DEVIL IN THE PARAMETERS

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ABSTRACT

Several different credit portfolio models have been recently developed, and proposed for use as tools in measuring and managing credit risk. These models differ in both their calculation techniques and their parameters. Koyluoglu and Hickman (1998) reveal that all of the proposed models share the same underlying intuition, and their calculation techniques yield very similar results if the estimates for their input parameters are harmonized. This paper studies the effect of parameter inconsistency by employing parameter estimates for three commercially available software packages – JP Morgan's *CreditMetrics*, KMV's *Portfolio Manager* and CSFP's *CreditRisk+* – which are generated from their "natural" data sets for *CreditMetrics* and *Portfolio Manager*, and from historical default rate volatility for *CreditRisk+*. Two single factor / single parameter models are also included for comparison. The results demonstrate that, under these conditions, the models produce significantly different results for identical portfolios, at both the aggregate and contributory risk levels. Obviously, such differences imply different recommendations for credit risk management, risk-based pricing and portfolio optimization. In combination with the earlier Koyluoglu / Hickman paper, the results of this study suggest that users seeking to select a model should focus more on the comparative quality of a model's parameter estimates than the nature of its calculation procedures**.**

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CONTENTS

I. INTRODUCTION

Proposed credit risk portfolio modeling techniques range from microeconomic models (e.g. JP Morgan's *CreditMetrics*[1], KMV's *PortfolioManager*[2]) to top-down macroeconomic and actuarial models which estimate risk directly for a sub-portfolio of borrowers (e.g. Credit Suisse Financial Product's *CreditRisk+*[3]). The models use different calculation procedures, and use different parameters to quantify joint-default behavior of obligors. For example, while joint-default is determined from the asset correlations between companies in *CreditMetrics* and *PortfolioManager, CreditRisk+* quantifies joint-default using default rate volatilities.

Recently Koyluoglu and Hickman (1998) introduced a generalized credit risk portfolio framework, which places credit risk modeling techniques from three different sciences – Finance (Merton-based[4]), Econometrics and Actuarial Science - into a single framework. A single framework is very helpful in reconciling the differences among these techniques in a structured fashion as it facilitates "apples-to-apples" comparisons. There are three main components in the generalized framework:

- Treatment for joint-default behavior
- Conditional default distribution for independent defaults
- Aggregation / convolution

Modeling differences in the latter two components were found to be immaterial. A deeper analysis of the treatment of joint default behavior revealed that Merton-based (e.g. *CreditMetrics, PortfolioManager*), Econometric (e.g. *CreditPortfolioView*[5]) and Actuarial (e.g. *CreditRisk+*) models in fact have a remarkable consensus in the underlying intuition. All of the models explicitly or implicitly assume that default rates vary over time, intuitively as a result of varying economic conditions; when conditions are favorable, fewer borrowers default, and vice versa, as shown in Figure 1. In addition to Koyluoglu and Hickman's study, other research papers[6] show that Merton-based and Actuarial modeling techniques are similar in apples-to-apples comparisons.

These common grounds to model default risk[7] facilitate parameter harmonization[8] - a means of calibrating joint-default parameters across different models to achieve

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[1] see Gupton, Finger and Bhatia (1997), see web site of JP Morgan

[2] see Vasicek (1987) and Kealhofer (1997), see web site of KMV

[3] see web site of Credit Suisse Financial Products

[4] see Merton (1974)

[5] see Wilson (1997)

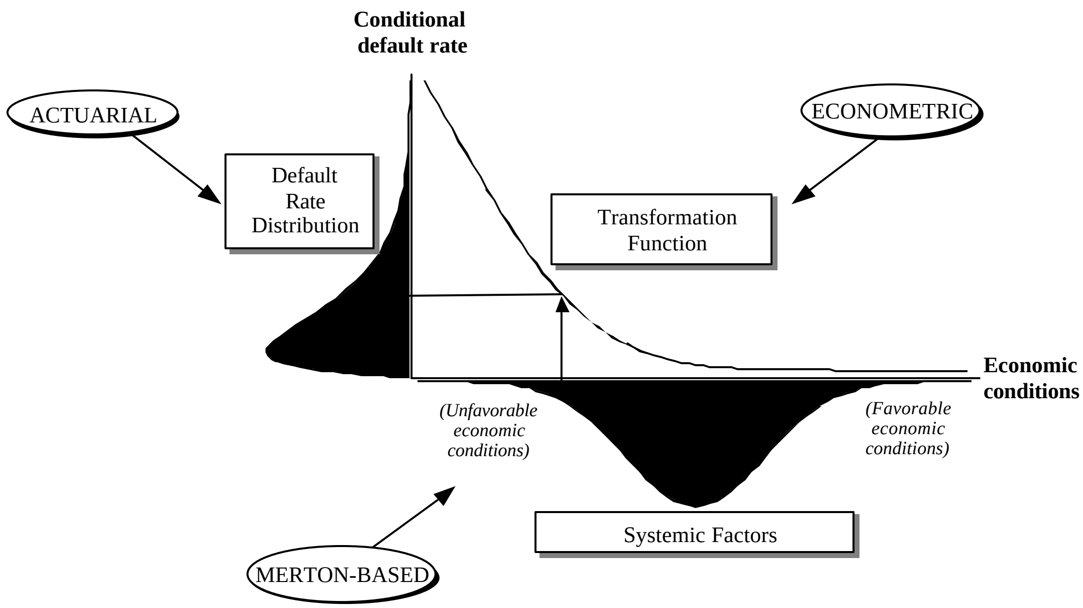
[6] Gordy (1998) explains the similarity in mathematical structure and runs simulation exercises to compare the techniques. Finger (1998) points out that these techniques model the same phenomenon in an intuitive story, then illustrates the similarity in cumulative conditional default probability

[7] This paper, as well as other research papers (Gordy (1998), Finger (1998), Koyluoglu and Hickman (1998)), examine only the default component of portfolio credit risk, and concentrate on loss distribution, not the change in value distribution. In a value framework, one should also incorporate ratings migration and changes in credit spread

[8] Koyluoglu and Hickman (1998) give the harmonization equations for homogeneous sub-portfolios. For non-homogeneous portfolios, harmonization is not trivial, one could apply different criteria to achieve parameter consistency

parameter consistency. If input parameters are harmonized in this way, the different credit portfolio models will yield the same result at the standard deviation level, and give similar results at the tail of the loss distribution.

FIGURE 1: CONDITIONAL DEFAULT RATE AS A FUNCTION OF ECONOMIC CONDITIONS



In practice, however, parameter inconsistency is not trivial to overcome. An end-user might end up with different risk results when the same portfolio is analyzed using such dissimilar models parameterized with their "natural" datasets[9]. This paper studies the effect of parameter inconsistencies on both the portfolio and contributory risk results using different applications of the Merton-based and Actuarial techniques. The analysis primarily focuses on the magnitude and nature of differences when sample portfolios with different risk and exposure characteristics are analyzed using three different commercial software packages - *CreditMetrics, PortfolioManager* and *CreditRisk+,* and two simple closed form models – a Markowitz's approach and a simplified Merton-based model with homogeneous correlation structure. In this study, we use parameter estimates for each model based on its "natural" data set – rather than intentionally harmonizing the parameters. The portfolio level comparisons are based on Expected Loss, Unexpected Loss and Economic Capital defined at various solvency standards. Contributory risk comparisons are performed by analyzing the numerical differences in Unexpected Loss Contributions which are attributed to individual exposures.

In addition to parameter inconsistencies, model mis-specification may be another source of difference. For instance, Merton (1974) model assumes that a firm defaults when its asset value falls below its liabilities. Its return and causality assumptions, which are used to relate asset correlations to default correlations may not be appropriate for all kinds of obligors leading to possible model errors at the Unexpected Loss and Economic Capital levels. Likewise, the Gamma distribution assumption for the default rates in Actuarial technique might not be accurate at the

[9] Several papers compare portfolio level risk results for sample portfolios using commercial software packages, e.g. ISDA (1998), Crouhy and Mark (1998)

tail of the default rate distribution for all sectors. Note however that Actuarial technique is model error proof at the Unexpected Loss level for a well diversified portfolio, as the standard deviation of default rates can be directly estimated from the default rate time series. Parameter inconsistencies across models are likely to dominate the differences at the Unexpected Loss level, while model specification inaccuracies could be more apparent at the tails of the loss distribution.

