Collateralised Debt Obligations

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Abstract

This chapter explores the market of CDOs and synthetic CDOs and their use in bank balance sheet management. We first review different types of CDOs used in capital markets and their economic rationals and then discuss the growth in synthetic CDOs under structural and balance sheet management perspectives. Following this we analyse the CDO equity piece and how it can be used in portfolio management, and then, we offer a structuring example: using with the Moody's Binomial Expansion and Double Binomial Expansion Techniques we arrive at the *best* debt structure for a synthetic CDO. We conclude with a short introduction on the S&P CDO evaluator.

1 The CDO structure

A CDO is a special purpose company or vehicle (SPV), complete with assets, liabilities and a manager. Typically, the CDO's assets consist of a diversified portfolio of illiquid and credit-risky assets such as high yield bonds (CBO) or bank leverage loans (CLO)[2].

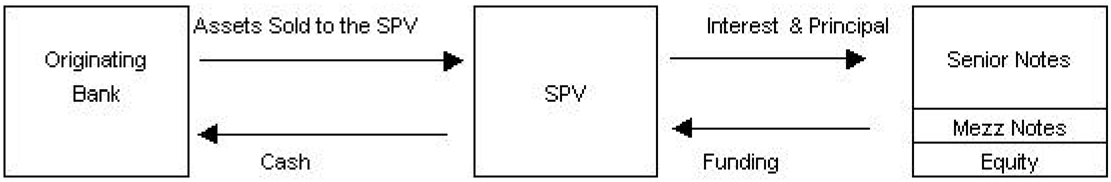


Figure 1: CDO Diagram

We have set up a typical CDO structure in Figure 1. The assets are transferred to the SPV that funds these assets, from cash proceeds of the notes it has issued.

The CDO structure allocates interest income and principal repayment from a pool of different debt instruments to a prioritised collection of securities notes called tranches.

Senior notes are paid before mezzanine and lower rated notes. Any residual cash flow is paid to the equity piece. This makes the senior CDO liabilities significantly less risky than the collateral.

[1] The content of this paper reflects personal view of the author and not the opinion of Royal Bank of Scotland.
[2] Most recently, CDO technology has been extended to emerging market debts, structured finance securities, commercial real estate-linked debt, distressed assets, and last to arrive, private equity funds.

On every payment date, equity receives cash distributions after the scheduled debt payments and other costs have been paid off. The equity is also called the “first-loss” position in the collateral portfolio. This is because it is exposed to the risk of the first dollar loss in the portfolio.

The CDO rating is based on its ability to service debt with the cash flows generated by the underlying assets. The debt service depends on the collateral diversification and quality guidelines, subordination and structural protection (credit enhancement and liquidity protection).
As we move down the CDO’s capital structure, the level of risk increases. The equity holders that bear the highest risk have the option to call the transaction after the end of the non-call period, which in most cases lasts three to five years.
The typical CDO consists of a ramp-up period, during which the collateral portfolio is formed, a reinvestment period, during which the collateral portfolio is actively managed, and an unwind period, during which the liabilities are repaid in order of seniority using collateral principal proceeds.

During the reinvestment phase, the equity class distributions consist of excess interest on the full portfolio, minus collateral interest income remaining after the payment of debt interest and other fees. The manager would reinvest collateral principal proceeds.
In the repayment period, excess interest payments gradually decrease as the collateral portfolio principal proceeds are used to repay the debt in order of seniority. After all the debt classes have been redeemed, and if the equity class has not elected to call the transaction, the remaining principal payments pass to the equity.

Figure 2 displays an example capital structure, where the high yield bonds collateralise CDO liabilities.

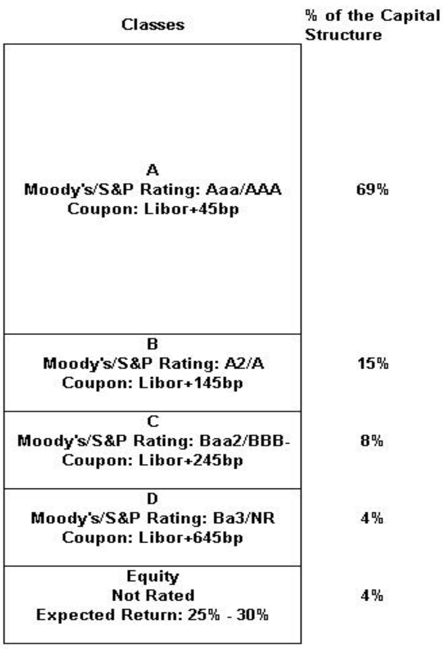


Figure 2: CDO Capital structure

1.2 Arbitrage and Balance Sheet CDOs

Most CDOs can be placed into either of two main groups: arbitrage and balance sheet transactions.
Figure 3 shows the conceptual breakdown between the two structures.

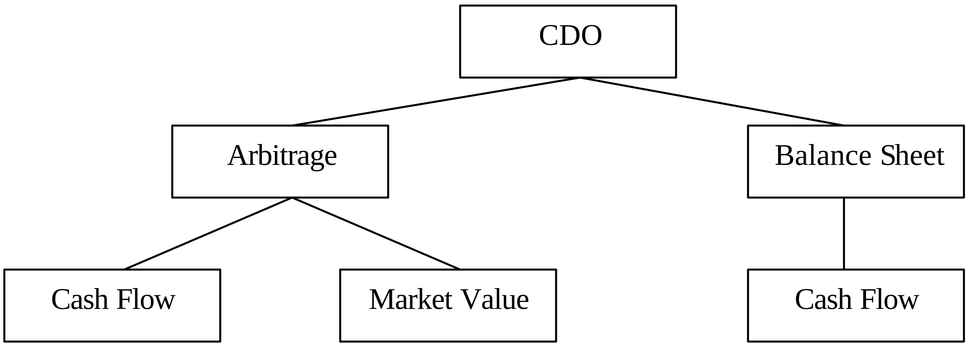


Figure 3: CDO structure

- **Cash flow CDOs**

A cash flow CDO is one where the collateral portfolio is not subjected to active trading by the CDO manager. The uncertainty concerning the interest and principal repayments is determined by the number and timing of the collateral assets that default. Losses due to defaults are the main source of risk.

- **Market value CDOs**

A market value CDO is one where the performance of the CDO tranches is primarily a mark-to-market performance, i.e. all securities in the collateral are marked to market with high frequency.
Market value CDOs leverage the performance of the asset manager in the underlying collateral asset class. As part of normal due diligence, a potential CDO investor needs to evaluate the ability of the manager, the institutional structure around him, and the suitability of the management style to a leveraged investment vehicle.

- **Balance sheet cash flows CDOs**

Balance sheet deals are structures for the purpose of capital relief, where the asset securitised is a lower yielding debt instrument. The capital relief reduces funding costs or increases return on equity, by removing, the assets that take too much regulatory capital, from the balance sheet.
These transactions rely on the quality of the collateral that is represented by guaranteed bank loans with a very high recovery rate.
The relative low coupon attached to these assets, results in a smaller spread cushion than the corresponding arbitrage structure. However, given their relative superior quality, they require less subordination when used in a CDO deal.
In the majority of the cases, the sold assets are loan-secured portfolios.
The size of a typical balance sheet CDO is in general very large, as the transaction must have an impact on the ROE of the institution looking for capital relief.

- **Arbitrage CDOs**

The aim of Arbitrage CDOs is to capture the arbitrage opportunity that exists in the credit-spread differential, between the high yield collateral and the highly rated notes.
The idea is to create a collateral with a funding cost lower than the returns expected from the notes issued.
Most arbitrage deals are private ones, where size is not large and the number of assets included in the deal are very limited compared to the cash flow type.

- **Arbitrage market value CDOs**

Arbitrage market value CDOs, unlike balance sheet CDOs where there is no active trading of loans in the portfolio, go through a very extensive trading by the collateral manager, necessary to exploit perceived price appreciations.
This type of CDO relies on the market value of the pool securitised, which is monitored on a daily basis.
Every security traded in capital markets, with an estimated price volatility, can be included in this type of CDO.
In fact, the primary consideration is the price volatility of the underlying collateral.
The important aspect is the collateral manager's capacity to generate a high total rate of return. The CDO manager has a great deal of flexibility in terms of the asset included in the deal. During the revolver period, the collateral manager can increase or decrease the funding amount that changes the leverage of the structure.

- **Arbitrage cash flow CDOs**

By their very nature, collateral assets have been purchased at market price and are negotiable instruments, therefore most assets are bonds. However syndicated loans, usually tradable, have been included in past transactions. As arbitrage deals, the collateral assets can be refinanced more economically by re-tranching the credit risk and funding cost in a more diversified portfolio. Unlike arbitrage market value CDOs, the collateral assets are not traded very frequently.

1.3 Credit enhancement in cash flow transactions

Senior notes in cash flow transactions are protected by subordination, over-collateralisation and excess spread.

The senior notes have a priority claim on all cash flows generated by the collateral, therefore, non-senior notes' performance is subordinated to the good performance of senior notes.

Over-collateralisation provides a further protection to senior notes by imposing a minimum collateral value with two coverage tests: par value and interest coverage tests.
Par value test requires that the senior notes (and subsequently the other notes) are at least a certain percentage of the underlying collateral (for example 115%).
The par value test is applicable to lower rated notes (mezzanines). In this case, the trigger percentage below that fails the test is selected at a lower rate (for example 105%).
An interest coverage test is applied to ensure that collateral interest income is sufficient to cover losses and still make interest payment to the senior notes. This credit support is also known as excess spread.

Credit enhancement may also be in the form of a letter of credit from a highly rated institution, a cash collateral account, or a guarantee.

1.4 Credit enhancement in market value transactions – Advance rates and the OC test

Advance rates are the primary form of credit enhancement in market value transactions.

The advance rate is the maximum percentage of the market rate that can be used to issue debt. Rating agencies assign different advance rates to different types of collateral. They depend on the volatility of the asset return, and on the liquidity of the asset in the market. Assets with a higher return volatility and lower liquidity are given lower advance rates.

Table 1 shows a sample table that Fitch would apply to different asset classes.

For example, it is possible to issue *AA* debt with 95% of the market value of CD or CP as collateral asset. To issue *BB* debt, we could use 100% of the market value of the same instrument.

Asset Category	AA	A	BBB	BB	B
Cash and Equivalents	100%	100%	100%	100%	100%
CD and CP	95%	95%	95%	100%	100%
Senior Secured Bank Loans	85%	90%	91%	93%	96%
BB-High Yield Debt	71%	80%	87%	90%	92%
<BB-High Yield Debt	69%	75%	85%	87%	89%
Convertible Bonds	64%	70%	81%	85%	87%
Convertible Preferred Stock	59%	65%	77%	83%	86%
Mezzanine Debt, Distressed, Emerging Market	55%	60%	73%	80%	85%
Equity, Illiquid Debt	40%	50%	73%	80%	85%

Source: Fitch

Table 1: Fitch's Advance Rates

For market value transactions, there are usually multiple Over-collateralisation tests.

To illustrate how the test works, we introduce a simple example with the collateral and liability structure of Table 2

Collateral			Notes			
Assets	*Market Value (euro)*	%	*Tranche*	*Rating*	*Face Value (euro)*	%
CD and CP	£230	46%	Senior Facility	AA	£175	35%
BB-High Yield Debt	£225	45%	Senior Notes	AA	£200	40%
Convertible Bonds	£10	2%	Mezz Notes	BBB	£50	10%
Mezz. Debt	£25	5%	Sub. Notes	B	£25	5%
Equity	£10	2%	Equity	NR	£50	10%
Total	£500	100%	**Total**		£500	100%

Table 2: CDO market value transaction

After applying the *AA* advance rates, we can see from Table 3 that the senior advance amount exceeds the total *AA* debt, defined as Borrowing Amount Surplus, by £27 m. This is also the market value loss that the *AA* structure can sustain before breaching the OC test.

Collateral				
Assets	**Market Value (euro)**	**%**	**AA Advance Rates**	**Sr. Advance Amounts**
CD and CP	£230	46%	95%	£219
BB-High Yield Debt	£225	45%	71%	£160
Convertible Bonds	£10	2%	64%	£6
Mezz. Debt	£25	5%	55%	£14
Equity	£10	2%	40%	£4
Total	£500	100%		£402
Total AA Senior Debt Face Value (Senior Facility + Senior Notes)				£375
Borrowing Amount Surplus				£27

Table 3: *AA* Debt OC test

Tables 4 and 5 show respectively, the borrowing amount surpluses of the *AA+BBB* debt and the *AA+BBB+B* debt. As for Table 3, the borrowing amount surpluses of 23 and 25 are the market value losses that the *AA+BBB* and *AA+BBB+B* structures can respectively sustain before breaching their OC tests.

