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INTEGRATING INTEREST RATE RISK AND CREDIT RISK IN ASSET AND LIABILITY MANAGEMENT

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Introduction

A recent study by the Federal Reserve [1995] came to the startling conclusion that no interest rate risk variables were statistically significant in predicting bank failures in the United States. The new FIMS monitoring system discussed in this Fed study predicts failure based on 11 key variables collected from the Report of Condition submitted to bank regulators in the United States. Five of the eleven variables are related to the riskiness of commercial lending, and none of the others are related to interest rate risk.

The demise of much of the savings and loan industry in the early and mid-1980s when interest rates were high and volatile certainly suggests that interest rates should be a significant risk factor. A cynic might argue that the regulators did not collect a meaningful interest rate risk measure from reporting banks until very recently.[2] Nonetheless, the FIMS research is an

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[2] Historically, regulators have collected interest rate sensitivity gap information with various assets and liabilities assigned maturities in a fairly arbitrary way.

indication that many market participants, including various software vendors[3], seem to believe that credit risk can be analyzed without the consideration of interest rate risk.

In stark contrast, the traditional approach to fixed income analysis (see Fabozzi and Fabozzi [1989]) assumes that only interest rate risk, and not credit risk, is the important factor in pricing corporate debt. This approach utilizes the standard techniques of duration and convexity hedging to risk manage a portfolio of corporate debt. These techniques are in common use by the industry[4].

The purpose of this paper is to critically analyze these two contrasting approaches to pricing credit risk. Using a unique data set, we provide an empirical analysis – a case study – of which risk, interest rate or credit (or perhaps both), is most important in the pricing of risky debt. The data set is unique because it consists of weekly quotes on a bank's (primary) debt offerings, for various maturities, over an 8-year observation period. Two standard models are compared in terms of their hedging performance. One is Merton's risky debt model, which assumes that interest rate risk is non-existent. The second is the traditional fixed income duration/convexity approach, which assumes credit risk is non-existent.

The hedging results are quite intriguing. The traditional fixed income approach dominates Merton's model, indicating that interest rate risk is significantly more important than credit risk, in the pricing of corporate debt. The implication, of course, is that for pricing and hedging purposes, if one risk needs to be ignored, it should be credit risk. But, this is not the final conclusion.

The results also indicate that the traditional fixed income approach to valuation still leaves a significant component of the bank's debt unhedged. We attribute the remaining hedging error to the omission of credit risk. The punch line is that the newer models, those that include both interest rate (market) and credit risk, are needed for more accurate pricing and hedging (see Jarrow and Turnbull [1995] and Jarrow, Lando, Turnbull [1997]).

This chapter is divided into four sections. Section 1 provides an overview of the credit risk problem and how it relates to valuation, pricing, and hedging. Section 2 provides an introduction to the Merton's risky debt model, which will be tested against the traditional fixed income approach in Section 3. Section 4 concludes by discussing the need to integrate both interest rate (market) and credit risk for enterprise-wide risk management.

1. An Overview of the Credit Risk Problem

The objectives of the credit risk process in asset and liability management are varied but closely related to each other. The objective is not just to know whether to make a particular loan or not. Nor is it merely to know the probability of default of a particular borrower over a particular time period. These are just two of many important questions, all of which should be

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[3] An example is J. P. Morgan's Credit Metrics.
[4] An example is the rich set of traditional fixed income analytics displayed by the Bloomberg financial information service.

answered by a comprehensive approach to credit and interest rate risk management. The questions are:

- Should I make this loan or not?
- What is the probability of default by Company XYZ?
- What are the major risk factors driving the value of my loan portfolio?
- Am I as diversified as I could be?
- What is the market value of my portfolio?
- How can I hedge the risks of the portfolio?
- How should this loan be priced?
- How much value-added does the loan business create for the firm?
- From a credit policy perspective, how should I view the risk of the bank's loan portfolio given that economic conditions have recently changed?
- What should my loan loss reserve be?
- Do I have enough capital in the bank?
- Do I have enough capital in this business unit?

In looking at various models of credit analytics, as emphasized above, we have to recognize that the objective is not just to avoid bad loans or to measure credit quality. Asset and liability management also relates to minimizing the loss when a good loan turns bad through hedging the price shocks. It relates to accurate pricing and continued marking-to-market so that capital is allocated efficiently. Finally, it relates to the correct measurement of risks so that the proper capital reserves can be determined (the ultimate protection) to conserve scarce capital resources.

A good risk management model provides answers to all of these questions. The key ingredient to any solution is a valuation model that accurately prices and hedges corporate debt. Indeed, if the model accurately prices and hedges corporate debt, then the model has accurately accessed the relevant risks, including the likelihood of default and the credit quality of the loan. The purpose of this paper is, thus, to critically analyze the two leading credit risk models along the indicated dimension of accurate pricing and hedging.

2. An Introduction to Merton's Risky Debt Model

Robert Merton won the Nobel Prize in 1997 in part for his insights in recognizing that the pricing of corporate debt is related to the options model introduced in 1973 by Black and Scholes (see Merton [1974]). Merton's model of risky debt rests upon a number of assumptions:

1. Interest rates are constant.
2. The firm issues only one type of debt and that is a zero-coupon bond.
3. The usual perfect market assumptions apply (frictionless and competitive markets).
4. The assets of the company are perfectly liquid.

Under these conditions, on the maturity date of the debt, if the value of the firm's assets is worth less than the amount due on the debt, then the equity is worthless. Equity has a positive value only if the corporate assets are worth more than the maturing debt. In this case, the value of equity is the residual firm value after paying off the debt.

Equity has the same cash flow as a "call option" (an option to buy) on the assets of the firm at a strike price equal to the amount due on the debt and with a time to exercise equal to the maturity of the debt. Therefore, the value of debt equals the value of firm assets less the value of a call.

Because of the constant interest rate assumption, the value of the firm's debt has zero correlation with interest rates. Furthermore, since interest rates aren't random, this debt pricing model has no interest rate risk.

This model depends on two parameters, the firm's asset value and its volatility. These two parameters can be calibrated to fit market observables, in particular, they can be calibrated

- to fit historical default frequencies, and
- to fit an observable yield curve for risky debt.