The rest of the paper is organized in five further sections. Section II describes the parameters of Merton-based and Actuarial techniques, and reviews applications of these at different levels of granularity in terms of joint-default parameter specification. Possible inconsistencies among models are identified to construct a structure for the sample portfolio runs. Section III explains the sample portfolio runs. Sections VI and V analyze the results. Section VI presents conclusions.

II. PARAMETERS AND PARAMETER INCONSISTENCIES

The fundamental parameter in default risk analysis is the default probability, also called expected default frequency (EDF). The expected loss (EL) of an individual loan is given by the product of three variables: EDF, Severity of loss in the event of default (SEV=1-Recovery Rate), and exposure level (EXP). The portfolio level Expected Loss (ELp) is given by the simple sum of the expected losses of loans. The ELp represents the mean of the credit default loss distribution for a portfolio of exposures.

Assuming that EDF, SEV and EXP are independent random variables, and their standard deviations are known, the standard deviation of loss for an individual loan, also called Unexpected Loss (UL), can also be calculated. Unexpected Losses, however, are not simply additive unless all risks are fully correlated. In a simple Markowitz framework, default/loss correlations are the parameter estimates needed to calculate the Unexpected Loss of the portfolio (ULp). Merton-based and Actuarial modeling techniques use different parameters - respectively asset correlations and default rate volatilities – as the basis for modeling joint default behavior. These three dissimilar parameters contain equivalent information, thus can be related to each other[10]. Table 1 below illustrates the relationships between EL and UL at both the individual loan and portfolio level.

TABLE 1: EXPECTED LOSS, UNEXPECTED LOSS CALCULATIONS

INDIVIDUAL COUNTER-PARTY	***PORTFOLIO OF N LOANS***
$EL_i = EDF_i \cdot SEV_i \cdot EXP_i$	$EL_p = \sum_{j=1}^{N} EL_j$
$UL_i = \sqrt{EDF_i (1 - EDF_i)}\, SEV_i \cdot EXP_i$ For simplicity, SEV and EXP are assumed to be deterministic in the above formula	$UL_p = \sqrt{\sum_{j=1}^{N}\sum_{k=1}^{N} UL_j UL_k \rho_{jk}}$ where ρ_{jk} is the default correlation between j^{th} & k^{th} loans

This paper will evaluate parameter inconsistency across models arising from inconsistencies in assigning EDFs and inconsistencies in assigning joint default parameters.

[10] The mapping from asset correlation to default correlation is explained in detail at several papers, see e.g. Kealhofer (1995), Gupton, Finger, Bhatia (1997). Koyluoglu and Hickman (1998) give the harmonization equations across Merton-based (asset correlation), Actuarial (default rate volatility), Econometric (regression coefficients) and Markowitz (default correlation) frameworks for homogeneous sub-portfolios

II. 1. EDF Inconsistency

The creditworthiness of a company is related to its financial health, which is reflected in its financial statements, the level and volatility of its market price (if it is publicly traded), and other quantitative and qualitative information. EDF quantifies the default probability of an obligor, most typically over a one-year period. EDF of an obligor is not a directly observable quantity. In practice, there are several techniques for determining the EDF of an individual obligor:

- Using a Merton-based model of default (as in KMV's *CreditMonitor* and *PrivateFirmModel*) to directly estimate EDF
- Using externally available grades assigned to the obligors by rating agencies such as Moody's and / or S&P, and calibrating them to EDF via historical default risk tables published by the rating agencies
- Mapping an internal grading system to EDFs using historical default experience
- Using results generated from commercially available scoring models (e.g. Zeta Services and Alcar) along with their calibration to EDF

An end-user might estimate a different EDF for the same obligor using these four different approaches. Small differences might accumulate or wash out at the portfolio level. This is clearly a fundamental issue, which is investigated in Section V. 2.

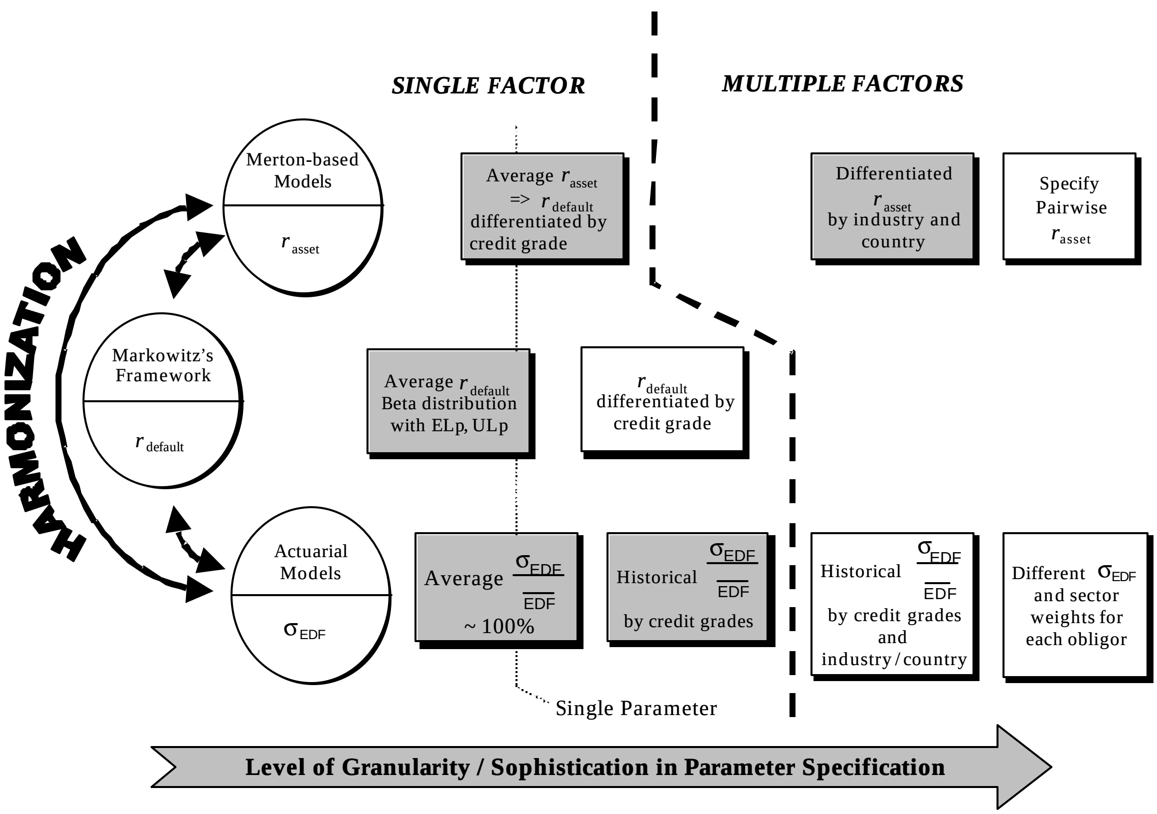
II. 2. Inconsistency in Joint-default Parameters Within a Modeling Technique

There are two distinct sources of parameter inconsistency within a modeling technique: numerical estimation procedure, and granularity of parameter specification. As an example, equity correlation estimates, which are used in Merton-based models to estimate asset correlations, vary with the length / duration of the equity time series, and the time step size used. Although statistical theory indicates that using more data is better as long as the data is clean and relevant, there is no theoretical rule for the selection of exact time step. Moreover, asset correlations may vary from time to time, increasing significantly during a crisis. Clearly, asset correlation estimates used within the available versions of Merton-based commercial software will vary due to differences in correlation estimation procedures.

