Credit Switch

by

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Abstract

A credit switch is the simultaneous purchase of credit protection on one asset and the sale of credit protection on another asset. This article provides a model for valuing this credit derivative whose payoff depends on the identities of a given list of credit events, such as defaults. The survival probabilities are modeled as a stochastic intensity process under a risk neutral framework. Closed form solutions are provided when the intensity process follows a popular diffusion process. A numerical example illustrates the effectiveness of a credit switch as a corporate risk-management solution.

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Introduction

Recently we have witnessed a surge in demand for credit derivative products[2]. One such derivative is a *credit switch* that involves the simultaneous purchase of credit protection on one asset and the sale of credit protection on another asset (Figure 1). For a corporation with a portfolio of loans that are concentrated in a particular industry, a "switch" may improve the risk-return profile by diversifying credit risk[3]. This paper provides a simple model for valuing a credit-switch whose payoff depends on the identities of the given list of credit events. Closed form solutions are provided when the underlying state variable follows a popular diffusion process. A numerical example illustrates the effectiveness of a credit switch as a corporate risk-management solution.

When a credit event occurs, an agent is unable to collect money that is promised at maturity. If a corporation has a large loan exposure and the consequences of a credit event are severe, it may be willing to diversify this risk. For a large loan exposure that cannot be switched in one whole transaction, several participants exchange the credit risk of a portion of the loan (also refereed to as a multi-party switch). This multi-party credit switch gives an improved risk and return profile by reducing the probability of large losses while increasing the probability of smaller losses. It also allows different parties to take opposing views on the possibilities of default on reference assets.

In order to explain a credit-switch with maturity *T*, suppose there are $j = 1,2,....,n$ assets and let t_j denote the occurrence of the associated credit event *j*. These t_j are

[2] Chapter 15 of Das (1998) provides an overview of the development of credit derivative markets.

random events, also referred to as default times. Let I_F denote the random variable defined as follows :

$$I_F = \begin{cases} j, if t_1 > T, t_j \le T \\ 1, if t_1 \le T, t_j > T \\ 0, otherwise \end{cases} \quad (1)$$

Suppose further that the payment S_j is received by a claim-holder when the event $\{I_F = j\}$ occurs, is positive and the claim-holder receives this payment at maturity T of the contract (this may be likened to the difference between the loan and the recovered amount). Then, the payoff to a claim holder from a switch between assets 1 and 2 is given by:

$$X_2 = F_1 S_1 1_{\{I_F = 1\}} - F_2 S_2 1_{\{I_F = 2\}} \quad (2)$$

where 1_A denotes the indicator function meaning that $1_A = 1$ if event A is true and is 0 otherwise. Note that we assume that there is no exchange of funds only if both the assets are in default. A corporation (claim holder) with a large face value of an asset may have several parties that participate in the switch. Each party assumes credit risk for a portion of the asset while selling credit risk on some other asset. Then the payoff to the claim holder is given by:

$$X_n = F_1 S_1 1_{\{I_F = 1\}} - \sum_{2}^{n} F_j S_j 1_{\{I_F = j\}} \quad (3)$$

Thus a credit switch payoff depends on the identity of the asset that defaults before maturity. The continuous-time financial theory has developed extensive tools to value

[3] A switch may have a lower cost and preferred tax treatment compared to loan sales, credit default swaps or securitizations (Reyfman and Toft (2001)).

such credit derivatives (Section II provides a literature overview). The approach in this paper is to identify the joint survival probabilities, denoted by:

$$S(t_0, t_1, ..., t_n) = P\{t_0 > t_0, t_1 > t_1, ..., t_n > t_n\}. \qquad (4)$$

Given the survival probabilities, we can evaluate the probabilities of the events $\{I_F = j\}$ in equation (2). Throughout this paper we fix the risk-neutral probability space $(O, F, ?)$ and denote the expectation operator by *E*. The conditional probability measure given the filtration F_t is denoted by P_t and the associated conditional expectation operator is E_t.

Several motivations are possible for the use of a credit switch:

- For a given level of debt, hedging can reduce the probability that a firm will find itself in a situation where it is unable to pay its debts. Thus if *financial distress* is costly, and there is an advantage to having debt in the capital structure, a credit switch is useful as a means to reduce the chance of distress.
- There is a corresponding reduction in the conditional variance of cash flows and earnings when a switch is used. Stulz (1984) argues that while outside shareholders ability to diversify will effectively make them indifferent to the amount of hedging activity under-taken, the same cannot be said for managers who may hold a relatively large portion of their wealth in the firms' stock. Thus *risk averse managers* with large stakes in a corporation will find a switch useful.
- Tax planning and investment decisions are better accomplished if the variance of cash flows is reduced. Smith and Stulz (1985) argue that if *taxes* are a convex function of earnings, it will be optimal for firms to hedge (a credit switch reduces the conditional variance of cash flows). Another rationale for hedging is given in

Froot, Scharfstein and Stein (1989) and Stulz (1990) that hedging can add value by reducing *investment distortions* associated with debt finance.

This article is organized as follows- Section 1 provides background information on credit derivative models, Section 2 describes the default model that underlies the derivations. Section 3 develops pricing formulae in a general set up and Section 4 presents a closed form solution when the default process follows a commonly used diffusion processes. Section 5 has examples that elicit the usefulness of a credit-switch and Section 6 concludes the article with a discussion of risk management applications of a credit-switch.