Collateral				
Assets	**Market Value (euro)**	**%**	**BBB Advance Rates**	**Sr. Advance Amounts**
CD and CP	£230	46%	95%	£219
BB-High Yield Debt	£225	45%	87%	£196
Convertible Bonds	£10	2%	81%	£8
Mezz. Debt	£25	5%	73%	£18
Equity	£10	2%	73%	£7
Total	£500	100%		£448
Total AA-BBB Debt Face Value				£425
Borrowing Amount Surplus				£23

Table 4: *AA+BBB* Debt OC test

Collateral				
Assets	**Market Value (euro)**	**%**	**B Advance Rates**	**Sr. Advance Amounts**
CD and CP	£230	46%	100%	£230
BB-High Yield Debt	£225	45%	92%	£207
Convertible Bonds	£10	2%	87%	£9
Mezz. Debt	£25	5%	85%	£21
Equity	£10	2%	85%	£9
Total	£500	100%		£475
Total AA-BBB-B Debt Face Value				£450
Borrowing Amount Surplus				£25

Table 5: *AA+BBB+B* Debt OC test

A collateral manager must ensure that the market value tests are not violated due to fluctuations in the underlying prices. A breach of the OC test is quite serious, and when it happens, the collateral manager must remedy it within a cure period that is usually between two to ten business days.

There are usually two options:

- to sell security/ies with a lower advance rate and buy one/more with a higher advance rate,

- or to sell security/ies with a lower advance rate and repay the debt starting with the more senior notes.

The first action is preferred when the OC test is slightly out of compliance. The second is a drastic cure. If the collateral manager cannot comply with the OC test, the debt holders have the power to take control of the fund and liquidate the portfolio in an event of default.

1.5 Credit enhancement in market value transactions – Minimum Net Worth test

The Minimum net worth test is also designed to offer credit protection to the senior notes holders, by creating an equity cushion. This is achieved by imposing that the excess market asset value, minus the debt notes is equal or greater than the equity face value, times a percentage

MAV - Debt >= % * Equity.

In cases where the test is breached, the manager has a cure period to bring the CDO into compliance, by either

- redeeming part or all of the senior notes,
- by generating enough capital gains by selling some assets.

The latter is preferable since the manager would not de-leverage the deal.

If the collateral manager cannot comply with the minimum net worth test, and an event of default occurs, the debt holders have the power to call the deal.

1.6 The Manager

The manager of the CDO is responsible for the credit performance of the collateral portfolio and for ensuring that the transaction meets the diversification, quality and structural guidelines specified by the rating agencies. In return for managing the collateral portfolio, the manager receives a fee, typically divided into base and incentive components. During the reinvestment period, the CDO manager continuously evaluates the state of the collateral portfolio and of the overall market. He trades out positions at risk for credit deterioration, and takes advantage of appreciation opportunities.

The key to a successful market value CDO is the manager's ability to generate high risk-adjusted returns through research, market knowledge and trading ability. The return performance of CDO equity hugely depends on the long-horizon returns of the underlying portfolio realised by the manager.

Today, successful CDO management franchises are found in a variety of asset management organisations, including mutual fund groups, insurance companies, banks, private equity firms and hedge funds.

Different managers stress different strategies to generate high risk-adjusted returns. For example, an insurance company may depend on its portfolio risk management system, a mutual fund group may use its size and market knowledge, and a private equity sponsor may rely on its knowledge of leveraged companies. The market value CDO is typically only open to managers who have established track records and who have demonstrated a high level of organisational commitment to the CDO business. Most successful CDO managers consider CDO issuance to be an integral component of their overall business development strategy.

CDOs are also a powerful asset-gathering tool that locks in management mandates for a fixed term, providing managers with exposure to a larger and more diverse pool of investors. For example, a traditional high-yield manager will form relationships with asset-backed AAA investors such as banks or structured investment vehicles, BBB buyers such as insurance companies, and alternative investment buyers who purchase the equity. In this way, broader client exposure helps the growth of the overall management franchise, without straying from core competency.

Additionally, by locking the structure to a financing rate and fixed term, the manager is free to focus exclusively on long-term horizon management rather than worry about short-term liquidity issues.

Although during the pay down phase of the CDO, the manager's ability to reinvest principal proceeds is limited, the manager is still responsible for avoiding problem credits in the portfolio.

2 The Economic rational for CDOs

CDOs make most economic sense for collateral securities in markets where there is limited information (inefficient) with the possibility of high risk-adjusted returns through active management.

Risky assets, such as the debt of leveraged corporations, are often difficult to analyse and value, thus limiting their potential investor base and creating a gap in the economy between the demand and supply of risky finance. As result, corporate debts are relatively illiquid in the secondary market.
The CDO structure addresses this market inefficiency by bringing a specialised manager to the transaction and allocating much of the risk, in the form of a liquidity premium in the equity class.

The CDO cash flow structure acts as a cushion and hedges the debt from defaults and the direct impact of mark-to-market changes in the value of the collateral.

In trying to reach its economic target, an issuer would have two main constraints: to minimize the total cost of notes (i.e. the floating or fixed rate attached to each note) and to minimize the size of the subordinated notes (among them the equity piece).
Normally, the seller would retain 2% of the structure, the *first loss*, by keeping the equity piece. He would also fund the Cash Collateral Account (CCA), a cash deposit that re-enforces the credit protection, usually in the range of 1% of the structure.
The originator's return is given by the excess spread of the notes (average rate of the loan portfolio minus the average rate of the notes) over the funding cost of his collateral. His maximum loss is also known and given by the first loss.
Figure 4 shows an example of ROE before and after a CDO.

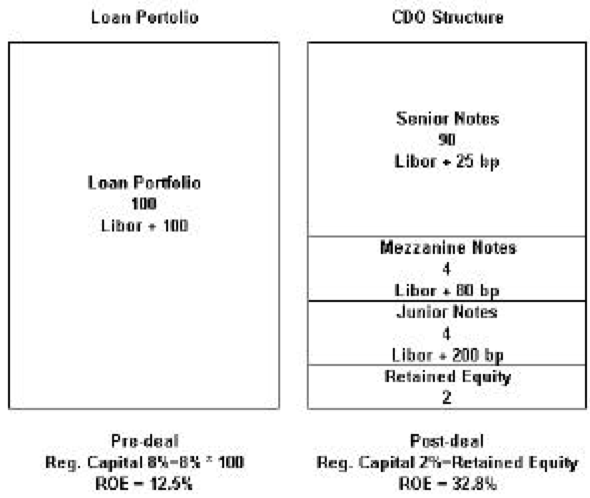


Figure 4: Regulatory Capital relief and ROE

In the example of Figure 4 the bank has a portfolio of loans of 100 million Euros on its balance sheet (left box), for which the average spread over the Libor is 100 bps. The loans receive a risk weight of 100% where the Regulatory Capital is 8%, and the loan portfolio ROE is of 12.5%.

With the CDO structure in Figure 4 (right box) the bank retains only 2% of the original loan portfolio (the loss piece) and securitises the remaining 98%. In this example, the bank would receive a huge relief of regulatory capital. This would drop from 8 million euros to 2 million euros. The new ROE is 32.8%[3]. If this transaction had a bigger volume it would hugely affect the ROE of the overall bank.

Alongside this, the bank would continue its commercial activity on its lending portfolio, and would carry out a review of its IT and scoring systems.

[3] This is calculated by dividing the gross return from the loan portfolio of 100 bps minus the average cost of the three notes (100 bps – 34.4 bps) by 2, which is the amount of regulatory capital.

3 Synthetic Collateralised Synthetic Obligations

In most conventional *cash flow* CDOs, assets are actually transferred into the SPV. However, the process of transferring loans to the SPV requires significant up front work. A loan-by-loan analysis is necessary to check it complies with the securitisation programme and to verify that there are no special clauses attached to any loan limiting its transfer.

The first stage of evolution of the conventional CDO, arrived when the credit risk was transferred into the SPV through a credit default swap[4], and when the underlying credit ownership of the underlying pool remained in the originator's book. For this the term *synthetic* is used, since the risk was synthetically transferred out of the originator' balance sheet.

With synthetic CDO's, the big advantage is that sensitive client relationship issues arising from loan transfer notification, assignment provisions and other restrictions can be avoided. Also, client confidentiality can be maintained. Not to mention that it takes less time to complete the transaction.

3.1 Fully funded synthetic structures

Historically, the fully funded CDO was the first to be used as an alternative to the more traditional structure.

In a fully funded synthetic CDO, the SPV issue notes for approximately 100% of the reference portfolio. The proceeds of these notes are generally invested in high quality securities used as collateral that have a 0% risk weight.

In order to hedge its credit risk exposure in its loan portfolio, the originating bank enters into a Credit Default Swap (CDS) with either the same SPV or with an OECD bank. With the CDS the originator buys credit protection in return for a premium.

The premium received is then added to the interest notes received by the note investors.

The mechanics are described in the Figures 5 and 6.

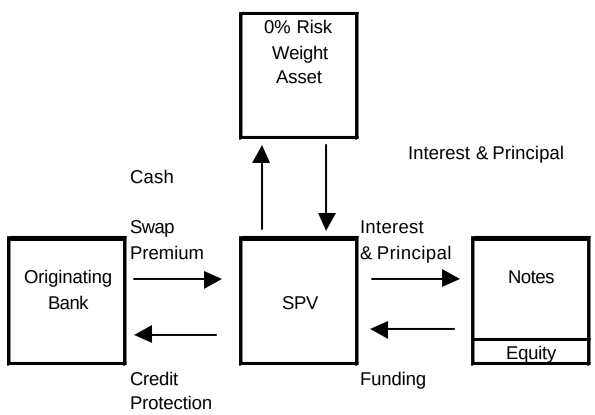


Figure 5: Fully funded synthetic CDO with CDS with an SPV

[4] Swiss Bank brought Glacier Finance 1997-1 and 1997-2 to market in late 1997. This was a master trust structure where Swiss Bank transferred the credit risk via a portfolio of credit linked notes.

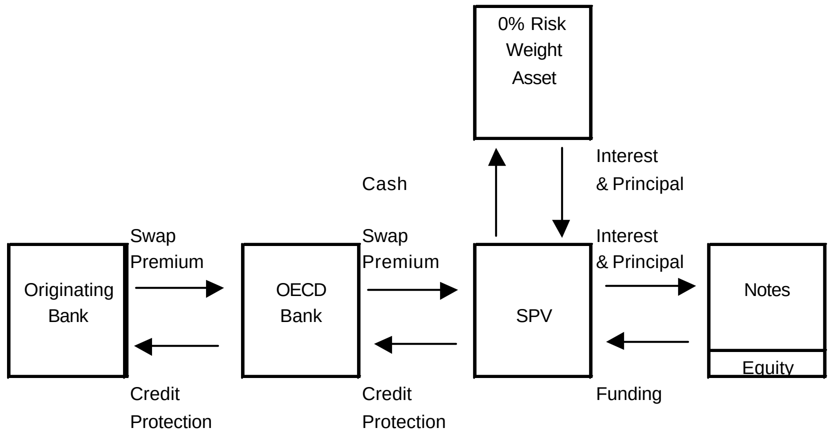


Figure 6: Fully funded synthetic CDO with CDS with an OECD bank

The equity retained by the originator brings a 100% risk weight. Therefore, as with the example in Figure 5, the bank would achieve a first capital release of 6%. An additional regulatory capital would depend on the presence of an OECD bank in the structure.

If the CDS is directly with the SPV (Figure 5), and if the note proceeds are invested in 0% risk weighted assets, no more regulatory capital is added to the transaction.

If the CDS is directly with an OECD bank (Figure 6), the regulatory capital on the CDS is 1.6% (i.e. 20% * 8%) of the notional amount of the same swap.

If the CDS has a notional equal to reference portfolio the total regulatory capital charge of this transaction would be 3.6%.