The other important source of inconsistency arises due to differences in granularity of parameter specification for both Merton-based and Actuarial models. Figure 2 shows several applications of the models highlighting their differentiation in sophistication / granularity of parameter specification. Merton-based models follow Merton's model of a firm's capital structure: a firm defaults when its asset value falls below its liabilities. A borrower's default probability then depends on the amount by which assets exceed liabilities, and the volatility of those assets. Joint default events amongst borrowers in the portfolio are related to the extent that the borrowers' changes in asset value are correlated. The asset correlations of Merton-based models can be set equal to an average correlation for all assets in a single factor / single parameter approach which yields default correlations differentiated by EDF or credit grade. Asset correlations can be further differentiated by industry and country, and even further refined to the obligor level. Both *CreditManager* and *PortfolioManager* use this increased level of granularity – however, asset correlations for obligors / segments still differ between these two models due to differences in parameter estimation procedure as discussed above, and industry / country segmentation.

Figure 2 also illustrates the spectrum of choices in the modeling of default rate volatility at different granularities. Actuarial models capture joint default behavior through the use of default rate volatilities and sector weights. The volatilities of default rate exhibit a strong relationship to the unconditional default rates. Therefore, the ratio of default rate volatility to the unconditional default rate (the coefficient of variation) is usually estimated instead of the volatility itself. In the simplest case, a uniform coefficient of variation is taken for the default rate. For better accuracy, the default rate volatility can be differentiated by credit grades. Additional granularity can be obtained by slicing the default data by country and / or industry, or assigning differentiated volatilities at the obligor level.

FIGURE 2: SOPHISTICATION/GRANULARITY IN PARAMETER SPECIFICATION (THE TYPES OF MODELS STUDIED ARE SHADED BELOW)



II. 3. Inconsistency in Joint-default Parameters Across Different Modeling Techniques

The parameter inconsistency across different modeling techniques is a tricky issue. Basically, asset correlations in Merton-based model, default rate volatility estimates in the actuarial model and default correlations in the Markowitz approach should indicate the same joint-default information. Harmonization equations allow us to derive implied parameters of an approach from the parameters of another approach. When the directly estimated parameters of one approach is compared to the implied parameters obtained from another approach, they usually don't match. This inconsistency stems from the use of totally different data sets to estimate parameters, e.g. default rate time series is used to estimate default rate volatility, equity/index time series is used to estimate asset correlations. If the naturally estimated parameters are similar to the implied ones, a practitioner will have more confidence in the model outputs. If these are different, then a sophisticated practitioner will have the opportunity to select the more reliable set of parameters. In this respect, in addition to experience-based judgement, the smallest standard error in the estimates[11] could be used as a criterion. Parameter inconsistencies across different modeling techniques require further research.

Clearly, joint-default parameter inconsistencies exist within and across modeling techniques. The degree to which these differences affect the risk results is investigated in Sections IV and V.

[11] e.g. see Stuart and Ord for more information on standard error of correlation / parameter estimates

III. SAMPLE PORTFOLIO RUNS

We have run sample portfolios with different risk and exposure characteristics using five models: *CreditManager*, *PortfolioManager*, *CreditRisk+*, a simplified Merton-based model with homogeneous asset correlation structure, and a simple Markowitz approach with homogeneous default correlation structure. These five models are henceforth referred to as *CM, PM, CR+, SMM* and *SMA* respectively. As explained earlier, we examine only the default component of portfolio credit risk, and concentrate on loss distribution, not the change in value distribution. For that reason, all the value / credit migration related features are turned off. Both *SMM* and *SMA*, assume that the portfolio loss distribution follows a Beta distribution. The sample portfolio runs are designed only to illustrate the issues identified in the previous section, not to make general statements about the performance of the models. *The direction and magnitude of differences in the risk results across models should not be assumed true for all portfolios.*

III. 1. Structure of Portfolio Runs

III. 1. A. Portfolio Composition

We have constructed two portfolios that represent two different credit-quality regimes - a high quality portfolio (to represent an investment grade, large corporate book) and a low quality portfolio (to represent a book of sub-investment grade middle market loans). The first portfolio is constructed with 180 obligor names that constitute an overlap of the borrower coverage of CM and PM12. The selected portfolio represents a large corporate loan portfolio of high credit-quality with an average EDF of 20 basis points, as measured by KMV's CreditMonitor. Most of the obligors (46%) are U.S.-based; there are also a significant amount of Japanese (13%), English (12%) and Canadian (11%) companies in the first portfolio. The second portfolio is constructed of lower grade obligors selected from KMV's database with an average EDF of 240 basis points. The obligors in the low credit-quality portfolio spanned six different industry segments and eight countries, such that CM data set matched an equity index for each of the industry / country combination represented in the portfolio. This made the selection of each obligor's industry and country weights a straightforward process for running the *CM* model. The combined portfolio with 360 obligors[13] has an average EDF of 128 basis points. Portfolio mix in terms of Moody's grades is given in Table 2.

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[12] It should be noted that *CR+* does not provide any of the input parameters, hence it does not constrain data selection

[13] Although 360 obligors is a small number compared to the numbers in a large commercial bank's loan book, we think that it is sufficient to illustrate the issues studied in this paper. Obviously, though, there is still room for diversification - the exposure normalized portfolio risk results presented in this paper can further be reduced as the number of obligors is increased

TABLE 2: PORTFOLIO MIX IN TERMS OF CREDIT GRADE

MOODY'S GRADE	***HIGH QUALITY PORTFOLIO***	***LOW QUALITY PORTFOLIO***	***COMBINED PORTFOLIO***
Aaa	15	0	15
Aa	46	0	46
A	66	0	66
Baa	30	41	71
Ba	17	98	115
B	6	41	47
TOTAL	**180**	**180**	**360**

III. 1. B. Joint Default Parameters

For *CM* and *PM*, we used the asset correlations specified in the Version 1.0 for *CM* and Version 4.32a for *PM.*

For *CR+* we used three base assumptions for default rate volatility, which attempt to mirror the choices which might be made by a potential user:

1. Using a default rate volatility equal to default rate (default σ = default μ). "All Corporate" default experience from 1970-1995 tabulated by Carty & Lieberman (1995) reports an unconditional default rate of 116bp and a standard deviation of default rate of 90bp. This implies a coefficient of variation of default rate to be 78 %. The same reference shows that coefficient of variation increases with higher credit quality. Incorporating the fact that the higher credit quality companies have less weight in the "All Corporate" default experience, we take a uniform coefficient of variation of 100 % for the least granular case.
2. Using differentiated coefficient of variation of default rate by credit grade as given in Table 3. Moody's Grades are mapped to coefficient of variation of default rates based on historical default rates published in Carty and Lieberman (1995) and extrapolations for the better credit grades.

TABLE 3: COEFFICIENT OF VARIATION OF DEFAULT RATE DIFFERENTIATED BY CREDIT GRADE

MOODY'S GRADE	*s/m*
Aaa-Baa1	300 %
Baa2	240 %
Baa3	170 %
Ba1	130 %
Ba2	100 %
Ba3	80 %
B1	73 %
B2	63 %
B3	55 %

3. Setting the coefficient of variation of default rate (σ/μ) equal to a forced constant *k*, such that ULp calculated by *CR+* matches the average ULp calculated by the average of *CM* and *PM*[14].

The above three variations of the *CreditRisk+* framework will be henceforth referred to as *CR+*, *CR+h*, and *CR+f* respectively.

We have also analyzed this portfolio using the simplified Markowitz (S*MA)* and simplified Merton (*SMM)* models. We chose parameters based on our experience which would approximately match results at the ULp level with the commercially vended models. The average default correlation for the combined portfolio is assumed to be 3% in *SMA*. The default correlation is increased to 4% for the low credit quality portfolio, and decreased to 1% for the high credit quality portfolio when these are run as independent sub-portfolios. Asset correlation is set to be 20% in all *SMM* runs.

III. 1. C. Exposure Characteristics

In our base set of runs, all exposures were set to be US$1MM. We then varied exposure amounts at the obligor level and tested the sensitivity of model differences to these changes. Figure 3 summarizes the variations from the base set of runs.