This simple model is elegant from a theoretical perspective. Does it work in practice? That is the subject of the following sections.

3. Testing the Hedging Performance of Various Credit Models

As argued in section 1 above, the proper way to evaluate a credit model is to analyze its pricing and hedging performance. In practice, however, calibration of a model's parameters to market observables, like the historical default frequency or the initial zero-coupon price curve, always guarantees accurate pricing[5]. For example, in the traditional approach to fixed income analysis, the bond's price can always be calibrated to market quotes by choosing the parameters of the model to match the initial zero-coupon price curve. As mentioned earlier, Merton's model can also be calibrated to match historical default frequencies and market prices. Hence, in practice, the only dimension that one can use to differentiate these models (in fact, any model) is their hedging performance.

To compare the two models, we test their hedging performance on a unique data set. The best possible data set would cover a long period of time and relate to one or more issuers whose credit quality has been volatile over the observation period. The frequency and quality of the debt prices is crucial. One source for this data is secondary market bond prices reported by large securities companies (e.g. the Wisconsin Fixed Income Data Base is provided by Lehman Brothers). The problems with secondary market prices (none of which should rule out their use) are:

[5] In theory, pricing and hedging are equivalent characteristics of an options model. This is due to the fact that arbitrage pricing theory values by synthetic replication. Synthetic replication is equivalent to hedging. So, to test accurate pricing and hedging, one only needs to test accurate hedging.

- the data is often collected infrequently (e.g. a month),
- there is little economic incentive for dealers to quote realistic levels or bid-offered prices, and
- insufficient debt issues are available to get accurate yield curves.

For hedging purposes, the frequency of the data is important, as hedging theory is based on the notion of "continuous" trading. Monthly observation intervals are too long to approximate "continuous trading". Weekly or daily price observations are best. Finally, it is obvious that accurate price quotes are necessary to provide a reasonable assessment of a model's hedging ability. Because of liquidity considerations, secondary market prices often fall short on this dimension. The data set we employ was constructed to avoid these problems.

a. The First Interstate Data Set

Our data set is a collection of weekly quotes on primary issuance spreads over U.S. Treasuries for First Interstate Bancorp, a major debt issuer, whose credit quality varied considerably over the observation period – January 1986 to August 1993.

First Interstate Bancorp, which was recently acquired by Wells Fargo & Co., collected this data for risk management purposes. First Interstate Bancorp, where one of the authors served as treasurer from 1984 to 1987, was one of the ten largest bank holding companies in the United States in the middle 1980s. In spite of the large number of bank failures at the time, First Interstate was an AA issuer of debt and one of the most frequent issuers of debt in both the United States and the Euro markets. First Interstate was the world's first issuer of Euro medium term notes and the first issuer of bank medium term notes. It was also one of the most active early dealers in interest rate swaps and fixed income options, ranking at one time in the top 10 dealers in the United States. As such, First Interstate represents exactly the type of institution for which the measurement of credit risk would be necessary and desirable.

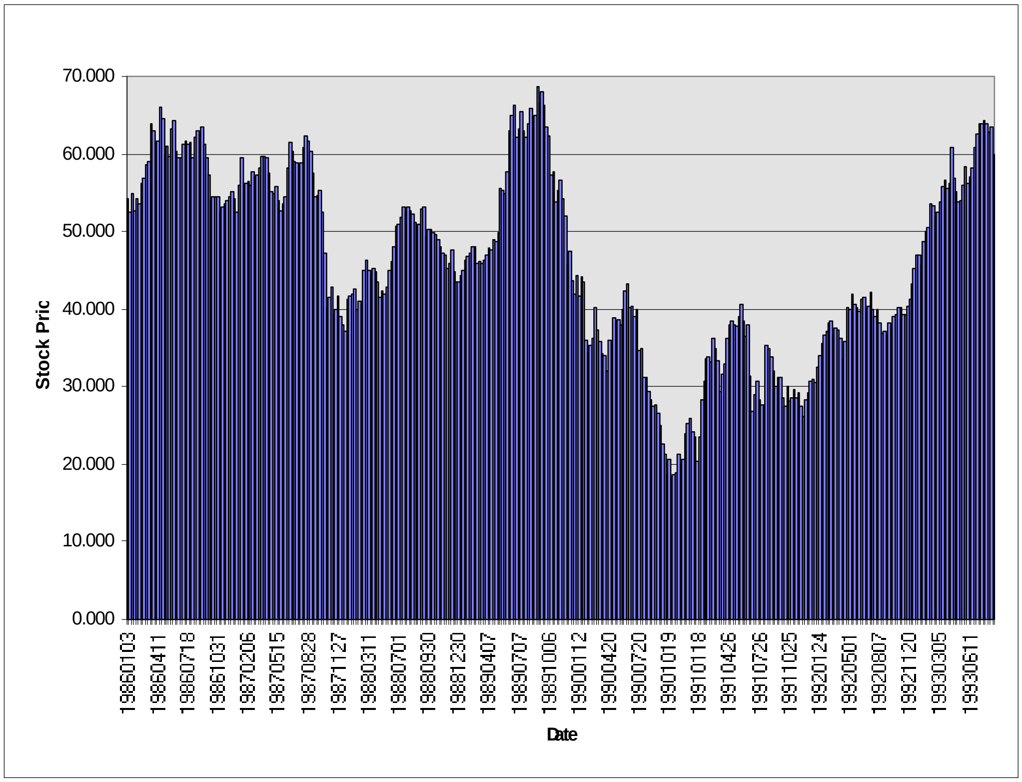
In 1984, First Interstate's treasury department began polling leading investment banking firms regarding spreads to U.S. Treasury bonds for a new issue of $100 million of non-callable bonds at the "on the run" maturities of 2, 3, 5, 7, and 10 years.

The data series collected by First Interstate consists of quotations taken each Friday from six investment banking firms. The high and low estimates were eliminated and the remaining four quotes averaged. Because the spreads represent the "on the run" maturities, there is no need to engage in yield curve smoothing to extract the "on the run" spreads to Treasuries from odd dates, a practice often necessary when using secondary market quotes for corporate debt issues.

