Modeling Correlated Interest Rate, Exchange Rate, and Credit Risk in Fixed Income Portfolios

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Abstract

Risk assessment methodologies seek to assess the maximum potential change in the value of a portfolio of financial instruments with a given probability over a pre-set horizon resulting from changes in market factors, credit risk, and liquidity risk. Current methodologies separate the portfolio analysis of market and credit risk and thus misestimate security and portfolio risk. We propose a new methodology that integrates interest rate, interest rate spread, foreign exchange rate, and credit risk analysis in one financial risk assessment for a fixed income portfolio. This risk assessment accounts for the correlation between these significant risk factors and allows improved portfolio risk measurement and management. The methodology is also shown to produce reasonable credit transition probabilities and prices for bonds with credit risk.

I. INTRODUCTION

We develop a diffusion-based methodology for assessing value-at-risk (VaR) of a portfolio of fixed income securities with correlated interest rate, credit, and exchange rate risk. Appropriately calibrated for the volatility of the period and firms to be studied the simulation methodology developed in this paper is also shown to produce reasonable credit transition probabilities and reasonable valuations for bonds with credit risk. In addition it provides the increased flexibility of assessing correlated interest rate, interest rate spread, FX, and credit risk for both individual assets and portfolios of assets including an ability to quantify the marginal impact of each risk factor.

The risk in owning a portfolio of risky fixed income securities is a function of changes in the risk-free term structure (interest rate risk), changes in macroeconomic or market conditions which affect the overall risk premium of an asset class (spread risk), changes in foreign exchange rates (FX risk), and changes in the credit quality of the individual assets in the portfolio (credit risk). VaR and credit risk models have been developed to provide measurements of these risks by providing a measure of how frequently the value of an asset or a portfolio may fall below certain thresholds. These risk measurement tools are then used for assessing capital adequacy requirements, capital-at-risk measures, and credit derivative pricing.

Two general methodologies have been developed to price debt instruments subject to correlated credit and interest rate risk. The first methodology models the asset value of the firm as a stochastic process and prices the debt as an option on the value of the firm (Merton (1974) and Longstaff and Schwartz (1995)). In the diffusion models, the value of a firm's bond is a function of the underlying asset, the total firm value, and the term structure. While this is a theoretically tractable methodology, it does not produce results consistent with the observed short-term credit spreads. The advantage of the diffusion based methodologies is that they rely on a theoretical pricing model, and the underlying pricing model can be easily expanded to factor into account correlated risk across assets in a portfolio. The inability of the diffusion based models to produce realistic short-term spreads lead to the development of a second general methodology, referred to as reduced form models or hazard rate models (Jarrow, Lando, and Turnbull (1995), Das and Tufano (1997), Madan and Unal (1998) and Duffie and Singleton (1999)). The reduced form models assume a stochastic process for the credit quality and recovery rate in the event of default. While these models produce realistic short-term credit spreads, there is no underlying theoretical model driving the bond prices.

Credit risk methodologies assess the potential change in the value of a financial instrument or a portfolio of instruments with a given probability over a pre-set horizon resulting from changes in the underlying asset's credit quality. Altman and Saunders (1998), Jarrow, Lando, and Turnbull (1997), CreditMetrics™ (Gupton, Finger, and Bhatia (1997)), CreditRisk+™ (1997), and ValueCalc™ (Barnhill (1998)) have all developed methodologies for assessing credit risk. These methodologies rely on analytical solutions, Jarrow et al., Altman and Saunders, and CreditRisk+™; or on numerical simulation methodologies, CreditMetrics™ and ValueCalc.

VaR and credit risk methodologies seek to assess the potential change in the value of a financial instrument or a portfolio of instruments with a given probability over a pre-set horizon resulting from changes in market variables (i.e. interest rates, exchange rates, equity prices, and commodity prices). VaR and credit risk have typically been treated as separate analyses in a portfolio setting. However, given the correlated nature of VaR and credit risk, it is difficult to determine one overall financial risk measure by aggregating the VaR and credit risk numbers.

The purpose of this paper is to provide a methodology to integrate VaR and credit risk into an overall financial risk assessment for a portfolio of fixed income securities. This is accomplished by simultaneously simulating both the future financial environment in which financial instruments will be valued and the credit rating of specific firms. The fundamental basis of this methodology is the contingent claims analysis (CCA) proposed by Merton (1974) with a stochastic default-free interest rate. For the purpose of this paper, we focus on fixed income instruments and thus on changes in interest rates and exchange rates. Changes in the interest rate on risky debt is a function of changes in the risk-free term structure and in the spread between a rating category and the risk-free security. We refer to interest rate risk plus FX risk as market risk. Interest rate volatility has been well explored and a number of one and two factor methodologies for modeling the evolution of risk-free interest rates have been proposed[1].

The financial risk model developed in this paper integrates market risk and credit risk. Financial risk in this model is a function of six types of underlying correlated and uncorrelated stochastic variables including interest rates, interest rate spreads, FX rates, returns on an equity market indices (i.e. systematic risk), firm specific equity returns (i.e. unsystematic risk), and default recovery rates. Therefore, we use a numerical simulation methodology. Given the correlated nature of credit and market risk (Fridson, Garman, and Wu; 1997), the importance of an integrated risk assessment methodology seems apparent.

In this paper, we first focus on the valuation and risk analysis of a single bond having correlated credit and interest rate risk. Next, we demonstrate the tractability of this numerical approach as a financial risk assessment for portfolios of bonds with correlated credit and market risk. In principle, a similar methodology could be applied to portfolios containing mortgages, variable rate loans, other fixed income securities, equities, and derivative securities.

As an overview, both the future financial environment in which the asset will be valued and the credit rating of specific firms are simulated. The financial environment is represented by eight correlated (approximately) arbitrage free term structures of interest rates (United States Treasury, Aaa… Caa-C), a single FX rate (e.g. Japanese Yen), and a set of twenty-four equity market indices representing various sectors of the economy (in practice any number of term structures, FX rates, and equity indices could be simulated). The correlated evolution of the market value of the firm's equity, its debt ratio, and credit rating are then simulated in the context of the simulated financial environment. The structure of the methodology is to select a time step (Δt) over which the stochastic variables are allowed to fluctuate in a correlated random process. The firm specific equity returns and security specific default recovery rates are assumed to be uncorrelated with each other and the other stochastic variables. For each simulation run a new financial environment (interest rate term structures, FX rate, and market equity returns) as well as

[1]Some examples of interest rate models include: Vasicek (1977); Cox, Ingersoll, and Ross (1985), Ho and Lee (1986), Jamshidian (1989), Hull and White (1990a, 1993, 1994), Black and Karasinksi (1991), and Heath, Jarrow, and Morton (1992).

firm specific market value of equity, debt ratio, credit rating, and default recovery rates are created. This information allows the value of financial assets to be estimated, and after a large number of simulations, a distribution of values is generated and analyzed.

The paper is organized in the following manner. First, a review of current credit risk and market risk analysis methodologies is provided. Second, the model for integrating market and credit risk is developed as well as discussion of how the parameters necessary for the model are empirically estimated. Third, the simulated credit transitions for representative bonds are compared to historical transition matrixes, bond valuation tests are performed, and the simulation methodology is used to assess integrated credit and market risk for various portfolios. Finally the conclusions are given.

II. CREDIT RISK

Credit risk analysis is a methodology to assess the effect of stochastic changes in credit quality (including default) on the value of a fixed income security or a portfolio of fixed income securities. To do this, credit risk methodologies estimate the probability of financial assets migrating to different risk categories (bond rating) over a pre-set horizon. The values of the financial assets are then estimated for each possible future risk category using forward rates from the term structure appropriate for each risk class. There are currently a number of different packages available to assess credit risk including CreditMetrics™, Credit Risk+™, Credit View, Loan Analysis System (LAS), and ValueCalc™. Altman and Saunders (1998) develop an analytical model that relies on Altman's (1993) z″ score to determine the probability of default over time. Jarrow, Lando, and Turnbull (1997) develop a model based on historical transition probabilities, which follow a Markov process, to examine the pricing of bonds. Jarrow et al. (1997) price fixed income instruments but this methodology can also be applied for credit risk assessment.