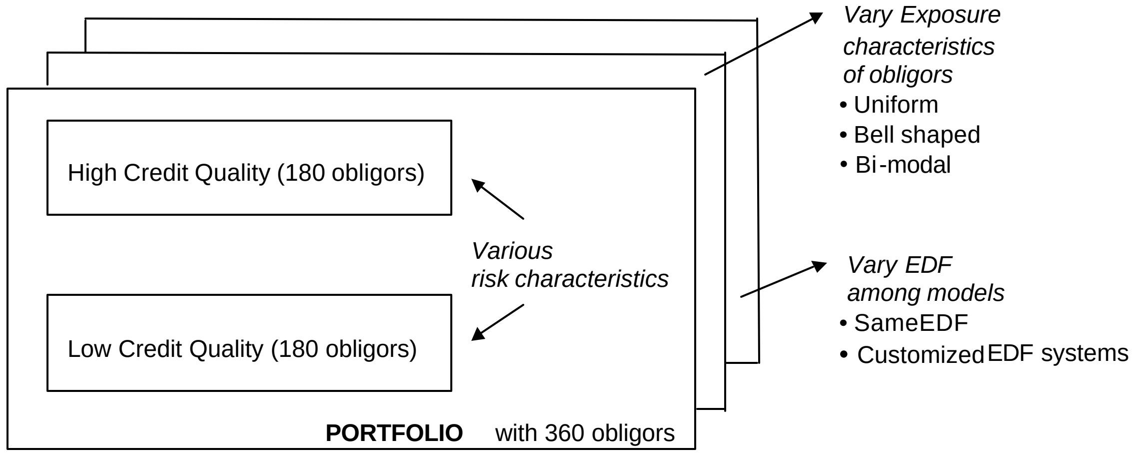
III. 1. D. EDF Parameters

We have also varied EDF calibration systems as summarized in Figure 3. In our base set of runs, EDFs obtained from KMV's *CreditMonitor* used for all models, and the severity of losses was set to a constant value of 40% so that all models would yield the same Expected Loss. This set of assumptions isolates the effect of parameter

[14] This is done by no means to make a correction to the risk results produced by *CreditRisk+*. It merely stands out to show that a forced match in ULp does not necessarily mean an exact match in the tail of the distribution. The flexibility of *CreditRisk+* in parameter specification is the main reason for applying such an adjustment to it, rather than adjusting *CM* and / or *PM*

inconsistencies among joint-default parameters in the variability of risk results. We then studied the differences in risk results when the EDF calibrations systems are different in section V.2.

FIGURE 3: VARIATIONS ON PORTFOLIO CHARACTERISTICS



IV. COMPARING THE RESULTS: "BASE" RUNS

IV. 1. Comparing Merton-based Approaches

First, we look at how the three applications of Merton-based approaches, namely *CM*, *PM* and *SMM* perform relative to each other, and also compare them with the simplified Markowitz approach.

IV. 1. A. Aggregate Risk Results – Full Portfolio

ULp and Economic Capital (defined at 99.90% confidence level) of the combined portfolio are shown in Table 4 as a percentage of total portfolio exposure. Economic Capital at 99 % solvency standard is also presented.

TABLE 4: PORTFOLIO ULp AND EC OF COMBINED PORTFOLIO; BASE CASE RUNS

MODEL TYPE	COMBINED PORTFOLIO WITH 360 OBLIGORS		
	% ULp	EC (99.9%)	EC (99%)
CM	0.73%	6.5%	3.2%
PM	0.70%	5.3%	2.7%
SMM	0.63%	4.1%	2.4%
SMA	0.62%	3.9%	2.3%

ULps calculated by *SMM*, *CM* and *PM* are matching more closely than Economic Capital values. Larger portfolio ULp estimates by *CM* and *PM* compared to *SMM* indicate that these two have assigned more than 20% asset correlations on the average for the studied portfolio. Much larger differences in economic capital are due to higher sensitivity of the tail characteristics to differences in asset correlations and model differences. The beta assumption for the portfolio loss distribution used in *SMA* and *SMM* creates differences in Economic Capital that are much larger than differences in ULp. The match in ULp of *SMA* and *SMM* means that the correlation parameters match on the average. Indeed, the implied average default correlation, calculated from 20% asset correlation, is 2.8%, matching closely with the 3% default correlation assumed in *SMA*.

IV. 1. B. Aggregate Risk Results – Separate High and Low Credit Quality Portfolios

ULp and EC obtained from various models for both high quality and low quality portfolios are given in Table 5.

TABLE 5: ULp AND ECONOMIC CAPITAL FOR HIGH AND LOW CREDIT QUALITY PORTFOLIOS; BASE CASE RUNS

MODEL TYPE	HIGH CREDIT QUALITY PORTFOLIO		LOW CREDIT QUALITY PORTFOLIO	
	% ULp	EC (99.9%)	% ULp	EC (99.9%)
CM	0.32%	3.7%	1.32%	11.3%
PM	0.28%	3.3%	1.20%	8.7%
SMM	0.20%	1.77%	1.13%	7.1%
SMA	0.19%	1.75%	1.13%	7.1%

ULps of *CM* and *PM* are very similar for high and low credit quality portfolios. At the Economic Capital level, there is less difference among these two applications of Merton-based models for high credit quality portfolios than for low credit quality portfolios. The large variation in Economic Capital of the low credit quality portfolio shows that *CM* and *PM* do not use very similar asset correlations for some risky companies. Asset correlations among small companies (which tend to be higher risk) are usually low, (about 0.15 – 0.25), compared to large corporates (about 0.30 - 0.40). A difference of 0.05 in asset correlation estimates between two models would make double the difference for smaller high risk companies compared to large corporates, that is 0.05/0.15 is twice of 0.05/0.30.

ULps calculated by *CM*, *PM*, *SMM* and *SMA* for the low credit quality portfolio are very close. This means that the low credit quality portfolio has an average asset correlation close to 20% and average default correlation of about 4% in *CM* and *PM*, and that the Beta assumption for the portfolio loss distribution is reasonable for the studied case. Large variations for high credit quality portfolio are attributed to correlation parameter inconsistencies.

IV. 1. C. Contributory Risk Results: Full Portfolio

There are different definitions of the risk contribution of an asset to the standard deviation of the portfolio.

- Some practitioners take an incremental approach where risk contribution of an asset to the portfolio standard deviation is defined as the change in the portfolio standard deviation due to the addition of the asset in aggregate to the portfolio. *CM* uses this approach to calculate risk contributions.
- Others define risk contribution as the change in the portfolio standard deviation due to a small percentage change in the size of the asset in the

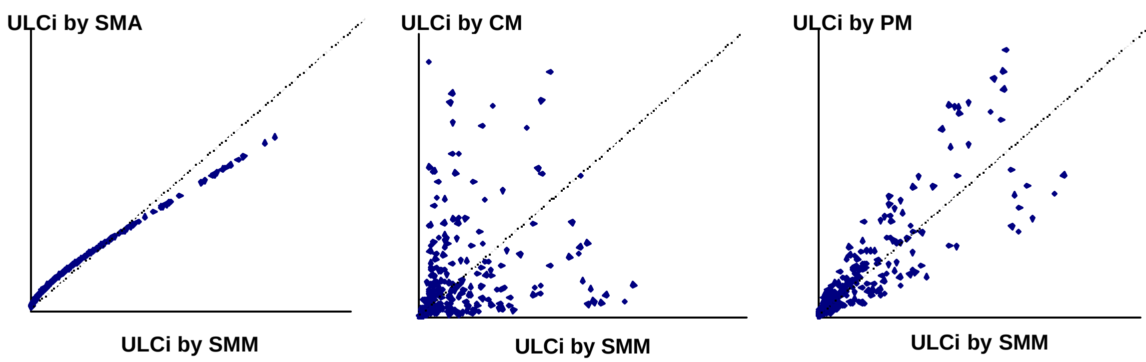
portfolio multiplied by the size of the asset. The advantage of the second approach compared to the incremental one is that it provides a mathematical decomposition of standard deviation in such a way that the contributions add up to the total portfolio standard deviation. *PM* and *CR+* use this approach to calculate risk contributions. We also used this approach in *SMA* and *SMM* runs.

We refer to these two definitions of risk contributions as discrete and continuous marginal risk contributions, respectively. These marginal contributions can be used complementary to each other in practice. While continuous one shows the "snapshot" of risk contributions adding up to the portfolio risk, discrete one is very useful to quantify the effects of buy/sell decisions on the portfolio standard deviation. In what follows, the comparisons are made for continuous marginal risk contributions, referred to as Unexpected Loss Contributions (ULCs). The discrete risk contributions obtained from *CM* runs are adjusted using a closed form formula, which links the discrete and continuous definitions of risk distributions.