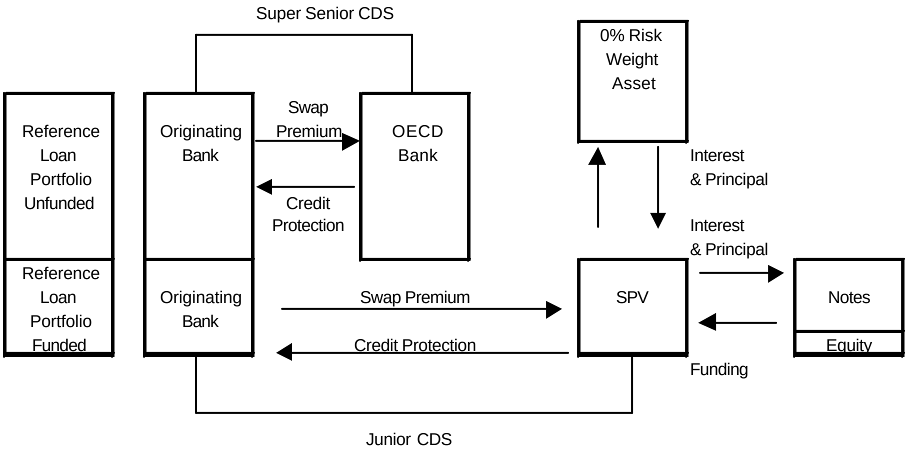
3.2 Partially funded structures

In fully funded CDOs, the bank originator is far from achieving an efficient capital use. Fully funded CDO-CLO may sometimes be a relatively expensive programme. However, it is also true that as *term funding* debt, a CDO-CLO programme remains less exposed to the risk that credit spreads may widen.

The structure behind a partially funded CDO transaction is very similar to that of a fully funded one. The originator bank buys credit protection directly from an SPV (Figure 7) or from an OECD bank (Figure 8). The difference is that the SPV issues a lower amount of notes because it guarantees a lower amount of collateral. What really characterises this structure is the un-funded piece called the *Super Senior.* This is a very high quality financial paper, virtually with a zero probability of being exposed to a credit loss.

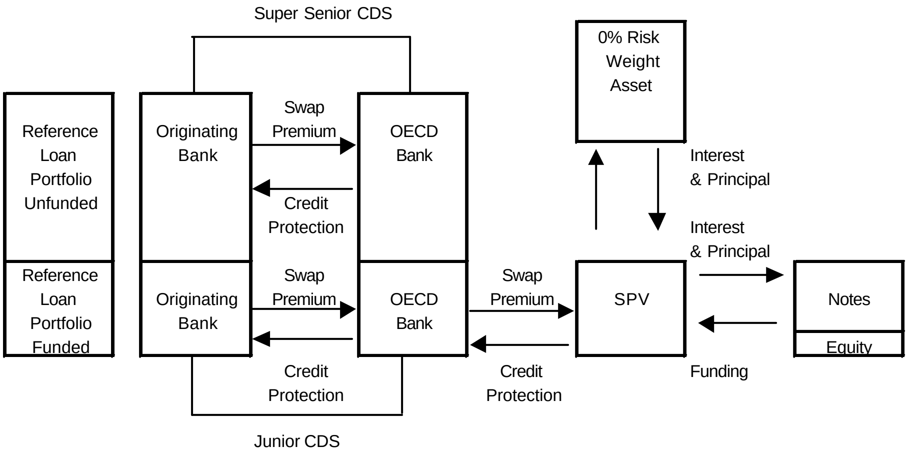
The originating bank enters in a CDS (super senior CDS) with an OECD bank for the amount of the super senior tranche.

Figures 7 and 8 illustrate the mechanics of this type of CDOs



Source: Investing in Collateralized Debt Obligations, Frank J. Fabozzi, Laurie S. Goodman

Figure 7: Partially funded synthetic CDO with CDS with an SPV



Source: Investing in Collateralized Debt Obligations, Frank J. Fabozzi, Laurie S. Goodman

Figure 8: Partially funded synthetic CDO with CDS with an OECD bank

The treatment of regulatory capital for European banks is currently still different from jurisdiction to jurisdiction.

The Federal Reserve Bank has issued several interpretations that apply only to US banks. Some conditions are necessary to receive a better treatment of regulatory capital. If these conditions are met, and if the credit risk is transferred to another OECD bank with a CDS, the super senior piece receives a risk weight of 20% with the capital charge of 8%.

The regulatory capital rules on the equity piece and on the junior CDS, are the same as those applied on the fully funded CDO's. Therefore, the last step is to add the regulatory capitals required on the funded and un-funded part of the structure.

If we apply those percentages to the portion of the super senior piece (87%), the total regulatory capital is 3.4% (4% for the Super Senior CDS and 2% for the Equity piece).

3.3 Balance Sheet Management with CDS

All banks constantly seek least expensive funding cost. Therefore, it does not come as a surprise that banks have a preference towards partially funded programmes.

If we compare Figure 8 with 1, the difference in funding between the two structures becomes clear.

An example makes this even clearer.

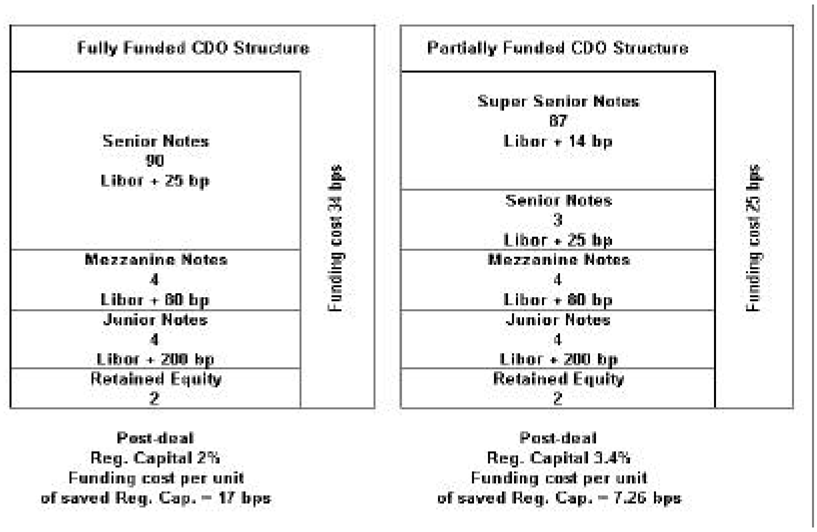


Figure 10: Funding costs with a fully and partially funded synthetic CDO

Figure 10 contains the CDO structure of Figure 5, plus a new and more convenient structure for the originator: partially funded CDO.

With the partially funded structure, we have achieved a reduction of the funding cost: the overall transaction cost has dropped by 9 bps[5]. Furthermore, for one unit of equity used in the partially funded structure, the originator would pay 7.26 bps Vs 17 bps for the fully funded.

Assuming now that all the loans in Figure 5 are *Baa1* loans, with a maturity of 6 years, a cumulative default probability of 37% [6] and a recovery rate of 65%, the expected loss on this portfolio is 0.48% = (1-65%)*37%. Consequently, the expected losses have to rise by a factor of four before hitting the junior notes.

The Figure 11 shows the statistical distribution of losses that might occur on this transaction.

[5] (34 bps – 25 bps) = 9 bps

[6] 1.37% corresponds to the 6 year cumulative probability of default as calculated by Moody's.

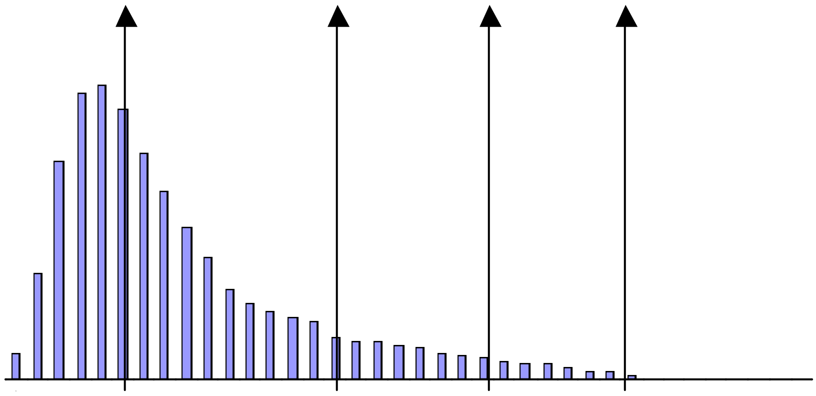


Figure 11: Distribution of Expected Losses

The structure is also free of any interest rate mismatch, i.e. it pays Libor and receives Libor.
The average spread of 100 bps compensates the bank for taking the credit risk of expected losses. The credit spreads, ranging from 25 to 200 bps, compensate the notes investors for taking different risks.
Therefore, we can remove the Libor and leave the spreads only as in Figure 12.

CDO Structure

Assets	%	Spreads	Liabilities	%	Spreads
Loan 1	2%	100 bps	Super Senior Notes	87%	15 bps
Loan 2	2%	100 bps	Senior Notes	3%	25 bps
Loan 3	2%	100 bps	Mezzanine Notes	4%	80 bps
……….	…	100 bps	Junior Notes	4%	200 bps
Loan 50	2%	100 bps	Retained Equity	2%	Dividend

Figure 12: CDO structure with hedged interest rate risk

In general terms, a CDS is designed to mimic the credit behaviour of a floating rate note, such as the loans in Figure 12. The loan spread, that is constant until the loan matures, is equivalent to the fixed leg of a CDS. In fact, the CDS seller, who seeks credit exposure, receives X basis points, i.e. a spread, per year until the credit reference matures or defaults. The constant spread is the fixed leg of the CDS.
As a consequence, we can remove the loans and add the CDS on the asset side.

CDO Structure

Assets	%	Spreads	Liabilities	%	Spreads
CDS 1	2%	100 bps	Super Senior Notes	87%	15 bps
CDS 2	2%	100 bps	Senior Notes	3%	25 bps
CDS 3	2%	100 bps	Mezzanine Notes	4%	80 bps
……….	…	100 bps	Junior Notes	4%	200 bps
CDS 50	2%	100 bps	Retained Equity	2%	Dividend

Figure 13: CDO structure with hedged interest rate risk and with CDS in place of loans

With the CDO in Figure 13 the bank is now exposed to the credit risk of 50 synthetic assets. To hedge its position, the bank borrows via four different credit risk notes. Retaining the equity, gives it the right to a possible dividend.

Some of the loans in Figure 13 may be in the same industry and same country. Thus, it is safe to assume that they may be affected by the same risk factors. As a consequence, we may treat them as one loan with a notional equal to the sum of their notionals.

CDO Structure					
Assets	**%**	**Spreads**	**Liabilities**	**%**	**Spreads**
pool of N		100 bps	Super Senior Notes	87%	15 bps
equal notional CDS		100 bps	Senior Notes	3%	25 bps
diversity score: 30		100 bps	Mezzanine Notes	4%	80 bps
average rating: Baa1		100 bps	Junior Notes	4%	200 bps
		100 bps	Retained Equity	2%	Dividend

Figure 14: CDO structure with a basket CDS

Figure 14 shows, on the asset side, a basket CDS with equal notional and a diversity score of 30, on a reference pool with average rating of *Baa1*.

The Diversity Score in Figure 14 indicates that the 50 loans almost behave as 30 uncorrelated loans.

Viewed from this angle, a CDO is a hedged portfolio. The assets are a portfolio of synthetics, the liabilities are the tranches with different ratings. By hedging its balance sheet from credit risk (and from interest rate risk), the bank is trying to achieve a higher return than investing in risk-less treasury bonds. By partially funding the CDO structure, the bank has also achieved a leverage position, with potentially huge returns.

However, the hedge is not perfect: the risk of expected losses may corrode equities up until the senior notes.

5 Valuing a CDO

There are several factors that affect the value of all CDO tranches (debt and equity pieces).
But the most important is the credit quality of the underlying portfolio, which depends on the individual securities' default probabilities and diversification.
Improvement or deterioration in the credit quality of collateral securities has a pronounced impact on the equity, since it represents a leveraged position.

Changes in the level of portfolio diversification bring the appearance of large, individual positions, and expose the CDO to the concentration risk.
When diversification and collateral quality guidelines specified by the rating agencies are violated, market participants react by lowering the CDO price.
The collateral assets underlying the CDO are perceived as riskier and influence the attractiveness of a newly issued CDO. For example, in a competitive return-risk environment the expected returns are no longer achievable with the old quantity of risk, and a new CDO formed with a less risky collateral becomes a better choice.

Similarly, changes in the cost of funding in the CDO liabilities market, affects the value of a CDO whose collateral return is fixed.

Finally, changes in perception of the CDO manager's skills are reflected in the valuation of the CDO itself.

5.1 CDO as an option

The latest approach is to value a market value CDO as a derivative instrument where the collateral portfolio is the underlying.
We can start with a very simple capital structure similar to the one reported in Figure 15, where there is only one asset as collateral, such as a corporate bond, and where the liability is given by one zero-coupon tranche, plus the equity piece.