Jarrow et al. (1997) decomposes fixed income instruments into zero-coupon bonds. By assuming the bond is held to maturity, the authors collapse the problem of credit risk into only two states of nature, default or not default. If the payoff in default is known with certainty, the value of a zero-coupon risky bond can be represented in continuous time as:

$$V_t = p_t(C_t e^{-rt}) + (1 - p_t)(D_t e^{-rt}) \quad (1)$$

Where

V_t = the value of the zero-coupon bond at time t,

p_t = the probability of not defaulting at time t,

C_t = the cash flow, principal repayment, on the zero-coupon bond at time t,

r = the interest (discount) rate at time t, and

D_t = the value of the bond in default at time t.

Since p_t and D_t are assumed to be certain, the appropriate discount rate is the risk-free rate at time t.

A. Credit Event

Credit risk is sometimes thought of as the probability of default. However, this definition of credit risk views the bond in only two states: defaulted or not defaulted. In the more complex setting necessary to price bonds that may be sold before maturity, credit risk is a continuum with multiple states with each state representing an associated probability of default. Hence, temporal credit risk is a function of the probability of a change in the value of the bond associated with a transition in the probability of default over time, and credit risk can be either a positive or negative shift.

A positive credit change decreases the likelihood of the bond defaulting and is commonly related to an increase in the bond's rating, an upgrade. An upgrade lowers the required yield on the bond driving up the bond's price. Given the prevalence of embedded call options in bonds, care must be used in assessing the upside potential of a bond. The embedded call options effectively limit the upside. This is true even when the bond has call protection for some period of time.

A negative credit event is related to either default or a downgrade, which can lead to a significant loss in the value of the bond. However, even in the event of default, bondholders typically receive some compensation for the fixed income instrument in reorganization. The discussion following on recovery rates examines the effect of default on the value of a bond.

A credit event is normally associated with a change in the bond's rating. Credit ratings provide a general assessment of the firm's credit quality and serve as a means of grouping bonds into discrete categories with similar credit qualities and are commonly used as measures of risk for fixed income securities. The significance of credit ratings is evident by the importance third parties and financial regulators place on them for assessing the risk of financial institutions, mutual funds, and pension funds.

Clearly some caution is in order when utilizing credit ratings. First, credit rating changes lag market pricing (Ederington, Yawitz, and Roberts (1987)). Second, credit ratings attempt to assess the overall credit risk of a fixed income security, and to do this they combine both the probability and severity of default into a single measure. This impedes the comparison of bonds across seniority classes and can lead to some confusion. For example, a senior secured Ba bond likely has a higher probability of default than a junior subordinated Ba bond. The senior secured bond has less credit risk in the event of default because on average it will have a higher recovery rate. Thus for the senior secured bond to be rated the same as the junior subordinated bond, the other component of credit risk, the probability of default, must be greater.

Generally, the rating agencies assign subordinated debt one minor rating classification lower than a comparable senior bond in the investment grade category and two minor rating classifications lower than a comparable senior bond for the noninvestment grade category. For example, a firm with senior debt rated Aa2 would normally have the subordinated debt rated Aa3, and a firm with senior debt rated Ba2 would normally have the subordinated debt rated B1.

B. Stochastic Credit Risk

Changes in bond ratings reflect changes in the perceived ability of the firm to meet its financial obligations. Such credit quality changes may result from changes in macroeconomic conditions or from changes in the unique financial conditions of the firm. The correlated impacts of macroeconomic factors on the credit quality of many firms imply a correlation in credit risk across firms in various industries.

An analysis of the effect of a shift of one rating category on the value of a bond is provided in Table 1. Noncallable term structures, estimated for 12/31/98, are used in this example. The significance of a credit event on the value of a bond is apparent. This is especially true as credit rating declines. A credit migration from Aaa to Aa for a five-year bond decreases the value of the bond by 0.96% while a credit migration from B to Caa decreases the value of the bond by 16.53%. A comparison of the effect of credit migration between the five and ten year bonds, as well as the discrepancy of the change in the price based upon an upgrade versus a downgrade, demonstrates the effect of duration and convexity on credit risk.

C. Transition Matrixes

To assess credit risk each possible credit transition must be associated with a probability. One method to project future transitions is to rely on historical transition probabilities, e.g., Jarrow et al. (1997) and CreditRisk+. Moody's Investor Service and Standard & Poor's are two of the most prominent firms that compile historical probabilities of credit transition by rating category. For this study, Moody's transition matrixes[2] are utilized for comparison to our simulated transition probabilities to demonstrate that the methodology produces reasonable transition estimates. Given Carty and Lieberman's (1996) finding of no systematic bias in the withdrawn category between upgrades and downgrades, the transition matrixes given in Table 2 are adjusted to eliminate the withdrawn category.

Utilizing a historically calculated transition matrix to assess the credit risk of current bonds has certain problems. For example, Fridson et al. (1997) found a relation between macroeconomic conditions and default probability. Thus credit transition probabilities differ considerably during economic recession and expansion. In addition, we believe that to accurately assess overall financial risk a methodology must account for the correlations between interest rate and credit rating changes across an entire portfolio of assets.

D. Recovery Rates in the Event of Default

As well as assessing the probability of default, the distribution of the amount recovered in the event of default must also be modeled. One method to assess recovery rates is to examine historical results. Carty and Lieberman (1996) and Altman and Kishore (1996) have studied the recovery rate in the event of default. These studies conclude that average recovery rates increase with the seniority and security of the bonds. However, within a seniority class there is a wide distribution of realized recoveries. Additionally, Altman and Kishore (1996) found some indication that recovery rates may be a function of industry.

[2] See Carty and Lieberman (1996).

E. Utilizing Transition Matrixes and Recovery Rates to Value Bonds before Maturity

In credit risk assessment, if the bond defaults the recovery rate can be modeled as either a deterministic or stochastic process. The Jarrow et al. (1997) model assumes that the recovery rate is deterministic. If the recovery rate is assumed to be deterministic, the value of the bond at any time step, $t = j$, can be easily calculated by multiplying the probability of ending up at each possible state (i.e. credit rating) by the value of the bond if it ends up at that state. The calculation of the expected value of a bond with credit risk at time $t = j$ is found in equation 2.

$$E(V_j) = \sum_{i=1}^{n} r_{i,j} V_{i,j} \tag{2}$$

where:

$E(V_j)$ = the expected value of the fixed income instrument at time j,

$r_{i,j}$ = the probability of being in state i at time j, and

$V_{i,j}$ = the value of the instrument at time j in state i as shown in equation 3.

$$V_{i,j} = \sum_{t=0}^{n} c_{i,t} \mathrm{e}^{R_{i,t}} \tag{3}$$

$C_{i,\mathrm{t}}$ is equal to the cash flow in state i at time t and $R_{i,t}$ is the continuously compounded spot interest rate to time t ($t \geq j$) appropriate for state i.

The variance and standard deviation of V_j is equal to:

$$s_{V_j}^{2} = \sum_{i=1}^{n} r_{i,j} \left[V_{i,j} - E\left(V_{i,j}\right)\right]^2 \tag{4}$$

$$s_{V_j} = \sqrt{s_{V_j}^{2}} \tag{5}$$

The cash flows from the bond after the first year are equal to the known coupon payments plus the face value of the bond at maturity. These payments are known with certainty unless the bond defaults. Nevertheless if the bond's credit quality has changed, then it must be revalued using the appropriate new term structure.

Table 3 shows an example of this type of calculation for a ten-year B-rated bond trading with an initial PAR value of $1,000. The value of the cash flows from the bond (price of the bond at t = 1 plus the coupon payment) is calculated at a one-year time step assuming the implied forward rates are the actual arrived at spot rates. The distribution of possible values multiplied by the probability of arriving at that credit quality is the mean expected value of the bond at the end of one year, $1,054.66 in this example. The standard deviation of the bond's value at the end of one year can then be easily calculated using equation 4 and 5, $174.12 in this example. Confidence levels can also be easily calculated in this framework by determining the level at which a cumulative percentage exceeds the confidence level. The cumulative percentage exceeds 95% when the bond is rated Caa ($619.50) and 99% when the bond is in default ($340.00).

Similar to Jarrow et al. (1997), this type of analysis assumes that the recovery rate in default is deterministic. The importance of modeling the stochastic nature of recovery rates increases as the bond's rating decreases, as there is a higher likelihood of default.

Given the large standard deviation of recovery rates[3], in our proposed simulation model default recovery rate is modeled as a stochastic variable drawn from a beta distribution. For this study, all bonds are assumed to be senior subordinated with a mean recovery rate of 34% and a standard deviation of 25% (Altman and Kishore, 1996).