1. Literature Overview

The credit derivative market has grown rapidly over the last few years. Credit derivatives are financial instruments whose payoffs are linked to the credit characteristics of reference asset values. Besides a credit switch, other examples of credit derivatives include total-return swaps, credit swaps and credit-spread options (e.g., Das (1995), Longstaff and Schwartz (1995), Das and Tufano (1996), Davis and Mavrodis (1997), Duffie (1998), Duffie and Gârleanu (2000), Kijima and Muromachi (2000)). There is a parallel stream of literature that analyzes the impact of default on financial contracts and the potential uses of credit derivative products (e.g., Kim, Ramaswamy and Sundaresan (1993), Nielson and Ronn (1995), Hull and White (1995))[4].

There are two basic approaches to model default risk. One approach, pioneered by Black and Scholes (1973) and Merton (1974) and extended by Black and Cox (1976), Longstaff and Schwartz (1995), and others, explicitly models the evolution of firm value observable by investors. This approach is commonly referred to as the "structural approach" and has been applied in Geske (1977), Merton (1974), Smith and Warner (1979), Cooper and Mello (1991), Hull and White (1992), Abken (1993), Leland and Toft (1997) and Zhou (1998) among others. In these structural models, the authors assume that the firm value follows a given stochastic process. A firm defaults on its debt if the firm value falls below the nominal value of outstanding debt. The distribution of default times in these models depend on the underlying diffusion specifications.

A second approach to modeling default is adopted by Jarrow and Turnbull (1995), Jarrow, Lando and Turnbull (1997), Madan and Unal (1998), Duffie and Singleton (1999), Kijima and Muromachi (2000) wherein the authors do not consider the relation between default and firm value in an explicit manner. This approach is called the reduced form approach. In contrast to the structural approach, the reduced form approach treats default as an exogenous event. In keeping with the reduced form approach we model the intensity process to obtain our results. From an implementation perspective, the reduced form approach has gained acceptance amongst practitioners.

2. The model

We now formally outline the default model used in the derivations. The outline follows the assumptions of Duffie (1998) and Kijima and Muromachi (2000). The first part considers the default of a single entity while the second part considers default by multiple entities.

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[4] A comprehensive bibliography of papers on the subject is available at Gupton's web

2.1 Default by a Single Entity

Let t be a stopping time associated with the filtration F_t, and assume that t represents a default time in our setting. The stochastic process $h(t)$ is called the intensity process for t if:

$$P_t\{t \le t + \Delta t \mid t > t\} = h(t)\Delta t \tag{5}$$

for sufficiently small $\Delta t > 0$. That is, the intensity process $h(t)$ is the conditional rate of default just after time *t* given all the information available up to that time. In this respect, we may call $h(t)$ a default process. The cumulative default process is denoted by:

$$H(t) = \int_o^t h(s)ds, \qquad t \ge 0. \tag{6}$$

Since $h(t) \ge 0$ by definition, the cumulative default process $H(t)$ is non decreasing in *t*. Suppose also that the default process $h(t)$ is bounded and that, for each fixed *T*,

$$Y(t) = E_t\left[\exp\left\{-\int_t^T h(s)ds\right\}\right], \qquad t \le T, \tag{7}$$

has no jumps almost surely. Then, it is shown by Duffie (1998) that $Y(t)$ defines the conditional survival probability, i.e.,

$$Y(t) = P_t\{t > T\} = e^{H(t)}E_t[e^{-H(T)}]. \tag{8}$$

It follows that:

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site : www.defaultrisk.com.

$$P\{t > t\} = E\left[e^{-H(t)}\right], \qquad t \geq 0 \text{ .}$$ [5] (9)

For the a constant value of $h(t) = l$ (Poisson Process), the survival probability is defined as:

$$P\{t > t\} = e^{-lt}, \qquad t \geq 0 \tag{10}$$

In addition to the default process assume the existence of a default free bond given by:

$$v_0(t) = E\left[e^{-\int_0^t h_0(s)ds}\right], \qquad t \geq 0\text{ ,} \tag{11}$$

where $h_0(t)$ denotes the default-free spot rate process.

2.1 Default by Multiple Entities

In this article, we are required to consider a scenario wherein there are n credit events and t_j denotes the occurrence of event j, $j = 1,2,....,n$. The corresponding default processes are $h_j(t)$, $j = 1,2,....,n$. In order to price some credit derivatives, we need to consider the joint evolution of these processes. For a deterministic default process $h_j(t)$, the joint survival probability is given by:

$$P\{t_0 > t_0, t_1 > t_1, ..., t_n > t_n\} = e^{\left\{-\sum_{j=0}^{n} H_j(t_j)\right\}}. \tag{12}$$

Therefore we have:

$$P\{t_0 > t_0, t_1 < t_1, ..., t_n > t_n\} = \left(1 - e^{-H_1(t_1)}\right) e^{-\sum_{\substack{j=0 \\ j \neq 1}}^{n} H_j(t_j)} \tag{13}$$

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[5] Note that the class of admissible default processes requires $h(t)$ to be continuous, bounded in any finite interval and satisfy $h(t) \geq 0$ and $\int_0^\infty h(t)dt = \infty$ almost surely.

because t_j are mutually independent. When the $h_j(t)$ are stochastic processes, they are not independent in general. Then the joint survival probability is given by:

$$P\{t_0 > t_0, t_1 > t_1, ..., t_n > t_n\} = E\left[e^{-\sum_{j=0}^{n} H_j(t_j)} \right]. \qquad (14)$$

A special case of interest in our setting is when asset 1 defaults and each of the other assets do not default by a certain time. The probability of this event is given by:

$$P\{t_0 > t, t_1 < t_1, ..., t_n > t_n\} = E\left[\left(1 - e^{-H_1(t_1)}\right) e^{-\sum_{j=0, j\neq 1}^{n} H_j(t_j)} \right] \qquad (15)$$

3. Valuation of a Credit Switch

Given the underlying model of default outlined in the previous section, we derive formulae for the valuation of a credit switch. These formulae are derived for general cases and then a specific parametric model is discussed in Section 4.