Single factor, *SMA* and *SMM,* and multiple factors, *CM* and *PM,* models differ significantly when ULCs at the obligor level are compared. Results across models are compared in figures 4, 5, and 6. Figure 4a illustrates the ULC differences between the two single-factor/single-parameter credit risk models. The single asset correlation applied in *SMM* leads to default correlations differentiated by credit grade. As a result of this *SMM* assigns smaller default correlation, and therefore less risk contribution to low risk loans; and larger default correlation, thence more risk to risky obligors. By contrast, SMA assigns default correlations uniformly to all borrowers. If there were no ULC differences, then all scatter plots would have been on the no-difference line - the straight line (with slope 1) shown in Figure 4a and all of the following ULC scatter plots.

Figures 4b and 4c compare ULCs obtained by *SMM* to ULCs obtained from multi-factor *CM* and *PM.* As seen, there is a lot of scatter due to the nonhomogeneous correlation matrices used in *CM* and *PM* compared to *SMM.* Low scatter in Figure 4c compared to Figure 4b indicate that *SMM* agrees more with *PM* compared to *CM* in terms of contributory risk for the studied portfolio.

FIGURE 4: ULC SCATTER PLOTS; ALL 360 OBLIGORS:
***A)* SMM *VERSUS* SMA, *B)* SMM *VERSUS* CM, *C)* SMM *VERSUS* PM**



Figures 5 and 6 compare the ULCs of CM and PM. CM seems to assign substantially larger correlations compared to PM for a subset of the high-risk companies. This is seen in the ULC scatter plots in Figure 5 outside of the box - there are a lot of risky loans where CM assigns much more contributory risk than PM. The differences in ULCs are observed to be as large as 35 times for relatively large contributions – ULCs exceeding 2% of its exposure. The differences at the obligor level also account for the large differences in aggregate EC between CM and PM illustrated earlier. While Figure 5 shows differences in ULC for large contributions, Figure 6 displays the differences for small contributions, as it is a log-log plot.

***FIGURE 5: ULC SCATTER PLOT,* CM *VERSUS* PM**

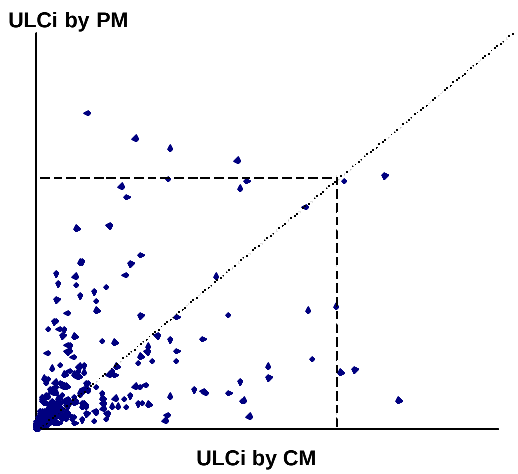
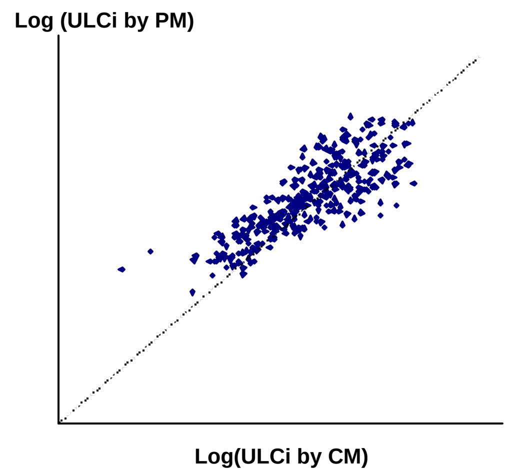


FIGURE 6: LOG (ULC) SCATTER PLOT,* CM *VERSUS* PM, *TO SHOW THE DIFFERENCES IN SMALL CONTRIBUTIONS



IV. 1. D. Concentration Index and Asset Level Decisions

A quantitative metric to illustrate that these models have different views on contributory risk is the Concentration Indicator (CI) defined as

$$CI_i = Sign\left[\left(\frac{ULC_i}{UL_i}\right) \div \left(\frac{UL_p}{\sum UL_k}\right) - 1\right]$$

CI measures relative reduction in UL at the asset level compared to portfolio level, and can be used to determine whether a loan relatively concentrates the portfolio. A positive CI means that the loan relatively concentrates the portfolio and vice versa. Within the 360 obligors, *CM* and *PM* give different views on the number of obligors, which concentrate the portfolio. As shown in Table 6, *CM* and *PM* agree that 71 obligors have a positive CI. According to *CM* results, there are a total of 71+36=107 obligors which concentrate the portfolio, whereas *PM* results show that there are 71+15=86 obligors which concentrate the portfolio. There are 51 obligors (36+15) on which using *CM* and *PM* would result in different conclusions, i.e. one model believes that the asset creates concentration while the other indicates that it diversifies the portfolio. We have analyzed the CI for the top 50% and 25% risky obligors in the portfolio and found that the proportion on which the models disagree is about 10-20% of the overall sample, see Table 6.

TABLE 6: CONCENTRATION INDICATOR RESULTS

		CM					
		ALL PORTFOLIO		***TOP 50% IN ULi***		***TOP 25% IN ULi***	
		+	**-**	**+**	**-**	**+**	**-**
PM	**+**	71	15	*64*	*8*	*42*	*7*
	-	36	238	25	83	2	39

IV. 1. E. Contributory Risk Results: Separate Sub-portfolios

The contributory level risk results show variability with respect to riskiness of the portfolio. A scatter measure is defined as the standard deviation of differences in ULC normalized with respect to average ULp in Table 7 to analyze the variability in contributory risk results. The scatter in differences in ULCs between *CM* and *PM* is much smaller for sub-portfolios compared to the combined portfolio. This is also seen when the ULC scatters in Figures 7 and 8 are compared to the one in Figure 5. Intuitively, it is surprising to see how the directional and quantitative agreement in contributory risk between *CM* and *PM* is distorted when high and low credit quality portfolios are combined. This shows that *CM* and *PM* have less agreement in cross correlations between higher and lower risk companies. In general, the differences are smaller for high quality than low quality portfolio, which shows that *CM* and *PM* correlations are closer for large corporations.

TABLE 7: STANDARD DEVIATION OF DIFFERENCES IN ULC CALCULATED BY **CM** ***AND*** **PM*****, NORMALIZED WITH RESPECT TO AVERAGE ULp***

COMBINED PORTFOLIO OF 360 OBLIGORS	***HIGH CREDIT QUALITY PORTFOLIO OF 180 OBLIGORS***	***LOW CREDIT QUALITY PORTFOLIO OF 180 OBLIGORS***
0.40 %	0.27 %	0.29 %

FIGURE 7: ULC SCATTER PLOT,* CM *VERSUS* PM *FOR HIGH CREDIT QUALITY PORTFOLIO

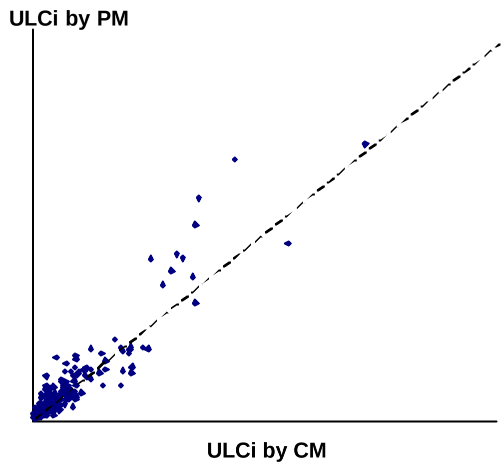
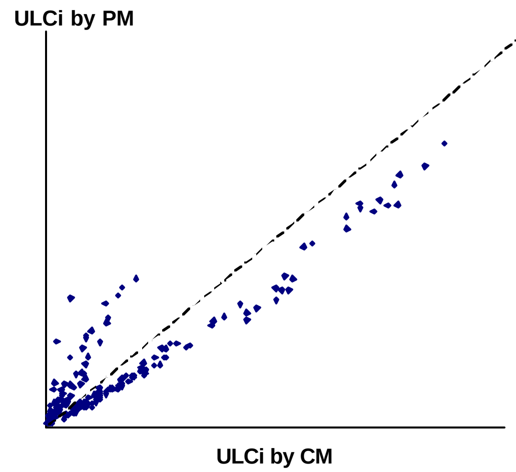


FIGURE 8: ULC SCATTER PLOT,* CM *VERSUS* PM *FOR LOW CREDIT QUALITY PORTFOLIO



IV. 2. Comparing Merton and Actuarial Approaches

IV. 2. A. Aggregate Results: Full Portfolio

All of the *CreditRisk+* runs of this paper are based on a single factor implementation. As seen in Table 8 below, *CreditRisk+* results for ULp and EC exhibit some sensitivity to the choice of the default rate volatility parameters.