III. AN INTEGRATED MODEL OF CORRELATED MARKET AND CREDIT RISK

A number of methodologies for pricing risky debt have been developed: Merton (1974), Longstaff and Schwartz (1995), Das and Tufano (1996), Duffie and Singleton (1997, 1999), and Madan and Unal (1998). In these frameworks, there are two principal components: a risk-free component plus a premium for default risk[4]. The risk-free term structure represents compensation for interest rate risk, and the default risk component reflects the probability of default as well as the recovery in the event of default. The yield or credit spread, the premium over the yield of the Treasury security with a corresponding maturity, is the compensation to the

[3] See Carty and Lieberman (1996) and Altman and Kishore (1996)

risk-averse investors for accepting default risk. The credit spread changes over time due to a change in the perception of the probability or severity of default.

In this section, the model for integrating market and credit risk in a correlated simulation framework is developed. We believe it is necessary to simulate the future financial environment in which bonds will be valued and the correlated evolution of the credit quality of the financial instruments to fully evaluate the risk characteristics of instruments and portfolios. This model is an extension of the diffusion models developed by Merton (1974) and Longstaff and Schwartz (1995), but applied in a multi-asset portfolio.

The price of a fixed income security is a function of the term structure for that asset. For current demonstration purposes, we have eight mutually exclusive asset classes (Aaa,…, Default) into which a bond may fall. The term structures for each asset class (excluding the default category) is a stochastic variable.

The simulation of bond credit rating is undertaken in a reduced form of the contingent claims analysis (CCA) framework. As developed by Black and Scholes (1973) and more explicitly by Merton (1974) the firm's stockholders hold a call option on the firm and the debt ratio is a measure of how far the call option is in the money.

In Merton's (1974) CCA framework the following assumptions are made:

A.1.1 There are no transaction costs or taxes and assets are infinitely divisible.

A.1.2 There are sufficiently many investors with comparable wealth levels so that each investor believes that he can buy and sell as much of an asset as he wants at the market price.

A.1.3 There is an exchange market for borrowing and lending at the same rate of interest.

A.1.4 Short sales of assets, with full use of the proceeds, is allowed.

A.1.5 Continuous trading is allowed.

A.1.6 The Modigliani-Miller theorem that the value of the firm is invariant to its capital structure holds.

A.1.7 The term structure is "flat" and known with certainty.

A.1.8 The dynamics for the value of the firm, V, through time can be described by a diffusion-type stochastic process with the stochastic differential equation

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[4]Even though liquidity and tax affects can affect both the risk-free and default risk component (e.g. Kamara 1994, Green and Odegaard 1997, and Elton and Green 1998), these factors are not directly incorporated into the pricing models.

$$dV = (\alpha V - C)dt + \sigma V dz$$

where:

α = the instantaneous expected rate of return on the firm per unit time,

C = the total dollar payout by the firm per unit of time to either its shareholders or liabilities holders,

σ^2 = the instantaneous variance of return on the firm per unit of time, and

dz = a standard Gauss-Wiener process.

Given that the purpose of our paper is to model the stochastic evolution of a firm's debt over time, we relax and modify some of the standard assumptions found in the CCA framework and make some additional assumptions as follows:

A.2.1 The value of debt in the debt ratio refers to the face value of the debt, which is the cash flow due at maturity of the bond.

A.2.2 The default-free interest rate is stochastic.

A.2.3 The firm's debt ratio (D/V) and volatility (σ) can be used to determine the appropriate risky term structure (AAA,.., Default) and thus the required rate of return from which the bond's cash flows will be discounted. If the bond defaults, the recovery rate is stochastic with a known mean and standard deviation.

A.2.4 The firm's expected return on equity and firm specific equity return volatility can be estimated using a one factor CAPM model (multi-factor models would also be feasible).

A.2.5 The expected growth rate in the market value of the firm's common stock is equal to the firm's expected return on equity minus its dividend yield.

A.2.6 The dividend yield is constant over the time period simulated.

A.2.7 The firm has an expected growth rate in assets and a target debt ratio that is constant.

Our goal is to model the stochastic changes in the market value of a bond. The factors that cause stochastic shifts in a bond's price are correlated interest rate, interest rate spread, exchange rate, and credit rating changes (including default). Default risk refers to the ability of the firm to meet set cash payments, which is in reference to the face value of the debt (book value), and the default recovery rate if the payments are not meet. Merton's assumption (1.7) of a flat yield curve is relaxed, and the default-free rate is modeled as a stochastic variable. Work by Ogden (1987) and Barnhill and Maxwell (1998) suggests that assumption 2.3 is reasonable as debt ratios can be used to reasonably map bond ratings if the industry specific nature of business risk is taken into account. Given assumption 2.7, we assume that a firm has a fixed financing plan (i.e. equity and debt sales or repurchases) over the simulation period and that variation in the debt to value ratio reflect changes in the market value of the firm's equity. Hence, the firm's equity return lead the bond returns. This is consistent with the findings by Kwan (1996) and Gebhardt (1999). They find that stock returns lead bond returns in reflecting firm-specific information over a short-term horizon (Kwan, 1996) and over a longer-term horizon (Gebhardt, 1999).

A. Simulating Stochastic Term Structures

For this study, the Hull and White extended Vasicek model (Hull and White; 1990a, 1993, 1994) is used to model stochastic risk-free (e.g. U.S. Treasury) interest rates. In the Hull and White model interest rates are assumed to follow a mean-reversion process with a time dependent reversion level. The simulation model is robust to the use of other interest rate models.

The Hull and White model for *r* is:

$$Dr = a\,(\frac{q(t)}{a} - r)Dt + sDz \qquad (6)$$

where

Dr = the risk-neutral process by which *r* changes,

a = the rate at which *r* reverts to its long term mean,

r = the instantaneous continuously compounded short-term interest rate,

$q(t)$ = "Theta" is an unknown function of time which is chosen so that the model is consistent with the initial term structure and is calculated from the initial term structure as: $q(t) = F_t\,(0,t) + aF\,(0,t) + \frac{s^2}{2a}(1 - e^{-2at})$

$F\,(0,t)$ = the forward interest rate at time t as calculated at time 0.

$F_t\,(0,t)$ = the derivative of the forward interest rate with respect to time.

Dt = a small increment to time,

s = "sigma" the instantaneous standard deviation of *r*, which is assumed to be constant, and

Dz = a Wiener process driving term structure movements with Dr being related to Dt by the function $Dz = e\sqrt{\Delta t}$.

Given a simulated future value of r, the initial term structure, and the other parameters of the model a complete term structure of risk-free interest rates can be calculated and financial assets can be re-valued at time step Dt. The Hull and White (1990a, 1993, 1994) model (equation 6) utilized to simulate the risk-free rate incorporates both a mean reversion rate and volatility as well as a time dependent mean interest rate. The mean reversion and volatility rates can be estimated from a time series of short-term interest rates or implied from cap and floor prices. The mean reversion rate and volatility estimates used in this study are estimated from a time series of short-term interest rates over the 1993-1998 period (Table 4).

Once the risk-free term structure has been estimated then the Aaa term structure is modeled as a stochastic lognormal spread over risk-free, the Aa term structure is modeled as a stochastic spread over Aaa, etc. The mean value of these simulated credit spreads are set approximately equal to the forward rates implied by the initial term structures for various credit qualities (e.g. Aaa). This procedure insures that all simulated credit spreads are always positive and that the simulated term structures are approximately arbitrage free.

The first step in modeling the eight different term structures is to determine the appropriate initial yield curves. For this study term structures estimates for United States Treasury securities, Aaa, Aa, A, Baa, Ba, and B bonds were taken from Standard & Poor's *CreditWeek*, while the Caa term structure was estimated from the Lehman Brothers Bond Database (Table 4). In addition a time series of short-term yields for the various credit ratings was estimated from 1993-1998. This time series was used to estimate the volatility of the various credit spreads (e.g. Aa vs. Aaa,.., B vs. Ba, etc.). Table 4 gives the estimated volatilities for the various interest rate spreads.

B. Correlation of Interest Rates, Foreign Exchange Rates, and Equity Returns

Fridson et. al. (1997) find a relation between interest rates and default rates. This is consistent with the implied relation in a contingent claims analysis as increasing interest rates are correlated with lower asset returns. The historical correlation structure between the change in interest rates, the return on various equity indices, and the U.S. dollar / Japanese Yen exchange rate are found in Table 5. Changes in interest rates are negatively correlated with returns on the S&P 500 Index. For example, the correlation coefficient between the change in the short U.S. Treasury rate and the S&P 500 is a negative 0.33. The correlation between interest rates and economic sector equity returns is a function of the interest rate sensitivity of the sector. Firms and industries would have higher or lower correlation structures based upon their sensitivity to interest rates.