There was considerable economic pressure on the investment banks to provide accurate quotations, for a spread that was too high ran the risk of causing a bond underwriting to be missed by the investment banker. Conversely, a quoted spread that was too low could lead to a "prove it" request by First Interstate to underwrite the issue at the level. Finally, consistently inaccurate quotes relative to the mean spread of all the underwriters polled had an adverse impact on the relationship of the investment banking firm with First Interstate.

First Interstate provides a challenging test for any credit model, since its credit rating fell from AA to BBB+ in January 1990, to BBB in January 1991, but rose back to A- by January 1993. Furthermore, First Interstate's stock price also showed considerable variation over the sample period. The stock price varied from below $20 per share to almost $70 per share (See Figure 1).

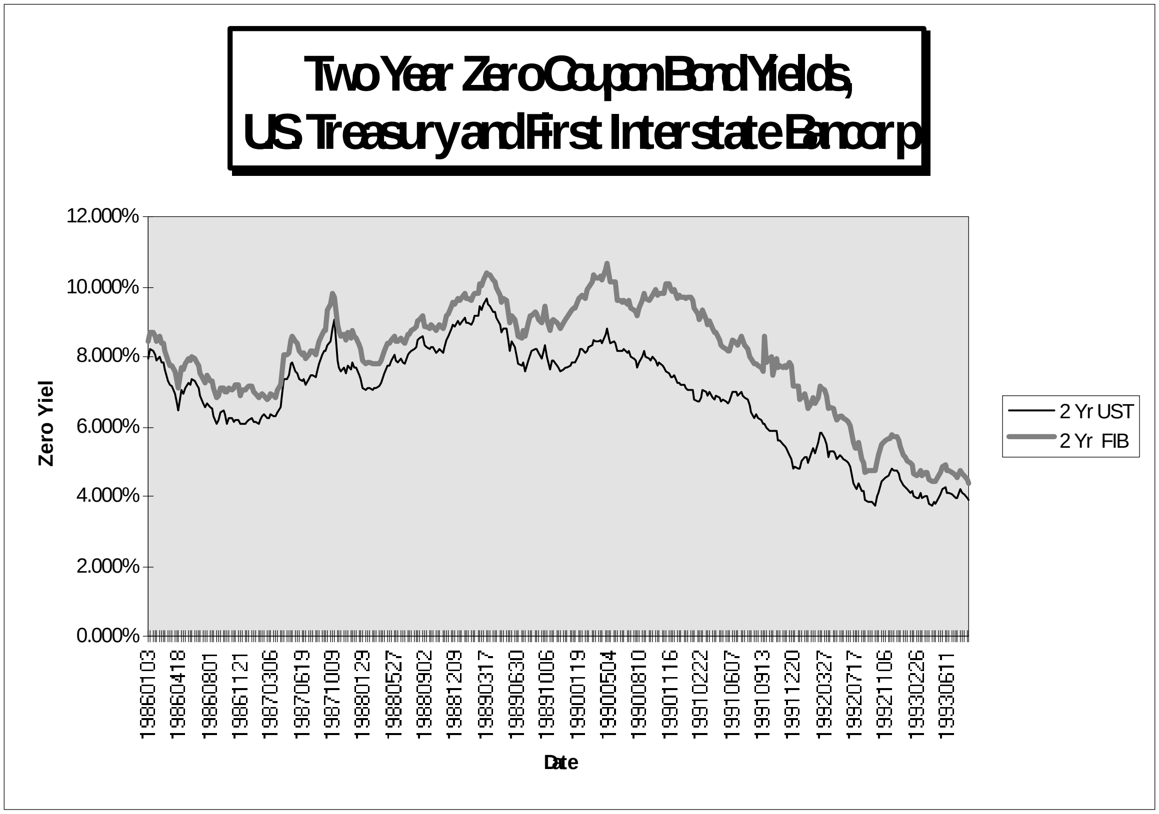
Figure 1



In what follows, we will concentrate on the First Interstate two-year spread over Treasuries. Two years is close to the average maturity of the typical financial institution's liability, thereby providing a useful comparison. The two-year zero-coupon bond yield for First Interstate showed a 93.6% correlation with U.S. Treasury zero-coupon bond yields (See Figure 2).

