

A Model of Corporate Bond Prices with Dynamic Capital Structure

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Abstract

This paper presents an analytical model of corporate discount bond prices. The critical assumption of the model is that the dynamics of the firm's debt ratio revert toward a long-term target debt ratio. Default is triggered at high values of the debt ratio. The model predicts that the levels of the credit spreads of long-term bonds are more sensitive to the firm's target debt ratio than to its current debt ratio. The case is the opposite for bonds with shorter maturities. The credit spreads predicted by the model are mean-reverting. The model outperforms that of Longstaff and Schwartz (1995) on bonds from Boise Cascade Corporation.

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I. Introduction

Black and Scholes (1973) and Merton (1974) demonstrate that corporate bonds may be viewed as options on the firm's assets. This contingent-claims approach to the pricing of corporate debt is now an integral part of finance.¹ However, empirical application of the contingent-claims approach has not been equally successful. Jones, Mason and Rosenfeld (1984) and Kim, Ramaswamy and Sundaresan (1993) find that the approach generates credit spreads for corporate bonds which are too low. Subsequent research focuses on finding a solution to this problem by introducing Poisson processes (Mason and Bhattacharya (1981) and Zhou (1997)), allowing for costly bankruptcy and deviations from priority rules in bankruptcy (Kim *et al.* (1993), Anderson and Sundaresan (1994), Leland (1995), Longstaff and Schwartz (1995a) and Leland and Toft (1996)), modeling agency costs of debt (Anderson *et al.* (1994) and Leland (1998)) and making default-free interest rates stochastic (Nielsen, Saá-Requejo and Santa-Clara (1993), Shimko, Tejima and van Deventer (1993), Longstaff and Schwartz (1995a), Izvorski (1996) and Briys and de Varenne (1997)).²

This paper presents a new solution to this problem while addressing a related problem that is common to all existing models. Specifically, these models predict that the credit spreads on corporate bonds drift downward over time as the value of the firm's assets drifts upward (non-negative drifts may be generated under some parameter values). This moves the firm increasingly further away from default because the firm's amount of debt (or capital structure) at which default occurs is constant. This is problematic, because the prediction that the credit spreads drift downward contradicts the evidence that they revert toward long-term averages (Duffee (1998a) and Taurén (1999)). Moreover, the assumption of a constant capital structure is unrealistic over the typical maturities of corporate bonds.³

¹An alternative approach to the pricing of risky debt called the reduced-form method has been applied in several recent papers. See Duffie and Singleton (1999) and the references therein.

²KMV Corporation calculates default probabilities by mapping market-value debt ratios to historical default frequencies (Crosbie (1998)). Their conceptual base is Merton's (1974) approach.

³The potential problems associated with this assumption were first pointed out by Black and Cox (1976) according to whom "... it assumes that the fortunes of the firm may cause its value to rise to an arbitrarily

In this paper, I derive a new analytical model of corporate bond prices. The model is able to generate credit spreads whose levels and dynamics are realistic. In the model, the dynamics of the firm's debt ratio are assumed to be governed by a mean-reverting stochastic process. The debt ratio is defined as the ratio of the book value of the firm's debt to the market value of its assets. This assumption makes the debt ratio gradually approach a long-term target debt ratio. It is consistent with the empirical findings of Taggart (1977), Marsh (1982), Jalilvand and Harris (1984), Auerbach (1985) and Opler and Titman (1995). These studies document that companies tend to gradually adjust their capital structures toward target debt ratios. Furthermore, the debt ratio dynamics are consistent with the dynamics of the value of the firm's assets assumed in Black and Cox (1976).

The parameters that specify the dynamic adjustment in the firm's debt ratio: the adjustment speed and the target debt ratio are assumed to be exogenously determined.⁴ Many of the remaining assumptions of the model parallel those made by Black and Cox (1976) and Longstaff and Schwartz (1995a). Specifically, default is assumed to be triggered when the firm's debt ratio reaches an upper threshold called the default boundary. This is consistent with the event of bankruptcy being associated with abnormally high levels of debt relative to the market value of the firm's assets. The recovery rates of the bonds in default are assumed to be stochastic. Their means can be made to depend on the priority of the bond. Finally, the model allows for stochastic default-free interest rates.

Under these assumptions, I derive an analytical solution to the price of a corporate discount bond. This solution is without loss of generality, because coupon bonds can be priced as portfolios of discount bonds. Since corporate bond prices are typically expressed

high level or dwindle to nearly nothing without any sort of reorganization occurring in the firm's financial arrangements". Consequently, Brennan and Schwartz (1984) and Fischer, Heinkel and Zechner (1989) apply the contingent-claims approach in constructing numerical models of optimal dynamic capital structure. Leland (1995, 1998), Leland and Toft (1996) and Fan and Sundaresan (1998) derive predictions about an optimal *constant* capital structure in a contingent-claims pricing framework. This paper is different in that it obtains an analytical solution to the corporate bond price while allowing for a dynamic capital structure.

⁴The pricing framework is a perfect capital market in which the Miller and Modigliani (1958) capital structure irrelevance theorem holds. Thus, the debt ratio dynamics are necessarily exogenous rather than endogenously determined as a solution to a dynamic capital structure optimization problem.

in terms of credit spreads, I derive expressions for the credit spreads and their dynamics. The model makes the following new predictions about the levels of the credit spreads on corporate bonds

- The model is able to generate credit spreads which are comparable to those on actual corporate bonds regardless of whether rated investment grade or speculative grade.
- The credit spreads are increasing in the firm's target debt ratio. This relationship is more important for long-term bonds. On the other hand, the firm's current debt ratio is relatively more important for bonds with short maturities.
- The credit spreads on corporate bonds are decreasing in the speed at which the firm adjusts its capital structure toward the long-term target debt ratio.

The fact that the capital structure dynamics are realistic is important for the model to be able to predict reasonable credit spread levels. This argument is supported by the evidence of Titman and Torous (1989). They find that the yields on commercial mortgage bonds can be explained by a model that assumes a constant capital structure. This assumption is realistic, because commercial real estate tends to be mortgaged by a single bond.

The model also makes the following new predictions about the dynamic properties of the credit spreads on corporate bonds

- The credit spreads on long-term corporate bonds exhibit mean-reversion toward long-term averages determined by the firm's target debt ratio.
- The mean-reversion in the credit spreads is faster at high capital structure adjustment speeds, at high credit spread levels and at short maturities than otherwise.
- The credit spreads are more volatile for short maturities versus long maturities and for high credit spread levels versus low credit spread levels.

These predictions follow directly from the assumption that the firm's debt ratio, to which the credit spreads are positively related, is mean-reverting. Longstaff and Schwartz (1995b), Stevens, Clinebell and Kahl (1998) and Morris, Neal and Rolph (1998) find that the credit spreads calculated from corporate bond yield indices are mean-reverting. Duffee (1998a) and Taurén (1999) document mean-reversion in the credit spreads of individual bonds.