C. Simulating the Asset Return

The equity indices and FX rate returns are simulated as stochastic variables correlated with the simulated future risk-free interest rate. Hull (1997) describes a procedure for working with an *n*-variate normal distribution. This procedure requires the specification of correlations between each of the *n* stochastic variables. Subsequently n independent random samples ε are drawn from standardized normal distributions. With this information the set of correlated random error terms for the *n* stochastic variables can be calculated. For example, for a bivariate normal distribution,

$$e_1 = x_1 \tag{7}$$

$$e_2 = r x_1 + x_2 \sqrt{1 - r^2} \tag{8}$$

where

x_1, x_2 = independent random samples from standardized normal distributions,

r = the correlation between the two stochastic variables, and

e_1, e_2 = the required samples from a standardized bivariate normal distribution.

The model utilized to simulate the value of the equity market indices and FX rate (*S*) assumes that (*S*) follows a geometric Brownian motion where the expected growth rate (*m*) and volatility (σ) are constant (Hull, 1997, p. 362). The expected growth rate is equal to the expected return on the asset (μ) minus its dividend yield (*q*). For a discrete time step, Δt, it can be shown that

$$S + \Delta S = S \exp\left[\left(m - \frac{\boldsymbol{s}^{2}}{2}\right)\Delta t + s e \sqrt{\Delta t}\right] \tag{9}$$

ε = a random sample from a standardized normal distribution.

The return on the market index (K_m) is estimated as

$$K_m = ((S + \Delta S)/S) + q \tag{10}$$

The return for the individual firm's equity is simulated using a one-factor model.

$$K_i = R_F + Beta_i\,(Km\text{-}RF\,) + s_i Dz \tag{11}$$

where

K_i = the return on equity for the firm_i

R_F = the risk-free interest rate,

$Beta_i$ = the systematic risk of firm_i,

K_m = the simulated return on the equity index from equation 10,

s_i = the firm specific volatility in return on equity, and

Dz = a Wiener process with Dz being related to Dt by the function $Dz = e\sqrt{\Delta t}$.

The next step is to estimate the parameters necessary to simulate the market return. In the simulations where bonds are priced in a risk neutral framework the expected return on the equity index was set equal to the risk-free rate. In the simulations undertaking integrated market and credit risk analysis on portfolios of bonds the expected return on the equity indices was set equal to the risk free rate plus a long-term average risk premium of eight percent. The average dividend yield on the S&P 500 from 1993 to 1998 of approximately 2.6% (source: DRI) was used as the market dividend yield. To model the stochastic component in equation 9, the 1998 equity return volatility for the S&P 500 of 23% is utilized as the estimate for market volatility.

After simulating the market return, the return on the individual stock is estimated in the CAPM framework (equation 11). The first step in calculating the expected return on equity for a "typical" firm in a particular rating class is to understand how beta coefficients change with credit ratings as well as the unsystematic component of equity return risk. To do this, a cross-sectional time series was developed from Compustat to calculate the average firm's beta by bond rating for period 1993-1998. The bonds were then segmented within bond category into one of two categories: high or low volatility. Low volatility firms were defined to be those in the lower third of total equity return volatility. High volatility firms were defined to be the remaining two thirds of firms. Due to their inherent high volatility B and Caa rated firms were not divided into different volatility categories. Bonds were sorted by bond rating and characteristic lines are estimated to compute the firm's beta. The results are found in Table 6. The results are as expected. As bond rating declines, the firm's systematic risk (beta) increases. To simulate a firm's return on equity its unsystematic risk must also be determined. Table 6 also reports the unsystematic risk (the annualized root mean square error) of the estimated characteristic lines.

D. Mapping Debt Ratios into Discrete Asset Classes

The firm's return on equity is simulated as described above. These simulated equity returns are then used to estimate a distribution of possible future equity market values and debt ratios. The simulated debt ratios are then mapped into bond ratings.

The methodology described above assumes a deterministic relation between the firm's debt ratio and its bond rating over the time frame simulated[5]. In a contingent claims framework this is equivalent to assuming a constant volatility for the value of the firm. Hence, an empirical analysis of the distribution of debt ratio[6] by rating class was performed on all manufacturing firms with a Standard and Poor's bond rating tracked by Compustat on a quarterly basis from 1987 to 1998. We segmented the bonds by rating class into two categories, high and low volatility firms, based upon the historical volatility of their equity returns as described above. Debt ratio distributions were then analyzed by rating category and volatility structure. The results are found in Table 7. As expected, debt ratio increases as bond rating declines, and high volatility firms have lower average debt ratios. For the Caa-C and Default categories it is noted that there is very little difference in the distribution of debt to value ratios which are based on the first observation when a firm is reported to have entered these categories. For simulation runs reported later in this

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[5] Blume, Lim, and MacKinlay (1998) suggest that leverage ratios and bond ratings are not constant over time. However, their results are over a longer time frame than simulated in this framework.

[6] Merton (1974) defined leverage ratio as debt over equity. To simplify for comparison purpose, the algebraically equivalent debt over total market capitalization (i.e. debt ratio), is defined as [book value of debt/(book value of debt + market value of equity)], is utilized in this study.

study, we assume that debt ratios start at the mid point between the first and third quartiles for the assumed initial credit rating category. Credit ratings are generally assumed to change when simulated debt ratios cross the quartile boundaries. However due to the fact that the distribution of debt to value ratios of Caa-C and defaulted companies is very similar, the debt to value ratio at which firms were assumed to default was set at 0.77. This level is approximately equal to the mean for defaulting firms and results in simulated default rates on B and CCC rated firms consistent with historical levels.

Utilizing the mean beta and dividend yield (from 1993 to 1998) by rating category, the expected return on the firm's equity and the growth in its equity value can be calculated. Using the assumptions regarding asset growth rate, target capital structure, and the simulated return on the individual stock (equation 11) the simulated debt ratio of the firm is calculated. Using this debt ratio the simulated credit rating is determined.

After simulating the bond's future credit rating its value is calculated using the simulated term structure of interest rates appropriate for that risk class. If the bond is simulated to default, the recovery rate on the bond is simulated as a beta distribution[7] with a mean value of 34% and a standard deviation of 25%. If the bond is denominated in a foreign currency then its numeraire currency value is calculated by multiplying the simulated bond value by the simulated foreign exchange rate that by construction is a correlated stochastic variable.

To determine a probability distribution of simulated values, the simulation is run 10,000 times. The distribution of values is then used to determine test statistics and estimates for the 99%, 97.5%, and 95% confidence levels.

III. SIMULATION RESULTS

In this section, we demonstrate the methodology described previously to undertake various analyses. Unless otherwise noted, standard assumptions for the following analyses include the following:

[7] Utilizing a beta distribution allows the recovery rate to fall within 0% and 100% while maintaining the same mean and standard deviation.

A.3.1 All bonds are senior subordinated with a mean default recovery rate of 34% and a standard deviation of 25% drawn from a beta distribution.

A.3.2 The initial term structures for Unites States Treasury Securities, Aaa, Aa, A, Baa, Ba, B, and Caa-C were those estimated as of December 31, 1998. The estimates were obtained from Standard & Poor's *CreditWeek*. The Caa-C term structure was estimated from the Lehman Brothers Bond Database.

A.3.3 In order to price bonds in a risk neutral framework the expected return on the equity index was assumed to be the risk free rate (4.5%). For value-at-risk analyses the expected return on the equity indices was increased by an eight percent risk premium reflecting the long-run historical difference between returns on the S&P 500 and U.S. T-Bills.

A.3.4 The dividend yield on the equity indices was assumed to be 2.6%.

A.3.5 The volatility of the returns on the equity indices was assumed to be 23% reflecting the S&P 500 volatility for 1998.

A 3.6 The volatility of the FX rate used to assess FX risk was 10% reflecting the historical volatility of the Japanese Yen versus the U.S. Dollar over the 1987 to 1996 period.

A.3.7 Starting and target debt ratio for bonds in a particular credit rating was the midpoint between the first and third quartile values given in Table 7, and the break points between rating categories were the first and third quartile estimates. The debt ratio break point for defaulted bonds was set approximately equal to the mean (.78) for that category.

A.3.8 The firm specific parameters (i.e. beta, and unsystematic equity return volatility) were those given in Table 6.