3.1 Two Asset Switch

Proposition 1: Given the default processes $h_j(t)$, $j = 0,1,2$ and deterministic payouts on default S_j, a credit-switch between two assets, for maturity T, will involve the purchase of y_{12} units of face value credit protection on asset 1 in exchange for a sale of 1 unit of face value credit protection of asset 2 where:

$$y_{12} = \frac{S_2}{S_1} \frac{E\left[e^{-H_0(T) - H_1(T)} \left(1 - e^{-H_2(T)}\right) \right]}{E\left[e^{-H_0(T) - H_2(T)} \left(1 - e^{-H_1(T)}\right) \right]} \qquad (16)$$

Proof: A credit switch is the simultaneous purchase of credit protection on one asset and the sale of credit protection on another asset. Given the face values of the loans, the payoff to the claim holder is given by:

$$X_2 = F_1 S_1 1_{\{I_F=1\}} - F_2 S_2 1_{\{I_F=2\}} \quad (17)$$

The price of this claim is the expectation of the present value of the payoffs:

$$p_2 = E\left[e^{-H_0(T)} X_2\right] = E\left[F_1 S_1 e^{-H_0(T)} 1_{\{I_F=1\}} \right] - E\left[F_2 S_2 e^{-H_0(T)} 1_{\{I_F=2\}} \right].$$

Because the face values and recovery amounts are constants, we get

$$p_2 = F_1 S_1 E\left[e^{-H_0(T)} 1_{\{I_F=1\}} \right] - F_2 S_2 E\left[e^{-H_0(T)} 1_{\{I_F=2\}} \right]. \quad (18)$$

Most switches are structured so that there is no exchange of funds (or are revenue neutral). Suppose $y_{12} = \frac{F_1}{F_2}$ is the ratio of the face values of loans switched so that there is no exchange of funds. Substituting $p_2 = 0$ in equation (18) above:

$$y_{12} = \frac{S_2}{S_1} \frac{E\left[e^{-H_0(T)} 1_{\{I_F=2\}} \right]}{E\left[e^{-H_0(T)} 1_{\{I_F=1\}} \right]} \quad . \quad (19)$$

Equation (19) is equivalent to (using equations (12) to (15)):

$$y_{12} = \frac{S_2}{S_1} \frac{E\left[e^{-H_0(T)-H_1(T)} \left(1 - e^{-H_2(T)}\right) \right]}{E\left[e^{-H_0(T)-H_2(T)} \left(1 - e^{-H_1(T)}\right) \right]}.$$

A credit switch between asset 1 and 2 will involve the purchase of $y_{12} = \frac{F_1}{F_2}$ units of face value credit protection on asset 1 in exchange for a sale of 1 unit of face value credit protection on a second asset.

3.2 Multiple Assets

In practice a corporation may hold a large amount of asset 1 that is switched in parts amongst several other assets denoted by j=2,…n. Then the payoff to a claim holder is given by:

$$X_n = F_1 S_1 1_{\{I_F = 1\}} - \sum_{2}^{n} F_j S_j 1_{\{I_F = j\}} \qquad (20)$$

The price of this claim is given by (following equation (18) above):

$$p_n = E[X_n] = E\left[F_1 S_1 e^{-H_0(T)} 1_{\{I_F = 1\}}\right] - \sum_{j=2}^{n} E\left[F_j S_j e^{-H_0(T)} 1_{\{I_F = j\}}\right], \qquad (21)$$

When $F_1 = \sum_{j=2}^{n} y_{1j} F_j$, the left hand side of equation (21) is zero and there is no exchange of funds[6]. Therefore a multi-asset switch is no more than a combination of several two asset switches. However, a multi-party switch may achieve a superior credit-loss distribution. The distribution critically depends on both the correlations of the default process between asset 1 and each of the other assets and on the correlations amongst the switched assets themselves.

4. A Parametric Model

We assume that the default processes $h_j(t)$ follow a translated Vasicek (1979) model under the risk-neutral probability measure. The Vasicek model has been extensively used in the modeling of discount bonds and other related applications. Even though it does not

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[6] $y_{1j} = \frac{F_1}{F_j}$ is the units of face value credit protection of asset 1 exchanged for the sale of 1 unit of face value credit protection on asset j (equation (16)).

satisfy some the condition in footnote 4, it is practically useful and easy to apply. Suppose the default processes $h_j(t)$ satisfy the linear stochastic differential equations

$$dh_j = \left(f_j(t) - a_j h_j(t)\right)dt + s_j dz_j(t), \qquad t \geq 0; \quad j = 1,2,....,n, \qquad (22)$$

where $f_j(t)$ are deterministic functions of time t, a_j and s_j are positive constants, and the standard Brownian motions $z_j(t)$ under P are correlated as:

$$dz_i(t)dz_j(t) = r_{ij}dt, \quad i, j = 1,2,....,n. \qquad (23)$$

Therefore the value of the default process at time t is a random variable given by:

$$h_j(t) = h_j(0)e^{-a_j t} + \int_0^t f_j(s)e^{-a_j(t-s)}ds + s_j\int_0^t e^{-a_j(t-s)}dz_j(s), \qquad t \geq 0. \qquad (24)$$