The paper also compares the credit spreads implied by the model with the observed credit spreads of Boise Cascade Corporation over the period 1987–1994. The same comparison is made for the Longstaff and Schwartz (1995a) model. The debt ratios and the asset values are calculated from financial statement and stock price data. The results indicate that the Longstaff-Schwartz model overstates the volatility of credit spreads. The main reason for this is that it generates credit spreads which are too sensitive to changes in the value of the firm's assets. In my model, the mean reversion in the debt ratio makes credit spreads less sensitive to changes in the value of the assets. This implies a lower root mean-squared error. On the other hand, their model is better able to account for the negative relationship between credit spreads and interest rates.

The remaining parts of the paper are organized as follows. In section II, the underlying assumptions of the model are presented. Section III derives the price of a corporate discount bond under these assumptions. It also discusses the hedging of corporate bonds. In section IV, the credit spreads are analyzed as functions of the parameters of the model. The dynamic properties of the credit spreads are studied in section V. Section VI presents the results from the empirical comparison. Section VII concludes.

II. The Assumptions

For the purpose of risk-neutral pricing, the capital markets are assumed to be perfect and complete, and no arbitrage opportunities are allowed. Trading is continuous in the finite time interval $[0, T^*]$. The current time is denoted by 0. Uncertainty is characterized by the

probability space (Ω, \mathcal{F}, P) , where Ω is the state space, \mathcal{F} is a σ -algebra that represents measurable events in the state space and P is the probability measure. Information in the capital markets is described by the standard filtration $F = \{\mathcal{F}(t) : t \in [0, T^*]\}$.⁵

I describe the dynamics of the instantaneous default-free interest rate $r(t)$ by the following stochastic differential equation (Vasicek (1977))

$$dr(t) = [\zeta - \beta r(t)] dt + \eta dW_1(t), \quad (1)$$

where ζ, β and η are non-negative constants and dW_1 is the differential of a standard Wiener process. The following papers also make the assumption that default-free interest rates are stochastic: Shimko, Tejima and van Deventer (1993), Nielsen, Saá-Requejo and Santa-Clara (1993), Kim, Ramaswamy and Sundaresan (1993), Longstaff and Schwartz (1995a), Izvorski (1997) and Briys and de Varenne (1997). Equation (1) implies the following type of behavior for the default-free interest rates. They exhibit autoregression toward constant long-term means. The conditional distribution of the changes in the default-free interest rates is normal. Thus, the interest rates can become negative, but this probability is negligible at reasonable parameter values. The equation implies that the changes in the yields of default-free bonds are perfectly correlated across maturities.

The dynamic evolution of the value of the firm's assets denoted by $V(t)$ and its book value of debt denoted by $B(t)$ is described by the following equations

$$dV(t) = [\mu_V + \Phi(V, B, t)] V(t) dt + \sigma_V V(t) dW_2(t), \quad (2)$$

$$dB(t) = \phi(V, B, t) B(t) dt, \quad (3)$$

where μ_V denotes the expected return on the market value of the firm's assets, $\sigma_V^2 > 0$ is the volatility of the firm's asset value and $dW_2(t)$ is the differential of a standard Wiener

⁵All subsequently defined variables are assumed to be progressively measurable in this filtration. The time subscript is sometimes suppressed in the case of the current value of the variable.

process. The adjustment functions $\Phi(V, B, t)$ and $\phi(V, B, t)$ specify the dynamic adjustment in the firm's assets and debt. The instantaneous correlation between the value of the assets and the default-free interest rate is denoted by ρ_{Vr} .

The specification of the adjustment in the firm's assets and debt in Equations (2) and (3) is consistent with the Miller and Modigliani (1958) theorem. It is consistent with many financial transactions which have zero net present values by the same theorem.⁶ Previous studies make different assumptions about the firm's financial transactions. Merton (1974) assumes that $\Phi(V, B, t) = \phi(V, B, t) = 0$ (for the solution derived by Black and Scholes (1973) to apply). Black and Cox (1976) consider lumpy changes in the book value of the debt at exogenous upper and lower boundaries for the value of the assets. Brennan and Schwartz (1984) and Fischer, Heinkel and Zechner (1989) build numerical models which make predictions about optimal dynamic capital structure adjustment. This paper is different from the above mentioned papers in that it obtains an analytical solution to the corporate bond price while allowing for a dynamic capital structure.

The critical assumption is that the adjustment functions $\Phi(V, B, t)$ and $\phi(V, B, t)$ implicitly satisfy the following equation for the firm's debt ratio $D(t) \equiv B(t)/V(t)$

$$dD(t) = \kappa [\log \mu - \log D(t)] D(t) dt + \sigma D(t) dW(t). \quad (4)$$

In Equation (4), the constant $\kappa > 0$ determines the firm's speed of adjustment toward a constant target debt ratio μ , σ^2 is the volatility of the debt ratio and $dW(t)$ is the differential of a standard Wiener process. Appendix A shows that there exist $\Phi(V, B, t)$ and $\phi(V, B, t)$ that satisfy this equation, that $\sigma^2 = \sigma_V^2$ and that $dW(t) = -dW_2(t)$.

⁶The following examples illustrate how the adjustment functions describe the firm's financial transactions. For example, an equity issue whose proceeds of \$100 are invested in new assets (which may be either physical or financial) is consistent with $\Phi(V, B, t) = \$100$ and $\phi(V, B, t) = 0$. The fact that the value of the assets increases exactly by the proceeds of the equity issue is implied by the Miller-Modigliani theorem. If the proceeds of the issue were used to retire existing debt, the functions would equal $\Phi(V, B, t) = 0$ and $\phi(V, B, t) = -\$100$. Finally, a dividend of \$80 that is financed by selling assets for \$30 and by issuing debt at the market for \$50 is consistent with $\Phi(V, B, t) = -\$30$ and $\phi(V, B, t) = \$50$.