A. Credit Transition Matrixes

Utilizing the above models, data, and assumptions a firm's debt ratio and hence bond rating can be simulated over any time step. The results for 10,000 simulations for one, and three-year time steps are reported in Table 8 for both high and low volatility firms.

Comparisons of the simulated transition matrixes (Table 8) and Moody's historical transition matrixes (Table 2) show many similarities. In each case the most likely event is that the rating stays the same, the next most likely event is that the ratings move up or down by one category. Also the rating transitions become more dispersed as the time step increases (e.g. one-year versus three-years).

Moody's does not distinguish between low and high volatility companies thus there is no direct comparison for historical transition probabilities and the simulated ones for low and high volatility firms. However it is interesting to note that the simulated probabilities of the lower volatility firms staying in their initial rating category are consistently larger than those for the higher volatility firms. Also an average of the simulated transition probabilities for the low and high volatility firms would result in distributions somewhat more dispersed than the historical average. In addition the simulated default rates on B and CC rated firms are marginally higher than the historical averages for the three-year time step. These results are consistent with the volatile conditions prevailing in the markets during 1998 where the S&P 500 had a volatility of 23 percent versus 20 percent over the long-term. Over other selected periods (e.g. early to mid 1990's) market volatility and default rates have of course been lower. Finally it is important to note that the investment grade bonds generally had a zero or very low simulated default rate while Moody's shows some small percentage. This is a limitation of the proposed methodology. Possible explanations for these differences include, inaccuracies in the proposed model or its estimated parameters, non-normal equity return distributions, delays in bond rating changes by rating agencies, actions by some companies to maintain a target bond rating by adjusting investment and financing strategies, a dispersion of firm characteristics not captured by the standard assumptions used in the analysis, and occasional changes in firms' target capital structures (e.g. leveraged buyouts). Of course simulated default rates can be increased (decreased) by lowering (raising) the debt to value ratio at which default is assumed to occur.

Overall it was concluded that the model produces transition probabilities similar to the reported historical transitions. Importantly it also allows for the modeling of correlated changes in interest rates, exchange rates, and credit ratings as well as the correlated evolution of credit ratings on a portfolio of bonds.

B. Bond Valuation Tests

To test the ability of the model to value bonds, comparisons are made between analytical and simulated coupon bond prices.   Bond prices with a maturity of ten years are calculated from the known typical yield curves for each rating class as of December 1998. The values of the coupon bonds are then simulated out one, and three-years (t = 1, 3) and discounted back at the average simulated risk-free rate. For the model to be arbitrage-free, the known value at $t = 0$ ($100) should equal the simulated value (at t =1, 3*)* discounted back to t=0 at the risk-free rate. A deviation between the known value and the simulated value implies a mispricing in a risk-neutral valuation framework.

Using the standard simulation assumptions, the bond valuation tests were performed on bonds with assumed initial credit ratings of Aaa through Caa. The results are found in Table 9. The estimated error represents the difference between the mean simulated values and the analytical solutions. The results suggest that the simulation models are reasonably accurate for bond rating categories Aaa through Baa for both one and three-year time steps, where the models produces close to arbitrage-free estimates in most cases with no error exceeding two percent. For non-investment grade bonds the simulated prices are somewhat higher than the analytical values, particularly so for the three-year time step. The finding of a premium in a risk neutral valuation framework for non-investment grade bonds is consistent with Fons (1987), Altman (1989), and Jarrow et al (1997) findings. It should also be noted that December 1998 was a time of wide credit spreads which is consistent with a liquidity premium for holding such securities. Finally if the objective were to produce arbitrage free values for a particular type of bond (e.g. B-rated) then it could easily be accomplished by adjusting the debt to value ratio at which firms are assumed to default.

C. Risk Analysis

After examining the transition probabilities and valuation for a single bond, we next examine the model's ability to analyze interest rate and credit risk for a portfolio of bonds. The advantage of using a simulation model in the portfolio analysis is the ability to capture the correlated nature of credit risk. If historical transition probabilities were utilized to assess credit risk in a portfolio setting, the underlying assumption is that credit risk is uncorrelated across the assets which underestimates credit risk. Credit risk in this model is a function of changes in equity valuations, which allows for the model to incorporate correlated credit risk directly through equity correlations. A bond portfolio that is highly concentrated in one industry would have less credit risk diversification, while a bond portfolio which is diversified across a large number of industries will have diversified credit risk to a greater extent.

The risk analysis will first focus on a single bond and subsequently consider portfolios of bonds. The value of the bond is simulated at the end of the time period and includes the last coupon payment. The risk analysis for a ten-year B-rated bond at a one-year time step is found in Table 10. The single bond is assumed to face interest rate risk (i.e. volatility in the risk-free term structure), interest rate spread risk, credit risk, and FX risk. Initially the risk analysis is performed with only interest rate risk. Under this assumption the mean simulated value of the bond is $108,866 with a standard deviation of $3,215 and 95% confidence level of $103,723. The inclusion of interest rate spread risk has little impact on the mean value ($109,081) however the standard deviation doubles to $6,450 and the 95% confidence level declines to $97,180.

Next a risk analysis is performed on credit risk only. Credit risk reduces the mean simulated value to $104,148 (due to credit downgrades and default losses), sharply increases the standard deviation to $22,561, and sharply reduces the 95% confidence level to $60,544. In the extreme the minimum value of the bond falls to $29 reflecting the possibility of default with minimal recovery. The simulated standard deviation for bond value resulting from credit risk alone is somewhat higher than that calculated in Table 3 using a standard credit risk analysis ($17,412 for a $100,000 initial value). This difference is explained by the fact that the simulated probabilities for higher volatility firms migrating out of the B rating category at the end of 1998 were somewhat larger than Moody's average historical credit transition probabilities. The inclusion of interest rate risk, and spread risk along with credit risk has little impact on the mean value of the portfolio ($104,225 versus $104,148), however it marginally increases the standard deviation by $872 ($23,433 versus $22,561) and reduces the 95% confidence interval from $60,544 to $56,736. This increase in the standard deviation of bond value ($872) is of course much lower than that attributable to interest rate and spread risk taken alone ($6,450). The reason that interest rate and credit risk are not additive is that they are less than perfectly positively correlated. It is obviously important to account for the correlations in these various risk factors.

The inclusion of FX risk has little impact on the mean value of the bond ($104,426) however it further increases the standard deviation ($25,869), and reduces the 95% confidence level ($53,230).

To perform portfolio risk analyses, we form portfolios of 1, 2, 5, 7, 10, 12, 15, 17, 20 and 24 B rated bonds. The results are found in Table 11. All bonds are assumed to have a ten-year maturity, are noncallable, and face interest rate, interest rate spread, and credit risk. We also include one portfolio of 24 bonds that faces FX risk as well. We assumed that each bond added to the portfolio was from a unique industry with different correlation structures based upon the historical estimates found in Table 5. Hence, our resulting estimates are for the maximum diversification available for the number of bonds in the portfolio. The ending value of the portfolio was simulated out one year, 10,000 times. The mean, standard deviation, maximum, and minimum simulated portfolio value, and the 99%, 97.5%, and 95% confidence levels are the resulting output statistics from the model.

As can be seen from Table 11, as the number of bonds included in the portfolio increases there is little change in the mean portfolio value (i.e. $104,200 to $104,400). More importantly from a risk analysis perspective, as the number of bonds in the portfolio increases, the standard deviation decreases (from $23,406 to $8,678) and the minimum value and confidence levels increase (e.g. 95% confidence level increases from $56,625 to $88,766). Credit risk diversification clearly pays.

In an effort to compare our results for a portfolio of single B bonds with historical risk measures, we calculated the one year holding period returns (change in the flat price plus interest) on a monthly basis from 1987 through 1998 for the Lehman Brothers B-Rated Long-Term Bond Index. We then determined the 99%, 97.5% , and 95% confidence levels for a $100,000 investment over a one-year time frame from the actual distributions. From the actual distribution the 99%, 97.5%, and 95% confidence levels are $87,416, $89,855, $94,906. These analytical results are then compared to the simulation results found in Table 11 for a 24 bond portfolio with interest rate, spread risk, and credit risk. The simulated confidence levels are $80,786, $85,032, and 88,766 at the 99%, 97.5%, and 95% confidence levels respectively. The simulation results suggest a higher level of uncertainty. This higher level of uncertainty in the simulation results is appropriate for two reasons. First, the simulation results are for 24 bonds as compared to the large portfolio of bonds in the index. However, the simulation results suggest that this difference should be minimal. Second and more importantly, the Lehman Brothers Index is refreshed every month with single B bonds. Thus if a bond is downgraded, it would effect the index return for that month, but the bond is then removed from the index for the following month. Hence, the bond index only catches one downgrade, while in the simulation model multiple downgrades in a single year are possible. We believe the comparison between simulated results and the analytical results suggests that the simulation model produces results consistent with historical measures.