The conditional mean and variance of $h_j(t)$ are given by:

$$h_j(0)e^{-a_j t} + \int_0^t f_j(s)e^{-a_j(t-s)}ds \ \text{ and } \ \frac{s_j^2}{2a_j}\left(1 - e^{-2a_j t}\right). \qquad (25)$$

Recall from equation (6) that:

$$H(t) = \int_o^t h(s)ds, \qquad t \geq 0. \qquad (26)$$

Therefore substituting equation (27) in (29) we get:

$$H_j(T) = \int_o^T \left[h_j(0)e^{-a_j t} + \int_0^t f_j(s)e^{-a_j(t-s)}ds + s_j\int_0^t e^{-a_j(t-s)}dz_j(s) \right]dt \qquad (27)$$

It is well known that $H_j(T)$ is normally distributed[7]:

$$H_j(T) \sim N\left(\bar{h}_j(,T), \bar{s}_j^2(T)\right)$$

where:

[7] Refer to pages 356-358, Karatzas and Shreve (1988), for more details.

$$\overline{h}_j(T) = \int_o^T \left[h_j(0) e^{-a_j t} + \int_0^t f_j(s) e^{-a_j(t-s)} ds \right] dt = \frac{h_j(0)}{a_j}\left(1 - e^{-a_j T}\right) + \frac{1}{a_j}\int_0^T f_j(s) e^{-a_j(T-s)} ds \, , \tag{28}$$

and

$$\overline{s}_j^{\,2}(T) = s_j^{\,2} c_{jj}(T,T) \text{ where } c_{ij}(T,T) = \frac{r_{ij}}{a_i a_j}\left[T - \frac{1 - e^{-a_i T}}{a_i} - \frac{1 - e^{-a_j T}}{a_j} + \frac{1 - e^{-(a_i + a_j)}}{a_i + a_j} \right]. \tag{29}$$

Therefore the sum of the $H_j(T)$ is a normally distributed random variable with mean and its variance-covariance matrix given by:

$$\sum_{j=0}^{n} H_j(T) \sim N\left(\overline{h}_0 + \overline{h}_1 + \ldots\overline{h}_n, \Gamma\right) \text{ where} \tag{30}$$

$$\Gamma = \begin{bmatrix} s_0^{\,2} c_{00}(T,T) & . & . & . & s_0 s_n c_{0n}(T,T) \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ s_0 s_n c_{0n}(T,T) & . & . & . & s_n^{\,2} c_{nn}(T,T) \end{bmatrix} \tag{31}$$

We also need the result that for a zero mean, normally distributed random variable I:

$$E\left[e^{-I}\right] = \exp\left\{\frac{1}{2} V[I]\right\} \text{ where } V(I) = \Gamma\Gamma' \; . \tag{32}$$

Now given the results in equation (30) and (32) we can compute the value of a credit switch between two parties (equation (16)):

$$y_{12} = \frac{S_2}{S_1} \frac{E\left[e^{-H_0(T) - H_1(T)}\left(1 - e^{-H_2(T)}\right)\right]}{E\left[e^{-H_0(T) - H_2(T)}\left(1 - e^{-H_1(T)}\right)\right]}$$

Simplifying we get:

$$y_{12} = \frac{S_2}{S_1}\frac{E\left[e^{-H_0(T)-H_1(T)}\right] - E\left[e^{-H_0(T)-H_1(T)-H_2(T)}\right]}{E\left[e^{-H_0(T)-H_2(T)}\right] - E\left[e^{-H_0(T)-H_1(T)-H_2(T)}\right]} \tag{33}$$

Therefore using (30) and (32) we get:

$$y_{12} = \frac{S_2}{S_1}\frac{e^{-\sum_{j=0}^{1}\bar{h}_j(T)+\frac{1}{2}\sum_{i=0}^{1}\sum_{j=0}^{1}s_i s_j c_{ij}(T,T)} - e^{-\sum_{j=0}^{2}\bar{h}_j(T)+\frac{1}{2}\sum_{i=0}^{2}\sum_{j=0}^{2}s_i s_j c_{ij}(T,T)}}{e^{-\sum_{j=0,2}\bar{h}_j(T)+\frac{1}{2}\sum_{i=0,2}\sum_{j=0,2}s_i s_j c_{ij}(T,T)} - e^{-\sum_{j=0}^{2}\bar{h}_j(T)+\frac{1}{2}\sum_{i=0}^{2}\sum_{j=0}^{2}s_i s_j c_{ij}(T,T)}}. \tag{34}$$ [8]

Equation (34) is the main result of this section.

5. Example

We now analyze a credit-switch via a numerical example that uses the Gaussian model discussed in section 4. In our example there is one primary portfolio with a concentrated holding of an asset and four other smaller portfolios. Table 1 contains parameter values for the numerical example. Assume that the recovery amounts are not identical but depend on the seniority of the debt (these recovery values indicate the nature and the credit rating of the underlying asset).

Table 2 has the individual switch ratios between portfolio 1 and portfolio j=2,3 and 4 obtained by applying the result in equation (33). Note that these switch ratios vary from a minimum of 1.13 to a maximum of 2.33. The primary factors that determine the ratios are the mean default rates and the recovery ratios. Therefore a switch ratio of 1.12 implies a lower default rate and/or a higher priority structure than the asset with a switch ratio of 2.33. In order to get a better fix on the determinants of the switch ratio we analyze the case of a switch between assets 1 and 2 (Table 1).

[8] Note that equation (37) can easily be adapted to the case of a random walk with drift by substituting $a_j = 0$.

