On The Pricing of Credit Spread Options: a Two Factor HW-BK Algorithm

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Summary

In this article we describe what a credit spread option (CSO) is and show a tree algorithm to price it. The tree algorithm we have opted for is a two factor model composed by a Hull and White (HW) one factor for the interest rate process and a Black-Karazinsky (BK) one factor for the default intensity. Market data is used to calibrate the model to price an at the money (ATM) European CSO and then tested to price an out of the money (OTM) American CSO on a CDS.

1) Introduction

In recent years the market for credit derivatives has experienced an extremely huge growth. These instruments are being actively used not only for hedging purposes but also as a way to improve return on capital.

A bank might use credit derivatives to manage its portfolio of credit risk. Moreover with a credit derivative a bank can sell credit exposure and still keep a good relationship with an important client. For a bank credit (derivative) instruments have become the most efficient way to transfer credit risk. Moreover its use to optimize the allocation of regulatory capital might be seen as a way to arbitrage the regulators.

There are two main approaches to modelling credit derivatives. In the first approach a credit derivative is modelled as a contingent claim on the assets of the firm. These are the so called firm value models and were initiated by Black Scholes [25] and Merton [26], followed by Longstaff and Schwartz [27] and Das [28] among others. For a comparison among the different models we refer to Eom et alli [1].

The idea behind firm value models is that default occurs when the firm value process reaches a certain boundary (e.g. book value) which might be pre-specified or not. The problem with this

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approach is that the firm value is not a direct observable process. The assumptions of this model underlies the use of the multi-normal distribution function currently in use on the (gaussian) copula approach of basket pricing methodologies (see e.g. Garcia et alli [19]). It is generally thought that firm value models might give a good estimate of the pay-off (loss) distribution function of a process but the associated default probabilities will not necessarily comply with the ones observed in the CDS market. This should explain why the Merton model has been rather successfully used in the KMV commercial package for credit risk. One should also mention that the KMV package uses a very large database of historical defaults to complement the model.

In the second approach, known as intensity based models, the time of default follows a point process with deterministic or stochastic intensity where the jump (to default) time is totally unpredictable. This approach has been followed by Duffie and Singleton[2], Jarrow and Turnbull[3], Jarrow, Lando and Turnbull[4], Sconbucher [5] and Lando[6], among others.

Another way of distinguishing between the two approaches might be seen in the sort of numerical techniques used. In firm value models one finds problems similar to the ones used when trying to extend the BS formula to include American options, dividends, stochastic interest rates and so on. While intensity models are more akin to the term structure modelling problems.[2]

One of the most actively traded credit derivatives is a credit default swap (CDS). A CDS provides insurance in the event of default (called a *credit event*) of a particular company (called the *reference entity*). In this work we show an approximation to the pricing of a CSO option on an a CDS.

In this paper we built on the work of Schonbucher [7]. The additional features in this work are the following: i) we use the HW model for the interest rate and the BK model for the hazard rate process, while Schonbucher uses the HW model for both processes; ii) we discuss a way of calibrating the model; iii) we compare the results of the approximation with market prices for the case of a CSO on CDS and give some comments on the market for CSO's.

Although the model may accommodate a correlation function between the dynamics of interest rates and hazard rates we will consider that both process are not correlated. The fine tuning of a correlation function would add a degree of complexity that we judged undesirable in a first approximation (see Schonbucher [7]). In a recent study JP Morgan has come up with results that support the view that there is a straight correlation between short term interest rates and rates of default [JP Morgan Study].

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[2] This analogy has been taken from Lando [8]

In section 2 we will describe the model used for the credit process. In section 3 we describe a credit spread option (CSO) and how this instrument is related to the risk free and intensity rates processes. The integration of interest and hazard rates in a two factor Hull and White (HW) Black Karazinsky (BK) is briefly described in section 4. Section 5 contains numerical results for an approximation of American CSO on a CDS and section 6 has the comments on the results.

2) The Credit Event Process

Instead of modelling the firm value we will model directly the time of default. We will assume that the default process follows a Poisson process with stochastic intensities (hazard rates). In the literature this is called a Cox process (see Bremaud [13] or Lando [6] for more details).