TABLE 8: PORTFOLIO ULp AND EC OF COMBINED PORTFOLIO; BASE CASE RUNS

	MODEL TYPE	***COMBINED PORTFOLIO WITH 360 OBLIGORS***		
		% ULp	***EC (99.9%)***	***EC (99%)***
ACTUARIAL	***CR+***	0.56%	3.4%	2.0%
	CR+h	0.52%	2.9%	1.8%
	CR+f (k=1.3)	0.71%	4.7%	2.7%
MERTON	***CM***	0.73%	6.5%	3.2%
	PM	0.70%	5.3%	2.7%
	SMM	0.63%	4.1%	2.4%
	SMA	0.62%	3.9%	2.3%

ULps are much closer than the ECs in *CR+*, *CR+h* and *CR+f* runs. Both ULp and EC are smaller than the ones obtained from *SMA* in *CR+ CR+h* runs. This indicates that the default correlations implied from the default rate volatilities of *CreditRisk+* runs is lower than 3% for this portfolio. This could be due to various factors. Firstly, historically-based loss volatility estimates tend to understate true volatility due to the skewed nature of the loss distribution. Secondly, *CreditRisk+* default rate volatilities used herein are estimated from Moody's default data, which is made up only of U.S. companies, and our portfolio is an international one.

Both *CR+* and *CR+h* models give substantially lower estimates of ULp and EC compared to *SMA*, *SMM*, *CM* and *PM*. This is due to parameter inconsistencies between the asset correlations derived within the Merton model and the historically-based techniques used to estimate default volatility for the Credit Risk+ model. A comparison of *SMM* and *CreditRisk+* reveals that the implied average asset correlation in *CreditRisk+* runs is smaller than 20%.

Even after CR+ is forced to match average ULp of *CM* and *PM* as in *CR+f* runs, EC is still different as the Merton-based and Actuarial models differ in the tail of the loss distribution. This difference in the tail is mostly due to model specification differences arising from the return assumption of Merton model and / or the Gamma distribution assumption for the default rates in Actuarial model. These differences are highlighted in Koyluoglu and Hickman (1998).

IV. 2. B. Aggregate Results: Sub-portfolios

As expected, the results obtained from three different applications of *CreditRisk+* differ significantly from the Merton model for the high credit quality portfolio, but are much more similar for the low credit quality portfolio, see Table 9. The results for low credit quality portfolio are much more closer as the historical coefficient of variation of default rate differentiated by grade for low credit quality assets are all close to 100%. The coefficient of variation of default rate (parameter *k)* has been selected for *CR+f* model, such that it gives close estimates of ULp compared to Merton-style models. The forcing parameter *k* for low quality portfolio is 1.25 compared to the value of 3.3 for high quality portfolio. This is consistent with our earlier assertion that the default rate volatility is a higher multiple of unconditional default rate for high quality companies.

Both *CR+* and *CR+h* models give substantially lower estimates of ULp and EC compared to *CM* and *PM* for high credit quality portfolio. The results are much closer for the low credit quality portfolio, especially at the ULp level.

TABLE 9: ULp AND EC FOR HIGH AND LOW CREDIT QUALITY PORTFOLIOS (UNIFORM EXPOSURE AND SAME EDF SYSTEM)

	MODEL TYPE	HIGH CREDIT QUALITY PORTFOLIO		LOW CREDIT QUALITY PORTFOLIO	
		% ULp	EC (99.9%)	% ULp	EC (99.9%)
ACTUARIAL	CR+	0.16%	1.0%	1.05%	6.2%
	CR+h	0.19%	1.5%	0.92%	5.1%
	CR+f	0.30% (*k*=3.3)	3.0%	1.26% (*k*=1.25)	8.4%
MERTON	CM	0.32%	3.7%	1.32%	11.3%
	PM	0.28%	3.3%	1.20%	8.7%
	SMM	0.20%	1.77%	1.13%	7.1%
	SMA	0.19%	1.75%	1.13%	7.1%

IV. 2. C. Contributory Risk Results

The systematic differences between *SMA*, *SMM* and *CreditRisk+* runs are also observed in the ULC comparisons, see Figure 9. ULCs differ significantly for high and low risk companies, where *SMA* assigns lesser contributory risk to low credit quality companies and higher contributory risk to high credit quality companies. The conclusion from Figures 4a and 9 is that single parameter applications of *CreditRisk+* and *SMM* are closer to each other. Should one of these is to be used, both single factor *CreditRisk+* and *SMM* are better than *SMA* as they differentiate default correlation with credit grade, which is in accordance with the observed default data. The single factor *CreditRisk+* runs naturally lead to very different contributory results when compared to multi-factor *PM* and *CM*. As shown in Table 10, agreement on CI across different modeling techniques is about 70-80% compared to 80-100% agreement within one modeling technique. Moreover, ULCs differ significantly, and models don't have any consensus on the size of contributions, see Figure 10. *CreditRisk+* runs agree more with *PM* in terms of risk contributions. When the observations from Figures 4.c and 10.b are aggregated, we see that the contributory risk results obtained from single factor models (*SMA*, *SMM* and *CR+*) match more closely among each other and also with the ones calculated by *PM* compared to results of *CM*. This means that these models have more consensus on which obligors are concentrating the portfolio.

TABLE 10: MATCHING FREQUENCY IN CONCENTRATION INDICATOR

	CM	***PM***	***CR+***	***CR+h***
PM	86%			
CR+	72%	81%		
CR+h	71%	79%	95%	
CR+f	72%	79%	97%	98%

***FIGURE 9: ULC SCATTER PLOT A)* CR+ *VERSUS* SMM *AND* SMA**

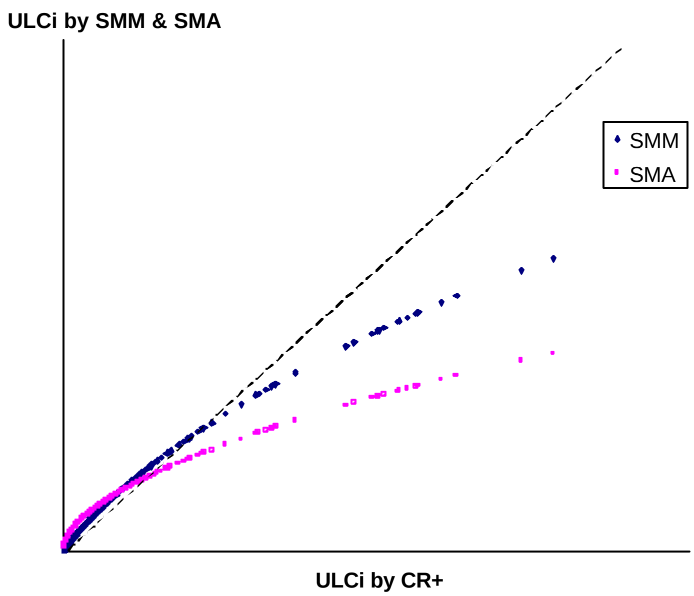
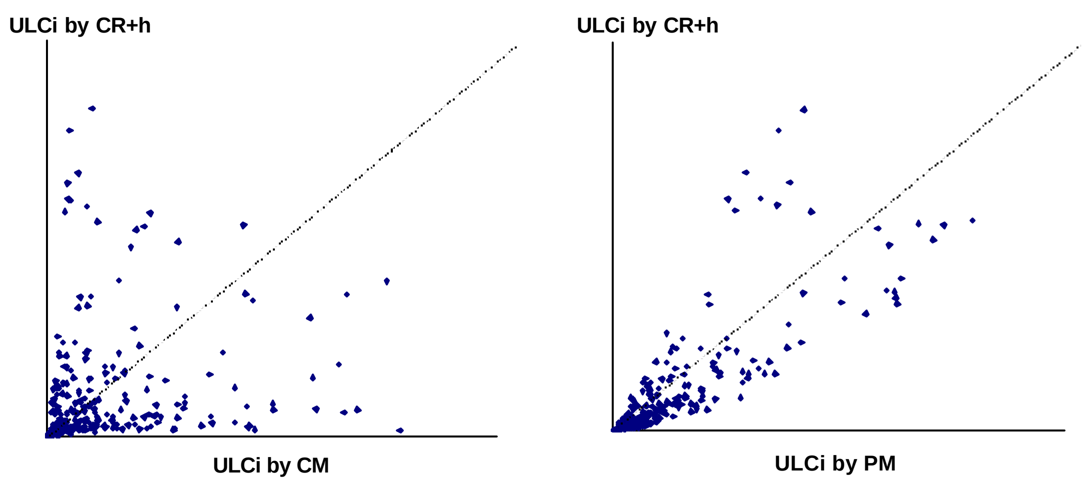