The stochastic differential equation (4) implies that the firm's debt ratio drifts downward at levels higher than the target debt ratio and upward at lower debt ratio levels. The firm is supposed to issue debt, pay dividends or repurchase equity, if its current debt ratio is less than the target debt ratio. Conversely, when the current debt ratio exceeds the target debt ratio, the firm is supposed to retire debt or issue equity.⁷ This description is consistent with the empirical findings of Taggart (1977), Marsh (1982), Jalilvand and Harris (1984), Auerbach (1985) and Opler and Titman (1995). These studies document that companies tend to gradually adjust their capital structures toward target debt ratios.⁸ The firm can implement the desired change in the debt ratio despite being unable to access external financing. This is because operating cash flow and proceeds from asset sales can be used to pay dividends (to increase the debt ratio) or to repay debt (to decrease it).⁹

The assumed debt ratio dynamics imply that the logarithm of the debt ratio follows an Ornstein-Uhlenbeck process that is conditionally normally distributed with mean

$$E_0 [\log D(t)] = \log \mu + e^{-\kappa t} [\log D(0) - \log \mu] \quad (5)$$

and variance

$$Var_0 [\log D(t)] = \frac{\sigma^2}{2\kappa} (1 - e^{-2\kappa t}). \quad (6)$$

The long-term mean and variance of the logarithm of the debt ratio are $\log \mu$ and $\sigma^2/(2\kappa)$, respectively. Equation (5) implies that the speed at which the firm approaches the target debt ratio is positively related to κ and the deviation of the debt ratio from the target debt ratio. Equation (6) implies that the variance of the debt ratio is increasing in the debt ratio volatility σ^2 and decreasing in the speed of debt ratio adjustment κ . Finally, the correlation

⁷Smith and Warner (1979) argue that highly leveraged firms have incentives to increase their debt ratios. This may not be feasible in practice, however, because highly leveraged firms are likely to face restrictions imposed by bond covenants on borrowing and dividend payments.

⁸Taggart (1977), Marsh (1982) and Jalilvand *et al.* (1984) report that it makes little difference to their results whether the target debt ratio is defined in terms of market or book value of equity.

⁹The adjustment in the assets and debt can be lumpy as long as the debt ratio changes smoothly. In the numerical model of Fischer *et al.*, the debt ratio jumps to a target debt ratio at upper and lower boundaries.

between the debt ratio and the default-free interest rate is $\rho = -\rho_{Vr}$.¹⁰

Default is assumed to occur when the debt ratio reaches an upper threshold $K \leq 1$ called the default boundary. This definition of default is consistent with bankruptcy, insolvency, material adverse restructuring of debt, bond covenant violation and failure to meet the payment of obligations when due. It allows the firm's bonds to be priced without specification of all the claims in its capital structure (Longstaff and Schwartz (1995a)). The default boundary is assumed not to exceed one to ensure that default occurs if the firm's total debt becomes due when the firm has more debt than assets (Briys and de Varenne (1997)). This implies, on the other hand, that the firm's assets are never worth less than the book value of its debt. Despite this, the bond holders may accept a payment in default that is worth less than the face value of their bonds. This is because the firm can threaten them with a costly liquidation of its assets, which would result in even lower proceeds to the bond holders (Anderson and Sundaresan (1996)). Franks and Torous (1994) document that the average market-value debt ratio for defaulted firms at the year-end preceding default was 0.81. This debt ratio is less than one and clearly exceeds the typical debt ratios of these firms as their stock prices fell on average by 76% in the two years that preceded default.

In default, the firm's bonds are assumed to be exchanged for default-free bonds whose maturities are identical to those of the original bonds. The face values of the new bonds are a positive stochastic fraction $1 - \tilde{w}$ of those of the original bonds. The fraction $1 - \tilde{w}$ is the recovery rate of the bond. The corresponding loss rate equals $\tilde{w} \leq 1$ and its expectation is w . The recovery rates are assumed to be independent of the default-free interest rate and the firm's debt ratio (which always obtains the same value in default). The uncertainty about their values is assumed not to be priced in the capital markets. The mean recovery rate can be made to vary according to the priority of the bond.¹¹ Its historical value for public debt,

¹⁰Empirical estimates of the model parameters can be obtained from Auerbach's (1985) study. He regresses annual changes in the market-value debt ratios of 143 firms in the period 1958-1977 on their debt ratios lagged by one year. The median estimate of the target debt ratio is 20.2% which is typical for firms with single A-ratings from Standard and Poor's (S&P CreditWeek). The median estimate of the adjustment speed (defined in terms of the first-order autocorrelation coefficient) is 0.44.

¹¹The specification also makes the recovery rates consistent with deviations from the absolute priority rule

calculated from market prices right after default by Carty and Lieberman (1998), varied in 1920-1996 being 0.63 for senior secured debt, 0.48 for senior unsecured debt, 0.38 for senior subordinated debt, 0.28 for subordinated debt and 0.15 for junior subordinated debt. The standard deviation of the recovery rate typically varied in the range 0.20 - 0.27.

III. The Corporate Bond Prices

This section derives the arbitrage-free price of a unit discount bond issued by the firm in the assumptions. The maturity date of the bond is $T \leq T^*$ and its priority is implicitly defined by the mean recovery rate $1 - w$. Coupon bonds can be valued as portfolios of unit discount bonds. The assumption that the face value of the bond is constant can be relaxed.¹² The price of the discount bond is contingent upon the firm's debt ratio, the default-free interest rate and the term-to-maturity of the bond: $P(D, r, T - t)$. It satisfies the following Lemma

Lemma *The price of the corporate unit discount bond with term-to-maturity T satisfies the following partial differential equation (PDE)*

$$P_D[\kappa(\log \mu - \log D) - \lambda] D + \frac{1}{2} P_{DD} \sigma^2 D^2 + P_r \left[\zeta + \frac{\mu_B}{C(m)} - \left(\frac{1}{C(m)} + \beta \right) r \right] + \frac{1}{2} P_{rr} \eta^2 + P_{Dr} \rho \sigma \eta D + P_t - r P(D, r, t) = 0, \quad (7)$$

where $\lambda \equiv \mu_B - \mu_V$, μ_B is the expected return on a default-free bond with a term-to-maturity of m and $\rho = -\rho_{Vr}$. The payoff of the bond is subject to the following boundary condition at maturity: $P(D, r, 0) = 1 - \tilde{w} 1_{\tau \leq T}$, where $1_{\tau \leq T}$ denotes an indicator function that obtains the value 1 if the first-passage-time of D to the boundary K , defined as $\tau = \inf\{t : D \geq K\}$, is at most T and 0 otherwise.

in bankruptcy court resolutions (Longstaff and Schwartz (1995a)).

¹²There is a positive probability for the event that the book value of the firm's debt $B(t)$ is less than the face value of the bond. This probability should be small in practice, however.

Proof: In the Appendix.