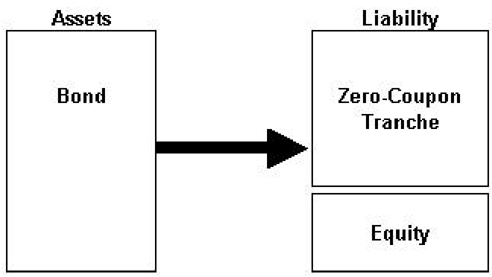


Figure 15: Simple CDO balance sheet

We can write the asset value as:

Assets = Zero-coupon Tranche + Equity (1)

At maturity, the collateral manager liquidates the asset. With the proceeds, he first pays off the zero-coupon tranche holders and then he pays the remaining to the equity holders.
The limited liability feature of the equity means that the equity holders have the right, but not the obligation, to pay off the debt holders and take over the remaining value of the asset. If at maturity the asset value exceeds the debt amount, the equity holders will exercise their options by paying off the debt. However, if the asset value is less than the debt amount, the equity holders will prefer to default and hand over the remaining asset value to the debt holders.
Thus, the equity holder is in the same position as the holder of a call option on the same asset, with a strike value equal to face value (book value) of the zero-coupon tranche.

The zero-coupon tranche holders can be thought of as having purchased a debt obligation that cannot default and that returns the risk free rate, and as having sold a put option to the equity holders. Clearly, if the equity holders decide not to pay off their debt, they will deliver the asset to the debt holders at a strike price equal to the debt amount.

Therefore, we can write the asset value as:

$$A = PV(Z) - \max(Z - A, 0) + \max(A - Z, 0) \qquad (1)$$

where

A = value of the asset at maturity,

Z = the face value of the zero-coupon tranche,

$PV(Z)$ = the present value of the zero-coupon tranche.

The equation in (1) is nothing more than the call-put parity of Black-Scholes.
Therefore, we can treat the zero-coupon tranche as a zero-coupon bond plus a short position on a put option on the underlying asset with strike price equal to the face value of the zero-coupon tranche itself.

The probability of default is the same as the probability of exercising the put option. If the probability of default goes up, the value of the put option goes up too and brings down the investment value of the debt holders.
The probability of exercising the option can be determined using option pricing techniques.
As in the case of equity options, the volatility of the asset price is the key variable to pricing the option. We can use the same volatility to have some information on the probability of default. The volatility is indeed the propensity of the asset value to change during a certain period of time.
For example, if the asset value is $100, the debt amount to pay in one year is $50 and the volatility of the asset price is 15%, then a fall in value from $100 to $60 will trigger the default[7]. This is a 3.2 standard deviation event with a probability of default of 0.38%.

The equation (1) can be further modified. This will help to analyse the other CDO tranches.

[7] We have assumed that the asset value is *log-normal* distributed.

In fact, we have seen that the CDO is characterised by the issuance of several tranches. They have different ratings according to the type of protection offered to investors. They can still be seen as zero-coupon bonds with embedded put options. However, since they offer a different level of credit risk (according to their ratings), the strike prices and the probability of default are different.

For simplicity, we assume that the liability is made of a Senior note, a Mezzanine note and a Junior piece.

The payoff for the Junior piece investor can also be seen as:

$$\begin{aligned} P_E &= E - \min(E, L) \\ &= \max(E - L, 0) \\ &= Put(E, L) \end{aligned} \qquad (2)$$

where

L is the realised loss in the collateral,

E is the size of the equity piece, expressed as percentage of the liability, and

Put(E, L) is written on the equity piece *E*.

Therefore, if losses are greater then *E*, the Junior piece is exhausted and the difference is paid by the Mezzanine investor.

It is always possible, starting from the equation in (2) to recover the equity piece as the call option in equation (1) in the following fashion:

$$\begin{aligned} P_E &= A - Z - \min(A - Z, A - L) \\ &= A - Z + \max[A - Z, L - A] \\ &= A - Z + \max[Z, L] - A \\ &= A - Z + \max[A - Z, L - A] \\ &= \max[A - Z, 0] \end{aligned} \qquad (3)$$

with the same variables used in (1) and (2).

The payoff for the Senior piece investor is:

$$\begin{aligned} P_S &= A - E - \max(L - E, 0) \\ &= A - E - Call(E, L) \end{aligned} \qquad (4)$$

where

Call(E,L) is written on the equity piece *E*.

Let's look at the payoff of the Mezzanine investor[8]:

$$\begin{aligned} P_M &= \max[\min(M - E, M - L), 0] \\ &= \max[(M - E) + \min(E - L, 0), 0] \\ &= (M - E) + \max[\min(E - L, 0), -(M - E)] \\ &= (M - E) + \min(E - L, 0) + \max[0, -(L - E) - \min(E - L, 0)] \end{aligned} \qquad (5)$$

[8] We proceed as suggested in M. Esposito (2002).

$$= (M - E) - \max(L - E, 0) + \max[0, E - L - \min(E - L, 0)]$$

We can note that

$$(E - L) - \min(E - L, 0) = a > 0 \qquad (6)$$

if $L = M + a$.

With equation (6), we can write

$$\max[0, E - L - \min(E - L, 0)] = \max(L - M, 0) \qquad (7)$$

Going back to equation (5), we have

$$P_M = (M - E) - \max(L - E, 0) + \max(L - M, 0) \qquad (8)$$

$$= (M - E) + \Pi(M, E)$$

where

$$\Pi(M - E) = \max(L - M, 0) - \max(L - E, 0) \qquad (9)$$

and

M is the size of the mezzanine piece, expressed as percentage of the liability.

Thus, with (8), the Mezzanine payoff is the same as a portfolio of a zero-coupon bond, plus a portfolio of two call options. One long call option on the exercise price *M* and one short call option on the exercise price *E*. Since the short call is more in the money than the long call, in relative terms, its price dominates in (8).

In more complex situations, the call and put options are written on a basket of different types of assets: corporate bonds, equities, etc. As we saw in Figure 14, the asset size can be seen as a basket CDS, with equal or different notional, a diversity score and an average rating. The importance has become how to measure the level of diversification in the underlying portfolio, i.e. default correlations. The study of risk profile will be covered in the next article.

6 CDO equity piece

The CDO equity piece is a truly hybrid security. It exhibits the features of a coupon bond, a corporate equity, a call option on the collateral and a managed fund.
As a coupon bond, CDO equity is issued at or near par and has a final maturity date.
Like with convertible bonds, payments are not contractually specified, although the range of expected distributions is established at the time of issuance.
In a similar way to a call option, the value of CDO equity increases with the price and volatility of the underlying assets.
As with any actively managed investment, the contribution of the manager is a crucial determinant of CDO equity performance.

6.1 The CDO Equity piece performance

The equity of a CDO represents a leveraged investment in the underlying asset class and in the asset management skills of the CDO manager. The leverage is achieved by issuing investment and sub-investment-grade debt as term[9] asset-backed securities.
Credit losses are the obvious drivers of the CDO equity piece performance and can affect investors in two ways. First, as collateral shrinks because of defaults (in cash flow CDO's) or realised price deterioration (in market value CDO's), the amount of underlying assets reduces and with it the size of received interest payments. Second, if the par size of the collateral falls below a trigger point (OC and Interest Cover tests) specified by the rating agencies, the excess interest that normally passes to the equity holders is redirected to pay down the senior liabilities, thereby de-leveraging the CDO.
Equity payments resume only after the ratio of collateral par to liabilities is restored above the trigger level. Redirection of equity distributions can also be triggered by a drop in the interest income relative to the interest cost of the transaction.

In performing CDO transactions, the remaining interest that needs allocating to equities, is equivalent to excess spread often in the range of 2.5% to 3%, implying a 25% to 30% running return on the equity.

6.2 The CDO embedded option

Depending on the collateral asset type and the timing of the transaction, the call option embedded in CDO equity may be quite valuable. Figure 16 shows the historical spread over Libor on the Goldman Sachs Single B Bond Index and the estimated cost of funding of CDO liabilities[10]. The wider the gap between the income from the assets and the cost of the liabilities, the greater the investment incentive for CDO equity.

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[9] Term is used to differentiate the term *securitisation* where the assets are bonds, from *conduit* where the assets are commercial papers.
[10] Olberg E., Nartey M., Takata H. and S. Shah (2001).

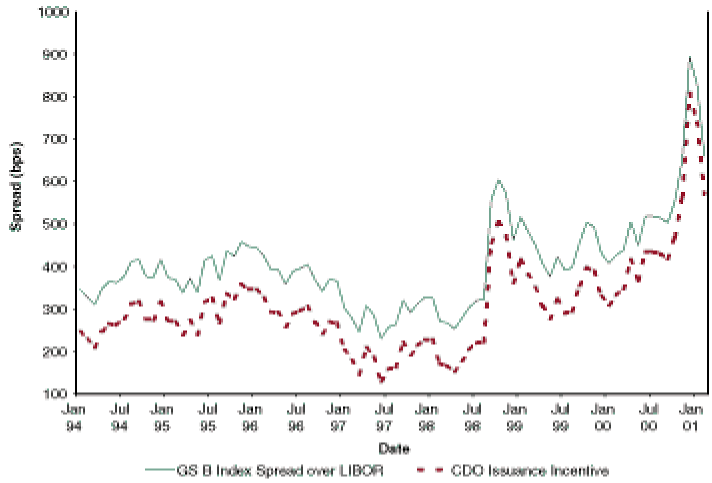


Figure 16: GS Single B Bond Index over Libor and the bond-backed CDO issuance incentive[11]

The upside from calling the transaction hugely depends on the type of collateral, making the distinction between CBO and CLO imperative.

Those CDOs where the collateral is represented by bonds purchased at low prices (when interest rates were high) and where the structure is financed through cheap term notes (current low interest rates) offer most benefit of a possibility of significant capital appreciation.

Floating-rate collaterals such as leveraged loans can easily be refinanced. The underlying borrowers can prepay outstanding loans and refinance at a lower spread. For this reason, a manager of a loan-backed CDO will be in a very difficult position to generate outsized capital appreciation. In other words, they do not offer as much potential for significant appreciation.

As the remaining expected returns fall, the equity holders are likely to exercise their option during the repayment period, either to take advantage of potential appreciation in CBOs, or to minimise the impact of a difficult credit environment with CLOs.

6.3 Investing in CDO equity

For long-horizon investors such as pension plans, endowments and insurance companies, portfolio diversification is an important investment consideration. In principle, diversification across asset classes lowers portfolio volatility without altering expected returns.

Traditionally, investments in real estate and foreign securities have been seen as effective diversification strategies. More recently, as volatilities in financial markets have increased, asset investors have also turned to more illiquid asset types such as private equity, hedge fund investments, commodities, insurance risk securities and ultimately CDO equity. CDO equities are perceived to have a lower correlation if compared with the traditional asset classes of which they are made of. This is not surprise since the CDO cash flow structure hedges the equity investment against short-term liquidity or technical fluctuations in the value of the collateral.

[11] This chart is from Olberg E., Nartey M., Takata H. and S. Shah (2001).

Indeed, the combination of non-generic collateral and active management should result in a low correlation between CDO equity returns and returns on benchmark asset classes, such as public equity, investment-grade corporate liabilities and government debt. Low long-term horizon return correlations, along with high expected returns, should lead asset investors such as insurance companies, pension plans, endowments and foundations, to consider investment in CDO equity as an effective diversification strategy and "alternative investment" bucket in the portfolios of long-term horizon investments.

The problem is that historical data on CDO equity returns is unavailable because the market is relatively new and remains a very private one.

E. Orberg at all[12] have looked at how to measure the correlation of CDO equities Their route has been to look at the underlying collateral markets as a starting point for thinking about correlations between CDO equity returns and other benchmark asset classes.

Historical (12/89 – 12/00) asset class return statistics*

	ML single B	LB Govt	LB Credit	S&P 500	Russell 2000
Average (%)	9.3	7.62	8.21	15.48	12.85
Standard deviation (%)	21.48	14.17	16.26	47.98	63.64

Correlations

	ML single B	LB Govt	LB Credit	S&P 500	Russell 2000
ML single B	1.00	0.31	0.47	0.49	0.57
LB Govt	0.31	1.00	0.95	0.34	0.15
LB Credit	0.47	0.95	1.00	0.43	0.26
S&P 500	0.49	0.34	0.43	1.00	0.69
Russell 2000	0.57	0.15	0.26	0.69	1.00

Table 6: Historical (12/89 – 12/00) asset class return statistics and correlations*

Table 6 shows the historical annualised monthly return averages, standard deviations and correlations for the Merrill Lynch Single B Index, the Lehman Brothers Government Index, the Lehman Brothers Credit Index, the S&P 500 Index and the Russell 2000 Index.

As expected, the high-yield Merrill Lynch Single B Index returns display the highest correlation with the small-cap Russell 2000 Index and lowest correlation with the Lehman Brothers Government Index.

The return correlation of the underlying asset type with other assets is an estimate of an upper bound for CDO equity return correlation.

If a fund invests in Government bonds and does not want to loose the appreciation given by investing in equities, it would be more beneficial to diversify into CDO equity (correlation of 0.1463).