Comparative statics on equation (34) give us some added insight:

$$\frac{\partial y_{12}}{\partial S_2} = \frac{1}{S_2} y_{12} > 0 \text{ and } \frac{\partial y_{12}}{\partial S_1} = -\frac{1}{S_1} y_{12} < 0 \qquad (35)$$

Not surprisingly, the switch ratio critically depends on assumptions on payoffs (or recovery amounts in the event of default). The relationship between switch ratios and payouts is graphically depicted in Figure 2. Note that there is almost a linear relationship between payout and the switch ratio.

Other partial derivatives of equation (33) are as follows:

$$\frac{\partial y_{12}}{\partial \bar{h}_1} < 0,\ \frac{\partial y_{12}}{\partial \bar{h}_2} > 0,\ \frac{\partial y_{12}}{\partial s_1} > 0,\ \frac{\partial y_{12}}{\partial s_2} < 0 \text{ and } \frac{\partial y_{12}}{\partial r_{12}} > 0 \qquad (36)$$

The impact on switch rations of each of the underlying parameters in equation (36) are graphically depicted in Figures 3, 4 and 5. Figure 3 shows that a correlation between the default intensities is positively related to the switch ratio. However the change is not very significant (the ratio increases from around 1.12 to 1.13 for the entire domain that is graphed). In contrast the standard deviation and initial default intensities are key determinants of switch ratios. Figure 4 shows that the switch ratio from 1.05 to 1.6 as the standard deviation of the default intensity increases for asset 1. Similarly figure 5 shows an increase in the switch ratio from 0.5 to 2.5 as the initial default intensity increases from 0.3 to 0.21. In summary, the key determinants of switch ratios are the payouts, initial default intensities and the standard deviation amongst the default intensities (note that the correlation is of importance insofar as it has an impact on the credit-loss distributions).

A credit loss distribution provides us with an evaluation of the beneficial impact of a switch. A credit loss distribution provides a graphical overview of losses in the event of default against the ex-ante probability of the default. For evaluating a credit-loss distribution in multi-party switch we also need the correlation coefficients amongst the switched assets themselves. Assume the following sample matrix of correlations between $j = 0,1,2,3$ and 4:

$$\Sigma = \begin{bmatrix} 1 & & & & \\ 0 & 1 & & & \\ 0 & 0 & 1 & & \\ 0 & 0 & 0 & 1 & \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Figure 6 illustrates the risk-neutral loss distribution before and after a credit-switch where a portfolio comprising a notional amount of $100 is equally switched amongst assets 2,3 and 4. The Y axis denotes the risk-neutral probability of losses and the X axis indicates the amount of the loss in the event of default. Before a credit-switch is initiated, there is a high chance of a large loss (22%). A credit-switch reduces that high probability of 22% to less than 1%. However at the same time a switch increases the probability of a loss in the mid-range. Therefore the impact of the credit-switch is to shift probability weight from the tails of this distribution to the center. The introduction outlines the theories and motivations for use of credit switches. As shown here, a redistribution of probability weights results in a loss distribution that provides a better fix on the expected loss. Therefore it is likely that the corporation can make accurate loan loss provisions and provide for better planning by the involved parties.

6. Conclusion

A credit switch is a simultaneous purchase of credit protection on one reference entity and the sale of credit protection on another reference entity, uncorrelated to the first. This paper provides a simple model for valuing this credit derivative whose payoff depends on the identities of the given list of credit events. Closed form solutions are provided when the intensity process follows two popular diffusion processes. Numerical examples illustrate the effectiveness of the credit switch as a risk-management solution. We outline the common motivations for hedging and the manner in which a credit-switch is useful within an overall risk management strategy.

References

Abken, P., "Valuation of default risky interest rate swaps", *Advances in Futures and Options Research* 6 (1993), 931-116.

Black, F. and M. Scholes, "The pricing of options and corporate liabilities", *Journal of Political Economy* 81 (1973), 637-659.

Black, F. and J. C. Cox, "Valuing corporate securities: Some effects of bond indenture provisions", *Journal of Finance* 31 (1976), 351-367.

Breeden, D. and S. Viswanathan, "Why do firms hedge? An asymmetric information model", working paper (1996), Fuqua School of Business, Duke University.

Copper, I. A. and A. S. Mello, "The default risk of swaps", *Journal of Finance* 47 (1992), 597-620.

Das, S., "Credit derivatives", *John Wiley and Sons* (1998).

Das, S. R., "Credit Risk Derivatives", *Journal of Derivatives* Spring (1995), 7-23.

Das, S. R. and P. Tufano, "Pricing credit-sensitive debt when interest rates, credit ratings, and credit spreads are stochastic", *Journal of Financial Engineering* 5 (1996), 161-198.

Duffie, D., "First-to-default valuation", working paper (1998), Stanford University.

Duffie, D. and N. Gârleanu, "Risk and valuation of collateralized debt obligations", forthcoming, *Financial Analysts Journal.*

Duffie, D. and K.J. Singleton, "Modeling term structures of defaultable bonds", *The Review of Financial Studies* 12 (1999), 687-720.

Froot, K., D. Scharfstein, and J. Stein, "Risk management: Coordinating investment and financing policies", *The Journal of Finance* 48 (1993), 1629-1658.

Geske, R., “The valuation of corporate liabilities as compound options”, *Journal of Financial and Quantitative Analysis* 12 (1977), 541-552.

Gupton, G. M., www.DefaultRisk.Com - a resource for credit risk modeling.

Hull, J. and A. White, “The impact of default risk on the prices of options and other derivative securities”, *Journal of Banking and Finance* 19 (1995), 299-322.