The following Proposition presents the unique solution to the PDE in the Lemma

Proposition 1 *The price of the corporate unit discount bond with term-to-maturity T is*

$$P(D, r, T) = A(r, T) [1 - w Q(D, T)], \quad (8)$$

where the price of a similar default-free unit discount bond is

$$A(r, T) = \exp[E(T) - Y(T) r] \quad (9)$$

and where $E(T) = [C(T) - T][\beta\zeta + \mu_B/C(m) - \eta^2/2]/\kappa^2 - \eta^2 C(T)^2/(4\kappa)$, $Y(T) = C(T) + \beta/[C(m)\kappa^2]$, $C(T) = [1 - e^{-\beta T}]/\beta$ and $Q(D, T) \equiv \lim_{n \rightarrow \infty} Q(D, T, n)$ is the risk-neutral default probability that is obtained from

$$Q(D, T, n) = \sum_{i=1}^n q_i, \quad (10)$$

where

$$q_1 = N \left(\frac{M(\log D, iT/n, T) - \log K}{\sqrt{V(iT/n)}} \right), \quad (11)$$

$$\begin{aligned} q_i &= N \left(\frac{M(\log D, iT/n, T) - \log K}{\sqrt{V(iT/n)}} \right) \\ &\quad - \sum_{j=1}^{i-1} q_j N \left(\frac{M(\log K, iT/n, T) - M(\log K, jT/n, T)}{\sqrt{V(iT/n) - V(jT/n)}} \right) \end{aligned} \quad (12)$$

if $i \geq 2$ and

$$\begin{aligned}
M(X, t, T) = & e^{-\kappa t} X + \left[e^{-\beta(T-t)} - e^{-\beta T - \kappa t} \right] \frac{\rho \sigma \eta}{\beta(\beta + \kappa)} \\
& + \left(\frac{1 - e^{-\kappa t}}{\kappa} \right) \left(\kappa \log \mu - \frac{\sigma^2}{2} - \lambda - \frac{\rho \sigma \eta}{\beta} \right), \tag{13}
\end{aligned}$$

$$V(t) = \frac{\sigma^2}{2\kappa} (1 - e^{-2\kappa t}) \tag{14}$$

and where $\lambda \equiv \mu_B - \mu_V$, $\sigma^2 = \sigma_V^2$ and $\rho = -\rho_{Vr}$.

Proof: In the Appendix.

Proposition 1 says that the corporate discount bond price is equal to the price of a similar default-free discount bond less a discount for default risk. The size of the discount equals the present value of the expected dollar loss in default $A(r, T)$ multiplied by the risk-neutral probability of default $Q(D, T)$. This is the probability that the firm's debt ratio D reaches the default boundary K before the maturity date T (under the risk-neutral probabilities).¹³ The expression for this probability is presented in Equations (10) to (14) as an infinite sum of standard normal distribution functions. The results are sufficiently accurate for practical purposes when $n \geq 200$. The risk-neutral default probability is a function of two variables: the debt ratio D and the term-to-maturity of the bond $T - t$. The default probability is not a function of the default-free interest rate r (which is an implication of the Vasicek model), but it is correlated with it when the value of ρ is non-zero. The remaining parameters, which determine the risk-neutral default probability, are: the target debt ratio μ , the debt ratio

¹³These risk-neutral probabilities are defined by the risk-neutral probability measure Q which has the following properties: 1) discounted asset prices are martingales under Q and 2) Q is equivalent to P in the sense that for any event $A \in \mathcal{F}$, $P(A) = 0$ implies $Q(A) = 0$ and vice versa. In practice, the risk-neutral probabilities can be obtained by subtracting a risk premium ("the market price of risk") from the drift of the dynamics of the relevant underlying stochastic variable. The resulting dynamics then define the relevant probabilities under the risk-neutral probability measure Q .

adjustment speed κ , the debt ratio risk premium $\lambda \equiv \mu_B - \mu_V$, the volatility of the debt ratio $\sigma^2 = \sigma_V^2$, the instantaneous correlation between the debt ratio and the default-free interest rate $\rho = -\rho_{Vr}$, the parameters of the default-free interest rate process, β and η , and the default boundary K . These parameters are separately identifiable.¹⁴

The firm's actual probability of default can be recovered from the risk-neutral probability by setting the value of the debt ratio risk premium λ equal to zero. By applying the results in Nobile, Ricciardi and Sacerdote (1985) (Remark 3.3), the actual probability of default can then be shown to approach one as time passes: $\lim_{T \rightarrow \infty} Q(D, T)|_{\lambda=0} = 1$. A sufficient condition for a certain default is that the dynamics of the debt ratio be stationary. Default need not be certain when debt is constant. For example, in Longstaff and Schwartz (1993), $\mu_V < \sigma_V^2/2$ implies that the default probability increases as time passes, but may not approach one. The result that default becomes certain makes long-term corporate discount bond prices equal the present value of their expected recoveries in default.

Corporate bond positions can be hedged by constructing a portfolio of hedge assets that dynamically replicates the payoffs of the bond. Two hedge assets are required, because the bond price is a function of two stochastic variables: D and r . The hedge portfolio is assumed to be composed of default-free discount bonds with prices of $G(r, m)$ and maturities of m years and firm-value replicating assets with prices of H dollars. The hedge ratios are given by Equations (15) and (16) (see the Proof of the Lemma)

$$h_1(D, r, T) = P_r \frac{1}{C(m)G(r, m)}, \quad (15)$$

$$h_2(D, r, T) = P_D \frac{D}{H}. \quad (16)$$

The number of the default-free bonds required for the hedge is $h_1(D, r, T)$. This number is typically negative, implying a short position. The derivative P_r is the modified duration of the bond when multiplied by $-[1 + r(t)]$. Because modified duration assumes that the term

¹⁴To reduce the number of parameters, substitute D/K for D and let $K = 1$.

structure of interest rates is flat, the adjustment term $1/[C(m)G(r, m)]$ is needed. Other modified duration measures are derived by Chance (1990) and Nawalkha (1996). Ilmanen, McGuire and Warga (1994) report that the use of modified duration eliminates only about a third of the total risk in corporate bonds rated A or Baa by Moody's. This poor performance is because modified duration does not account for changes in the credit risks of the bonds. The credit risk can be hedged by a short position in $-h_2(D, r, T)$ firm-value replicating assets. The firm's stock can be used as the hedge asset in practice, because changes in stock prices and asset values are almost perfectly correlated. This approach has been implemented by Bookstaber and Jacob (1986), Grieves (1986), Ramaswami (1991) and Skinner (1998).

IV. The Credit Spreads

In this section, I discuss the properties of the term structures of credit spreads on discount bonds implied by the model. Coupon bonds make payments before their maturity. Thus, their credit spreads are slightly lower than those of discount bonds when the term structure is upward-sloping. On the other hand, when the term structure is downward-sloping, their credit spreads are higher than those of discount bonds. The shapes of the term structures for coupon and discount bonds are identical. Proposition 1 implies that the credit spread on a corporate discount bond with term-to-maturity T is

$$S(D, T) = -\frac{1}{T} \log[1 - w Q(D, T)], \quad (17)$$

where $Q(D, T)$ is the risk-neutral probability of default in Equation (10). Equation (17) demonstrates that the credit spread on a corporate discount bond is an increasing function of the expected loss rate of the bond w and the risk-neutral probability of default $Q(D, T)$. Through this probability, the credit spread depends on the variables D and $T - t$ and the parameters $\mu, \kappa, \lambda, \sigma^2, \rho, \beta, \eta$ and K , but not on the default-free interest rate r .

