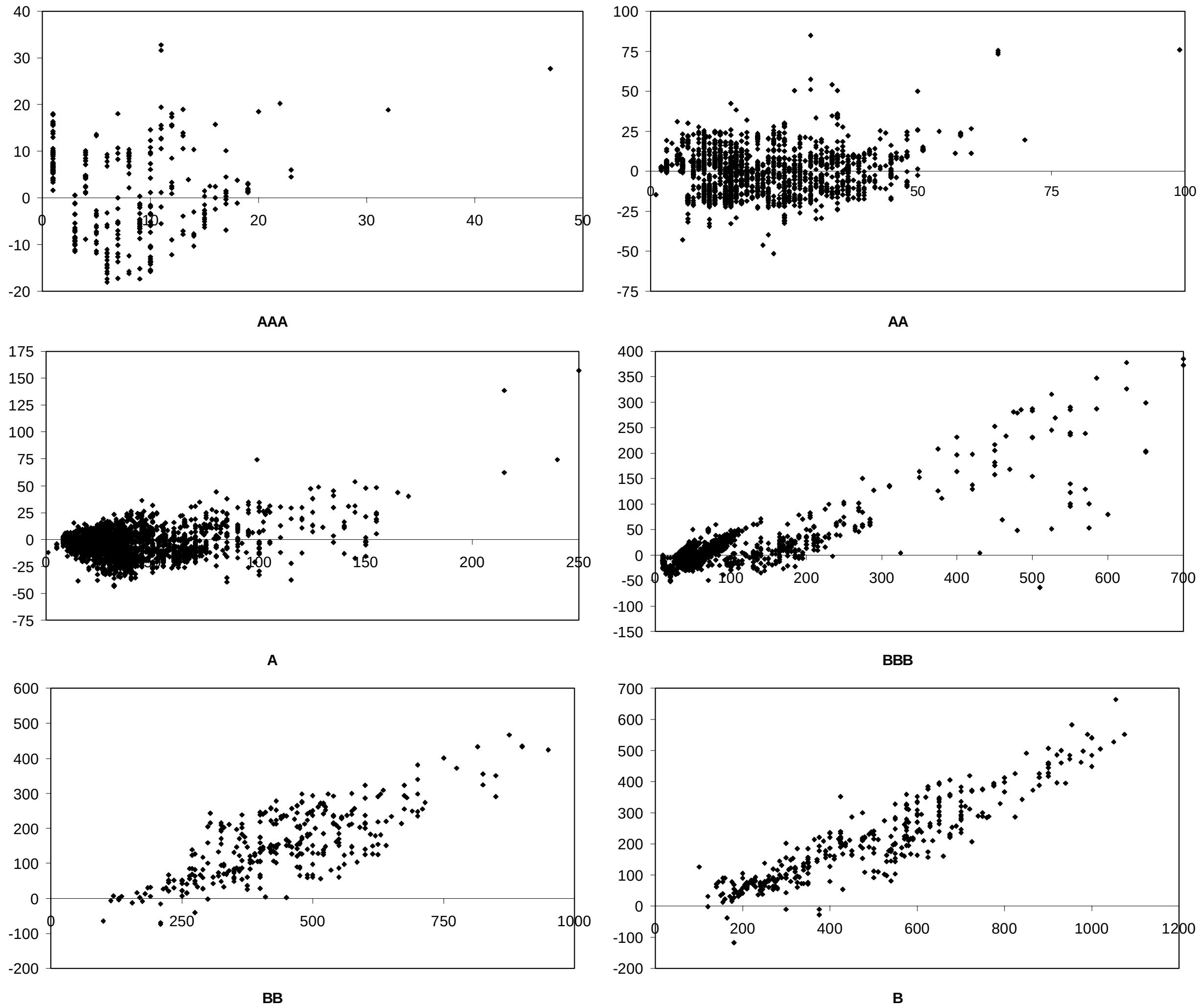

Figure 4. Scatter plots of pricing errors versus default swap premiums per rating.
The premium of a default swap written on a specific entity is approximated by the yield difference (spread) between a defaultable bond issued by that entity and an equivalent, but default-free bond. The default-free curve is approximated by the swap curve. To make pairs of bond spreads and default swap premiums, we use the interpolation method: find two bonds, one whose maturity is smaller than, but at most twice as small as, the default swap's maturity, and one whose maturity is larger than, but at most twice as large as, the default swap's maturity, and linearly interpolate their spreads.
Each time a pair can be formed of a default swap premium and an interpolated bond spread, we calculate two pricing errors. One by subtracting bond bid spreads from default swap ask quotes, and the other by subtracting bond ask spreads from default swap bid quotes.
The graphs depict scatter plots of pricing errors versus default swap premiums per rating. The horizontal (vertical) axis corresponds to the default swap premiums (pricing errors).

	AAA	AA	A	BBB	BB	B	CCC	NR	*All*
I: By rating									
	12.0	24.5	43.4	87.9	269.5	483.4	1957.5	55.7	*70.6*
	(1794)	(8321)	(18613)	(13187)	(1595)	(1118)	(10)	(2182)	*(46820)*
II: By rating and deal type									
bid	9.8	20.7	38.4	81.5	236.0	431.9	1985.0	55.7	*66.4*
	(882)	(3724)	(8672)	(6484)	(868)	(592)	(5)	(880)	*(22107)*
ask	14.2	27.6	47.8	94.1	309.5	541.3	1930.0	55.8	*74.4*
	(912)	(4597)	(9941)	(6703)	(727)	(526)	(5)	(1302)	*(24713)*
III: By rating and currency									
dollar	13.7	25.0	47.1	89.8	269.7	482.1	1957.5	59.9	*79.7*
	(1325)	(5193)	(14910)	(12543)	(1589)	(1106)	(10)	(1775)	*(38451)*