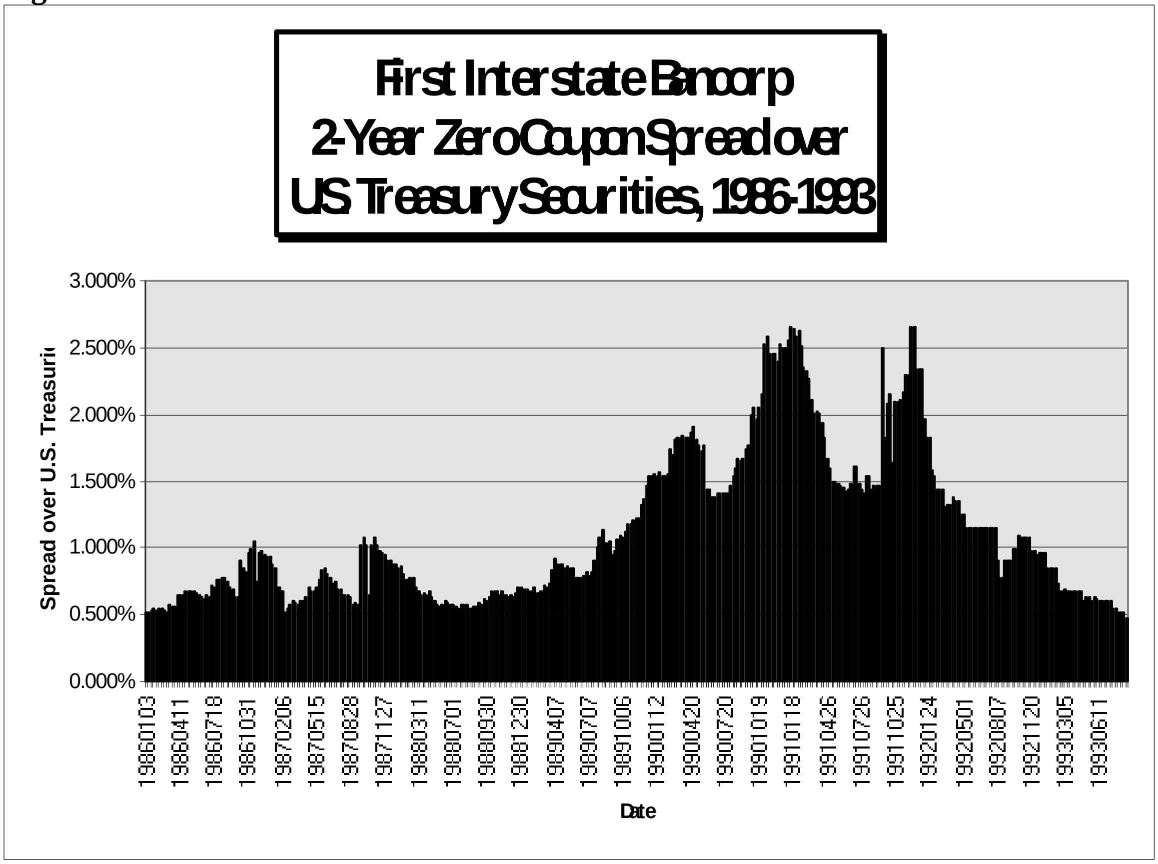
Figure 2

Figure 3 shows that the spreads for the two-year zeros over Treasuries exhibited tremendous



variation over the sample period. This was partly due to the facts that over this observation period, First Interstate went through a failed attempt to acquire BankAmerica Corporation, was the subject of numerous merger rumors itself, and suffered from serious credit quality problems.

Figure 3



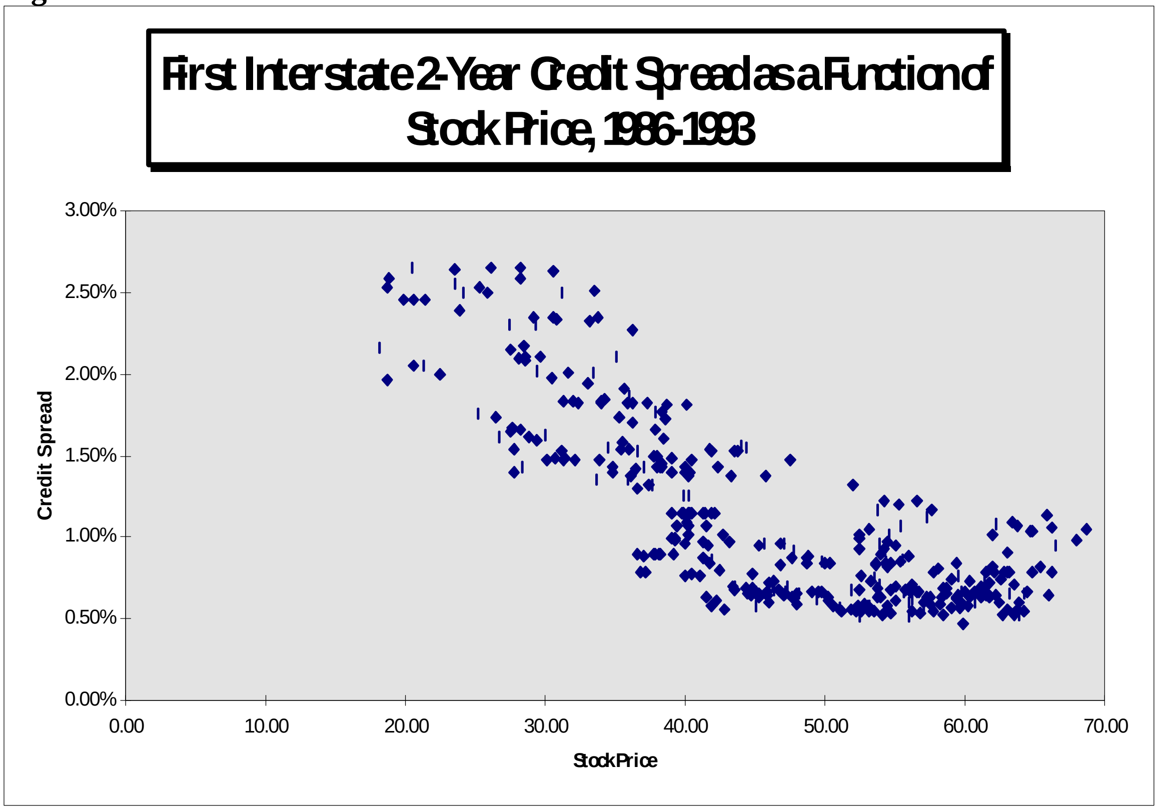
Only two weeks of data (2 out of 377 observations) were omitted as outliers. The outliers were determined by screening the data for unusual values. On August 14, 1992, the First Interstate 2-year spread over Treasuries exhibited such a value. It jumped from 90 basis points in the previous week to 350 basis points, and then back to 88 basis points on August 21. In contrast, over the same period, the stock price did not exhibit the same unusual movements – the three relevant values being $38.125 to $37.75 and then to $37.25. So, the week of August 14 was omitted from the data, and because much of the subsequent analysis concerns weekly changes in prices, the August 21 observation was necessarily omitted as well.

b. Relationship between Stock Price and Credit Spread

Before testing hedging performance, it is instructive to perform some preliminary data analysis to get a sense for the relationships involved among interest rates, credit risk, and stock prices.

Figure 4 plots First Interstates 2-year credit spread versus the stock price over the observation period. The graph produces a downward sloping pattern that bends as stock price increases, something that seems, at least at first glance, to be consistent with the Merton model of risky debt. Merton’s risky debt model implies that the debt’s value is negatively related to the stock price, and hence, the credit spread is non-linearly and negatively related to the stock price as well.