Consider the term structures displayed in Figure 1 that assumes the following values of

the current debt ratio D : 12%, 33% and 46%. These values are similar to the market-value debt ratios of firms whose senior debt ratings from Standard and Poor's are AA, BBB and BB, respectively. The default boundary K is assumed to be 81% because this is the average market-value debt ratio of firms in default in Franks and Torous (1994). The depicted term structures exhibit steep upward slopes at short maturities. At longer maturities, their shapes are either flat or upward-sloping for investment grade bonds (AA and BBB). Intuitively, this is because the probability that the firm reaches the default boundary increases over time. For speculative grade bonds (BB-rated), the term structure is downward-sloping for maturities longer than a few years. The phenomenon of high credit spreads at short maturities is called "crisis-at-maturity" by Johnson (1967).¹⁵ This downward slope is generated by high initial default probabilities which are expected to decrease over time as the firm survives (these curves are drawn conditional upon the survival of the firm).

The displayed term structures of credit spreads for corporate bonds are similar to their empirical counterparts studied in Sarig and Warga (1989) and Duffee (1998a). However, Helwege and Turner (1999) find evidence that the term structures for speculative grade bonds are upward-sloping rather than downward-sloping. On the other hand, the historical default experience for corporate bonds reported by Fons (1994) supports a negative slope. The term structure graphs are surprisingly similar to those obtained from models in which the firm's capital structure is constant (Merton (1974) and Leland and Toft (1996), for example). This suggests that allowing for a dynamic capital structure does not change the basic shapes of the term structures of credit spreads. Nevertheless, their levels and dynamics are affected. The credit spreads depicted in the graph are comparable to the credit spreads for similarly rated corporate debt (that is, AA, BBB and BB).¹⁶

¹⁵Johnson observes that low quality bonds have very high credit spreads at short maturities. He explains this phenomenon by arguing that probabilities of default are high near maturity, because there is uncertainty about the ability to refinance the bond. This explanation assumes, as Johnson points out, that firms are unable to accumulate cash before maturity for debt repayment before maturity.

¹⁶The prediction that credit spreads are close to zero at very short maturities contradicts the findings of Fama (1986) from commercial paper credit spreads. The problem of downward-biased credit spreads at short maturities is common to all contingent-claims pricing models which assume continuous dynamics. Therefore,

The credit spreads in Figure 1 are positively related to the firm's current debt ratio D . This is because the firm is closer to default at high debt ratios. Figure 1 indicates further that credit spreads are more sensitive to the current debt ratio at short maturities than at long maturities. The expected recovery rate of 48% that is assumed in the graphs is typical for senior unsecured debt. Senior secured debt would have credit spreads which are 10–70 basis points lower than those depicted. This reflects its higher average recovery rate of 63%. On the other hand, the credit spreads on subordinated bonds would be 10–100 basis points higher than those depicted, which is consistent with their lower average recovery rate of 28% (the recovery rates are from Carty and Lieberman (1998)).

The relationship between credit spreads and the firm's target debt ratio μ is depicted in Figure 2. The credit spreads are an increasing function of the target debt ratio regardless of the maturity of the bond. Nevertheless, the target debt ratio is a relatively more important determinant of the credit spreads of long-term bonds than those of short-term bonds (this is evident from Figure 1). Figure 3 shows that credit spreads are negatively related to the debt ratio adjustment speed κ . This is because firms which adjust faster toward their target debt ratios are less likely to reach the high debt ratio levels at which default occurs (when $\mu < K$). From Figure 4, credit spreads are negatively related to the debt ratio risk premium λ . Negative values of λ imply that credit spreads exceed the expected percentage default losses to the bond holders during the lifetimes of the bonds. Figure 4 also shows that firms with identical actual default probabilities (implied by $\lambda = 0$) can have different credit spreads. According to Figure 5, credit spreads are an increasing function of the volatility of the firm's debt ratio σ^2 (that equals the volatility of its assets). The Figure assumes volatilities which are similar to those estimated by Jones, Mason and Rosenfeld (1984).

Finally, the effect of the correlation between the value of the firm's assets and the default-free interest rate ρ is displayed in Figure 6. The credit spreads are negatively related to ρ ,

criticism of the models based on low credit spreads at short maturities is that of the underlying distributional assumptions. Mason and Bhattacharya (1984), Zhou (1997) and Madan and Unal (1998) derive models in which the short-term credit spreads are non-trivial.

but its effect is typically only 5–15 basis points. The other parameters of the default-free interest rate process, η and β , are secondary determinants of credit spreads and their effects are not graphed. To conclude, allowing for stochastic interest rates has little effect on credit spreads. This conclusion was also reached by Kim, Ramaswamy and Sundaresan (1993) and Leland and Toft (1996). However, credit spreads and interest rates can be negatively correlated if the debt ratio and interest rates are negatively correlated.

V. The Credit Spread Dynamics

The dynamics of the credit spreads can be obtained by applying Itô's Lemma to Equation (17) and then by substituting in the dynamics of D . They are given by Proposition 2

Proposition 2 *The dynamics of the credit spread on the corporate unit discount bond with term-to-maturity T are governed by the following stochastic differential equation*

$$dS(t) = \left\{ S_D \kappa [\log \mu - \log D(t)] D(t) + \frac{1}{2} S_{DD} \sigma^2 D(t)^2 - S_{T-t} \right\} dt + S_D \sigma D(t) dW(t), \quad (18)$$

where $S_D = w Q_D / \{[1 - w Q(D, T)] T\}$, $S_{DD} = \{Q_{DD} [1 - w Q(D, T)] + w Q_D^2\} / \{[1 - w Q(D, T)]^2 T\}$ and $S_{T-t} = -\log[1 - w Q(D, T)] / T^2 + w Q_T / \{[1 - w Q(D, T)] T\}$, and where $Q(D, T)$ is the risk-neutral default probability in Equation (10), and $dW(t)$ is the differential of the standard Wiener process in Equation (4).

The dynamics of the credit spreads implied by Proposition 2 are illustrated graphically because of the infinite sum in the risk-neutral default probability $Q(D, T)$. The expected annual rate of change in credit spreads is plotted in Figure 7. The Figure assumes that the current debt ratio of the firm D is 12% or 46% (credit ratings of AA and BB, respectively). Its target debt ratio μ is 33% (a credit rating of BBB). Figure 7 shows that the credit spreads on bonds with short maturities tend to approach zero with a hump at low qualities.¹⁷ The

¹⁷This is implied by the low credit spreads at short maturities which contradict Fama's (1986) findings (see footnote 15).

credit spreads on long-term bonds exhibit mean reversion: they drift downward at debt ratios which are above the target debt ratio and upward at debt ratios which are below the target debt ratio. The long-term mean spreads are implied by the target debt ratio.