euro	7.5	23.7	28.6	51.7	223.0	598.3		37.5	*28.7*
	(469)	(3128)	(3703)	(644)	(6)	(12)		(407)	*(8369)*
IV: By rating and maturity									
(0,1]	12.1	18.4	31.9	100.8	199.1	407.6		61.9	*120.2*
	(55)	(208)	(790)	(1191)	(306)	(359)		(124)	*(3033)*
(1,2]	9.4	22.3	30.3	108.1	290.2	432.8	2900.0	52.0	*133.5*
	(24)	(80)	(721)	(788)	(295)	(183)	(4)	(71)	*(2166)*
(2,3]	9.6	26.4	37.2	74.5	242.4	501.0		45.7	*75.6*
	(387)	(562)	(2142)	(1750)	(306)	(222)		(279)	*(5648)*
(3,4]	11.8	27.3	45.3	87.0	330.9	567.9	1016.7	50.6	*65.8*
	(24)	(453)	(1242)	(686)	(32)	(47)	(3)	(335)	*(2822)*
(4,5]	11.8	24.1	44.6	82.3	281.6	561.2	425.0	60.4	*58.4*
	(675)	(6072)	(11743)	(6710)	(555)	(230)	(1)	(1217)	*(27203)*
(5,-)	14.0	26.3	51.3	103.0	419.0	622.1	2250.0	45.2	*75.5*
	(629)	(946)	(1975)	(2062)	(101)	(77)	(2)	(156)	*(5948)*
V: By rating and date									
Q2-1999	13.4	19.7	56.0	103.5	259.6	768.1		98.2	*90.7*
	(265)	(839)	(2374)	(2529)	(345)	(105)		(110)	*(6567)*
Q3-1999	13.4	24.2	58.8	104.1	300.0	698.4		134.7	*106.8*
	(211)	(642)	(2106)	(2258)	(253)	(217)		(110)	*(5797)*
Q4-1999	12.2	25.5	42.4	100.1	301.7	545.3	2585.7	46.6	*89.4*
	(77)	(622)	(1835)	(1052)	(280)	(104)	(7)	(90)	*(4067)*
Q1-2000	10.4	23.5	34.9	63.1	172.8	356.3		50.6	*58.4*
	(32)	(355)	(1367)	(953)	(118)	(99)		(185)	*(3109)*
Q2-2000	11.9	27.8	37.8	73.1	271.5	481.8		47.1	*53.7*
	(109)	(837)	(2990)	(1636)	(99)	(56)		(570)	*(6297)*
Q3-2000	9.5	25.2	35.9	67.9	299.5	353.8	491.7	46.5	*60.0*
	(393)	(2489)	(4321)	(2893)	(304)	(415)	(3)	(618)	*(11436)*
Q4-2000	11.9	23.6	41.4	93.7	211.9	342.1		53.3	*52.8*
	(619)	(2271)	(3166)	(1566)	(186)	(118)		(464)	*(8390)*
Q1-2001	17.6	30.6	58.9	114.4	220.5	518.8		61.4	*66.7*
	(88)	(266)	(454)	(300)	(10)	(4)		(35)	*(1157)*