Jarrow, R. A. and S. M. Turnbull, “Pricing derivatives on financial securities subject to credit risk”, *Journal of Finance* 50 (1995), 53-85.

Jarrow, R. A., D. Lando, and S. M. Turnbull, “A markov model for the term structure of credit risk spreads”, *Review of Financial Studies* 10 (1997), 481-523.

Karatzas, I. and S. E. Shreve, “Brownian Motion and stochastic calculus”, *Springer Verlag* 1998, New York.

Kijima, M. and Y. Muromachi, “Credit events and valuation of credit derivatives of the basket Type”, *Review of Derivatives Research* 4 (2000), 55-79.

Kim, J., K. Ramaswamy and S. Sundaresan, “Does default risk in coupons affect the valuation of corporate bonds? A contingent claims model.”, *Financial Management* 22 (1993), 117-131.

Lando, D., “Modeling Bonds and Derivatives with Default Risk.” *Mathematics of Derivative Securities*, M. Dempster and S. Pliska, eds. Cambridge, England: Cambridge Univ. Press (1997), 369-393.

Leland, H. E. and K. B. Toft, “Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure and Credit Spreads”, *Journal of Finance* 51 (1996), 987-1019.

Longstaff, F. and E. Schwartz, “A simple approach to valuing risky fixed and floating rate debt.” *Journal of Finance* 50 (1995), 789-819.

Madan, D. and H. Unal, "Pricing the risks of default." *Review of Derivatives Research* 2 (1998), 121-160.

Madan, D. and H. Unal, "A two-factor hazard model for pricing risky debt and the term structure of credit Spreads", *Journal of Financial and Quantitative Analysis* 35 (2000), 43-65.

Merton, R.C., "On the pricing of corporate debt: The risk structure of interest rates". *Journal of Finance* 29 (1974), 449-470.

Nielsen, S. and E. Ronn. "The valuation of default risk in corporate bonds and interest rate swaps." Working Paper (1995), Univ. of Texas at Austin.

Reyfman, A. and K. Toft, "Credit Derivatives- A risk management tool for non-bank corporations", forthcoming, *Risk*.

Smith, C. W. and R. Stulz, "The determinants of firms' hedging policies", *The Journal of Financial and Quantitative Analysis* 28 (1985), 391-405.

Smith, C. W. and J. B. Warner, "On financial contracting: An analysis of bond covenants", *Journal of Financial Economics* 7 (1979), 117-161.

Stulz, R., "Optimal hedging policies", *Journal of Financial and Quantitative Analysis* 19 (1984), 127-140.

Stulz, R., "Managerial discretion and optimal financing policies", *Journal of Financial Economics,* 26 (1990), 3-28.

Vasicek, O., "An equilibrium characterization of the term Structure. "*Journal of Financial Economics,* 5 (1997), 177-188.

Zhou, C. S. "A jump-diffusion approach to modeling credit risk and valuing defaultable securities." Working Paper (1998), Federal Reserve Board.

Table 1
Parameter values for the numerical example.

This table reports parameter values for four defaultable assets and a risk free asset. The recovery rates are assumed to be constant. The conditional mean default rates, correlations and volatility values are listed here.

j	S_j	f_j, a_j	s_j	r_{0j}	$h_j(0)$
0	-	0, 0	0.015	1	0.05
1	1-0.64	0, 0	0.05	0	0.08
2	1-0.64	0, 0	0.06	0	0.09
3	1-0.48	0, 0	0.07	0	0.10
4	1-0.39	0, 0	0.08	0	0.11

Table 2
Credit switch ratios

This table contains switch ratios for a credit-switch between claimholder 1 and parties $j=2,3,4$. The conditional mean default rates, correlations and volatility values are listed in Table 1. The switch ratios are obtained by evaluating equation (34).

j	$y_{1,j}$
2	1.12
3	1.86
4	2.33

Figure 1
A Credit Switch

A credit switch is the simultaneous purchase of credit protection on one reference asset and the sale of credit protection on another reference asset.

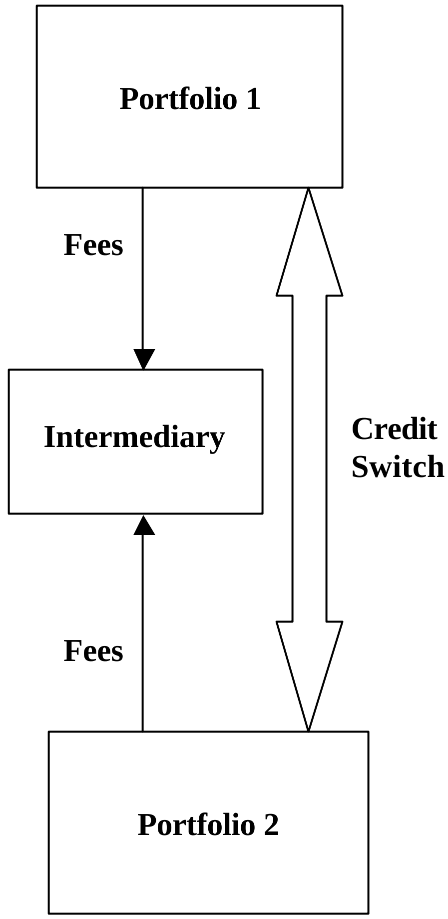

Figure 2
Switch Ratio Vs. Payout

This figure illustrates the impact on the switch ratio $y_{1,2}$ because of a change in the payout on default, S_j. The switch ratio is computed using equation (34) with parameter values given in Table 1. The line denoted "Asset 1" (Asset 2) is computed when the payout from asset 1 (asset 2) is changed and all other parameters are kept constant.