Figure 7 indicates further that the speed of the mean reversion is positively related to the speed at which the firm adjusts toward the target debt ratio κ , negatively related to the maturity of the bond and positively related to the level of the credit spread. These predictions are conditional upon no default occurring. The prediction that credit spreads are mean-reverting is consistent with the empirical findings of Longstaff and Schwartz (1995b), Stevens, Clinebell and Kahl (1998) and Morris, Neal and Rolph (1998) for corporate bond yield indices and those of Duffee (1998a) and Taurén (1999) for individual corporate bonds. Credit spreads do not exhibit mean reversion in models with a constant capital structure.

The volatilities of the credit spreads are examined in Figure 8. This volatility is due to uncertainty about the value of the firm's assets, because the book value of the debt is deterministic (conditional upon the value of the assets) and the credit spread is not a function of the default-free interest rate. Figure 8 shows that credit spreads are more volatile at high levels than at low levels of the credit spread. This is consistent with the findings of Taurén (1999). The volatilities are positively related to term-to-maturity at short maturities, but a decreasing function of term-to-maturity at more typical maturities.

The instantaneous correlation between the changes in the credit spreads and the default-free interest rate equals $\rho = -\rho_{Vr}$. This correlation is typically positive, because interest rates tend to be negatively correlated with asset values. However, the results from the studies of Kwan (1996), Longstaff and Schwartz (1995a) and Duffee (1998b) show that changes in credit spreads and interest rates are negatively correlated. Moreover, this correlation is negatively related to the credit quality of the bond. This suggests that there exists a relationship between the default risk in corporate bonds and interest rates that is not taken into account by the model. Other models of corporate bond prices which incorporate stochastic interest rates, like that of Longstaff and Schwartz (1995a), are consistent with a negative

relationship between credit spreads and default-free interest rates.

VI. An Empirical Comparison

This section compares the credit spreads predicted by the model with the observed credit spreads of the bonds of Boise Cascade Corporation. The same comparison is made for the Longstaff and Schwartz (1995a) model. This model provides a good point of comparison with my model because it generates realistic credit spreads, has an analytical solution and most of its parameters have counterparts in my model. The default boundary in the Longstaff-Schwartz model is recalibrated quarterly to reflect changes in the firm's amount of debt. This makes their model, that assumes a constant capital structure, perform much better.

The methodology parallels those of Jones, Mason and Rosenfeld (1984) and Titman and Torous (1989). Specifically, I calculate the debt ratios and asset values from stock price and financial statement data. The other parameters are calibrated. I compare the credit spreads predicted by the two models to the observed credit spreads of Boise Cascade in the period 1987–1994. This comparison evaluates the abilities of the models to generate realistic credit spreads when their parameters cannot be implied from bond prices, which is when the models are most useful. The model errors are overstated due to errors in the parameter values used. This is not a problem when the models are compared with each other, because they have similar parameters, but it may considerably reduce their fit to the data.

A. The Credit Spread Data

The bonds of Boise Cascade are: 11.875% January 1993, 8.375% August 1994 (callable at 100%), 9.625% July 1998 (callable at 100%) and 9.900% March 2000. The bonds were issued between 1985 and 1990 in sizes of \$100 million. They are straight, senior and unsecured. The data are bid prices recorded by the traders of Salomon Brothers at month-ends in the

period January 1987 to December 1994.¹⁸ The prices include accrued interest. I calculate the credit spreads of the bonds as differences between their yields (obtained from the bid prices) and those of U.S. Treasuries with similar maturities. There are 269 credit spreads. They fluctuate in a range of 53 and 576 basis points.

Descriptive statistics of the credit spreads are presented in Table I. The average credit spread is 188 basis points which is typical for corporate bonds rated BBB or BB like these bonds. The decaying pattern of the positive autocorrelations suggests that the credit spreads are mean-reverting.¹⁹ The credit spreads are graphed in Figure 9. The shapes of the term structures of the credit spreads in Figure 9 are relatively flat. This is quite surprising, because the bonds had speculative grade ratings for much of the period. Notice also that the spreads of the 1993 and 1994 bonds moved in opposite directions in 1991–1993. This makes sense only if the bid quotes for the 1993 bonds were lower than the actual prices of the bonds.

B. The Model Implementation

The model credit spreads are computed using the expression for the credit spread in Equation (17).²⁰ The risk-neutral default probability $Q(D, T)$ in Equation (10) is used for my model. For the Longstaff-Schwartz model, I use the risk-neutral default probability in their Equation (6). The computation of the credit spreads requires estimates of the underlying parameters of the models. My model has the following parameters: $D \equiv B/V, T, \mu, \kappa, \lambda, \sigma^2 = \sigma_V^2, \rho = -\rho_{Vr}, \beta, \eta$ and K . Their model uses the parameters: $V, T, \sigma_V^2, r, \rho_{Vr}, \beta, \eta$ and K' , where K'

¹⁸The empirical studies of Nunn, Hull and Schneeweis (1986) and Warga (1981) compare the bond price quotations from the traders of Merrill Lynch and Lehman Brothers, respectively, to those from the New York Stock Exchange. Both studies conclude that the trader quotations reflect the true market prices better. This seems reasonable, because much of the trading on the NYSE is driven by odd lot transactions. Most of the trading in corporate bonds takes places on OTC markets between banks and institutional investors.

¹⁹It is also consistent with a unit root (Dickey and Fuller (1979)) and a survivorship bias (Taurén (1999)). Therefore, I do not compare the models on the basis of their implications for credit spread dynamics. Duffee (1999) and Taurén (1999) find that the credit spreads on corporate bonds exhibit mean reversion.

²⁰By pricing the coupon bonds as if they were discount bonds the computations become much faster. This is because the model needs to be evaluated only once for each observation rather than as many times as the bond has coupons. The number of the coupons can be large. For example, the March 2000 bond had 24 coupons in March 1988. On the other hand, this method introduces error into the credit spreads. The error is likely to be small in practice, however.

is the default boundary for the value of the assets. The critical difference between these two parameter sets is that my model requires the target debt ratio μ , the speed of debt ratio adjustment κ and the risk premium λ whereas their model requires the default-free interest rate r . For both models, it is assumed that $n = 200$.