Figure 4



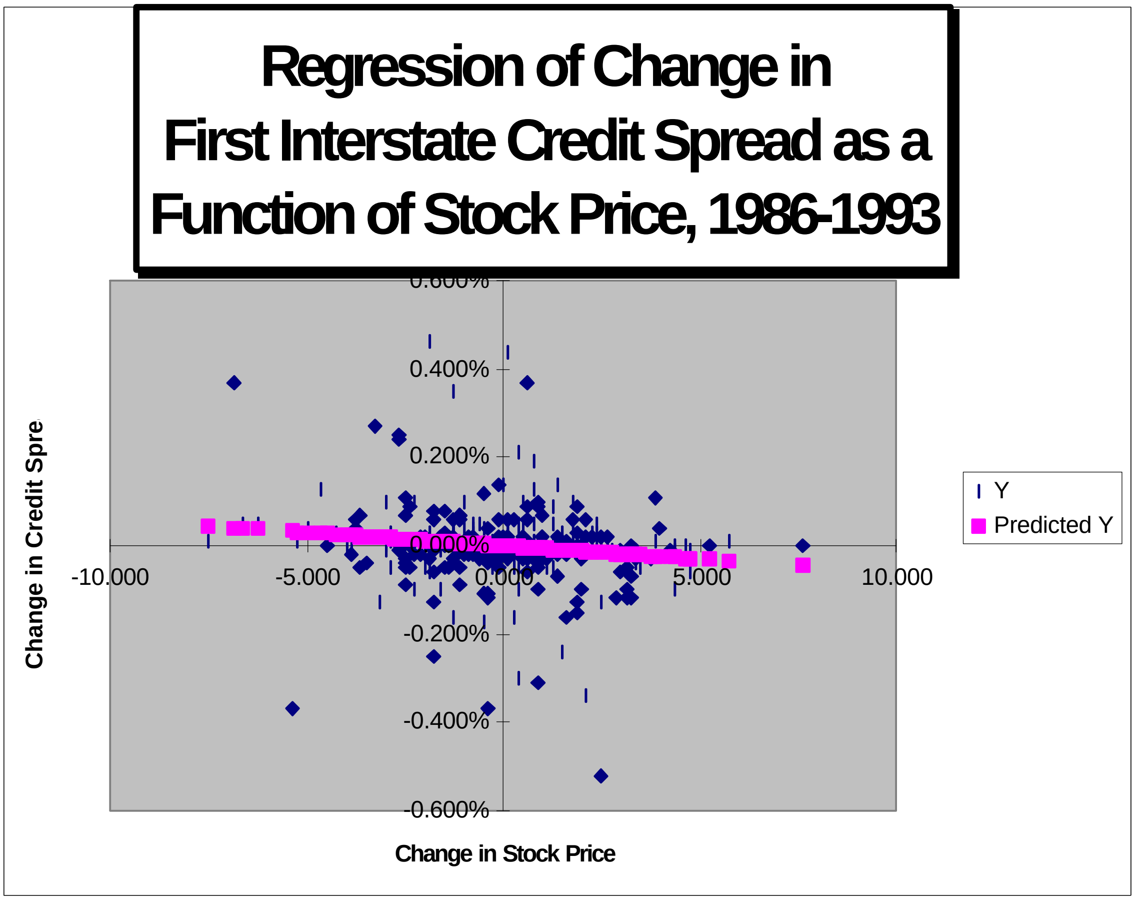
As a result, one would expect to see only three possible combinations of stock price and credit spread movements:

- stock prices rise when credit spreads fall,
- stock prices fall when credit spreads rise, and
- stock price remains unchanged when credit spreads are constant.

Accordingly, a graph of changes in credit spreads versus changes in the stock price provides a quick check on this relation. Figure 5 provides this comparison. Figure 5 graphs the weekly changes in the credit spread/ changes in stock price pairs.

If Merton's model is valid, we would expect too see all of the data points clustered either in the upper left hand quadrant (lower stock prices and higher credit spreads) or lower right hand quadrant (higher stock prices and lower credit spreads). Casual observation indicates that this is not the case. Only 42% of the data points in the 375 weeks of First Interstate data are consistent with this relationship. More importantly, 58% of the data points are not. A regression analysis of the change in credit spreads on changes in the stock price is downward

Figure 5



Sloping, as the Merton model predicts, but the explanatory power is very, very low. T-scores are given in parentheses below the coefficients.

Change in Credit Spread as a
Function of Change in Stock Price

Y = 0.000000 – 0.000058 X
(-0.0039) (-1.0518) Adjusted R^2=0.000283

c. Constructing the Hedge

The hedging test was constructed as follows.

- At the start of the observation period, we simulate the purchase of $1 million principal amount of First Interstate 2-year zero-coupon bonds.
- Simultaneously, we construct the appropriate hedge with U.S. Treasury 2-year bonds, First Interstate common stock, or both. The exact hedge ratios (discussed below) differ for each of the models tested.

- After one week, the position is liquidated. The First Interstate bonds are "sold" and all hedging positions closed out. The sale price of the First Interstate bonds and the U.S Treasury bonds correctly recognizes the fact that they now have one year and 358 days to maturity. The yields used for pricing the First Interstate bonds, however, are the prevailing 2-year zero-coupon yields quoted at the end of the one-week holding period.
- The net profit or loss from this one-week strategy is calculated, stored, and then the process is repeated 375 times.

Since the net investment in the hedged portfolio is non-zero, if the model is correct, the hedged portfolio will be riskless and therefore earn the weekly Treasury rate (for a week). We examined changes in the value of the hedged portfolio (profits or losses) over the week. For comparison purposes, this change should be nearly constant across time and approximately zero[6].

To avoid computing the return on the net investment, we computed the standard deviation of the hedged position's weekly profits. For all practical purposes, computing the standard deviation of the profits across time eliminates this expected (riskless) profit on the net investment from the analysis. This is because the expected profit is approximately constant (and small), therefore, its value is incorporated into the computation of the standard deviation.

Thus, the "best" credit model using this hedge is the one with the lowest standard deviation of weekly profit numbers over the sample period. The following models were tested:

- Do nothing (no hedge)
- 1 to 1 U.S. Treasury hedge
- (Macauley) Duration hedge
- Constant hedge ratio
- Merton's risky debt model hedge

The first hedge is doing nothing. This provides an upper bound on the standard deviation that is useful for comparison purposes.