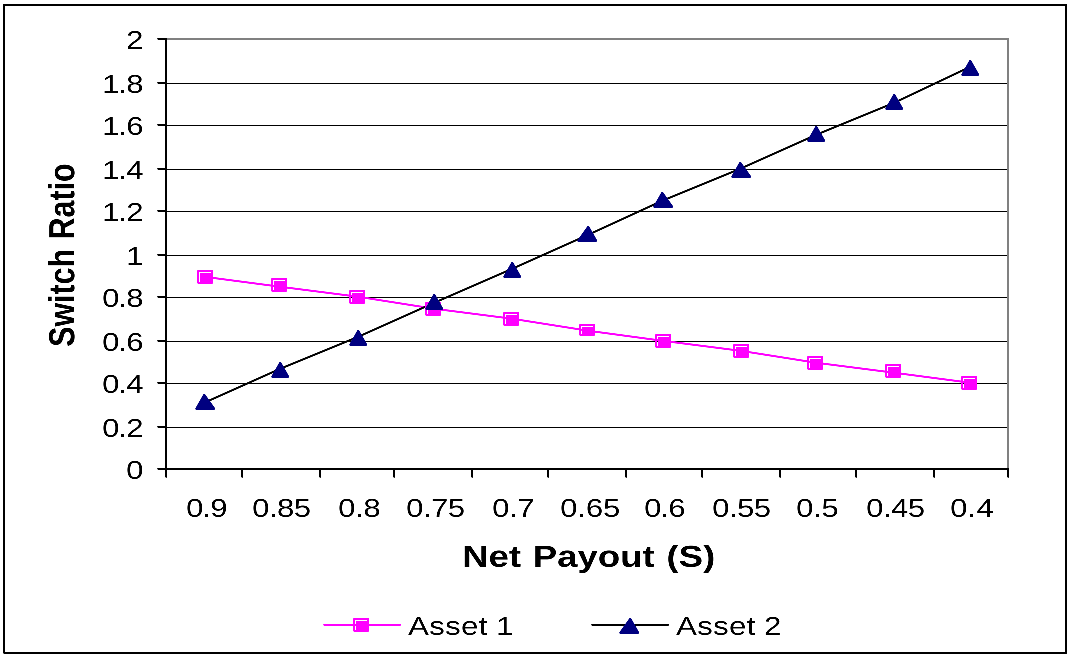

Figure 3
Switch Ratio Vs. Correlation

This figure illustrates the impact on the switch ratio $y_{1,2}$ because of a change in the correlation of default intensities $h_j(t)$ (between assets 1 and 2). The switch ratio is computed using equation (34) with parameter values given in Table 1.

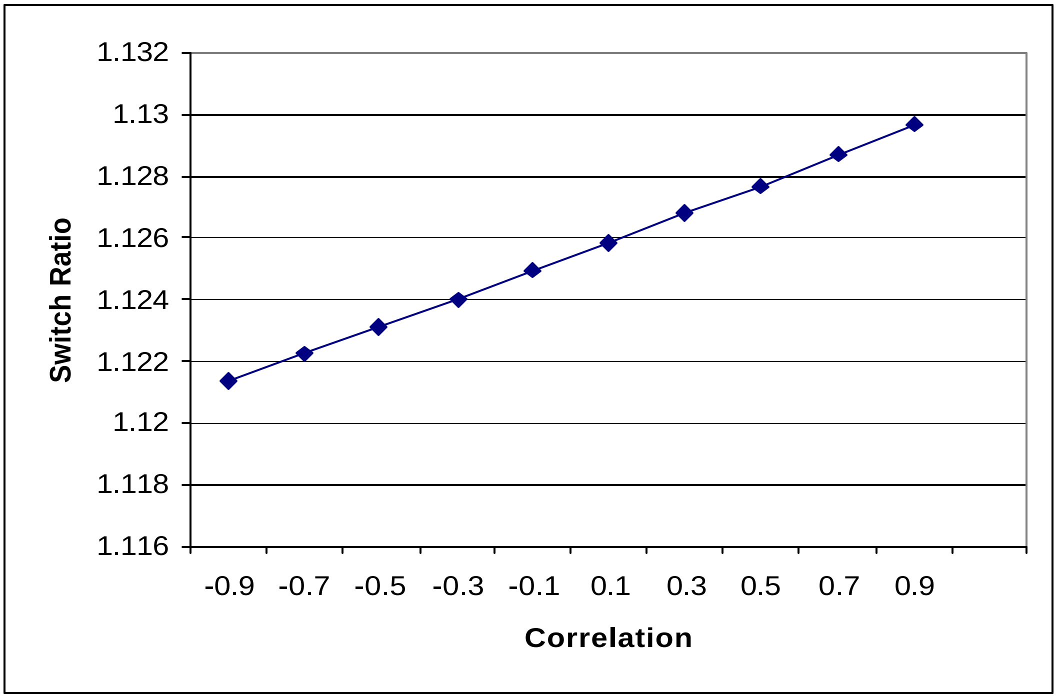

Figure 4
Switch Ratio Vs. Standard Deviation

This figure illustrates the impact on the switch ratio $y_{1,2}$ of a change in the standard deviations of default intensities $h_j(t)$ (for assets 1 and 2). The switch ratio is computed using equation (34) with parameter values given in Table 1. The line denoted "Asset 1" (Asset 2) is computed when the standard deviation for the intensity asset 1 (asset 2) is changed and all other parameters are kept constant.