[12] Olberg E., Nartey M., Takata H. and S. Shah (2001).

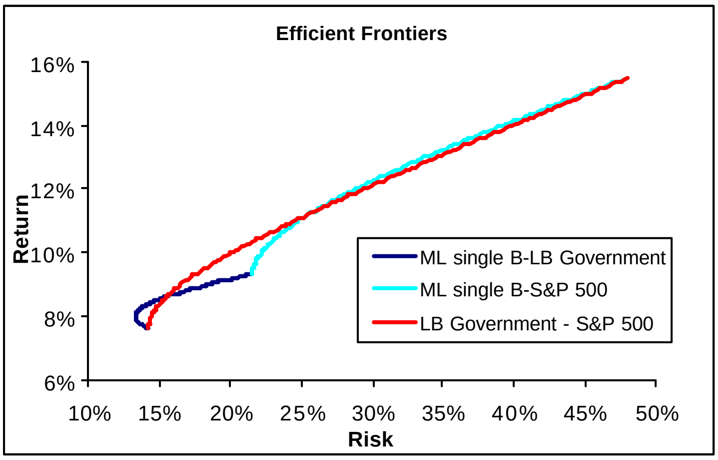


Figure 17: Three efficient frontiers: ML single B - LB Governments, ML single B -S&P and S&P - LB Governments.

Figure 17 contains the efficient frontiers of three portfolios: ML single B - LB Governments, ML single B -S&P and S&P - LB Governments. In the three portfolios the first name has the initial weight of 100%.
We can see that below a risk (standard deviation) of 15% it is efficient to diversify into single B's and government bonds. However, with a risk greater than 15% the most efficient portfolio is investing in government bonds and equities.

Active management of the underlying collateral portfolio and the cash flow structure of CDOs should insulate short-term horizon CDO equity returns from the returns on the underlying asset class. With the result that returns on CDO equity over three to five years will be most affected by underlying defaults and long-term collateral price moves.

6.4 The price of CDO equity

The price of CDO equity is expected to have a natural downward path as soon as the principal begins to be redeemed[13]. Figure 18 shows the cash flow profile of equity distributions over time for a CBO transaction. The distributions are per quarter. The same CBO is fully analysed in the next article.
We have created the example equity distributions under the assumption that the underlying collateral portfolio experiences a constant annual default rate of 3%.
The equity distributions only receive interest until quarter 17. The remaining principal is received in the last 4 quarters. In generating this payment time path, we have assumed that the equity holders do not call the transaction. In reality, equity investors are likely to call a well-performing transaction when leverage falls, usually between six and eight years.

[13] The price denotes the present value of future cash flows over the initial CDO equity principal.

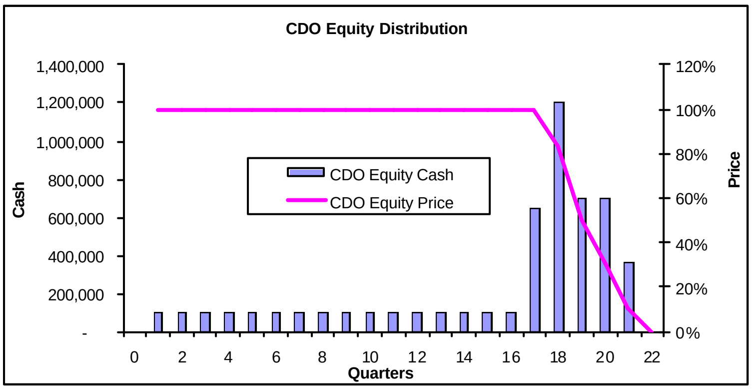


Figure 18: Price and Cash distribution of a CDO equity tranche.

We also expect the price of CDO equity to change over time. Other effects, such as changes in the value of the collateral portfolio, the value of the call option and changes in the floating rates attached to the notes will also affect the price path of equity over time.

Since the CDO equity is a call option on the collateral, we expect the CDO equity price to go down as final maturity approaches.

The change in the floating rates affect the yield spread between the income from the collateral and the funding cost of the issued notes. An increase in the floating rates compresses the interest margin in the transaction that can be used to cover losses.

Also, we can expect the leverage to affect the return profile of the equity piece.

More highly leveraged deals have steeper return profiles.

Figure 19 shows the returns of two CDO equity pieces with various loss rates: the more highly levered equity piece yields more until 6% losses but looses more after that.

Besides, the deal with greater leverage would also have tighter OC levels, which would trigger sooner and de-lever the structure.

To generate the returns we have used the same CDO structure as before.

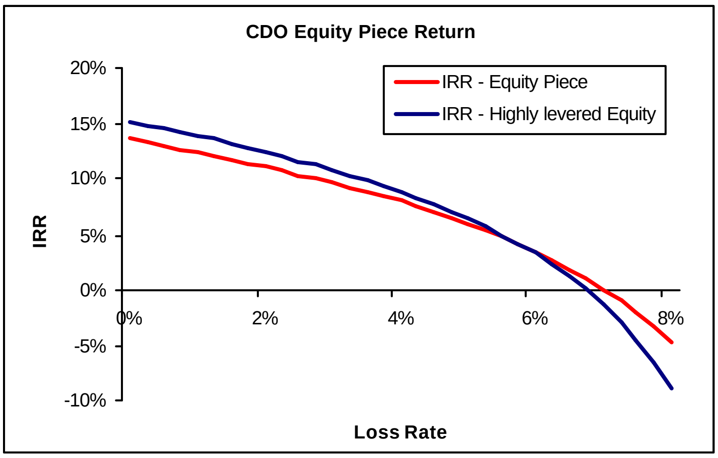


Figure 19: CDO equity investor return with different loss rate scenarios.

7 How to use the Moody's BET to structure a synthetic CDO

7.1 The Binomial Expansion Technique

Moody's use the Binomial Expansion Technique (BET) to determine the amount of credit risk present in the collateral.

The BET reduces the actual pool of collateral assets with correlated default probabilities, to a homogenous pool of assets with uncorrelated default probabilities via the Diversity Score. The Diversity Score D, provides the number of uncorrelated bonds or loans that mimic the behaviour of the original pool.

For example, at maturity, one of the D bonds may or may not have defaulted, i.e. there are only two outcomes. Furthermore, the probability that one particular bond defaults is independent on the probability that any other bond defaults. The consequence of such an assumption is the probability that N of the D bonds default can be calculated with the Binomial Distribution $P \sim Bi(D, N, p)$

$$P_N = \binom{D}{N} p^N (1-p)^{D-N} \qquad (1)$$

where p is the average probability of default of the pool, stressed by the appropriate factor.

Once the collateral risk is calculated, it is compared to the credit protection offered by the structure to arrive at the correct rating of all CDO tranches.

At default, the losses first hit the junior notes, then the mezzanine and finally the senior notes. The calculation is performed via simulating the number of defaults that the transaction can experience through its life. Starting with the initial state of no default, each *homogeneous* bond is taken to its maturity through binomial branches of default with probability p and no default with probability 1- p. The expected loss that hits the CDO structure is calculated and mapped against the Moody's Idealised Cumulative Expected Losses in the Table 7. For example, from a collateral with average maturity of 5 years, the maximum amount of cumulative expected loss for a *Aaa* Senior note with the same maturity, must not be greater than 0.002%.

	Year									
	1	2	3	4	5	6	7	8	9	10
Aaa	0.000%	0.000%	0.000%	0.001%	0.002%	0.002%	0.003%	0.004%	0.005%	0.006%
Aa1	0.000%	0.002%	0.006%	0.012%	0.017%	0.023%	0.030%	0.037%	0.045%	0.055%
Aa2	0.001%	0.004%	0.014%	0.026%	0.037%	0.049%	0.061%	0.074%	0.090%	0.110%
Aa3	0.002%	0.010%	0.032%	0.056%	0.078%	0.101%	0.125%	0.150%	0.180%	0.220%
A1	0.003%	0.020%	0.064%	0.104%	0.144%	0.182%	0.223%	0.264%	0.315%	0.385%
A2	0.006%	0.039%	0.122%	0.190%	0.257%	0.321%	0.391%	0.456%	0.540%	0.660%
A3	0.021%	0.083%	0.198%	0.297%	0.402%	0.501%	0.611%	0.715%	0.836%	0.990%
Baa1	0.050%	0.154%	0.308%	0.457%	0.605%	0.754%	0.919%	1.085%	1.249%	1.430%
Baa2	0.094%	0.259%	0.457%	0.660%	0.869%	1.084%	1.326%	1.568%	1.782%	1.980%
Baa3	0.231%	0.578%	0.941%	1.309%	1.678%	2.035%	2.382%	2.734%	3.064%	3.355%
Ba1	0.488%	1.111%	1.722%	2.310%	2.904%	3.438%	3.883%	4.340%	4.780%	5.170%
Ba2	0.858%	1.909%	2.849%	3.740%	4.626%	5.374%	5.885%	6.413%	6.958%	7.425%
Ba3	1.546%	3.030%	4.329%	5.385%	6.523%	7.419%	8.041%	8.641%	9.191%	9.713%
B1	2.574%	4.609%	6.369%	7.618%	8.866%	9.840%	10.522%	11.127%	11.682%	12.210%
B2	3.938%	6.419%	8.553%	9.972%	11.391%	12.458%	13.206%	13.833%	14.421%	14.960%
B3	6.391%	9.136%	11.567%	13.222%	14.878%	16.060%	17.050%	17.909%	18.579%	19.195%
Caa	14.300%	17.875%	21.450%	24.134%	26.813%	28.600%	30.388%	32.174%	33.963%	35.750%

Table 7: Moody's Idealised Cumulative Expected Losses (with a recovery rate of 45%).

The Loss of one of the D homogeneous assets defaulting is calculated as the loss in the present value of cash flows associated with the defaulted bond, adjusted by recovery.

The probability of this event is

$$EL_1 = P_1 * L_1 \qquad (2)$$

The Expected Losses of the pool are calculated by taking the sum of all losses under all the scenarios, *N = 0, 1, 2,…, D*.

$$EL = \sum_{N=0}^{D} P_N * L_N \; . \qquad (3)$$

and the Unexpected Losses are

$$UL = \sum_{N=0}^{D} P_N * (L_N - EL)^2 \qquad (4)$$

Thus to use the BET, we need to calculate the following collateral variables: the default probability, the losses and the diversity score.

1. Default Probability

The default probabilities are calculated using the ratings of the collateral assets. When public ratings are not available, Moody's determines *shadow* ratings.

The default probabilities are then adjusted by taking into account the underlying asset maturities to give the cumulative default probabilities.

The collateral cumulative default probability is calculated as the weighted average of the assets cumulative default probabilities where the weights are the assets par values,

$$CDP = \frac{\sum_{N=0}^{M} CP_N * A_N}{\sum_{N=0}^{M} A_N}$$

where,

CP_N is the cumulative default probability of bond N

A_N is the par value of bond N

M is the total number of assets.

2. Losses

Loss severity depends on the assumed recovery value and time of recovery. Moody's assumes that the recoveries are not affected by the asset rating, but they depend on the seniority and security of the obligation. Moody's also assumes that the base case recovery rate is a minimum of 30% of the market value or 25% of par value. For Emerging Markets the recovery rates drops to a minimum of 20% of the market value or 15% of par value.

Loan/Bond	Recovery Rates
Senior Secured Loans	70%
Senior Unsecured Loans	-
Senior Secured Bonds	52%
Senior Unsecured Bonds	49%
Subordinated Bonds	33%

Table 8: Recovery rates used by Moody's for different seniority.

3. Diversity Score

Moody's has solved the problem of estimating default correlation through the Diversity Score. This measures the number of uncorrelated assets in the pool that would experience the level of default in the original pool. Since default correlation is higher in poorly diversified portfolios, a low diversity score value is a sign of a riskier portfolio.

To calculate the Diversity Score the industry classification of Table 9 is used.

Industry Classification
Aerospace and Defence
Automobile
Banking
Beverage, Food and Tobacco
Buildings and Real Estate
Chemicals, Plastics and Rubber
Containers, Packaging and Glass
Personal, Nondurable Consumer Products
Diversified / Conglomerate Manufacturing
Diversified / Conglomerate Service
Diversified Natural Resources
Ecological
Electronics
Finance
Farming and Agriculture
Grocery
Healthcare, Education and Childcare
Furnishings, Houseware Durable Consumer Products
Hotels, Motels Inns and Gaming
Insurance
Leisure, Amusement, Motion Pictures, Entertainment
Machinery
Mining, Steel, Iron and Non-precious Metals
Oil and Gas
Personal, Food and Misc Services
Printing, Publishing and Broadcasting
Cargo Transport
Retail Stores
Telecommunications
Textiles and Leather
Personal Transportation
Utilities

Table 9: Industry Classification

The industry concentrations are calculated using bond par values as weights. Once the concentration is measured the Diversity Score is calculated by using the values of Table 10 in the "Diversity Score" column.