FIGURE 10: ULC SCATTER PLOT, A) CM VERSUS CR+h, B) PM VERSUS CR+h



The scatter measure defined in Section IV.1.E. is used to compare Merton-based and Actuarial modeling techniques in Table 11. As expected, the differences across modeling techniques are much larger due to parameter inconsistencies and single / multiple factor applications. The scatter decreases in the more diversified combined portfolio.

TABLE 11: STANDARD DEVIATION OF DIFFERENCES IN ULC NORMALIZED WITH RESPECT TO AVERAGE ULp

	COMBINED PORTFOLIO OF 360 OBLIGORS		***HIGH CREDIT QUALITY PORTFOLIO OF 180 OBLIGORS***		***LOW CREDIT QUALITY PORTFOLIO OF 180 OBLIGORS***	
	CM	***PM***	***CM***	***PM***	***CM***	***PM***
PM	0.40 %		0.27 %		0.29 %	
CR+h	0.55 %	0.25 %	0.39 %	0.30 %	0.69 %	0.41 %

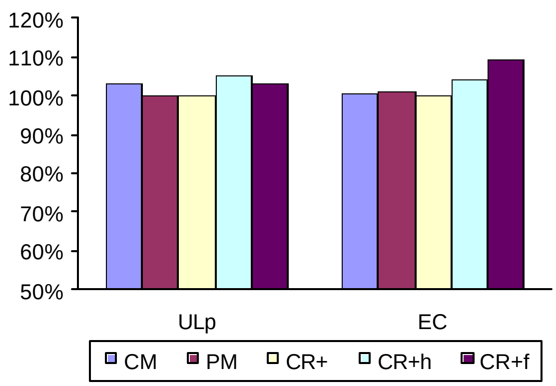
V. VARIATIONS OF THE "BASE" RUNS

V. 1. Effect of Loan Size Distribution

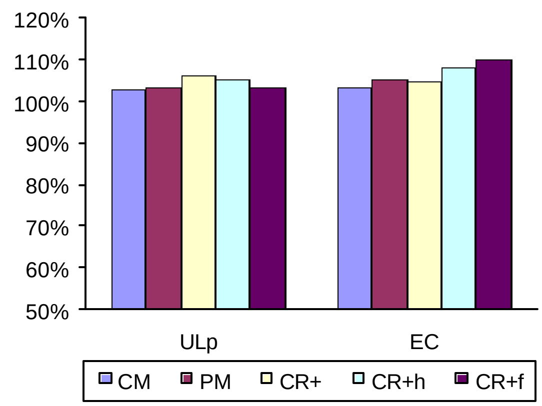
The results presented so far were based on a uniform exposure distribution of US$ 1MM each. We repeated the model calculations with two variants of the exposure profile. In the first variant, all loan exposures are assumed to be normally distributed around a mean exposure of $1 MM and a standard deviation of $0.1MM. The results for this profile are reported under "bell-shaped". In the second variant, we created a bi-modal distribution of exposure with half the portfolio distributed around $0.5MM and the other half around $1.5MM. A normal distribution probability function was used to assign exposures with a standard deviation of $0.1MM for each mode, and the results are reported under "bi-modal exposure". Figure 11 shows the results for different exposure runs normalized with respect to uniform exposure results for the high credit quality portfolio. The magnitude of differences between the three types of exposure profiles seems to be minor for all the models. EC seems to be more sensitive to exposure variation compared to ULp. These findings also apply to low credit quality portfolio and are consistent with the results of Gordy (1998).

FIGURE 11: ULp AND EC NORMALIZED TO CORRESPONDING ULp AND EC; BASE CASE RUNS

A) HIGH CREDIT QUALITY BI-MODAL EXPOSURE



B) HIGH CREDIT QUALITY BELL-SHAPED EXPOSURE



V. 2. Effect of Varying EDF systems

Next, we explore the effect of using different EDF systems on the results. This is to simulate a case in which a portfolio is analyzed by different risk managers who use different customized EDF systems and different credit risk portfolio models. We designed the following customization in order to realize the differences in EDF systems to explore the impact of varying EDF systems on the risk results:

- Run *PM* with the EDFs provided by KMV's *CreditMonitor*
- Run *CM* with the Moody's 6-state rating system that was calibrated to EDFs using historical default data and extrapolations of it for investment grades

- Run *CreditRisk+* with a hypothetical bank internal grade system R1-R7 as in Table 12

The EDF calibrations of Moody's grades and the hypothetical bank grade are shown in Table 12.

TABLE 12: EDF CALIBRATIONS OF MOODY'S GRADE AND HYPOTHETICAL BANK INTERNAL GRADE

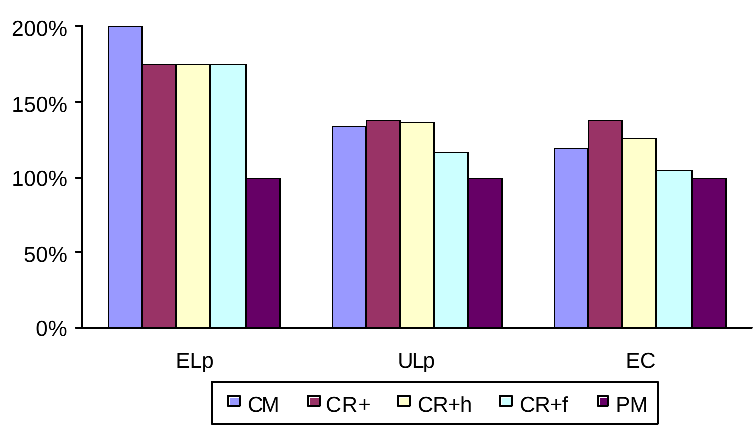
MOODY'S GRADE	***EDF***
Aaa	0.01%
Aa	0.03%
A	0.07%
Baa	0.20%
Ba	1.20%
B	7.54%

INTERNAL GRADE	***EDF***
R1	0.02%
R2	0.08%
R3	0.25%
R4	0.50%
R5	1%
R6	5%
R7	20%

V. 2. A. Aggregate Risk Results

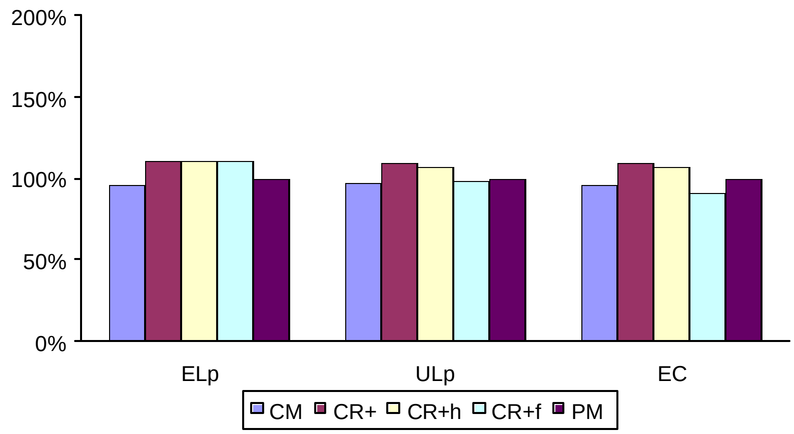
Figure 12 and 13 compare the portfolio level risk results for both high and low credit quality portfolios, when varying EDF rating systems are used. The results are normalized to corresponding results obtained for the base case runs, presented in Table 9.