I calculate time series of the debt ratio of Boise Cascade and its value of assets as follows. The monthly debt ratio D_t is the ratio of the most recent book value of debt to the sum of the book value of debt and the market value of equity. The book value of debt is obtained quarterly as the sum of the following items in the COMPUSTAT quarterly data base: debt in current liabilities, total long-term debt and deferred taxes and investment tax credit. The market value of equity is calculated monthly as the sum of the carrying value of preferred stock and the month-end closing stock price times the number of common shares outstanding. These data are obtained from CRSP. The value of the assets V_t equals the sum of the book value of debt and the market value of equity.²¹

The other parameters are assumed to have the following values. The target debt ratio μ is assumed to be equal to the average debt ratio of 32.7% for BBB-rated firms. The speed of debt ratio adjustment κ is assumed to be 0.15. These parameters would have large standard errors if they were estimated from the debt ratio data. The default boundary K is assumed to be 81% because this is the average debt ratio for defaulted firms in the sample of Franks and Torous (1994). The default boundary for the Longstaff-Schwartz model K'_t is recalibrated quarterly. It equals the book value of debt divided by 0.81 so that default occurs at a debt ratio of 81%. The volatility of the debt ratio that equals the volatility of the assets is calculated using debt ratio data from the sample period as follows:

²¹During the sample period, the debt ratio of Boise Cascade varied between 36% and 73% with a mean and a standard deviation of 50% and 7.6%, respectively. In the beginning of the period, the debt ratio varied between 36% and 40% (apart from a two-month peak in January/February 1988). This stable period lasted until June 1989 when the debt ratio started a gradual climb that resulted from a halving of the stock price and a simultaneous doubling of the firm's amount of debt. The debt ratio reached its maximum level of 73% in March 1991 after which it stayed quite stable until it started a gradual decline in January 1993. This decline was associated with a rise in the firm's stock price and a slow decline in the amount of its debt. The debt ratio was about 60% at the end of the sample period.

$12 \times \sum_{t=2}^{96} \left[\log \left(\frac{D_t}{D_{t-1}} \right) \right]^2 / 95 = 0.0328$. This figure is similar to the average estimate of the asset volatility in Jones, Mason and Rosenfeld (1984). The expected recovery rate $1 - w$ is assumed to be 0.52 that is the historical average for senior unsecured public debt in Carty and Lieberman (1998). I assume that the debt ratio risk premium λ is -3%. The default-free interest rate r is the monthly average yield on long-term U.S. Treasury bonds from S&P Bond Guides. For simplicity, the correlation between the debt ratio and the default-free interest rate ρ is assumed to be zero. Finally, the interest rate process parameters are: $\beta = 1$ and $\eta^2 = 0.001$.

C. The Results

Descriptive statistics for the credit spreads from both models and their root mean-squared errors (RMSE) are presented in Table II. First consider the statistics for the credit spread levels in Panel A of Table II. My model in (1) generates credit spreads which are downward-biased and whose standard deviation is lower than that of the observed credit spreads. Consequently, the model does not fit the data very well, which is demonstrated by its RMSE of 140 basis points. The Longstaff-Schwartz model in (3) fares somewhat better in comparison. Although its mean predicted credit spread of 109 basis points is lower than its empirical counterpart, the standard deviation of the credit spreads is quite similar. Based on the RMSE, the Longstaff-Schwartz model performs slightly better than my model. The standard deviations of the changes in the credit spreads in Panel B are similar for the two models and higher than the standard deviations of the observed credit spreads.

These results may be due to a poor choice of the parameter values. To examine this possibility, I choose new values for the following parameters: $\mu = 47.0\%$ and $\kappa = 0.20$ (formerly 32.7% and 0.15, respectively) and $\sigma_V^2 = 0.0450$ (formerly 0.0328). These parameter values are chosen to first match the mean of the observed credit spreads and then their standard deviation. They are reasonable, because the debt ratio of Boise Cascade fluctuated

considerably around its mean of 50% in the sample period. The method of choosing new values for the parameters to improve the fit of the model is similar to that of using parameter values which are implicit in the data. It is computationally less intensive, however.

Given the new parameter values, my model in (2) generates credit spreads which have a similar mean and standard deviation as the credit spreads in the sample (Panel A of Table II). Even though this is by design, it shows that the model is able to generate credit spreads whose levels are empirically realistic at reasonable parameter values. Moreover, the RMSE of the model has decreased to 95 basis points, which indicates a significant improvement in fit. The RMSE is lower than that of the Longstaff-Schwartz model in (3). The RMSE could be reduced further by iterating over more parameters.²² The credit spreads which are predicted by my model under the new parameter values are plotted in Figure 10.

The Longstaff-Schwartz model is unable to simultaneously fit the levels and standard deviations of the observed credit spreads. Even though the mean of the predicted credit spreads in (4) in Panel A of Table II is the same as that of the sample credit spreads, their standard deviation is too high at 155 basis points. Because of this, the RMSE of the model actually increases to 138 basis points. I conclude that the Longstaff-Schwartz model overstates the standard deviation of the credit spreads. This can also be seen in Figure 11 which graphs the credit spreads predicted by their model. The conclusion is stronger because the credit spreads of Boise Cascade were particularly volatile in the sample period. Figure 12 compares the credit spreads predicted by the models to the credit spreads observed for each bond. The same conclusion can be drawn from this Figure.

D. An Analysis of the Model Errors

I define a model error as the difference between the credit spread predicted by the model and the observed credit spread. The results from regressions of the model errors on the bond

²²This exercise would be computationally very intensive, because the model has many parameters with quite similar implications for credit spreads. Additionally, since the solution for a finite n is only approximate, the choice of n would affect the results.

maturities, the debt ratio and the default-free interest rate are in Table III.

The results for my model are in Panel A of Table III. The term structures of credit spreads predicted by the model have too steep downward slopes. This is because the model predicts downward-sloping term structures whereas the observed term structures are usually flat. This result is statistically insignificant. The model overstates the importance of the firm's current debt ratio. This is consistent with the high standard deviations of the credit spread changes in Panel B of Table II. There are three alternative explanations for this. First, there is measurement error in the debt ratio. Second, the assumed debt ratio adjustment speed κ is too low, which makes the credit spreads too sensitive to the current debt ratio. These explanations do not contradict the model. The third explanation is that the model fails to incorporate an effect that is negatively correlated with the effect of the firm's debt ratio. Table III shows further that the model does not account properly for a negative relationship between credit spreads and default-free interest rates.

Panel B of Table III presents the results of the error analysis for the Longstaff-Schwartz model. Their model generates term structures which have even steeper downward slopes. The coefficients of the debt ratio are higher than for my model, which suggests that their model overstates the effect of the current debt ratio to a greater extent. This is consistent with the absence of mean reversion in their model, because mean reversion makes the current debt ratio (or the value of the assets) less important. It also explains the upward-biased standard deviations of the credit spreads shown in Panel A of Table II. On the other hand, their model is better able to incorporate the negative effect of the default-free interest rate. This makes sense, because the model predicts credit spreads which are negatively related to default-free interest rates. Nevertheless, there remain significant interest-rate related errors in the Longstaff-Schwartz model.