Table 1. Characteristics of the default swap data set.
The data set consists of indicative bid and ask quotes on default swaps obtained from internet trading services *creditex* and *CreditTrade* and from daily price sheets posted by banks and brokers. The quotes are observed from May 1, 1999 to January 10, 2001. All contracts have a notional of 10 million (euros or US dollars) and a maturity of at most 10 years.
The table shows average default swap premiums by rating (Panel **I**), rating and deal type (**II**), rating and currency (**III**), rating and maturity (**IV**) and rating and quote date (**V**). The number of observations per cell is shown in parentheses.

	AAA	AA	A	BBB	*IG*	BB	B	*SG*	NR	*All*
Matching										
Obs.	1058	2168	1951	1188	*6365*	441	297	*738*	40	*7144*
Swap	5.9	-1.4	-4.9	9.4	*0.7*♯	129.7	174.9	*148.0*	0.2*	*16.0*
	(9.1)	(14.6)	(11.4)	(34.3)	*(16.4)*	(137.0)	(187.2)	*(157.3)*	(27.0)	*(31.0)*
Repo	1.7	-5.8	-9.7	4.4	*-3.8*	124.9	170.3	*143.3*	-5.5*	*11.4*
	(8.3)	(15.9)	(13.5)	(34.3)	*(17.3)*	(133.6)	(183.0)	*(153.5)*	(29.1)	*(31.5)*
Government	-31.1	-32.6	-37.1	-15.8	*-30.6*	106.6	151.3	*124.7*	-29.1	*-14.5*
	(31.4)	(34.4)	(37.8)	(41.0)	*(36.1)*	(118.5)	(165.6)	*(137.5)*	(41.8)	*(46.7)*
Interpolation										
Obs.	292	1839	2260	1067	*5458*	316	387	*703*	61	*6222*
Swap	0.8*	-1.6	-3.7	16.6	*1.2*	154.1	200.5	*179.6*	-4.3*	*21.3*
	(8.2)	(11.1)	(10.6)	(29.5)	*(14.3)*	(156.0)	(201.5)	*(181.1)*	(28.6)	*(33.3)*
Repo	-3.4	-6.0	-8.7	11.6	*-3.5*	149.4	196.0	*175.1*	-9.9♯	*16.6*
	(8.2)	(12.0)	(12.8)	(29.2)	*(15.5)*	(151.8)	(197.2)	*(176.8)*	(32.0)	*(33.9)*
Government	-33.9	-33.4	-31.7	-7.5	*-27.6*	177.2	133.6	*157.6*	-32.6	*-6.8*
	(34.3)	(34.2)	(32.5)	(34.8)	*(33.6)*	(178.7)	(137.2)	*(160.0)*	(49.5)	*(48.1)*