Consider that Q(t,T) is the cumulative default probability viewed at time t for the period [t, T]. In a Cox process Q(t,T) is given by:

$$Q(t,T) \equiv 1 - \exp\left(- \int_t^T \lambda_t(u) \cdot du \right) \qquad (1)$$

where $\lambda_t(u)$ is the instantaneous forward rate of default at time u viewed at time t (the intensity of the Cox process). If one has Q(t,T) then $\lambda(t,T)$ will be given by:

$$\lambda(t,T) \equiv -\frac{\partial}{\partial T}\left(\ln Q(t,T)\right) \qquad (2)$$

In the following section we show how the hazard rate and the cumulative default probability are used to price a defaultable bond.

3) Credit Spread Options

Credit spread options (CSO) are designed to give protection in case of spread widening. As the bond market is less liquid than the CDS market, instead of buying a put option on a bond one might want to buy a call option on a CDS. One should keep in mind that for the moment CSO's are still rather exotic instruments traded over the counter only. Moreover the contracts are very rich in detail and we will be presenting one sort of structure only.

In the next two sub-sections we describe the CSO on a bond and on CDS.

3.1) Credit Spread Put Options on a Bond

Consider $B_D(t,T)$ the price at time t of a defaultable bond with maturity T. Assume that at time t the bond is being traded at a yield spread of y_t above the yield on a risk free identical bond ($B_{RF}(t,T)$). A credit spread put option with expiry date T_{exp} ($T_{exp} < T$) gives the holder the right to sell the bond for a pre-specified yield spread K (the strike of the option) in case the yield spread y_t goes higher than K.

In what follows we will be considering that at the time of default the bond will be worth a recovery factor (L) multiplied by the notional. Any accrued interest is supposed to be included in the recovery factor. For a description of alternative models of recovery we refer to Schonbucher [15].

Consider that r(s) is the continuous short term interest rate at time s, and as before $\lambda(s)$ is the hazard rate for the given entity at time s seen at time of pricing (for simplicity the second index has been omitted). The price of the defaultable bond is given by[3]:

$$B_D(t,T) = E_t\left[e^{-\left(\int_t^T (r(u)+L\cdot\lambda(u)du)\right)} \right] \quad (3)$$

If one assumes independence between the intensity process and the risk free rate process we have:

$$B_D(t,T) = B_{RF}(t,T)\cdot E_t\left[e^{-\left(\int_t^T (L\cdot\lambda(u)du)\right)} \right] \quad (4)$$

[3] In what follows expectations are taken with respect to the equivalent martingale measure. We have assumed that a lot of technical conditions are observed. In order to have more about those technical conditions we refer the interested reader to Musiela and Rutkovisk [16] in general or to Schonbucher [15] in the specific case of credit derivatives.

For the payoff of a CSO suppose K is the strike spread and the expiry date is at time T_{exp}. The payoff of the credit spread (put) option at the expiry date is given by:

$$CSO_{payoff}(T_{\exp}) = \left[e^{(-K\cdot(T-T_{\exp}))} \cdot B_{RF}(T_{\exp},T) - B_D(T_{\exp},T) \right]^+ \tag{5}$$

where the index (x-y)+ means the maximum between x-y and zero. The price of the option at time t is given by:

$$CSO(t) = E_t\left[CSO_{payoff}(T_{\exp}) \right] \tag{6}$$

where as before E_t is the expectation at time t under the equivalent martingale measure.

3.2) Credit Spread Option on a CDS

A call option on a CDS gives the holder the right to buy a CDS with a certain strike rate K at (or until, depending on the nature of the option) a certain date T_{exp} . With this instrument the buyer acquires the right of buying protection on the default of a general bond (which is detailed in the CDS contract).

In this study we will assume that one can build a CDS synthetically by taking positions in a defaultable and a risk free floating rate note (FRN). An option on a CDS could then be approximated by using the algorithm of the last section taking into account for FRN's instead of bonds. The results of the approximation are discussed in section 5.

A long position on a CDS can be synthetically built by a short position in a defaultable FRN and a long position in a risk free FRN. In order to see it consider the cash flows of each side of the position: in case there is no default the short side will pay the risk free forward rate plus the (CDS) spread, while from the long side one will receive risk free forward, generating a net position of the CDS spread. In case there is default the short side will deliver the recovered value of the defaulted FRN while receiving (in full) the notional, the net value for the short side is the loss in case of default (we refer to Schonbucher [15] for more details).