FIGURE 12: ULp AND EC NORMALIZED TO CORRESPONDING ULp AND EC OF BASE CASE RUNS: HIGH QUALITY PORTFOLIO



The EL, ULp and EC are significantly higher for both *CM* and *CreditRisk+* when the customized EDF systems are used for the high credit quality portfolio as the EDFs derived through Moody's/Internal grade calibrations are higher than KMV model-based EDFs for this sample of firms at this point in time. Moody's EDFs are retrospective and average the experience across a couple of business cycles. KMV's EDFs incorporate market prices and volatilities which should be forward looking and business cycle sensitive. The exposure weighted average EDF for the portfolio was 40 bp in Moody's calibrated system compared to 20bp from KMV's *CreditMonitor*(we used EDFs from 1997), a difference in line with the strong economic conditions and booming stock market at the point the KMV EDFs were measured. Figure 12 shows that the maximum differences occurred at the EL level, followed by ULp and EC. This shows the linear, square root and highly nonlinear sensitivities of respectively EL, ULp and EC with respect to EDF. The portfolio runs were repeated for the low-quality portfolio. In this case, the results show less variation when varying EDFs were used, see Figure 13.

FIGURE 13: ULp AND EC NORMALIZED TO CORRESPONDING ULp AND EC OF BASE CASE RUNS: LOW QUALITY PORTFOLIO



V. 2. B. Contributory Risk Results

The match in contributory risk results obtained from *CM* and *PM* is also significantly reduced when EDF systems are different, especially for the high credit quality portfolio. Table 13 shows the Concentration Indicator for high credit quality portfolio with the same EDF system and customized (different) EDF systems. All exposures are taken to be equal in these runs. While both *CM* and *PM* suggest that one sixth (30/180) of the obligors concentrate the portfolio in the same EDF runs, this ratio drops to less than one twelfth (14/180) in the runs with different EDF systems. The consensus in between the models has been decreased significantly at the top 50 and 25 percentile risky companies. The consensus is (30+107)/180 = 76% for the portfolio in the same EDF runs. This drops to (14+111)/180=69% when EDF systems are different. While both *CM* and *PM* see 18 companies out of 45 (18+6+8+13) concentrating the portfolio in the same EDF runs, they have a consensus on only 10 companies in the runs with different EDF systems. Herein, top 50% and 25% ULi is calculated using the EDF system of the *PM* runs so that the number of companies with positive CI is fixed for PM rows (e.g. 30+17=14+33, or 26+15=12+29).

TABLE 13: CONCENTRATION INDICATOR RESULTS

			CM					
			ALL PORTFOLIO		**TOP 50% IN ULi**		**TOP 25% IN ULi**	
			+	-	+	-	+	-
SAME EDF SYSTEM	**PM**	+	30	17	*26*	*15*	*18*	*6*
		-	26	107	15	34	8	13
DIFFERENT EDF SYSTEMS	**PM**	+	14	33	*12*	*29*	*10*	*14*
		-	22	111	14	35	9	12

V. 3. Relative Effects of EDF and Correlation Parameter Inconsistencies to the Differences in ULCs

Parameter inconsistencies will lead to differences in risk results. Both EDF and correlation parameter inconsistencies result in differences in ULp and ULCs. In order to contrast the effect of EDF differences with differences in joint default parameters (correlations), we compare the following three sets of models:

a) *Correlation effect*: Compare the risk results calculated by *PM* and *CM*, where both models use the same EDF system (from KMV's *CreditMonitor*)

b) *EDF effect:* Compare the risk results calculated by *CM* and *CM*', where the two versions of *CreditManager* use different EDF systems. *CM* uses the *CreditMonitor* EDFs, while *CM*' uses Moody's rating system

c) *Combined Effect:* Compare the risk results calculated by *PM* and *CM'*

Here set (a) represents a pair of models that differ in their estimation of joint default parameters. The set (b) respresents two models that differ in EDF inputs while they utilize the same joint default parameters. The two models in set (c) use different EDF and default correlations. The scatter measure introduced earlier is used to quantify the variance between the models. The measure is calculated by taking the standard deviation of the differences in ULC and normalizing it by the average ULp. Table 14 displays the results for high and low credit quality portfolios where all obligors are assumed to have equal exposure.

TABLE 14: STANDARD DEVIATION OF DIFFERENCES IN ULC NORMALIZED WITH RESPECT TO AVERAGE ULp (EQUAL EXPOSURE CASE)

SET		HIGH CREDIT QUALITY PORTFOLIO OF 180 OBLIGORS	LOW CREDIT QUALITY PORTFOLIO OF 180 OBLIGORS
a	Correlation Effect	0.3 %	0.3 %
b	EDF Effect	1.1 %	0.1%
c	Combined Effect	1.3 %	0.3%

For the studied cases, the effect of EDF differences is significantly (3.5 times) more dominant than the effect of differences in joint default parameters for the high quality portfolio consisting of exposures to large publicly traded companies. The opposite is true (correlation effect is 3 times the EDF effect) for the case of low credit quality borrowers. EDF and correlation parameter inconsistencies do not have additive effects on the risk results, as seen in Table 14, due to some wash out.

In practice, we expect the effect of EDF differences to be more dominant than the effect of differences in correlations for a bank portfolio, as exposures would be highly skewed towards high credit quality obligors.

VI. CONCLUSIONS

Recent studies have shown that sophisticated credit risk portfolio models can be analytically linked, share similar economic intuition and would yield very similar risk results, provided that the input parameters are harmonized. However, in practice, parameter inconsistency is a "fact of life". Potential sources of inconsistencies include dissimilar EDF calibration systems, differences in recovery rates, different views on the exact amount of exposure, and correlation parameter variation across commercially available and internally developed credit risk portfolio models. In addition to parameter inconsistencies, modeling details may be another source for differences, especially in the tail of the portfolio loss distribution.

In this work, we ran sample portfolios with different risk and exposure characteristics using Merton-based (*CreditManager*, *PortfolioManager* and a single parameter / single factor Merton-based model), Actuarial (*CreditRisk+*) modeling techniques, and a simple Markowitz approach. We demonstrate that the models yield significantly different portfolio and contributory level risk results for identical portfolios if parameter estimates used within the models correspond to their "natural" data set. Clearly, such differences imply different recommendations for credit risk management, risk-based pricing and portfolio optimization.

This study makes a case for a deeper analysis of credit risk parameter estimation methods. The quality of estimates available from different techniques should be compared across various sub-portfolios. For example, Merton-based asset correlations derived from equity price relationships might be most accurate for publicly traded companies, while default rate volatilities derived from historical experience might be most relevant for consumer portfolios. Moreover, alternative Merton-based correlation estimation techniques should be compared rigorously as there is wide dispersion of results even when the same framework is used (*CM* vs. *PM*). Since back-testing of loss statistics, which is routinely performed in market risk models, is not practical for credit portfolio models due to the small number of observations, practitioners should focus on best estimation of credit risk parameters. Once the best parameter estimation method is determined for each sub-portfolio, the resulting estimates at the sub-portfolio[15] level can be adjusted through harmonization equations to fit the form of the enterprise-wide portfolio model (i.e. default rate volatility estimates can be mapped to asset correlations for use in an enterprise-wide Merton model, vice versa). This approach makes the best use of the limited data available for credit portfolio analysis. Users should also employ sensitivity analysis to stress test significant portfolio pricing and optimization decision across a range of potential parameter estimates, given that there is substantial variation in these estimates across the currently available models and significant challenge in determining which is better.

[15] In this respect, in addition to data cleaning and experience-based judgement, the smallest standard error in the estimates could be used as a criterion

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