Number of Bond Issuers per Industry/Region	Diversity Score	Diversity Score for Latin America
1.00	1.00	1.00
1.50	1.20	1.10
2.00	1.50	1.25
2.50	1.80	1.40
3.00	2.00	1.50
3.50	2.20	1.60
4.00	2.30	1.65
4.50	2.50	1.75
5.00	2.70	1.85
5.50	2.80	1.90
6.00	3.00	2.00

Table 10: Diversity Score

Moody's also distinguishes diversity scores for bonds originated in Emerging Markets from all other regions. From Table 10 a pool of four EM bonds have a diversity score of two whereas with the same number of US high yield bonds the diversity score is three.

To arrive at the Latin America Diversity Score the following adjustment is used:

*LADS = 1 + (DS – 1) * 0.5*

4. Weighted Average Credit Rating

Moody's also requires the calculation of the collateral WACR.

For each rated asset of the collateral Moody's provide a rating factor (Table 11). The par value of each asset is multiplied by the corresponding rating factor. The result is divided by the total of the pool par value to calculate the WACR.

$$WACR = \frac{\sum_{N=0}^{M} RF_N * A_N}{\sum_{N=0}^{M} A_N}$$

where

RF_N the rating factor of bond N

A_N the par value of bond N.

Moody's rating factors	
Aaa	1
Aa1	10
Aa2	20
Aa3	40
A1	70
A2	120
A3	180
Baa1	260
Baa2	360
Baa3	610
Ba1	940
Ba2	1,350
Ba3	1,780
B1	2,220
B2	2,720
B3	3,490
Caa	4,770

Table 11: Moody's rating factors

7.2 The Double Binomial Expansion Technique

Occasionally, the collateral pool may be made of two (or more) highly uncorrelated assets, having different average properties.

Moody's models this case with a variation of the BET called Double BET.

With the Double BET, we approach the two pools as two independent pools. In this case the probability that a assets in pool A, and b assets in pool B default, are two independent events distributed as $P \sim$ $Bi(D_A, D_B, N_A, N_B, p_A, p_B)$

$$P_{a+b} = \binom{D_A}{a} p_A^a (1-p_A)^{D_A-a} \binom{D_B}{b} p_B^b (1-p_B)^{D_B-b} \qquad (4)$$

The Loss of having $a + b$ defaults can be calculated as the present value of the cash flows associated to those $a + b$ defaulted bonds, over the present value of all cash flows.

The Expected Losses of the combined pool is calculated by taking the sum of all the expected losses under all the scenarios, $a + b = 0, 1, 2, \ldots, N_A + N_B$

$$EL = \sum_{i=0}^{D_A} \sum_{j=0}^{D_B} P_{ij} L_{ij} \qquad (5)$$

and the Unexpected Losses

$$UL = \sum_{i=0}^{D_A} \sum_{j=0}^{D_B} P_{ij} \left(L_{ij} - EL\right)^2 \qquad (6)$$

7.3 Structuring Examples

We now structure the relative size and prioritisation of two CDO bond tranches: Senior and Mezzanine notes plus a Super Senior Swap with the BET and the Double BET.

The collateral bond portfolio is described in the following paragraph.

7.3.1 The Collateral

The collateral is formed of fifty-two bullet bonds with a total par value of *US$* 1bn.

The High Yield North America bonds (in US$) represents 78% of the pool, the remaining 22% are High Yield European bonds (in US$). The collateral composition per industry and the rating and maturity breakdown are shown in Tables 18 and 19 at the end of this chapter.

Each bond pays semi-annual cash flows to the CDO at its coupon rate, until maturity or default. At default, a quote is sold to the market and the recovery value is made available to the CDO.

Also, the proceeds from the notes are used to purchase U.S. Government Treasury Bonds.

The total interest proceeds available at period k, paid in the CDO is,

$$I_k = \sum_{i=1}^{40} \frac{c_i}{k} B_i + \frac{r_{3M}}{k} GTBonds$$

where B_i is the face value of bond i, c_i is the bond coupon rate, r_{3M} is the *US* 3-month default free rate, and *GTBonds* are the *US* Government Treasury Bonds.

In the event that the bond *i* pays less interest than scheduled, $U_k < I_k$, any difference is accrued at the bond coupon rate.

There may also be some prepayment of principal PP_k, some contractual unpaid reduction of principal UP_k, some contractual reduction of principal P_k, and some recovery of principal for those bonds defaulted R_k.

At maturity, the last coupon I_k, any unpaid accrued interest U_k, and principal P_k, are paid into the CDO.

The total actual payment received by the CDO in any coupon period k is

$$I_k + \sum_{i=1}^{m} \left(U_{k-1,i} - I_{k-1,i}\right)\frac{c_i}{2} + P_k + UP_k + PP_k + R_k ,$$

where m is the number of bonds missing the interest scheduled payment at the coupon period k - 1 and paying at the coupon period k.

7.3.2 Prioritisation

The Super Senior Swap and the three CDO tranches need to have their face values F_S, F_1, F_2 and F_3 determined. The floating coupon rate is the *US* 3-month Libor rate, plus the spreads s_1, s_2 and s_3 that depend on the note ratings and on their average lives. Also, p_S is the CDS premium the CDO pays to the hedge provider.

At the coupon period k, the CDO pays the collateral interest cash flow called the interest waterfall *IW*, pari-pasu to the Trustee and Administrative Fees and to the Senior Management Fees.

Following this, the CDO transfers the remaining collateral interest cash flow to pay the Super Senior Swap premium and the interest accrued on the CDO tranches according to the following priority scheme

- to the Super Senior Swap

$$Y_{S,k} = \min(I_{S,k,} IW_k)$$

- to the Senior tranche A

$$Y_{A,k} = \min(I_{A,k,} IW_k - Y_{S,k})$$

- to the Mezzanine tranche B

$$Y_{B,k} = \min(I_{B,k,} IW_k - Y_{S,k} - Y_{A,k})$$

- to the Equity tranche C

$$Y_{C,k} = \min(I_{C,k,} IW_k - Y_{S,k} - Y_{A,k} - Y_{B,k})$$

where $I_{S,k}$ is the premium to pay to the Super Senior Swap, and $I_{A,k}, I_{B,k}$ $I_{C,k}$ are the interests to pay to the notes *A*, *B* and *C*.

In the same fashion, the principal waterfall PW plus any remaining IW is transferred to the CDO tranches with the following priority scheme

- to the Super Senior Swap

$$PR_{S,k} = \min(P_{S,k,} PW_k) \ ,$$

and only after the Super Senior Swap has been fully redeemed,

- to the A tranche

$$PR_{A,k} = \min(P_{A,k\,,} PW_k)$$

- to the B tranche

$$PR_{B,k} = \min(P_{B,k,} PW_k - PR_{A,k})$$

- to the C tranche

$$PR_{C,k} = \min(P_{C,k\,,} PW_k - PR_{A,k} - PR_{B,k})$$

where $P_{S,k}$ is the notional of the Super Senior Swap, $P_{A,k\,,} P_{B,k}$ and $P_{C,k}$ are the principal to pay to the notes *A*, *B* and *C*.

Any excess cash flow from the collateral is deposited in a reserve account earning a 3-month default-free interest rate r_k .

In case the OC test for the *A* tranche is breached, the *B* and *C* tranche will not receive any interest until the OC test for the *A* tranche is cured. The interest waterfall is redirected to buy *AAA* rated assets. In this fashion the numerator of the OC ratio is increased and the OC test is cured.

The OC test also works for the *B* tranche and when the OC test for the *B* tranche is breached the interest waterfall is redirected to buy *AAA* rated assets.

7.3.3 The BET and DBET results

In this section we calculate the size of the Super Senior Swap and the Senior Note that is consistent with the target of idealised expected losses.

As noted earlier, the main assumption is that the risk analysis of the CDO can be conducted by assuming that the performance of the collateral can be approximated by the performance of a *comparison* portfolio.

Table 12 shows some information of the *comparison* portfolio: WACR, Diversity Score, Cumulative Default Probability and Recovery Rate.

Collateral Information	
No of Loans	52
Balance (000,000)	1,000
WAM	2.30%
Max Maturity	4.83 yr
WAL	3.00 yr
Rating level	Baa3
WA P(D)	2.28%
Diversity score	25
Recovery rate	45%

Table 12: Collateral Summary Information

We proceed in the following manner:

- Create a cash flow model where the waterfall is the one suggested in the Prioritisation section and where the bonds amortise according to their contractual profile,

- Use twenty six scenarios (since the diversity score is twenty five) where the number of defaulted bonds goes from zero to twenty five,
- Record the losses[14] hitting the Super Senior Swap, the Senior and the Mezzanine notes (as the loss in their Present Value[15]),
- Calculate the expected losses of the twenty six scenarios,
- Increase the notional of Super Senior Swap until the amount of expected losses that hit the Super Senior Swap itself reaches the target loss. This iteration selects the maximum amount of $900m (90%).
- Increase the size of the Senior note and the Mezzanine note until the expected losses that hit the above notes reach the target loss. The iteration selects $20m (2%) as the size of the Senior note rated at Aa1 and $40m (4%) as the size of the Mezzanine note rated as Baa1. This leaves a Junior piece not rated of $40m (4%) retained by the originator.

A summary is contained in Table 13. The first column shows the number of homogeneous bonds defaulted, the second shows the binomial default probabilities calculated using the binomial formula in (1) and the third column reports the loss as a % of the Super Senior Swap notional.

The stress factor is 1.5 and takes the probability of default to 3.36% from 2.284%.

The loss distribution of the Senior and Mezzanine Notes are reported from columns four to seven.

In the scenario where three bonds default, the cumulative losses do not hit either the Super Senior Swap or the Senior note, but cause a loss in the present value of the Mezzanine of 8.14%. The Junior piece is insufficient to provide enough cushion.

The performance of the Senior note changes in the scenario of six bonds defaulting. The loss in the PV is 52.85%.

With seven bonds defaulting, the Super Senior Swap is hit with losses.

Therefore, as long as the defaults are not greater than 20%[16] the Super Senior note investors will not suffer any loss.

[14] The losses are front loaded 50% in the first year. The remaining 50% are equally distributed between year two and six.

[15] We have applied a constant risk free interest rate of 4% per annum.

[16] 20% is calculated as 5 bonds / 25 (homogeneous bonds).

	Stressed Probability of Default 3.36% Super Senior		Stressed Probability of Default 3.36% Senior Note		Stressed Probability of Default 2.73% Mezzanine Note	
Default Number	Binomial Prob Of Default	Losses %	Binomial Prob Of Default	Loss in the PV	Binomial Prob Of Default	Loss in the PV
0	41.83%	0.000%	41.83%	0.00%	49.34%	0.00%
1	37.10%	0.000%	37.10%	0.00%	35.35%	0.00%
2	15.79%	0.000%	15.79%	0.00%	12.16%	0.00%
3	4.30%	0.000%	4.30%	0.00%	2.67%	8.14%
4	0.84%	0.000%	0.84%	0.00%	0.42%	22.54%
5	0.12%	0.000%	0.12%	0.00%	0.05%	63.36%
6	0.01%	0.000%	0.01%	52.85%	0.00%	83.43%
7	0.00%	1.494%	0.00%	83.56%	0.00%	88.63%
8	0.00%	3.551%	0.00%	83.56%	0.00%	97.68%
9	0.00%	5.862%	0.00%	83.75%	0.00%	100.00%
10	0.00%	8.216%	0.00%	83.99%	0.00%	100.00%
11	0.00%	10.548%	0.00%	84.24%	0.00%	100.00%
12	0.00%	12.875%	0.00%	84.65%	0.00%	100.00%
13	0.00%	15.189%	0.00%	85.33%	0.00%	100.00%
14	0.00%	17.502%	0.00%	85.98%	0.00%	100.00%
15	0.00%	19.816%	0.00%	86.63%	0.00%	100.00%
16	0.00%	22.129%	0.00%	87.21%	0.00%	100.00%
17	0.00%	24.442%	0.00%	87.41%	0.00%	100.00%
18	0.00%	26.756%	0.00%	87.41%	0.00%	100.00%
19	0.00%	29.069%	0.00%	87.41%	0.00%	100.00%
20	0.00%	31.360%	0.00%	87.53%	0.00%	100.00%
21	0.00%	33.656%	0.00%	88.43%	0.00%	100.00%
22	0.00%	35.640%	0.00%	89.52%	0.00%	100.00%
23	0.00%	35.422%	0.00%	90.50%	0.00%	100.00%
24	0.00%	35.204%	0.00%	91.11%	0.00%	100.00%
25	0.00%	36.668%	0.00%	91.42%	0.00%	100.00%

Table 13: Expected Losses of 26 scenarios for the Super Senior Swap, Senior and Mazzanine notes.