In this way a call on a CDS rate is identical to a put on a defaultable FRN note, and we use the eq. 5 above just that in place of a bond we have an FRN. In this case the call on the CDS is then given by:

$$CSO_{payoff}(T_{\exp}) = \left[e^{(-K\cdot(T-T_{\exp}))}\cdot FRN_{RF}(T_{\exp},T) - FRN_D(T_{\exp},T)\right]^+ \quad (7)$$

In this work the risk free and the intensity processes will be modelled using a HW and a BK process respectively. In the next section we describe the algorithms.

4) Hull – White and Black - Karazinsky Models

In order to price a credit derivative security one in general needs at least a two-factor model: one for the interest rate and the other for the intensity process. In what follows we first show how to build the risk free and the intensity process separately, then we show how the two are integrated in a three dimensional like tree.

In some cases the assumption of independence between interest rates and intensity makes it possible to de-couple the payoff of a derivative in a way that one would not need to build a tree for the interest rate process. This is not however the most general case. For this reason in what follows we show how to develop a model which involves both a tree for the interest rates and a tree for the intensity process.

The whole algorithm follows very closely the two factor process algorithm described in Hull and White [20]. In the next section we give a brief description of the HW and BK models and how they are integrated in a two factor model for credit derivatives (see Schonbucher [23] for more details).

4.1) The Risk Free Interest Rate Tree

In this article we will assume that the interest rate process will follow the HW model (basically an Ornstein Ullenbeck process [17]). I.e. we assume:

$$dr = \left(\theta_r(t) - a_r \cdot r\right)\cdot dt + \sigma_r \cdot dB \quad (8)$$

where r, a_r and σ_r are the (unobserved) instantaneous short rate, mean reversion and volatility respectively.

The algorithm builds the dynamic of the interest rate process in a recombining trinomial tree structure. For each time step of length Δt the short rate will assume values of the form $r(t+\Delta t) = r(t) + k \cdot \Delta r$, where k might be negative or positive integer and Δr is given by:

$$\Delta r = \sigma_r \cdot \sqrt{3 \cdot \Delta t} \qquad (9)$$

The branching probabilities at the nodes are evaluated by the use of three constraints: the first two moments of the process and the fact that probabilities add up to 1. One more constraint on the whole tree might be added: it may not grow infinitely otherwise probabilities might become negative (this last requirement affects the geometry of the tree which might not grow indefinitely).

The steps in the building of the tree are then the following:

a) suppose in eq. 8 that $\theta_r(t)$ is zero and build a symmetric tree for the r process;
b) evaluate the value of $\theta_r(t)$ to be added at each node such that one might price correctly zero coupon bonds;
c) evaluate the values of a_r and σ_r which would price correctly swaptions or caps/floors or any derivative which might be linked with the securities one needs to price.

The third step above is called calibration. Observe that by construction any values of mean reversion and volatility will lead to the correct prices of zero coupon bonds. The determination of the mean reversion and volatility to be used in each case is done by searching the values of the two parameters which give good prices of market available interest rate options.

In principle one could also use the HW model above proposed to model the evolution of the hazard rate (the default intensity). Indeed this was the approach proposed by Schonbucher [9]. The HW model however does not preclude the values in the tree nodes to become negative[4]. As default intensities are related to default probabilities (see eq.1) negative intensities would lead to negative default probabilities (which is not possible).

In the next section we (briefly) describe the BK tree model.

[4] The HW model is still used by some market participants despite the fact the it might give negative rate (they have indeed very low probabilities).

4.2) The Default Intensity Tree

The default intensity process will obey the following stochastic differential equation:

$$d\ln(\lambda) = (\theta_\lambda(t) - a_\lambda \cdot \ln(\lambda)) \cdot dt + \sigma_\lambda \cdot dB \qquad (10)$$

where as before a_λ and σ_λ are the mean reversion and volatility for the intensity process, λ is the intensity and *ln* is the natural logarithm.

The steps in building the tree are basically the same as described above and we refer to Hull and White [9] for details. There are however two important points worth mentioning.

The first is that (see item b) of 4.1 above) in order to evaluate $\theta_\lambda(t)$ *one needs the cumulative probability of non-default*[5] *curve*. In our case we implied this cumulative probability from the CDS spreads observed at the CDS market. The algorithm used is based on the work of Martin et alli [10] and we refer to Garcia et alli [11] for its use in present valuing CDS's.