Table 2. Performance of the direct comparison methods.
The premium of a default swap written on a specific entity is approximated by the yield difference (spread) between a defaultable bond issued by that entity and an equivalent, but default-free bond. We consider three curves to proxy default-free interest rates: swap, repo and government curves. Moreover, two methods are considered to make pairs of bond spreads and default swap premiums: (1) *matching*: find a bond whose maturity differs at most 10% from the default swap's maturity; (2) *interpolation*: find two bonds, one whose maturity is smaller than, but at most twice as small as, the default swap's maturity, and one whose maturity is larger than, but at most twice as large as, the default swap's maturity, and linearly interpolate their spreads.
Each time a pair can be formed of a default swap premium and a (matched or interpolated) bond spread, we calculate two pricing errors. One by subtracting bond bid spreads from default swap ask quotes, and the other by subtracting bond ask spreads from default swap bid quotes.
The first row of the table shows the number of (bond spread, default swap premium) pairs that could be formed for each pricing method. The remaining rows show mean pricing errors (MPE) and mean absolute pricing errors (MAPE, between parentheses) by rating, pricing method and proxy for the default-free curve. All MPE values are statistically significant at confidence levels above 99%, except for the ones marked with *, which are insignificant at confidence levels up to 95%, and the ones marked with ♯, which are insignificant at confidence levels up to 99%. The *italic* columns provide summaries for investment grade (IG) and speculative grade (SG) subsamples, and for the entire sample (All).

	Matching			Interpolation		
	Swap	Repo	Government	Swap	Repo	Government
INVESTMENT GRADE						
Matching						
Swap		-12.60	-28.08	6.87	4.41	-14.38
Repo			-27.41	9.56	7.32	-12.55
Government				29.27	29.28	7.63
Interpolation						
Swap					-9.54	-24.71
Repo						-25.73
Government						
SPECULATIVE GRADE						
Matching						
Swap		14.30	13.89	-4.13	-3.45	-1.46
Repo			12.21	-4.61	-3.93	-1.94
Government				-6.36	-5.68	-3.73
Interpolation						
Swap					31.28	22.72
Repo						17.26
Government						

Table 3. Paired Z-tests of the direct comparison methods.
The premium of a default swap written on a specific entity is approximated by the yield difference (spread) between a defaultable bond issued by that entity and an equivalent, but default-free bond. We consider three curves to proxy default-free interest rates: swap, repo and government curves. Moreover, two methods are considered to make pairs of bond spreads and default swap premiums: (1) *matching*: find a bond whose maturity differs at most 10% from the default swap's maturity; (2) *interpolation*: find two bonds, one whose maturity is smaller than, but at most twice as small as, the default swap's maturity, and one whose maturity is larger than, but at most twice as large as, the default swap's maturity, and linearly interpolate their spreads.
Each time a pair can be formed of a default swap premium and a (matched or interpolated) bond spread, we calculate two pricing errors. One by subtracting bond bid spreads from default swap ask quotes, and the other by subtracting bond ask spreads from default swap bid quotes.
The table shows t-values of paired Z-tests for all combinations of pricing methods and proxies for the default-free curve. The paired Z-test determines whether the mean absolute pricing errors of two competing methods significantly differ from each other. The tests are conducted on the investment grade and speculative grade subsamples.

	AAA	**AA**	**A**	**BBB**	**BB**	**B**	**NR**	***All***
Obs.	933	955	166	182	146	255	3	*2639*
Swap								
1	0.17	0.22	0.39	0.74	2.15	1.95	0.53	*0.88*
2	0.15	0.17	0.26	0.54	1.26	1.21	0.30	*0.55*
3	0.15	0.15	0.23	0.46	1.13	0.97	0.15	*0.46*
Repo								
1	0.17	0.22	0.39	0.74	2.14	1.95	0.54	*0.88*
2	0.15	0.17	0.27	0.54	1.26	1.20	0.30	*0.55*
3	0.15	0.15	0.23	0.46	1.13	0.96	0.16	*0.46*
Government								
1	0.37	0.38	0.51	0.87	2.29	2.10	0.61	*1.02*
2	0.18	0.17	0.27	0.55	1.25	1.21	0.31	*0.56*
3	0.16	0.15	0.24	0.46	1.12	0.95	0.17	*0.46*

Table 4. Goodness of fit of the reduced form credit risk models.
For each issuer and for each day, for which we have at least five bond quotes and one default swap quote, we estimate a reduced form credit risk model with a constant recovery rate of 50%. The integrated hazard function is modelled as either a linear, quadratic or cubic polynomial of time to maturity. The default-free term structure of interest rates for each day is approximated by either zero-coupon swap, repo or government curves. The model parameters are estimated using non-linear squares estimation.
The first row of the table shows the number of issuer-days categorized by rating on which we have at least one default swap quote and at least five bond quotes. The remainder of the table shows, for each model, the average model fit per rating measured by the root mean squared error (RMSE) of the bond residuals. Each model is characterized by the proxy for the default-free curve and the degree of the integrated hazard function.