As discussed previously interest rate and spread risk taken alone produce a portfolio standard deviation of $6,450. With 24 bonds credit risk taken alone produced a portfolio standard deviation 0f $6,810. With 24 bonds interest rate, spread, and credit risk produced a standard deviation of $8,878. Clearly interest rate, spread, and credit risk are not additive. The importance of integrated risk analysis that accounts for the correlations in these risk factors is apparent.

The results also have implications for the number of bonds necessary to diversify away credit risk, assuming each bond is from a different industry. There was a significant reduction in risk as bonds were added to the portfolio. However the gains from diversification are small after 10 bonds and remain essentially unchanged after 17 bonds. The implication is that a bond investor diversifies away risk with 17 bonds in 17 different industries in this simulation model.

Adding FX risk again has little impact on the mean value of the portfolio ($104,282). However it substantially increases portfolio standard deviation ($13,825 versus $8,878) and reduces the 95% confidence level ($82,159 versus $88,766).

V. CONCLUSIONS

This paper provides a methodology to assess correlated interest rate and credit risk. The proposed simulation model allows for a better understanding of the risks in owning a portfolio of fixed income securities that have correlated credit risk.

Current portfolio risk estimation methodologies calculate interest rate risk and credit risk in separate analyses. Given the correlated nature of interest rate and credit risk, this additive risk assessment methodology tends to overestimate total financial risk. Such risk estimation errors have significant implications for many types of financial decisions including financial institution capital adequacy requirements.

Interest rate, credit, and FX risks are jointly estimated by simulating both the future financial environment in which financial instruments will be valued and the credit rating of specific firms. The fundamental basis of this methodology is a reduced form of the contingent claims analysis (CCA) proposed by Merton (1974).

The financial risk assessment methodology developed in this paper is a function of six stochastic variables (interest rates, spread volatility, systematic equity return risk, unsystematic equity return risk, default recovery rates, and FX rates). Interest rate risk is modeled using the

Hull and White extended Vasicek model for the evolution of instantaneous risk-free interest rates. Credit spreads are modeled as correlated lognormal variables with historically estimated volatilities. The parameters necessary to calculate interest rate risk and spread volatility are the mean reversion rate and volatility rates for each term structure and the correlation between the term structures. The correlation structure between interest rates and equity returns must also be estimated. Finally, the credit risk parameters must be determined (initial debt ratios, debt ratios breakpoints, firm specific systematic risk, firm specific unsystematic risk, and firm specific dividend payout ratios). Given the number of stochastic variables, the results must be simulated.

The viability of the model is tested by comparing simulated credit rating transition probabilities to historical transition probabilities and by comparing simulated and analytical bond prices. Simulated credit rating transition probabilities are shown to closely approximate historical patterns, but the model does underestimate the frequency of large jumps in credit ratings over a one-year time frame. The bond valuation tests show that the model works better for investment grade than non-investment grade bonds. However adjustments to the debt to value ratio at which bonds default could be made to produce simulated values that would be very close for a particular type of bond (e.g. B-rated).

The risk assessment methodology applied to a single bond demonstrates that while all four risk factors (interest rate, spread, credit, and FX risk) are important the most important for non-investment grade bonds is credit risk. Thus a crucial data requirement for any risk assessment is the credit quality of the security.

The portfolio analysis capabilities of the model highlight the importance of diversification of credit risk across a number of fixed income assets and sectors of the economy. The model can be extended to deal with other types of financial instruments such as mortgages, variable rate loans, equities, and derivatives. Potential applications extend beyond valuing and modeling bond portfolios to undertaking financial institution risk assessments, evaluating alternative hedging strategies, and assessing capital adequacy.

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Table 1.
Changes in Bond Values Resulting from Rating Changes

The change in the value of the bond is calculated by changing the required yield to maturity to that of the adjacent rating category. Bond yields to maturity were estimated as of December 31, 1998. Bonds are assumed to initially be priced at par (100). For example, a five-year Ba bond priced at 100 would be priced at 93.97 if downgraded to B, or it would be priced at 111.96 if upgraded to Baa.

Rating Category	Aaa	Aa	A	Baa	Ba	B	Caa
5 Yr. Bond Yld	0.0509	0.0531	0.0555	0.0600	0.0879	0.1034	0.1501
Downgrade to Next Category	99.04	98.96	98.07	88.75	93.97	83.47	34.00
% Change Downgrade	-0.96%	-1.04%	-1.93%	-11.25%	-6.03%	-16.53%	-66.00%
Upgrade to Next Category	na	100.96	101.05	101.95	111.96	106.25	118.16
% Change of Upgrade	na	0.96%	1.05%	1.95%	11.96%	6.25%	18.16%
10 Yr. Bond Yld	0.0543	0.0575	0.06	0.0649	0.0966	0.1143	0.21
Downgrade to Next Category	97.56	98.12	96.39	79.53	89.35	56.38	34.00
% Change Downgrade	-2.44%	-1.88%	-3.61%	-20.47%	-10.65%	-43.62%	-66.00%
Upgrade to Next Category	na	102.47	101.90	103.69	123.37	111.43	157.60
% Change Upgrade	na	2.47%	1.90%	3.69%	23.37%	11.43%	57.60%

Table 2.
Moody’s Transition Matrixes Adjusted for Withdrawn Ratings (1920 -1996)

To examine if credit transitions are Markov and as benchmark for the transition probabilities generated using a contingent claims analysis, Moody’s historical transition probabilities are reported (Carty and Lieberman, 1996). Carty and Lieberman find no bias in the withdrawn category. Thus, the transition probabilities are adjusted for bonds that have had their ratings withdrawn by Moody’s.

Probability of Rating after One Year

Initial Rating	Aaa	Aa	A	Baa	Ba	B	Caa-C	Default
Aaa	92.28%	6.43%	1.03%	0.24%	0.02%	0.00%	0.00%	0.00%
Aa	1.28%	91.68%	6.09%	0.70%	0.17%	0.02%	0.00%	0.06%
A	0.07%	2.45%	91.59%	4.97%	0.67%	0.11%	0.02%	0.13%
Baa	0.03%	0.26%	4.19%	89.41%	5.07%	0.66%	0.07%	0.30%
Ba	0.01%	0.09%	0.43%	5.09%	87.23%	5.47%	0.45%	1.23%
B	0.00%	0.04%	0.15%	0.67%	6.47%	85.32%	3.44%	3.90%
Caa-C	0.00%	0.02%	0.04%	0.37%	1.38%	5.80%	78.78%	13.60%

Probability of Rating after Three Years

Initial Rating	Aaa	Aa	A	Baa	Ba	B	Caa-C	Default
Aaa	81.64%	13.93%	3.26%	0.75%	0.36%	0.02%	0.00%	0.03%
Aa	3.09%	78.67%	14.54%	2.53%	0.76%	0.09%	0.02%	0.29%
A	0.18%	5.80%	80.42%	10.26%	2.19%	0.45%	0.07%	0.63%
Baa	0.08%	0.76%	10.26%	75.43%	9.55%	2.12%	0.26%	1.54%
Ba	0.05%	0.25%	1.62%	12.14%	69.19%	10.59%	1.44%	4.72%
B	0.01%	0.10%	0.44%	2.26%	13.67%	65.88%	5.60%	12.04%
Caa-C	0.00%	0.00%	0.03%	1.04%	3.88%	10.12%	56.79%	28.14%

Table 3.
Credit Risk Analysis for a Ten-year B Rated Bond

A sample of a standard credit risk analysis for a B rated bond with a ten-year maturity with an initial PAR value of 1,000 is provided. The probability transitions are from Moody's one-year transition matrix. The spot and implied forward rates are estimated from the 12/31/98 yield curve. The cash flows from the bond (price + coupon) are revalued at the end of the first year utilizing the implied forward rates as of December 31, 1998. Since the yield curve is upward sloping in this example, the value of the bond at the end of the first year is worth less than its original value even if the bond stays in the same rating category. The distribution of possible bond values plus coupon multiplied by the probability of arriving at that credit quality is the mean expected value of the bond at the end of one year, $1,054.66 in this example.