The second and not less important aspect is how to calibrate the model. One should remember that by construction any value of a_λ and σ_λ will reproduce the cumulative probability given. As in the interest rate case one still needs the option market to determine the values of the mean reversion and volatility (calibration). In here however there is no liquid option market for credit products. The approach we have used for the calibration is the following: we got from the market a price for an at the money (ATM) and an out of the money (OTM) CSO. The model was calibrated for the ATM and the parameters so determined have been used to price an OTM option. Later on we give a comment about this approach and we discuss an alternative methodology for the calibration (which is described in more details in Garcia et alli [24]).

In the next section we show how the two trees are integrated.

4.3) The Credit Tree (HW + BK)

The new integrated tree (called 3D tree in what follows) has the same number of time steps as the other two trees (called 2D trees). One should make sure that both 2D trees have the

[5] The probability in question is given by 1-Q(0,t) = exp(-λt) and it is the analogous of the discount factor curve used when building the risk free tree.

same time step interval. At each node in the new tree one may go to 9 possible nodes if there is no default or to one node in case of default (10 possible nodes in total).

A node in the 3D tree will be represented by $n_{3D}(x,y,z)$, where the first index represents time step and the other two indexes are such that the interest rate comes from the node $n_{RF}(x,y)$ in the interest rate tree, and the default intensity from the node $n_D(x,z)$ in the intensity tree.

The branching probabilities in the 3D tree are given in table 1 and the default probability in node $n_D(x,y,z)$ is given by:

$$P_{default}(x,y,z) = 1 - e^{-n_D(x,z)\cdot\Delta t} \qquad (12)$$

		Default Intensity Move		
		Up	Middle	Down
Interest Rate Move	Up	$p_{RF}^{up} * p_D^{up}$	$p_{RF}^{up} * p_D^{middle}$	$p_{RF}^{up} * p_D^{down}$
	Middle	$p_{RF}^{middle} * p_D^{up}$	$P_{RF}^{middle} * p_D^{middle}$	$p_{RF}^{middle} * p_D^{down}$
	Down	$p_{RF}^{down} * p_D^{up}$	$P_{RF}^{down} * p_D^{middle}$	$p_{RF}^{down} * p_D^{down}$

Table 1 Branching probabilities in the 3 D tree[6]

From table 1 it is clear we are assuming independence between the two process (see section 6 for a comment about it).

4.4) Using the Tree to price a CSO

In this section we go in more detail on how to price a CSO on a CDS. Consider one wants to evaluate a one-year call CSO with strike K on a six year CDS. As we have already mentioned we will approximate it by pricing a put option on a defaultable FRN. For simplicity consider that the notional of the contract is N, the recovery rate supposed fixed is α. In what follows we will call the interest rate and the default intensity trees as 2D trees.

The steps to be followed are the following:

a) build a tree for the six year risk free FRN. Consider that $n_{RF}(x,y,z)$ and $n_{RF}(x,y)$ represents the node of the 3D and the 2D risk free FRN trees. Then we have:

$$FRN_{RF}(x,y,z) = FRN_{RF}(x,y) \qquad (13)$$

in this way the value of the risk free FRN depends only on the values it has in the 2D tree nodes;

b) build a tree for the defaultable FRN (represented as FRN_D). Consider that T_{FRN} is the maturity of the FRN_D (in our case $T_{FRN} = 6$). At time T_{FRN} we have:

$$FRN_D(x_{T_{FRN}}, y, z) = N + C, \qquad \forall y, z \qquad (14)$$

where x_{TFRN} is the time step corresponding to time T_{FRN} (the maturity date of the FRN). C is the coupon of the floater. The remaining nodes in the tree are calculated by backward induction as follows:

$$FRN_D(x_n, y, z) = (e^{-\lambda(x_n,z)\cdot\Delta t}) \cdot \Big(\sum_{i,j} \left[p(x_n, y_i, z_j) \cdot FRN_D(x_{n+1}, y_i, z_j) \cdot e^{-r(x_n,y)\cdot\Delta t} \right] + I(x_n) \cdot C \Big)$$
$$+ (1 - e^{-\lambda(x_n,z)\cdot\Delta t} \cdot \alpha \cdot N) \qquad (15)$$

where $\lambda(x_n,z)$ and $r(x_n,y)$ are the values of the default intensity and risk free rate at nodes (x_n,z) and (x_n,y) in their respective trees. The $p(x_n,y_i,z_j)$ is in fact a short cut notation to the following more cumbersome notation $p((x_n,y,z)|\ (x_{n+1},y_i,z_j))$ which means the probability in node (y,z) at time x_n of going to node (y_i,z_j) at time x_{n+1}. $I(x_n)$ is the indicator function which is 1 if there is a coupon payment at time x_n and 0 otherwise. The first factor in the eq. 15 is the value of the bond in case there is no default while the second factor gives the value in case of default.