		AAA	AA	A	BBB	BB	B	NR
Swap								
1	λ_1	0.07	0.36	0.72	1.42	7.18	6.65	1.95
2	λ_1	0.10	0.27	0.46	1.05	4.83	4.52	0.43
	λ_2	-0.02	0.02	0.05	0.05	0.38	0.39	0.43
3	λ_1	0.10	0.27	0.42	1.16	5.46	2.98	-2.77
	λ_2	-0.02	0.02	0.07	0.00	0.18	1.02	2.47
	λ_3	0.00	0.02	-0.01	0.01	0.02	-0.06	-0.31
Repo								
1	λ_1	0.15	0.45	0.83	1.52	7.28	6.75	2.05
2	λ_1	0.18	0.36	0.55	1.14	4.93	4.61	0.53
	λ_2	-0.02	0.02	0.05	0.05	0.38	0.39	0.43
3	λ_1	0.18	0.36	0.52	1.23	5.55	3.08	-2.69
	λ_2	-0.02	0.01	0.07	0.01	0.18	1.02	2.48
	λ_3	0.00	0.02	-0.01	0.01	0.02	-0.06	-0.31
Government								
1	λ_1	0.75	1.04	1.36	1.98	7.66	7.23	2.59
2	λ_1	0.39	0.50	0.71	1.32	5.06	4.78	0.73
	λ_2	0.07	0.10	0.11	0.10	0.43	0.45	0.53
3	λ_1	0.26	0.45	0.61	1.29	5.61	3.15	-2.53
	λ_2	0.12	0.13	0.17	0.10	0.24	1.11	2.61
	λ_3	-0.01	0.02	-0.01	0.00	0.02	-0.06	-0.31

Table 5. Parameter estimates for the reduced form credit risk models.
For each issuer and for each day, for which we have at least five bond quotes and one default swap quote, we estimate a reduced form credit risk model with a constant recovery rate of 50%. The integrated hazard function is modelled as either a linear, quadratic or cubic polynomial of time to maturity with parameters $\lambda_i, i = 1, 2, 3$ (λ_0 is restricted to zero). The default-free term structure of interest rates for each day is approximated by either zero-coupon swap, repo or government curves. The model parameters are estimated using non-linear squares estimation.
The table shows average parameter estimates (times 100) per rating and model. Each model is characterized by the proxy for the default-free curve and the degree of the integrated hazard function.