The next three hedges relate to the traditional fixed income approach to bond pricing. The one-to-one US Treasury hedge is the most naive hedge. This hedge ignores duration considerations. The duration hedge is the one proposed by the traditional theory. But, because the duration of a two-year US Treasury note is close to 2 years, the 1 to 1 Treasury and the duration hedge will give similar results. The constant ratio hedge is a modification of the traditional approach, to take advantage of "in sample" estimation. By this we mean that the

[6] To see this, let I = the net investment. The theoretical change in the value of the hedged position is Ir(1/52) where r is the weekly rate on a per year basis. This change is less than (.07)(1/52)=.0013 times the net investment. For a net investment of 500,000 dollars, this change is 673 dollars. Since the net investment is approximately constant across time, so will be this expected profit.

constant hedge ratio was selected to minimize the profit's standard deviation. This hedge, therefore, provides the "best" possible performance for the traditional approach.

The final hedge is based on Merton's model. This is the most complex of the five hedges, and the technique we employ is discussed next.

d. Implementing the Merton's Model Hedge

To implement the hedge based on Merton's model, we need to estimate the strike price of the single zero-coupon bond issued by the firm, the value of the firm's assets, and the volatility of the firm's assets.

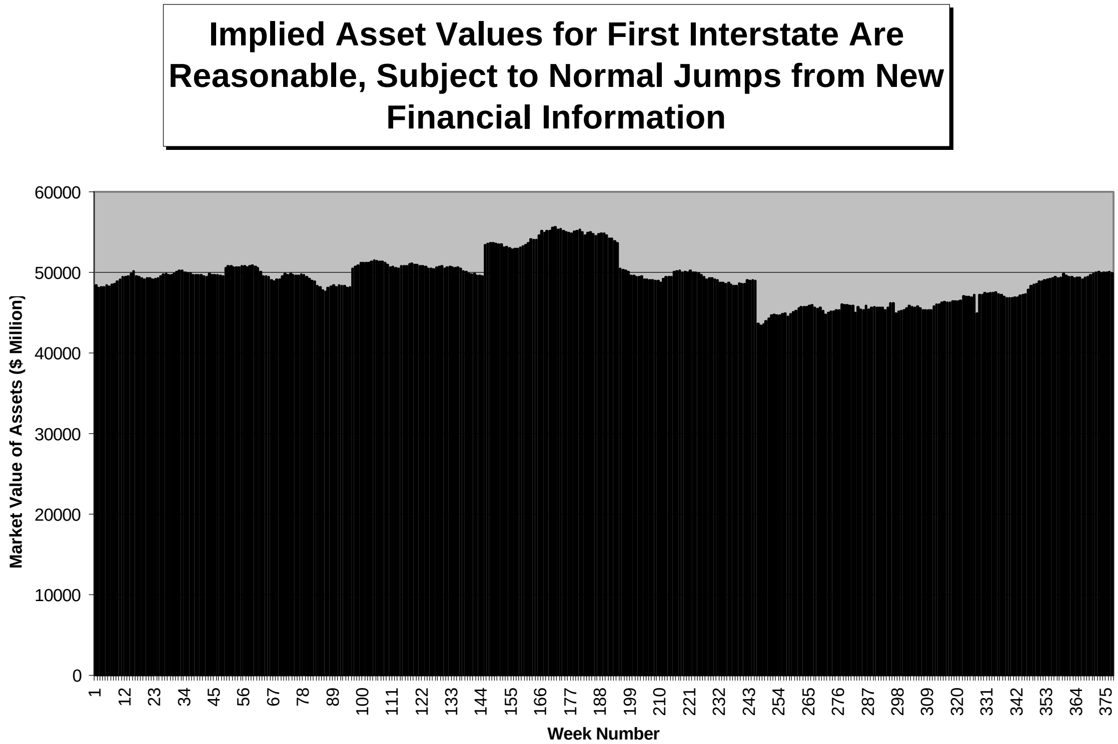
Estimating the strike price for the assumed zero-coupon bond liability structure is the most problematic of the three. This needs to be estimated from balance sheet data. First, First Interstate's quarterly financial statements were used to measure the "book value" of all of its liabilities[7]. Second, two years of interest expense, calculated by compounding the quarterly average cost of liabilities for two years, was added to this to account for the interest appreciation. The combined result is the estimate used.

The firm's asset values and the assets' volatility were estimated implicitly. The Merton model's parameters were calibrated weekly to fit the observable credit spread and stock price at the initiation of the trading position.

Figure 6 provides the estimates for the asset values. These values appear reasonable. Their values jump quarterly based on new financial information.

[7] Merton's model was treated more favorably than would be possible in practice. It was assumed that financial statements for a given quarterly were instantly known to the trader on the last day of the quarter, but in reality there would be a lag of some weeks before detailed balance sheets were publicly available.