	Probability Of Transition	Coupon	Bond Value t = 1	Bond plus Coupon Value t = 1	Prob. Weighted	Change from Mean
Aaa	0.00%	$117.61	$1,432.45	$1,550.06	$-	$495.39
Aa	0.04%	$117.61	$1,400.63	$1,518.23	$0.61	$463.57
A	0.15%	$117.61	$1,377.47	$1,495.07	$2.24	$440.41
Baa	0.67%	$117.61	$1,333.98	$1,451.59	$9.73	$396.92
Ba	6.47%	$117.61	$1,084.28	$1,201.89	$77.76	$147.22
B	85.32%	$117.61	$972.12	$1,089.73	$929.76	$35.06
Caa	3.44%	$117.61	$501.89	$619.50	$21.31	$(435.17)
Default	3.90%	-	-	340	$13.26	$(714.66)
				Average	$1,054.66	
				Std. Deviation	$174.12	
				99% Confidence Level	$340.00	
				95% Confidence Level	$619.50	

Table 4.
Term Structure by Bond Rating Class and Mean Reversion and Volatility of Term Structures by Bond Rating Class

The term structure was estimated from Standard & Poor's *CreditWeek* and the Lehman Brothers Bond Database. Mean reversion rates and volatilities of the short rates were estimated empirically over the January 1993 to December 1998 time period.

Term Structure Information: 12/31/98

	Time to Maturity			
Asset Class	1	5	10	15
Treasury	4.59%	4.39%	4.59%	4.89%
Aaa	4.96%	5.09%	5.43%	5.80%
Aa	5.00%	5.31%	5.75%	6.18%
A	5.17%	5.55%	6.00%	6.43%
Baa	5.53%	6.00%	6.49%	6.95%
Ba	7.41%	8.79%	9.66%	10.12%
B	8.78%	10.43%	11.43%	11.99%
Caa	12.00%	15.01%	21.00%	21.00%

Term Structure Parameter Estimates (Empirically estimated from 1/93 – 12/98)

	Treasury	Aaa	Aa	A	Baa	Ba	B	Caa-C
Mean Reversion Rate	0.048	0.061	0.062	0.058	0.084	0.171	0.069	0.142
Std. Dev. of the Short Interest Rate	0.007	0.010	0.010	0.010	0.011	0.014	0.010	0.039
Std. Dev. of the Interest Rate Spread (e.g. Ba-Baa)	n.a.	0.002	0.002	0.001	0.002	0.011	0.011	0.034

Table 5.
Correlation of the Change in Interest Rates, the Return on U.S. Industry Equity Indices, and the Yen/$U.S. Exchange Rate on a Monthly Basis from January 1987 to December 1996

	ΔTreasury	SP500	Auto	Bank	Chem	Bldg	Energy	Entmt	Health	Insur	Manuf	OilGas	Paper	Tech	Telcom	Textile	Util	Yen/$U.S.
ΔTreasury	1	-0.33	-0.02	-0.23	-0.15	-0.27	-0.23	0.02	-0.18	-0.35	-0.18	-0.03	-0.13	-0.02	-0.26	-0.05	-0.49	0.101
SP500		1	0.37	0.69	0.66	0.73	0.60	0.56	0.77	0.70	0.71	0.56	0.47	0.71	0.54	0.45	0.57	-0.062
Auto			1	0.41	0.47	0.54	0.22	0.37	0.05	0.19	0.45	0.22	0.43	0.31	0.27	0.15	0.07	0.022
Bank				1	0.52	0.54	0.41	0.49	0.38	0.55	0.49	0.35	0.39	0.51	0.44	0.24	0.32	0.073
Chem					1	0.48	0.51	0.31	0.50	0.32	0.61	0.43	0.64	0.44	0.42	0.30	0.15	-0.054
Bldg						1	0.41	0.57	0.42	0.47	0.71	0.46	0.43	0.51	0.37	0.46	0.39	0.025
Energy							1	0.21	0.33	0.36	0.33	0.46	0.08	0.35	0.25	0.22	0.57	-0.219
Entmnt								1	0.39	0.37	0.50	0.34	0.36	0.58	0.23	0.28	0.10	0.033
HlthCre									1	0.52	0.41	0.29	0.33	0.51	0.30	0.39	0.39	-0.014
Insur.										1	0.41	0.25	0.19	0.36	0.46	0.21	0.58	0.038
Manuf											1	0.53	0.57	0.61	0.35	0.48	0.24	-0.095
OilGas												1	0.21	0.48	0.20	0.37	0.23	-0.211
Paper													1	0.40	0.37	0.29	0.00	-0.084
Tech														1	0.29	0.32	0.18	-0.126
Telecom															1	-0.01	0.41	-0.003
Textile																1	-0.02	0.007
Util																	1	-0.068
Yen/$U.S.																		1

Table 6.
Equity Return Volatility for Low and High Volatility Firms by Bond Rating Category and Market Volatility

A cross-sectional time series was developed from Compustat to calculate the average firm's beta by bond rating for the period 1993-1998. The bonds were then segmented within bond category into one of two categories high or low volatility. Low volatility firms were defined to be those in the lower third of total equity return volatility. High volatility firms were defined to be the remaining two thirds of firms. Due to their inherent high volatility B and Caa rated firms were not divided into different volatility categories. Bonds were sorted by bond rating and characteristic lines are estimated to compute the firm's beta and unsystematic (firm specific) risk. The market volatility over the 1993-1998 time period is also displayed.

Low Volatility Firms with Bonds Rated	Mean Beta 1993-98	Mean Firm Specific Equity Return Volatility 1993-1998	High Volatility Firms with Bonds Rated	Mean Beta 1993-98	Mean Firm Specific Equity Return Volatility 1993-1998
Aaa	0.679	0.245	Aaa	0.682	0.317
Aa	0.649	0.249	Aa	0.757	0.363
A	0.699	0.222	A	0.864	0.412
Baa	0.864	0.292	Baa	0.994	0.507
Ba	1.019	0.425	Ba	1.131	0.729
			B	1.314	0.727
			Caa	1.301	0.954

Market Volatility

	1993	1994	1995	1996	1997	1998
S&P 500 Volatility	0.059	0.107	0.050	0.107	0.158	0.230

Table 7.

Debt Ratios and Bond Ratings for Firms Segmented into Low and High Volatility Firms

All non-financial firms with a Standard and Poor's bond rating which Compustat tracks over the period of 1987 to 1998 were identified. Quarterly data on debt ratios and bond rating was obtained. The debt ratio was defined as (book value of short and long-term debt / (book value of short and long-term debt + market value of equity)). Due to their inherent high volatility B and Caa rated firms were not divided into different volatility categories. For Caa and defaulted companies only the first observation in that category is utilized in the analysis. The descriptive statistics of this analysis are provided.

Low Volatility Firms

Rating	N	Mean	STD	MAX	Q3	Median	Q1	MIN
Aaa	57	0.141	0.127	0.988	0.171	0.121	0.075	0.051
Aa	293	0.241	0.122	0.489	0.334	0.278	0.117	0.011
A	989	0.319	0.132	0.606	0.419	0.350	0.221	0.020
Baa	509	0.341	0.171	0.747	0.463	0.352	0.200	0.018
Ba	723	0.472	0.186	0.943	0.589	0.460	0.333	0.060

High Volatility Firms

Rating	N	Mean	STD	MAX	Q3	Median	Q1	MIN
Aaa	286	0.144	0.145	0.748	0.157	0.101	0.048	0.015
Aa	1067	0.163	0.120	0.690	0.204	0.127	0.077	0.017
A	3646	0.240	0.140	0.821	0.340	0.212	0.131	0.012
Baa	4312	0.319	0.159	0.832	0.431	0.305	0.198	0.011
Ba	3500	0.397	0.207	0.960	0.554	0.386	0.226	0.012
B	3076	0.515	0.235	0.983	0.702	0.525	0.324	0.015
Caa	34	.0.729	0.262	0.984	0.931	0.819	0.615	0.117
Def	17	0.779	0.226	0.990	0.940	0.851	0.699	0.127

Table 8.
Simulated Credit Rating Transition Matrixes

Utilizing a CCA framework, simulated credit rating transition matrixes are estimated for low and high volatility firms by bond rating category. The transition matrices are a function of the volatility of the equity market indices (e.g.., S&P 500), the systematic risk of the company, the unsystematic risk of the company, the debt ratio of the firm, and the dividend yield. The volatility of the equity market index (0.23) was estimated for the year 1998. The firm specific parameters were estimated over the 1993 to 1998 period.