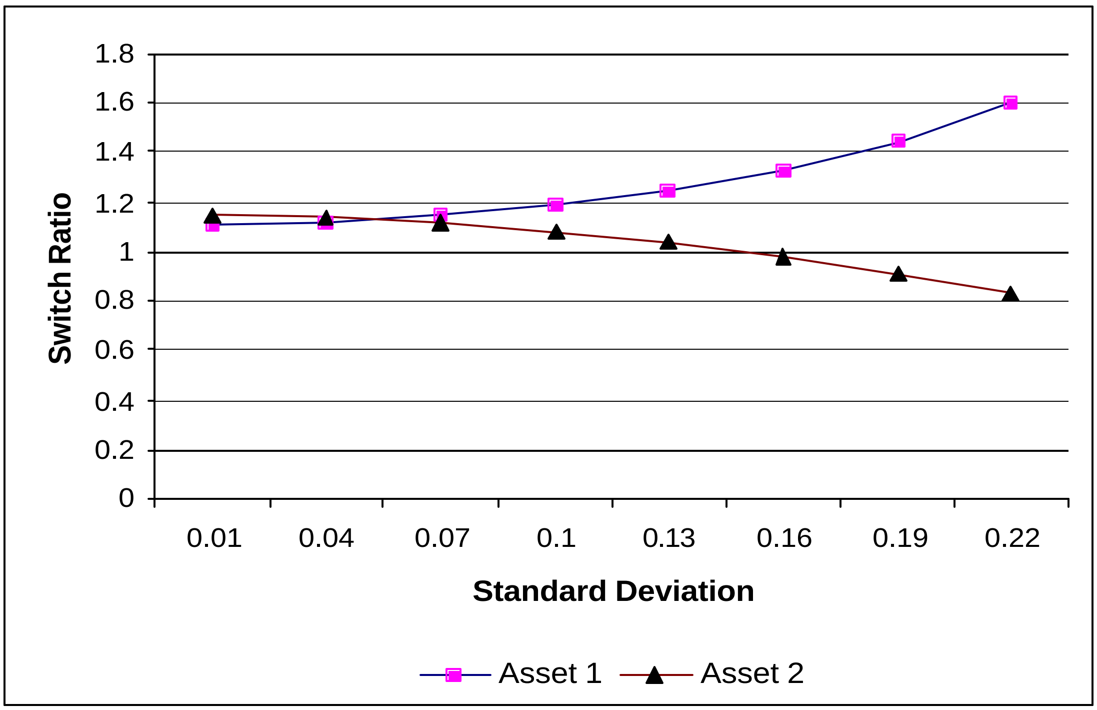

Figure 5
Switch Ratio Vs. Standard Deviation

This figure illustrates the impact on the switch ratio $y_{1,2}$ of a change in the standard the initial default intensities $h_j(0)$ (for assets 1 and 2). The switch ratio is computed using equation (34) with parameter values given in Table 1. The line denoted "Asset 1" (Asset 2) is computed when the initial intensity asset 1 (asset 2) is changed and all other parameters are kept constant.

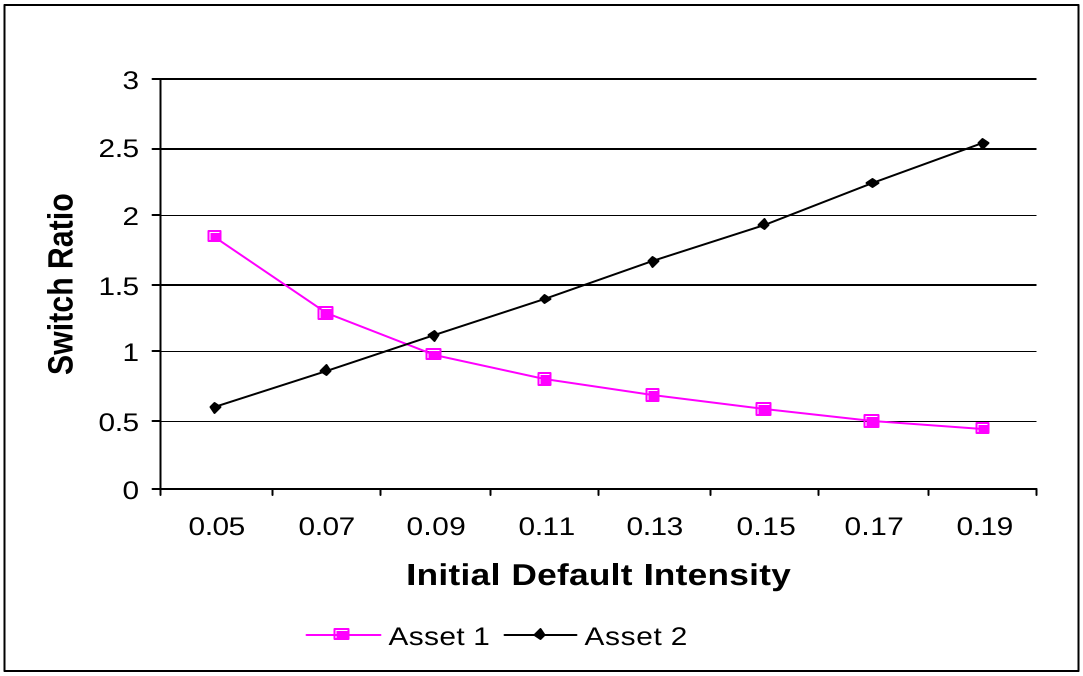

Figure 6
Credit loss distribution

This figure illustrates the change in the risk-neutral credit-loss distribution when claimholder 1 switches a portfolio between parties 2,3 and 4.