	AAA	**AA**	**A**	**BBB**	***IG***	**BB**	**B**	***SG***	**NR**	***All***
Swap										
1	1.8	0.6$^{\sharp}$	-0.4*	-3.6$^{\sharp}$	*-0.4**	60.3	88.0	*76.3*	12.5	*14.3*
	(4.5)	(8.2)	(11.7)	(24.7)	*(12.0)*	(110.8)	(152.4)	*(134.8)*	(12.5)	*(35.6)*
2	3.0	2.5	2.1	5.1	*2.8*	123.4	145.9	*136.3*	74.4	*30.2*
	(4.1)	(6.9)	(10.0)	(19.3)	*(10.1)*	(130.5)	(159.5)	*(147.2)*	(74.4)	*(38.2)*
3	3.7	4.0	2.8	7.4	*4.3*	114.0	168.7	*145.3*		*36.6*
	(4.9)	(7.9)	(9.3)	(20.6)	*(10.8)*	(137.2)	(170.7)	*(156.3)*		*(44.2)*
Repo										
1	-2.0	-3.1	-4.7	-8.5	*-4.5*	55.6	83.7	*71.8*	7.4	*9.7*
	(4.7)	(9.6)	(11.8)	(24.8)	*(12.6)*	(109.7)	(151.1)	*(133.6)*	(8.2)	*(35.1)*
2	-0.3*	-1.4	-2.3	0.3*	*-1.4*	118.7	141.6	*131.9*	69.3	*24.9*
	(3.7)	(7.8)	(10.4)	(18.8)	*(10.4)*	(126.7)	(156.4)	*(143.8)*	(69.3)	*(36.7)*
3	0.4*	0.0*	-1.4	2.6$^{\sharp}$	*0.0**	109.4	164.5	*140.9*		*30.3*
	(3.9)	(8.1)	(9.4)	(19.6)	*(10.4)*	(134.0)	(166.8)	*(152.7)*		*(41.1)*
Government										
1	-33.4	-36.6	-35.7	-29.9	*-34.8*	34.6	56.8	*47.4*	-18.6	*-19.5*
	(33.5)	(36.7)	(36.3)	(36.0)	*(36.1)*	(103.6)	(149.6)	*(130.2)*	(18.6)	*(53.7)*
2	-25.3	-28.7	-25.3	-17.5	*-25.4*	104.8	123.8	*115.8*	55.1	*0.5**
	(25.3)	(28.8)	(27.0)	(24.7)	*(27.2)*	(114.8)	(144.5)	*(131.9)*	(55.1)	*(46.4)*
3	-25.9	-28.4	-22.9	-13.0	*-23.5*	96.6	148.1	*126.1*		*6.4*
	(25.9)	(28.5)	(25.0)	(22.3)	*(26.0)*	(123.0)	(151.0)	*(139.0)*		*(48.6)*

Table 6. Performance of the reduced form credit risk models.
For each issuer and for each day, for which we have at least five bond quotes and one default swap quote, we estimate a reduced form credit risk model with a constant recovery rate of 50%. The integrated hazard function is modelled as either a linear, quadratic or cubic polynomial of time to maturity. The default-free term structure of interest rates for each day is approximated by either zero-coupon swap, repo or government curves. The model parameters are estimated using non-linear squares estimation.
For each day a model can be estimated, we calculate two pricing errors: one as the default swap ask market quote minus the model-implied default swap premium using parameters estimated from bond bid quotes; the other as the market bid quote minus the model premium using parameters estimated from bond ask quotes.
The table shows mean pricing errors (MPE) and mean absolute pricing errors (MAPE, between parentheses) categorized by rating and model. Each model is characterized by the proxy for the default-free curve and the degree of the integrated hazard function. All MPE values are statistically significant at confidence levels above 99%, except for the ones marked with *, which are insignificant at confidence levels up to 95%, and the ones marked with $^{\sharp}$, which are insignificant at confidence levels up to 99%. The *italic* columns provide summaries for investment grade (IG) and speculative grade (SG) subsamples, and for the entire sample (All).

	Swap			Repo			Government		
	1	2	3	1	2	3	1	2	3
INVESTMENT GRADE									
Swap									
1		4.50	0.70	-7.87	1.85	0.19	-30.40	-22.44	-18.78
2			-2.79	-7.11	-4.71	-3.48	-27.47	-24.04	-21.57
3				-3.06	0.43	-1.30	-24.48	-18.50	-22.26
Repo									
1					5.95	2.80	-29.94	-20.57	-16.82
2						-1.28	-27.42	-25.22	-21.63
3							-24.27	-19.14	-23.73
Government									
1								16.39	14.06
2									1.07
3									
SPECULATIVE GRADE									
Swap									
1		-1.43	-4.02	1.06	-0.29	-3.03	-0.87	3.03	-0.18
2			-5.04	1.44	19.22	-3.28	0.71	15.54	1.51
3				3.97	6.61	23.75	2.96	10.23	22.14
Repo									
1					-0.37	-3.01	-1.27	2.71	-0.27
2						-4.88	-0.13	12.82	0.01
3							2.15	8.61	17.54
Government									
1								2.49	0.14
2									-3.87
3									

Table 7. Paired Z-tests of the reduced form credit risk models.
For each issuer and for each day, for which we have at least five bond quotes and one default swap quote, we estimate a reduced form credit risk model with a constant recovery rate of 50%. The integrated hazard function is modelled as either a linear, quadratic or cubic polynomial of time to maturity. The default-free term structure of interest rates for each day is approximated by either zero-coupon swap, repo or government curves. The model parameters are estimated using non-linear squares estimation.
For each day a model can be estimated, we calculate two pricing errors: one as the default swap ask market quote minus the model-implied default swap premium using parameters estimated from bond bid quotes; the other as the market bid quote minus the model premium using parameters estimated from bond ask quotes.
The table shows t-values of paired Z-tests for all combinations of models and proxies for the default-free curve. The paired Z-test determines whether the mean absolute pricing errors of two competing models significantly differ from each other. The tests are conducted on the investment grade and speculative grade subsamples.