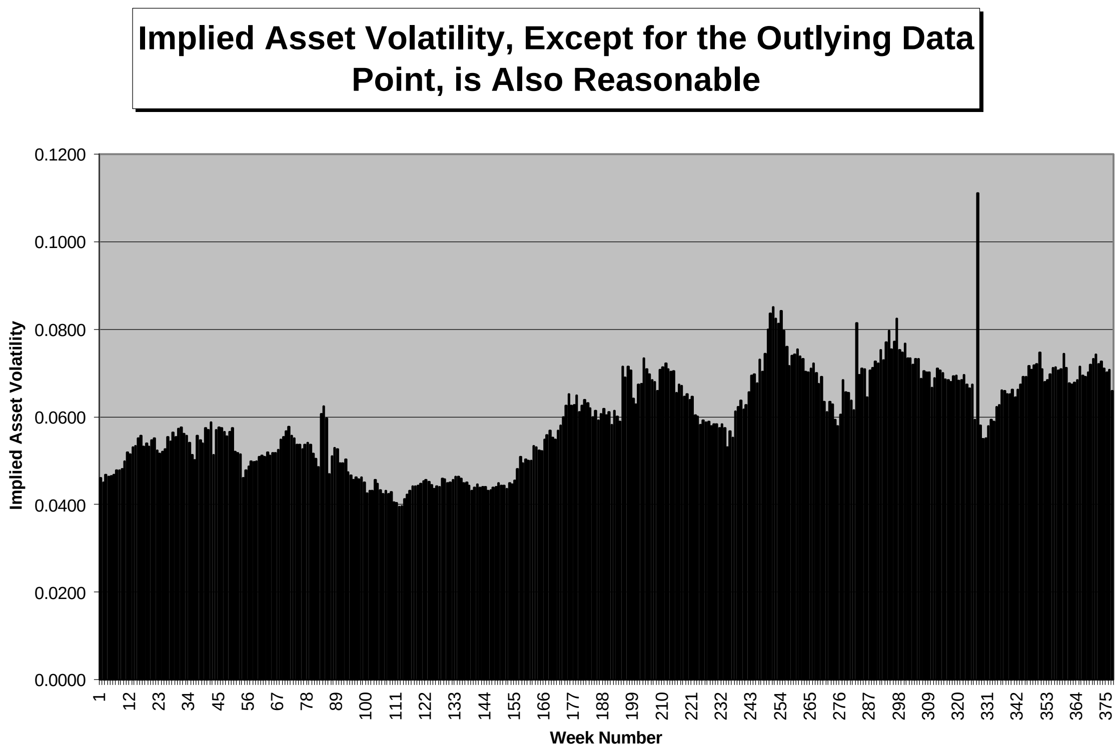
Figure 6



The values for the firm's implied asset volatility are contained in Figure 7. Except for the outliers, these estimates also appear reasonable.

Figure 7

The Merton's risky debt hedge consists of two securities, both stock and the U.S. Treasury note[8]. For the stock position, hedge ratios were calculated using the well-known "deltas" from the Black Scholes model (see Jarrow and Turnbull [1995] for this calculation)[9]. The U.S. Treasury hedge implied by the model was calculated by computing "rho," the derivative of risky debt with respect to changes in the riskless interest rate, and then neutralizing this derivative with a position in U.S. Treasury bonds.



e. The Results

The hedging performance of the two credit models is contained in Table 1.

As expected, for the traditional approach to fixed income security valuation, the optimal constant hedge ratio provides the lower bound and the 1-1 hedge the upper bound, with the duration hedge in the middle. The differences among the three hedging techniques are slight. They all reduce the standard deviation, from an unhedged position, by about 40 – 41 percent.

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[8] Although the model implies an interest rate hedge is unnecessary, we use a simple Taylor series expansion to extend the model to include an interest rate hedge. The expansion is: $\Delta D=(\partial D/\partial E)\Delta E+(\partial D/\partial r)\Delta r$. To use the Black-Scholes deltas, note that $\partial D/\partial E=(\partial D/\partial V)/(\partial E/\partial V)$. The ratios in the last expression can be obtained using the Black-Scholes formula.
[9] See footnote 8.

The ranking of the models produces the surprising result that Merton's risky debt model has the poorest performance. In contrast to the traditional approach to fixed income analysis, it reduces the standard deviation, from an unhedged position, by only about 20 percent.

Table 1

	Standard Deviation	Reduction In Hedge Error
Optimal Constant Hedge Ratio	1873.73	41.07%
Macauley Duration Hedge	1848.22	40.73%
1 to 1 Principal Amount Hedge	1850.52	40.66%
Merton Hedge	2488.41	20.20%
Unhedged Position	3118.28	0.00%

These hedging results indicate that for the pricing of First Interstate's two-year bond, interest rate risk is a more important component than is credit risk. The traditional approach provides a more accurate hedging model.

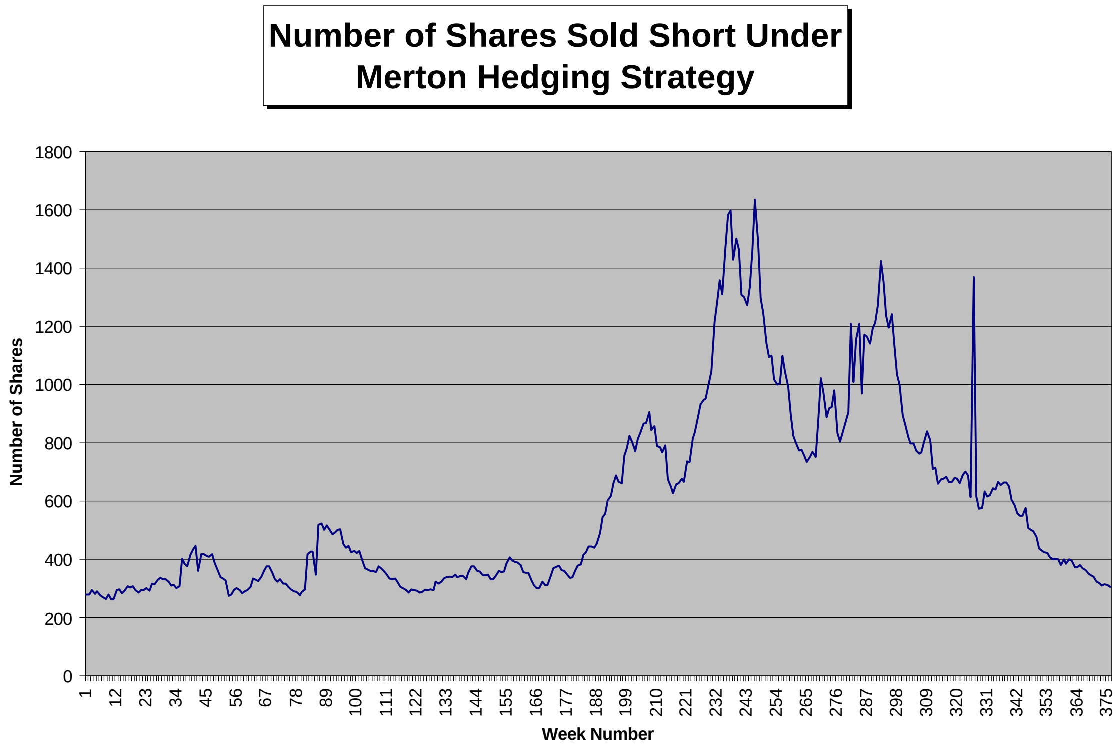
But, this is not the only implication from Table 1. Although the interest rate risk is the most important factor, a significant portion of the portfolio's standard deviation was unhedged, between 59 – 60 percent. The percentage unexplained is greater than the percentage explained. Combined with the fact that credit risk is ignored in this approach (and the fact that the Merton's hedge reduced the standard deviation), this is strong evidence that a model integrating both interest rate and credit risk is needed for more accurate pricing and hedging.

f. Reasons for the Merton Model's Performance

The poor performance of the Merton model compared to the traditional approach to fixed income analysis is surprising. It is useful to further investigate the reasons for this poor hedging performance.