[6] As will be seen later on when pricing each probability in the table will still be multiplied by the probability of non-default.

c) build a tree for the CSO. Consider that T_{CSO} ($T_{CSO} < T_B$) the expiry time of the option and that this corresponds to node x_{CSO} in the 3D tree. Assuming that the strike in the option is K at the expiry of the (put) option we have that:

$$CSO(x_{T\,cso}, y, z) = \left[e^{-K\cdot(T_B - T_{cso})} \cdot FRN_{RF}(x_{T\,cso}, y, z) - FRN_D(x_{T\,cso}, y, z)\right]^{+} \qquad (16)$$

where as before the + sign means the maximum between 0 and the value between

$$CSO^{eur}(x_n, y, z) = (e^{-\lambda(x_n,z)\cdot\Delta t}) \cdot \underset{i,j}{} \left[p(x_n, y_i, z_j) \cdot CSO(x_{n+1}, y_i, z_j) \cdot e^{-r(x_n,y)\cdot\Delta t}\right]$$
$$+ (1 - e^{-\lambda(x_n,z)\cdot\Delta t} \cdot \alpha \cdot N) \qquad (17)$$

brackets. Assuming that in case of default of the FRN_D the option holder gets the recovery rate on the FRN then the remaining nodes in the tree are calculated as:

for the case of an European option. If the option is American (or Bermudan) we have:

$$CSO^{am}(x_n, y, z) = (e^{-\lambda(x_n,z)\cdot\Delta t}) \cdot \max(\quad \left[p(x_n, y_i, z_j) \cdot CSO(x_{n+1}, y_i, z_j) \cdot e^{-r(x_n,y)\cdot\Delta t}\right],$$
$$e^{-K\cdot(T_B - T_{cso})} \cdot FRN_{RF}(x_{cso}, y, z) - FRN_D(x_{cso}, y, z)) + (1 - e^{-\lambda(x_n,z)\cdot\Delta t} \cdot \alpha \cdot N) \qquad (18)$$

In the above formulation one simplifying assumption has been made: the option premium is paid up front. We also decided to count the defaultable coupon as being the risk free forward rate plus the spread. A simplifying assumption is that the risk free forward rate will be evaluated using the current yield curve (it avoids the option to become path dependent). In this way the coupon of the FRN_D is given by:

$$C(t_n) = (f(0, t_{n-\&}, t_n) + K) \cdot (t_n - t_{n-1}) \qquad (19)$$

where t_{n-1} and t_n are the dates when the coupon rate is determined and paid respectively; $f(0,t_{n-1},t_n)$ is the forward rate observed at time zero for the period between t_{n-1} and t_n, and K is the CDS strike rate in the option.

In the next section we give results of the pricing of (Bermudan) CSO on a CDS with the model being calibrated to market prices.

5) Results

In this section we show the results of pricing a (Bermudan) CSO on a CDS. As already mentioned we have calibrated the HW interest rate tree for swaptions and then calibrated the intensity (BK) tree to an ATM American CSO and both prices have been taken from a market participant.

The mean reversion and volatility for the interest rate tree are respectively 0.012 and 0.009. The risk free discount curve of the day is shown in table 2.

Date	Discount Factor
16/8/2001	1.0000
17/08/2001	0.9998747
20/08/2001	0.9994978
27/08/2001	0.9986182
3/09/2001	0.9977382
20/09/2001	0.9956654
22/10/2001	0.9919294
22/11/2001	0.9886339
20/2/2002	0.9785173
20/05/2002	0.9695015
20/8/2002	0.9600748
20/08/2003	0.9205583
20/08/2004	0.8793688
22/08/2005	0.8371468
21/08/2006	0.7947228
20/08/2007	0.7521805
20/08/2008	0.7102778
20/08/2009	0.6695355

Table 2 Risk Free Discount Factors Used in the Evaluations

In order to build the BK intensity tree one will need to imply default probabilities. In table 3 we show the CDS credit spreads. The implied default probabilities are shown in table 4.