Low Volatility Firms

Probability of Rating after One Year								
Initial Rating	Aaa	Aa	A	Baa	Ba	B	Caa-C	Default
Aaa	93.50%	6.50%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Aa	0.09%	97.67%	2.23%	0.01%	0.00%	0.00%	0.00%	0.00%
A	0.00%	1.51%	94.30%	3.57%	0.62%	0.00%	0.00%	0.00%
Baa	0.00%	0.99%	7.17%	79.41%	12.19%	0.24%	0.00%	0.00%
Ba	0.01%	0.46%	2.40%	7.61%	76.44%	12.13%	0.88%	0.07%

Probability of Rating after Three Years								
Initial Rating	Aaa	Aa	A	Baa	Ba	B	Caa-C	Default
Aaa	79.58%	20.33%	0.08%	0.01%	0.00%	0.00%	0.00%	0.00%
Aa	4.08%	81.94%	11.11%	1.63%	1.20%	0.04%	0.00%	0.00%
A	0.09%	9.87%	72.41%	9.14%	8.11%	0.38%	0.00%	0.00%
Baa	0.33%	8.08%	11.42%	52.95%	21.38%	5.42%	0.38%	0.04%
Ba	0.65%	6.41%	6.22%	8.70%	52.21%	16.72%	4.88%	4.21%

High Volatility Firms

Probability of Rating after One Year								
Initial Rating	Aaa	Aa	A	Baa	Ba	B	Caa-C	Default
Aaa	80.75%	15.68%	3.56%	0.01%	0.00%	0.00%	0.00%	0.00%
Aa	3.66%	83.51%	12.72%	0.11%	0.00%	0.00%	0.00%	0.00%
A	0.05%	4.38%	82.99%	10.91%	1.66%	0.01%	0.00%	0.00%
Baa	0.01%	1.30%	9.70%	71.39%	15.21%	2.38%	0.01%	0.00%
Ba	0.14%	1.44%	6.47%	4.03%	72.22%	14.12%	1.22%	0.36%
B	0.00%	0.25%	0.80%	1.15%	9.15%	78.15%	6.60%	3.90%
Caa-C	0.01%	0.25%	0.78%	0.92%	4.82%	7.60%	74.27%	11.35%

Probability of Rating after Three Years								
Initial Rating	Aaa	Aa	A	Baa	Ba	B	Caa-C	Default
Aaa	67.69%	17.54%	14.07%	0.67%	0.03%	0.00%	0.00%	0.00%
Aa	13.37%	60.48%	22.79%	2.85%	0.51%	0.00%	0.00%	0.00%
A	3.05%	10.90%	61.05%	14.90%	8.36%	1.68%	0.04%	0.02%
Baa	1.99%	6.71%	12.42%	50.77%	17.71%	9.51%	0.64%	0.25%
Ba	2.87%	5.84%	9.11%	3.98%	55.89%	15.53%	2.81%	3.97%
B	1.16%	2.49%	5.05%	2.52%	10.66%	51.81%	5.18%	21.13%
Caa-C	1.24%	1.67%	3.30%	1.79%	7.04%	7.86%	46.13%	30.97%

Table 9.
Bond Valuation Tests

The value represents the value of a ten-year coupon bond simulated out one, and three-years in a risk neutral framework and then discounted back at the risk-free rate($V_0 = V_t e^{-rt}$). The simulation output contains the mean value and the standard deviation of the simulated values. The estimated error represents the over or under valuation of the simulated mean compared to the initial market value of $1,000.

	Aaa	Aa	A	Baa	Ba	B	Caa-C
	Simulated price at t = 1 discounted back at the risk-free rate						
High Volatility Firms							
Mean Value	997.15	1,001.83	999.09	980.25	1,037.15	1,012.21	1,025.80
Standard Deviation	42.10	40.24	50.36	90.73	119.97	199.83	490.15
% Pricing Error	-0.29%	0.18%	-0.09%	-1.98%	3.72%	1.22%	2.58%
Low Volatility Firms							
Mean Value	1,000.88	1,002.72	1,002.77	989.48	1,035.12	n.a.	n.a.
Standard Deviation	40.09	39.68	44.91	77.92	105.69	n.a.	n.a.
% Pricing Error	0.09%	0.27%	0.28%	-1.05%	3.51%	n.a.	n.a.
	Simulated price at t = 3 discounted back at the risk-free rate						
High Volatility Firms							
Mean Value	1,004.49	1,016.71	1,007.88	994.89	1,118.79	1,052.70	1,086.43
Standard Deviation	75.29	79.29	103.50	134.35	188.04	366.56	694.47
% Pricing Error	0.45%	1.67%	0.79%	-0.51%	11.88%	5.27%	8.64%
Low Volatility Firms							
Mean Value	1,008.46	1,016.54	1,009.54	998.03	1,104.48	n.a.	n.a.
Standard Deviation	74.10	81.15	102.48	126.84	205.10	n.a.	n.a.
% Pricing Error	0.85%	1.65%	0.95%	-0.20%	10.45%	n.a.	n.a.

Table 10.
Value-at-Risk Measures for a B-rated Bond

VaR measures are simulated for a B-rated bond with an initial value of $100,000 at one-year time step. The value of the bond is equal to the price at t=1 plus the coupon payment if the bond did not default.

One B-rated bond facing various risks.

Interest Rate Risk	Yes	Yes	No	Yes	Yes
Interest Rate Spread Risk	No	Yes	No	Yes	Yes
Credit Risk	No	No	Yes	Yes	Yes
FX Risk	No	No	No	No	Yes
Mean Value	108,866	109,081	104,148	104,225	104,426
Std. Deviation	3,215	6,450	22,561	23,433	25,869
Change in Std. Deviation	n.a.	3,235	16,111	872	2,436
Maximum Value	121,238	126,769	153,464	164,057	198,049
Minimum Value	98,037	72,129	29	21	18
VaR Confidence Levels					
99% Level	101,718	90,096	9,130	8,844	9,187
97.5% Level	102,790	94,406	29,164	29,791	29,427
95% Level	103,723	97,180	60,544	56,736	53,230

Table 11.
Portfolio Risk Analysis

To perform portfolio risk analyses, we form portfolios of 1, 2, 5, 7, 10, 12, 15, 17, 20 and 24 B rated bonds. All bonds are assumed to have a ten-year maturity, are noncallable, and face interest rate, interest rate spread, and credit risk. We also include one portfolio of 24 bonds that faces FX risk as well. We assumed that each bond added to the portfolio was from a unique industry with different correlation structures based upon the historical estimates found in Table 5. Hence, our resulting estimates are for the maximum diversification available for the number of bonds in the portfolio. The ending value of the portfolio was simulated out one year, 10,000 times. The ending value includes the value of the bonds plus the coupon payment. The mean, standard deviation, maximum, and minimum simulated portfolio value, and the 99%, 97.5%, and 95% confidence levels are the resulting output statistics from the model.

Distribution of values for a $100,000 initial value portfolio of B-rated Bonds at a one-year time step facing various risks.

Risk Included in Analysis												
Interest Rate Risk	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes	no	yes
Interest Rate Spread Risk	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes	no	yes
Credit Risk	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
FX Risk	no	no	no	no	no	no	no	no	no	no	no	yes
Number of bonds	1	2	5	7	10	12	15	17	20	24	24	24
Mean Value	104,225	104,373	104,425	104,291	104,320	104,284	104,285	104,342	104,232	104,279	104,093	104,282
Std. Deviation	23,433	17,860	11,948	10,733	9,686	9,465	9,026	8,968	8,995	8,878	6,810	13,825
Change in Std. Dev.	n.a.	(5,573)	(5,913)	(1,215)	(1,047)	(220)	(439)	(58)	26	(117)	(2,068)	7,015
Maximum Value	164,057	156,231	140,689	134,290	133,147	136,393	132,428	132,448	131,366	130,604	125,907	66,716
Minimum Value	21	3,591	48,410	50,843	51,979	64,391	64,674	64,635	64,684	66,434	69,835	51,741
VaR Confidence Levels												
99% Level	8,844	46,461	71,385	76,005	78,283	78,752	80,545	80,746	80,436	80,786	84,826	73,479
97.5% Level	29,791	57,522	77,336	80,258	83,542	83,824	85,323	85,011	84,827	85,032	88,761	78,729
95% Level	56,736	65,747	82,185	84,897	87,131	87,406	88,379	88,610	88,312	88,766	91,553	82,159