Moody's idealised cumulative losses are averages calculated from data collected in different economic conditions. Therefore, different rating stresses are applied to the collateral probability of default rate (2.28%) in order to cover the risk that the realised cumulative losses may be quite higher than the idealised.

Also, since the loss distribution is skewed, there is a significant chance of realising losses that are six to eight standard deviations in excess of the expected losses. The stress values should enforce the creation of enough room against the probability of very large losses hitting the CDO tranches.

In Table 14, we can notice that a stress factor of 1.5 on the *Aaa1* probability of default corresponds to multiplying the expected losses by a factor of 9.60. Likewise, a stress factor of 1.23 on the *Aa1* probability of default corresponds to multiplying the expected losses by a factor of 1.78.

Rating	Stress on the Default Probabilities	Stressed Expected Losses	No-Stressed Expected Losses	Stress on the Expected Losses
Aaa1	1.50	0.00909%	0.00095%	9.60
Aa1	1.23	0.34898%	0.19655%	1.78

Table 14: Default Probability Stresses.

7.3.4 The Double BET: Example

We can argue that since the bonds originate from two different geographic areas and have different averages, they should be modelled as two independent pools.

We have divided the North American bonds from the European bonds and formed two groups: A for North America, and B for Europe.

The two collateral diversity scores, WACR's, Cumulative Default Probabilities, Loss Given Rates, Recovery Rates, together with other information are shown in Table 15.

Collateral Information		
	US	Europe
No of Loans	41	11
Balance (000,000)	769.50	230.50
WAM	2.30%	2.30%
Max Maturity	4.83 yr	4.83 yr
WAL	2.83 yr	3.51 yr
Rating level	Ba1	Baa2
WA P(D)	2.60%	1.22%
Diversity score	21	6
Recovery rate	45%	45%

Table 15: North America and Europe pools -Default Probabilities, Diversity Scores and Recovery Rates.

To determine the Debt structure, as we did in the BET example, we study how the collateral losses are distributed among the super senior swap, senior, mezzanine and equity pieces. However, we must slightly adapt the BET cash flow model to incorporate the calculation of the joint default probabilities and the joint loss distribution. There are now 132 scenarios: the number of defaulted bonds goes from zero to twenty-one in the America pool, and from zero to six in the Europe pool.

With the same iteration used in the BET, we select the size of Super Senior Swap as $900m (90%). The iteration also selects $20m (2%) as the size of the Senior note rated at Aa1 and $40m (4%) as the size of the Mezzanine note rated as A3. This leaves a Junior piece (not rated) of $40m (4%) retained by the originator.

From Tables 21, 22 and 23 we see that in the scenario of one bond of group Europe and two bonds of group North America defaulting, losses do not hit either the Super Senior Swap or the Senior Note. However the loss in the Mezzanine present value is 6.20%.

In the scenario with two bonds of group Europe and four bonds of group North America defaulting, losses hit the senior piece. The senior piece suffers a loss of 22.46% of its par value. The Super Senior Swap does not remain immune from loss very long. In the scenario with two bonds of group Europe and five bonds of group North America defaulting, losses hit the Super Senior Swap and are 0.33%. Now, both mezzanine and senior notes are insufficient to offer enough cushion for losses.

The differences between the BET and Double BET are summarised in Table 16.

With the BET the collateral of Tables 19 and 20 can support 90% of Super Senior Swap rated *Aaa1*, 2% of Senior Note rated as *Aa1* and 4% of Mezzanine Note rated as *Baa1*. The Equity piece that the originator retains on its balance sheet is 4%.

With the Double BET the structure becomes 90% of Super Senior Swap rated *Aaa1*, 2% of Senior Note rated as *Aa1* and 4% of Mezzanine Note rated as *A3*. The Equity piece retained by the originator balance sheet remains at 4%.

Thus with the Double BET the originator would save some funding cost due to the better rating received by the mezzanine.

The Double BET methodology takes advantage of the increase in diversification as measured by the Diversity Score, where 25 in the BET becomes 27 in the Double BET, 21 for the pool North America and 6 for the pool Europe.

	BET			Double BET		
Notes	**Rating**	**Volume**	**%**	**Rating**	**Volume**	**%**
Super Senior Swap	Aaa	900.0	90.00%	Aaa	900.0	90.00%
Senior Note	Aa1	20.0	2.00%	Aa1	20.0	2.00%
Mezzanine Note	Ba1	40.0	4.00%	A3	40.0	4.00%
Equity	Unrated	40.0	4.00%	Unrated	40.0	4.00%
Totals		*1000.0*	*100.00%*		*1000.0*	*100.00%*

Table 16: CDO Structure with BET and DBET.

7.4 Structure Results

With this structure the originator hedges the credit risk of its US$ 1billion bond portfolio through a series of independent transactions:

1. It retains the first loss portion of US$ 40 million (4% of the portfolio),
2. It enters into a credit default swap (Super Senior Swap) with an OECD bank covering US$ 900 million (90% of the portfolio) paying a premium of 12 bps,
3. It enters into a credit default swap with the SPV covering US$ 100 million (10% of the portfolio) paying a premium of 25 bps.

The SPV issues the following tranches of 5-year note:

1. US$ 20 million Senior notes (rated *Aaa*) bearing interest at US Libor + 45 bps (2% of the portfolio),
2. US$ 40 million Mezzanine notes (rated *A3* with DBET or *Baa1* with BET) bearing interest at US Libor + 150-200[17] bps (4% of the portfolio),
3. US$ 40 million of Equity (Unrated - 4% of the portfolio) subscribed by the originator, with a ROE of 21.7% (calculated with zero losses).

The $100 million proceeds of the note issue are invested in US Government Bonds to collateralise the credit default swap with the originator.

[17] 150 bps in case the Mezzanine is rated *A3*, and 200 bps in case the rating is *Baa1*.

8 S&P CDO Evaluator

In rating CDOs S&P uses a *Monte Carlo simulation* proprietary model called CDO evaluator. The model estimates the default rate distribution in the collateral portfolio. It takes into consideration the borrower credit rating, the probability of default, maturity and correlation between each pair of assets. The result is the default rates probability distribution of the aggregate portfolio.

S&P does not use the Moody's diversity score to simplify the calculation of the default correlation between each pair of assets in the portfolio. Rather, it assumes an asset correlation between assets in different corporate sectors of 0.3, and between assets within the same corporate sector of 0.

The simulation engine draws a large number of multivariate normally distributed numbers $X \sim N(0,1)$. At each trial, the draw of asset i is compared to its default threshold, and if it is lower, the asset defaults. The default threshold is calculated given the asset default probability and maturity. The principal balances of all defaulted assets are summed up and then divided by the initial total portfolio balance to estimate the default rate in the j^{th} trial. After 100,000 trials the histogram of the probability distribution of default rate is derived.

Table 17 shows the credit information of the same assets analysed in the Moody's model calculated by the S&P CDO evaluator.

The assets have weighted average maturity of 2.99 years (three years in Moody's) and weighted average rating of BBB- (equal to Baa3 in Moody's). Their probability of default is 2.54% Vs 2.28% in Moody's.

Pool characteristic	
Number of assets	52
Number of obligors	52
Total principal balance	$1,000,000,000
Weighted Average Maturity (yrs)	2.993
Weighted Average Rating	BBB-
Expected Portfolio Default Rate (EPDR)	2.544%
Annualized Expected Portfolio Default Rate (APDR)	0.857%
Standard Deviation of Portfolio Default Rate (SD)	2.875%
Ratio of SD of Portfolio with Correlation to without	1.05
Weighted Average Correlation	0.482%

CORRELATION ASSUMPTIONS	
Between Corporate Sectors	0.0
Within Corporate Sector	0.3

Source: S&P CDO evaluator

Table 17: Assets statistics as calculated by the S&P CDO evaluator (with 10,000 simulations).

Table 18 contains the output of the CDO evaluator, the first three columns from left to right, plus four more columns that we have estimated to help understand the default rates distribution.

Desired Rating	Max Default Probability	Stressed Default Rates	Risk Factors	Default Rates	Credit Enhanc. (No Rec.)	Credit Enhanc. (with Rec.)
AAA	0.119%	20.88%	1.20	17.40%	79.12%	88.52%
AA+	0.143%	18.81%	1.14	16.50%		
AA	0.393%	16.33%	1.11	14.72%		
AA-	0.462%	15.44%	1.08	14.30%		
A+	0.499%	14.80%	1.05	14.10%		
A	0.540%	14.28%	1.02	14.00%		
A-	0.630%	13.37%	0.99	13.50%		
BBB+	0.909%	12.48%	0.96	13.00%		
BBB	1.179%	11.36%	0.93	12.22%		
BBB-	2.310%	9.58%	0.90	10.65%		
BB+	4.905%	7.48%	0.84	8.90%	13.40%	7.37%
BB	7.482%	6.08%	0.81	7.50%		
BB-	8.361%	5.85%	0.78	7.50%		
B+	11.053%	4.69%	0.75	6.25%		
B	18.565%	3.24%	0.72	4.50%		
B-	21.529%	2.76%	0.69	4.00%		
CCC+	28.659%	2.11%	0.66	3.20%		
CCC	35.788%	1.70%	0.63	2.70%		
CCC-	57.192%	0.90%	0.60	1.50%		
UR					7.48%	4.11%

Source: S&P CDO evaluator

Table 18: Probability Distribution associated with the assets in Tables 12 and 17.

The values in column *Max Default Probability* are fundamentals to be able to read the default distribution of the assets. For example, the probability of exceeding a 9.58% default rate (column *Stressed Default Rates*) is no greater than 2.3% (i.e. confidence interval of 97.7%), but the probability of exceeding an 18.8% default rate is no greater than 0.14% (i.e. confidence interval of 99.86%).

The values in column *Max Default Probability* are taken from the history of cumulative default rates that S&P has estimated from 1981 (see Table 19).

Stressed Default Rates are the default rates associated with the *Max Default Probabilities*. S&P adjusts the default rates by factors depending on the rating category. S&P does not publish these factors, and only reports the *stressed default rates* in the simulation output.

We have used the CDO evaluator with other assets to test the size and effect of these factors. We have chosen the assets so that their weighted average maturity does not diverge from three years. Our final estimates are in the *Risk Factors* column. If we apply the risk factors to the *Stressed Default Rates*, we can calculate the *Default Rates* associated with the probability distribution (see Figure 20 for the default histogram).

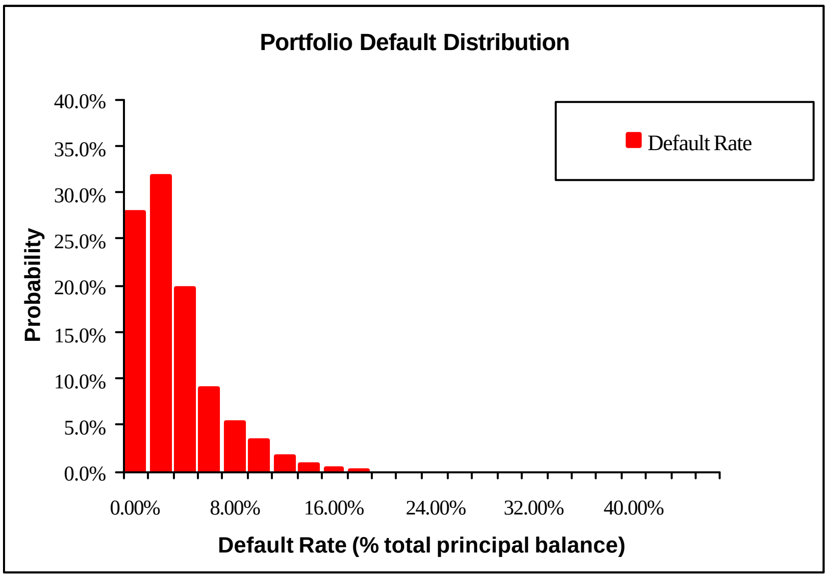
A consequence of this methodology is that in this portfolio, the probability of exceeding a 21% default rate (with no recovery and having being stressed by 1.2) is no greater than 0.12% (i.e. confidence interval of 99.88%). Thus, if we fix the size of the *AAA* tranche as 79% (100% -21%) the probability of exceeding a 0% default rate is 0. The last column shows the credit enhancement with a recovery rate of 45%, and the size of the *AAA* tranche is now 88%. The equity piece (UR) is 7.5% in case no recovery is allowed and 4.1% with a recovery rate of 45%.