Time (yr)	1	2	3	4	5	6	7	8	9	10
Spread (bp)	175	241	264	276	285	293	300	302	304	306

Table 3 CDS Rates Used in the Determination of the Default Probability Curve

Date	Default Probability
16/08/2001	0
16/08/2002	0.0342510
16/08/2003	0.0920711
16/08/2004	0.1474603
16/08/2005	0.1996514
16/08/2006	0.2505164
16/08/2007	0.3004684
16/08/2008	0.3487388

Table 4 Cumulative Default Probability Curve

The prices for the ATM and the OTM CSO got from the market and from the tree are shown in table 5. Both market prices are the bid prices[7] by a large financial institution.

[7] The CSO's are very exotic instruments traded only OTC. Market players are very reluctant to show information like bid offer spread unless they are very sure that the deal will be done. It took some time until a market player gave the quotes used in this study.

Strike (bp)	Market Price (bp)	HWBK Price (bp)
285 (ATM)	160	-
340 (OTM)	105	135

ATM = at the money(used for calibration), OTM = out of the money

Table 5 CSO prices used in evaluations

As it can be seen from table 5 the bid price from the model for the OTM CSO is considerably higher than the one given by the market. We see several possible reasons for it: a) bid prices are normally lower than fair value specially in such an illiquid instrument (the bid ask spread on the underlying CDS at the day of the quote was 30 bp); b) error in the approximation used in the evaluation of the FRN_D (the risk free part of its coupon); c) overvaluation by the model.

A problem related with credit derivatives market is that there are still large differences on the CDS rates given by banks. Although a trader has access to quotes via CDS web sites of some large banks they are for information purpose only. Moreover the CDS curve used for implying default probabilities might differ considerably from bank to bank. We should still mention that the algorithm we have used to imply default probabilities did not take into account that there are problems of liquidity among the quotes. For example the 5 year CDS rate is (normally) a lot more liquid than any other one in the table (see table 3). The degree of uncertainty on the other rates is rather large.

Even if the calibration step is in principle sound in practice it might also be put into question. We had only one ATM CSO available. This is linked to the difficulty in getting market quotes. Alternatively instead of getting market quotes (to be used for calibration) one could use the Black Scholes formula (see Schonbucher [12]) for european CSO's and calibrate the model to it. As one might expect, the BS model depends on default intensity volatilities. Those volatilities are not yet available in the market. In an internal study (not yet available for publication) we have implied default probabilities from historical CDS curves and from them we evaluated historical intensity default volatilities. The volatilities are then used in the European CSO formula and compared with market quotes. Again we observed differences of the order observed in this work.

Two other problems we have not mentioned yet are the uncertanties in the recovery rates and the correlation factor between the interest rate process and the intensity process. In this work we have supposed that correlation is zero (see Schonbucher [23] for details). In a recent study of JP Morgan it is shown that there a high correlation between short term rates and the rate of default. The impact of this new finding is an object of future research.

Our conclusion is that although the model might give a rough indication of where the price might be more research should be done in order to get it fine tuned. Absence of reliable data, be it the CDS curve or intensity volatilities, might be a big issue.

6) Comments

In this paper we have developed a two factor tree model to price Bermudan put CSO's on a CDS. Some approximations are done in considering the CSO as options on the spread between a defaultable and a risk free FRN. The model uses a HW tree for the interest rate dynamics and a BK tree for the intensity dynamic.

The HW tree has been calibrated to swaption prices.

In the construction of the BK tree we have used a default probability curve which was implied from observed market CDS rates at the time of pricing. Once the BK tree is constructed it is then calibrated to an (ATM Bermudan) put CSO market quote. The parameters so determined are then used to price an (OTM Bermudan) put CSO.

Although we have observed that the model overprices the bid price of the OTM option we can not say that the price is out of the bid offer spread. The CSO market is still very illiquid and prices might vary considerably from bank to bank. Several problems will be met when trying to model a CSO of which we will mention three: a) lack of reliable intensity volatilities or even CDS rates; b) uncertainties about recovery rates; c) uncertainties about correlation parameters between interest rate process and intensity process. The way these uncertainties affects the price of a CSO will continue an area for future research.

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