		Swap	Repo	Government
	R^2	58%	57%	46%
	constant	38.2	36.7	46.4
		(52.6)	(52.9)	(72.3)
Deal type	bid	5.4	5.7	5.6
		(8.4)	(8.9)	(9.2)
	ask	-7.0	-6.9	-6.4
		(-8.4)	(-8.9)	(-9.2)
Maturity	(0,1]	-48.1	-45.2	-37.7
		(-26.6)	(-24.5)	(-18.2)
	(1,2]	-26.6	-24.5	-18.2
		(-8.3)	(-7.8)	(-6.1)
	(2,3]	-8.7	-7.9	-7.5
		(-4.1)	(-3.9)	(-4.0)
	(3,4]	-2.7	-2.1	-1.4
		(-0.8)	(-0.7)	(-0.5)
	(4,5]	7.7	7.0	5.8
		(9.6)	(9.2)	(8.7)
	(5,-)	19.7	16.9	7.8
		(6.7)	(6.1)	(3.3)
Rating	AAA	-36.8	-33.9	-25.7
		(-10.5)	(-11.0)	(-10.7)
	AA	-36.5	-33.0	-23.3
		(-26.5)	(-25.6)	(-20.4)
	A	-30.6	-28.5	-21.1
		(-27.5)	(-26.9)	(-21.0)
	BBB	-22.2	-21.4	-19.8
		(-11.0)	(-11.0)	(-10.7)
	BB	102.6	98.9	79.2
		(38.9)	(38.4)	(31.9)
	B	138.3	135.1	113.2
		(60.3)	(60.3)	(52.3)
Currency	dollar	-0.9	-0.6	-3.2
		(-1.3)	(-0.9)	(-4.7)
	euro	2.1	1.3	6.0
		(1.3)	(0.9)	(4.7)
Date	Q2 1999	6.0	6.1	-1.0
		(2.7)	(2.8)	(-0.5)
	Q3,Q4 1999	0.4	1.0	7.0
		(0.2)	(0.4)	(2.7)
	Q1,Q2 2000	-6.4	-6.5	-4.9
		(-2.4)	(-2.6)	(-2.0)
	Q3,Q4 2000	-4.3	-5.1	-3.3
		(-1.7)	(-2.1)	(-1.4)
	Q1,Q2 2001	11.1	8.2	-0.9
		(1.9)	(1.5)	(-0.2)

Table 8. Analysis of absolute pricing errors from reduced form credit risk models.
For each issuer and for each day, for which we have at least five bond quotes and one default swap quote, we estimate a reduced form credit risk model with a constant recovery rate of 50%. The integrated hazard function is modelled as quadratic polynomial of time to maturity. The default-free term structure of interest rates for each day is approximated by either zero-coupon swap, repo or government curves. The model parameters are estimated using non-linear squares estimation.
For each day a model can be estimated, we calculate two pricing errors: one as the default swap ask market quote minus the model-implied default swap premium using parameters estimated from bond bid quotes; the other as the market bid quote minus the model premium using parameters estimated from bond ask quotes.
The table shows estimated coefficients and t-values (between parentheses) of regressions of absolute pricing errors on dummy variables for deal type (bid or ask), currency (euro or dollar), rating (AAA, AA, A, BBB, BB), maturity (1-year segments up to 5 years, and a segment from 5 to 10 years) and quote date (6-month periods). For each set of dummies, we set a linear combination of the coefficients to 0, where the weight of a coefficient equals the sample mean of the corresponding dummy variable.