The values in columns Credit Enhancement are only indicative. S&P requires a cash flow model and imposes several stress scenarios on the timing of default and recoveries, future interest rates and prepayment rates to fully understand the collateral performance. The final size of *AAA* tranche is selected through a reiteration where its size is increased as long as its losses (in present value terms) are lower than the *Stressed Default Rate* (21%) in column three.

	RATING																			
YEARS	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC+	CCC	CCC-	D
1	0.023%	0.023%	0.111%	0.136%	0.136%	0.136%	0.145%	0.225%	0.225%	0.544%	1.666%	2.772%	2.792%	3.667%	8.594%	9.563%	14.693%	19.824%	46.549%	100.000%
2	0.062%	0.071%	0.242%	0.290%	0.303%	0.317%	0.358%	0.532%	0.638%	1.357%	3.316%	5.265%	5.667%	7.535%	14.514%	16.626%	23.401%	30.176%	53.451%	100.000%
3	**0.119%**	**0.143%**	**0.394%**	**0.464%**	**0.501%**	**0.542%**	**0.632%**	**0.911%**	**1.182%**	**2.317%**	**4.916%**	**7.498%**	**8.380%**	**11.078%**	**18.594%**	**21.564%**	**28.696%**	**35.829%**	**57.219%**	**100.000%**
4	0.193%	0.239%	0.565%	0.659%	0.728%	0.808%	0.959%	1.352%	1.814%	3.344%	6.439%	9.489%	10.826%	14.122%	21.446%	24.962%	32.024%	39.086%	59.390%	100.000%
5	0.284%	0.357%	0.757%	0.875%	0.984%	1.111%	1.330%	1.841%	2.500%	4.387%	7.866%	11.255%	12.973%	16.655%	23.488%	27.316%	34.200%	41.083%	60.722%	100.000%
6	0.392%	0.497%	0.968%	1.113%	1.265%	1.448%	1.737%	2.368%	3.215%	5.415%	9.189%	12.817%	14.834%	18.735%	24.997%	28.985%	35.690%	42.394%	61.596%	100.000%
7	0.517%	0.656%	1.198%	1.372%	1.570%	1.814%	2.173%	2.921%	3.941%	6.410%	10.407%	14.197%	16.436%	20.438%	26.151%	30.208%	36.762%	43.317%	62.211%	100.000%
8	0.658%	0.835%	1.445%	1.650%	1.896%	2.204%	2.632%	3.492%	4.667%	7.360%	11.525%	15.419%	17.816%	21.840%	27.065%	31.141%	37.576%	44.010%	62.673%	100.000%
9	0.815%	1.033%	1.710%	1.946%	2.242%	2.614%	3.108%	4.074%	5.383%	8.261%	12.548%	16.503%	19.008%	23.004%	27.816%	31.883%	38.222%	44.562%	63.041%	100.000%
10	0.988%	1.247%	1.990%	2.259%	2.604%	3.041%	3.597%	4.661%	6.084%	9.112%	13.486%	17.470%	20.044%	23.984%	28.453%	32.497%	38.760%	45.023%	63.349%	100.000%

Source: S&P CDO evaluator

Table 19: S&P historic cumulative default rates.



Source: S&P CDO evaluator

Figure 20: Default rate distribution.

8 Appendix - Tables

Industry	**Concentration %**
European (US$)	
Telecommunications	5.78%
Utilities	4.80%
Finance	3.56%
Chemicals, Plastics and Rubber	2.67%
Oil and Gas	2.13%
Banking	1.78%
Electronics	0.89%
Sub Total	*21.61%*
North America (US$)	
Beverage, Food and Tobacco	12.92%
Buildings and Real Estate	10.64%
Leisure, Amusement, Motion Pictures, Entertainment	7.65%
Banking	7.25%
Finance	6.45%
Insurance	6.23%
Utilities	5.78%
Printing, Publishing and Broadcasting	5.51%
Retail Stores	3.25%
Chemicals, Plastics and Rubber	2.67%
Personal, Food and Misc Services	2.67%
Telecommunications	2.22%
Oil and Gas	2.05%
Mining, Steel, Iron and Non-precious Metals	1.78%
Furnishings, Houseware Durable Consumer Products	0.53%
Containers, Packaging and Glass	0.44%
Machinery	0.36%
Sub Total	*78.39%*
Total	100.00%

Table 17: Industry concentration

		Maturity					
		1 y	2y	3 y	4 y	5 y	***Totals***
Rating	Aaa	-	25,000,000	10,000,000	25,000,000	-	***60,000,000***
	Aa1	-	-	-	-	30,000,000	***30,000,000***
	Aa2	-	6,000,000	20,000,000	-	10,000,000	***36,000,000***
	Aa3	-	-	-	30,000,000	50,000,000	***80,000,000***
	A1	22,500,000	-	8,500,000	-	-	***31,000,000***
	A2	-	-	-	-	30,000,000	***30,000,000***
	A3	-	15,300,000	49,000,000	85,000,000	-	***149,300,000***
	Baa1	-	66,500,000	20,000,000	50,000,000	30,000,000	***166,500,000***
	Baa2	-	29,000,000	-	-	-	***29,000,000***
	Baa3	-	-	-	-	75,000,000	***75,000,000***
	Ba1	12,550,000	29,000,000	56,650,000	104,000,000	7,000,000	***209,200,000***
	Ba2	-	-	30,000,000	-	-	***30,000,000***
	Ba3	-	4,000,000	10,000,000	-	-	***14,000,000***
	B1	20,000,000	-	20,000,000	-	-	***40,000,000***
	B2	-	-	-	-	20,000,000	***20,000,000***
	B3	-	-	-	-	-	-
	Caa	-	-	-	-	-	-
	Totals	***55,050,000***	***174,800,000***	***224,150,000***	***294,000,000***	***252,000,000***	***1,000,000,000***

Table 18: Rating and Maturity concentration.

	Defaulted Bonds	Double Binomial Default Probabilities Class Super Senior Swap and Senior Note European Pool 0	1	2	3	4	5	6
US Pool	0	42.44%	6.26%	0.38%	0.01%	0.00%	0.00%	0.00%
	1	30.70%	4.53%	0.28%	0.01%	0.00%	0.00%	0.00%
	2	10.58%	1.56%	0.10%	0.00%	0.00%	0.00%	0.00%
	3	2.31%	0.34%	0.02%	0.00%	0.00%	0.00%	0.00%
	4	0.36%	0.05%	0.00%	0.00%	0.00%	0.00%	0.00%
	5	0.04%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%
	6	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	7	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	8	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	9	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	10	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	11	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	12	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	13	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	14	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	15	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	16	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	17	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	18	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	19	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	20	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	21	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

Table 19: Double Binomial distribution of the Super Senior Swap and the Senior Note.

	Defaulted Bonds	Double Binomial Default Probabilities Mezzanine Note European Pool 0	1	2	3	4	5	6
US Pool	0	44.98%	6.18%	0.35%	0.01%	0.00%	0.00%	0.00%
	1	30.30%	4.17%	0.24%	0.01%	0.00%	0.00%	0.00%
	2	9.72%	1.34%	0.08%	0.00%	0.00%	0.00%	0.00%
	3	1.97%	0.27%	0.02%	0.00%	0.00%	0.00%	0.00%
	4	0.29%	0.04%	0.00%	0.00%	0.00%	0.00%	0.00%
	5	0.03%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	6	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	7	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	8	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	9	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	10	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	11	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	12	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	13	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	14	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	15	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	16	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	17	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	18	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	19	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	20	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	21	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

Table 20: Double Binomial distribution of the Mezzanine Note

	Losses						
	Class Super Senior Swap						
	European Pool						
US Pool – Defaulted Bonds	0	1	2	3	4	5	6
0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.65%
2	0.00%	0.00%	0.00%	0.00%	0.00%	0.54%	2.71%
3	0.00%	0.00%	0.00%	0.00%	0.42%	2.60%	4.69%
4	0.00%	0.00%	0.00%	0.36%	2.49%	4.58%	6.88%
5	0.00%	0.00%	0.33%	2.38%	4.47%	6.77%	9.02%
6	0.00%	0.31%	2.26%	4.36%	6.67%	8.91%	11.15%
7	0.24%	2.15%	4.25%	6.56%	8.81%	11.05%	13.28%
8	2.04%	4.14%	6.46%	8.71%	10.94%	13.18%	15.40%
9	4.04%	6.35%	8.60%	10.84%	13.07%	15.30%	17.52%
10	6.24%	8.50%	10.74%	12.97%	15.19%	17.42%	19.64%
11	8.40%	10.63%	12.87%	15.09%	17.31%	19.54%	21.76%
12	10.53%	12.77%	14.99%	17.21%	19.43%	21.65%	23.88%
13	12.66%	14.89%	17.11%	19.33%	21.55%	23.77%	26.00%
14	14.78%	17.01%	19.23%	21.45%	23.67%	25.89%	28.11%
15	16.90%	19.13%	21.35%	23.57%	25.79%	28.01%	30.21%
16	19.02%	21.24%	23.47%	25.69%	27.91%	30.11%	32.33%
17	21.14%	23.36%	25.59%	27.81%	30.01%	32.22%	34.42%
18	23.26%	25.48%	27.70%	29.90%	32.12%	34.32%	35.59%
19	25.38%	27.60%	29.80%	32.02%	34.22%	35.59%	35.39%
20	27.50%	29.70%	31.92%	34.12%	35.60%	35.40%	35.19%
21	29.59%	31.82%	34.02%	35.61%	35.41%	35.20%	36.67%

Table 21: Loss distribution of the Super Senior Swap.

	Losses						
	Senior Note						
	European Pool						
US Pool – Defaulted Bonds	0	1	2	3	4	5	6
0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	39.25%
1	0.00%	0.00%	0.00%	0.00%	0.00%	35.05%	83.56%
2	0.00%	0.00%	0.00%	0.00%	30.85%	83.56%	83.56%
3	0.00%	0.00%	0.00%	26.65%	83.56%	83.56%	83.63%
4	0.00%	0.00%	22.46%	83.56%	83.56%	83.62%	83.85%
5	0.00%	18.26%	83.56%	83.56%	83.61%	83.84%	84.08%
6	14.06%	83.56%	83.56%	83.59%	83.83%	84.07%	84.30%
7	83.56%	83.56%	83.58%	83.82%	84.06%	84.29%	84.79%
8	83.56%	83.57%	83.81%	84.05%	84.28%	84.76%	85.39%
9	83.56%	83.80%	84.03%	84.27%	84.72%	85.36%	85.99%
10	83.79%	84.02%	84.26%	84.69%	85.33%	85.96%	86.58%
11	84.01%	84.25%	84.65%	85.30%	85.93%	86.55%	87.16%
12	84.24%	84.61%	85.27%	85.90%	86.53%	87.15%	87.41%
13	84.57%	85.24%	85.87%	86.50%	87.12%	87.41%	87.41%
14	85.21%	85.84%	86.47%	87.09%	87.41%	87.41%	87.41%
15	85.81%	86.44%	87.06%	87.41%	87.41%	87.41%	87.41%
16	86.41%	87.04%	87.41%	87.41%	87.41%	87.41%	87.79%
17	87.01%	87.40%	87.41%	87.41%	87.41%	87.75%	88.80%
18	87.38%	87.41%	87.41%	87.41%	87.70%	88.75%	89.79%
19	87.41%	87.41%	87.41%	87.65%	88.70%	89.75%	90.63%
20	87.41%	87.41%	87.61%	88.65%	89.70%	90.59%	91.15%
21	87.41%	87.60%	88.60%	89.65%	90.56%	91.13%	91.42%

Table 22: Loss distribution of the Senior Note.

	Losses						
	Mezzanine Note						
	European Pool						
US Pool – Defaulted Bonds	0	1	2	3	4	5	6
0	0.00%	0.00%	0.00%	7.03%	20.24%	54.58%	79.00%
1	0.00%	0.00%	6.61%	19.60%	52.62%	78.98%	88.63%
2	0.00%	6.20%	18.95%	50.65%	78.95%	88.63%	94.14%
3	5.79%	18.31%	48.69%	78.93%	88.63%	94.14%	100.00%
4	17.66%	46.72%	78.91%	87.64%	94.14%	100.00%	100.00%
5	44.75%	78.88%	85.98%	94.14%	100.00%	100.00%	100.00%
6	78.86%	84.32%	94.14%	100.00%	100.00%	100.00%	100.00%
7	83.43%	94.14%	100.00%	100.00%	100.00%	100.00%	100.00%
8	94.14%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
9	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
10	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
11	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
12	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
13	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
14	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
15	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
16	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
17	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
18	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
19	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
20	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
21	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%

Table 23: Loss distribution of the Mezzanine Note.