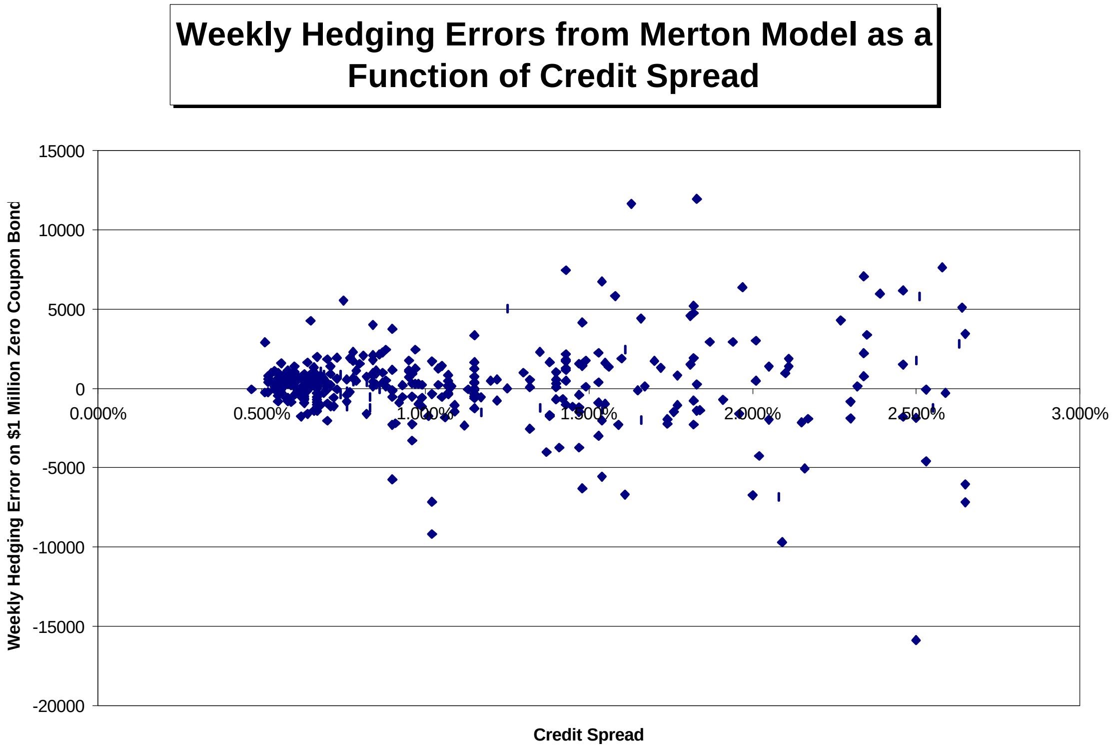
Figure 8 graphs the number of shares sold short under the Merton model. These hedge ratios looks reasonable at first glance, with the number of shares sold increasing as credit quality deteriorates.

Figure 8



However, in looking at hedging errors as a function of credit quality, one can see that the magnitudes of the hedging errors increased substantially as the credit quality deteriorates. So, the more "risky" the debt, the less well the hedge performed.

Figure 9



We then did an optimization, "in sample", on the hedge ratios prescribed by the Merton model to see whether hedging performance could be improved by scaling the Merton hedges up or down. The results were unexpected.

- The debt hedges should be reduced to 95% of the hedge ratio implied by the Merton model, and
- the equity hedges should be reduced to 21% of the Merton equity hedge ratio, and the hedge should be executed by ***buying the common stock instead of selling it***!

The second adjustment is noteworthy. On average, the direction of the Merton hedge appears to be incorrect. This is a result of the fact that 58% of the weekly data points in the sample had stock price and credit spreads moving in a direction opposite to that predicted by the model.

Regression analysis on the hedging errors of the Merton model shows that the change in stock price is a statistically significant explanatory variable. T-scores are given below the coefficient.

Hedging Error as a Function of Change in Stock Price

$Y = 208.97 + 710.95\ X$
(1.998) (13.873) Adjusted $R^2 = 0.339$

This result is certainly not a recommendation that one should buy common stock to hedge risky debt. Selling stock short, as recommended by the Merton's risky debt model, is almost certainly the correct strategy. The implication of the above analysis is that missing risk factors from the Merton model have caused the large hedging errors, and that stock hedge ratio was compensating for the fact that other hedging instruments were necessary, but missing from the hedge.

4. Conclusion

This paper compares two common modeling approaches to pricing and hedging credit risk. The first is the traditional approach to pricing fixed income securities. The second is Merton's risky debt model. The traditional approach ignores credit risk, and only prices interest rate risk. Merton's risky debt model ignores interest rate risk, and only prices credit risk. Both approaches are implemented in professional software and both approaches are used in practice.

A hedging comparison of the two models in performed using a unique data set. The data set consists of weekly quotes on First Interstate Bancorp's two-year bonds from January 1986 to August 1993. First Interstate experienced significant changes in its credit rating over this time period, providing a good case study for analysis.

The results are intriguing. The traditional approach to hedging fixed income securities dominates Merton's risky debt approach. So, if only one risk is to be included, interest rate risk appears to be the most important. But, the traditional fixed income approach's hedge eliminates only less than half of the portfolio's standard deviation. This indicates that significant risk remains still remains unhedged – the credit risk.

The conclusion of our investigation is that the newer models (see Jarrow and Turnbull [1995], Jarrow, Lando, Turnbull [1997]) are needed to more accurately price and hedge corporate debt. The existing approaches, although of some use, leave most of the risk of corporate debt unexplained and unhedged. This model misspecification is too large to be ignored, especially as these models become more relevant in the determination of capital requirements.

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