VII. Conclusions

This paper develops a new corporate bond pricing model that is consistent with the observed capital structure adjustment of companies. The critical assumption is that firms adjust their assets and debt such that their debt ratios exhibit gradual mean reversion toward target debt ratios. This description is consistent with the empirical findings of Taggart (1977), Marsh (1982), Jalilvand and Harris (1984), Auerbach (1985) and Opler and Titman (1995). The model is able to generate realistic credit spreads. The dynamics of the credit spreads are mean-reverting, which is consistent with the empirical evidence from Duffee (1998a) and Taur  n (1999). Moreover, the model implies that credit spreads are more sensitive to the firm’s current debt ratio than to the target debt ratio at short maturities. For bonds with long maturities, the target debt ratio is more important than the current debt ratio.

The performance of the model is compared against that of Longstaff and Schwartz (1995a) and the credit spreads of Boise Cascade Corporation over the period 1987–1994. The assumption that debt ratios are mean-reverting is supported by the data: credit spreads are less sensitive to the current debt ratio at long maturities than predicted by the competing model. This suggests that bond markets place less emphasis on current debt ratios in evaluating companies than predicted by their model. It also implies that financial ratio analysis for corporate lending purposes should put more emphasis on target debt ratios.

Some ideas about further research follow. Empirically estimated debt ratio dynamics (like in Shyam-Sundars and Myers (1999)) can be used. The corporate bond prices can then be solved for numerically. To reduce computation time, the analytical solution obtained in this paper can be used as a control variate. The debt ratio can also replace the value of the firm’s assets as the underlying variable in reduced-form models like that of Madan and Unal (1998). Finally, the inability of the derived model to generate credit spreads which are negatively correlated with default-free interest rates motivates further research.

Appendix

A Proof that Equation (4) is consistent with (2) and (3)

Apply Itô's Lemma to the definition of the debt ratio: $D(t) \equiv B(t)/V(t)$ to obtain the following partial differential equation

$$dD(t) = D_B dB(t) + \frac{1}{2}D_{BB}[dB(t)]^2 + D_V dV(t) + \frac{1}{2}D_{VV}[dV(t)]^2 + D_{BV} dB(t) dV(t). \quad (19)$$

Substitution of the dynamics of $dV(t)$ and $dB(t)$ from Equations (2) and (3), respectively, transforms this equation into

$$dD(t) = D_B \phi(V, B, t) B(t) dt + D_V [\mu_V V(t) + \Phi(V, B, t) V(t)] dt + D_V \sigma_V V(t) dW_2(t) + \frac{1}{2} D_{VV} \sigma_V^2 V(t)^2 dt. \quad (20)$$

Then substitute the partial derivatives of D with respect to B and V and simplify the resulting equation to have the following stochastic differential equation

$$dD(t) = [\phi(V, B, t) - \Phi(V, B, t) - \mu_V + \sigma_V^2] D(t) - \sigma_V D(t) dW_2(t). \quad (21)$$

This stochastic differential equation describes the dynamics of the firm's debt ratio in terms of the difference between the adjustment functions $\phi(V, B, t)$ and $\Phi(V, B, t)$. Equation (4) follows from the substitution of $\phi(V, B, t) - \Phi(V, B, t) = \kappa [\log \mu - \log D(t)] + \mu_V - \sigma_V^2$ for this difference and the changes of notation: $\sigma^2 = \sigma_V^2$ and $dW(t) = -dW_2(t)$. Q.E.D.

B Proof of the Lemma

I construct an arbitrage portfolio that consists of one corporate discount bond with a price of $P(D, r, T)$, $h_1(D, r, T)$ default-free discount bonds with maturities of m and prices of $G(r, m)$ and $h_2(D, r, T)H$ dollars invested in a firm-value replicating asset. The existence of the firm-value replicating asset is implied by the completeness of the capital markets. By design, its return is perfectly correlated with that on the firm's assets. I choose the hedge ratios $h_1(D, r, T)$ and $h_2(D, r, T)$ dynamically such that the portfolio is risk-free at any instant of time. The entire investment in the portfolio is financed by borrowing at the default-free interest rate, which implies a zero net investment in the portfolio.

The instantaneous dollar return on this arbitrage portfolio is

$$dP(D, r, T - t) + h_1 dG(r, m) + h_2 dH(t) - r(t)[P(D, r, T - t) + h_1 G(r, m) + h_2 H(t)]. \quad (22)$$

Application of Itô's Lemma to dP gives the following partial differential equation

$$\begin{aligned} P_D dD(t) + \frac{1}{2} P_{DD} [dD(t)]^2 + P_r dr(t) + \frac{1}{2} P_{rr} [dr(t)]^2 + P_{Dr} dD(t) dr(t) + P_t dt \\ + h_1 dG(r, m) + h_2 dH(t) - r(t) [P(D, r, T - t) + h_1 G(r, m) + h_2 H(t)]. \end{aligned} \quad (23)$$

Then substitution of the dynamics for r and D from Equations (1) and (21), respectively, the following dynamics from Vasicek (1977): $dG(r, m) = \mu_B G(r, m) dt - \eta C(m) G(r, m) dW_1(t)$, where μ_B denotes the expected return on the default-free discount bond, and the firm-value replicating asset dynamics: $dH(t) = \mu_V H(t) dt + \sigma_V H(t) dW_2(t)$ yields

$$\begin{aligned} P_D \kappa [\log \mu - \log D(t)] D(t) dt - P_D \sigma_V D(t) dW_2(t) + \frac{1}{2} P_{DD} \sigma_V^2 D(t)^2 dt \\ + P_r [\zeta - \beta r(t)] dt + P_r \eta dW_1(t) + \frac{1}{2} P_{rr} \eta^2 dt - P_{Dr} \rho_{Vr} \sigma_V \eta D(t) dt \\ + P_t dt + h_1 \mu_B G(r, m) dt - h_1 \eta C(m) G(r, m) dW_1(t) + h_2 \mu_V H(t) dt \\ + h_2 \sigma_V H(t) dW_2(t) - r(t) P(D, r, T - t) - h_1 r(t) G(r, m) - h_2 r(t) H(t). \end{aligned} \quad (24)$$

The arbitrage portfolio becomes risk-free when the coefficients of the Wiener process differentials $dW_1(t)$ and $dW_2(t)$ are zero. To achieve this, substitute the following values for the hedge ratios into Equation (24): $h_1(D, r, T) = P_r/[C(m)G]$ and $h_2(D, r, T) = P_D D/H$. The resulting equation is equal to 0 dt , because the return on a risk-free portfolio must be zero if the portfolio has no net investment. Divide the resulting expression by dt on both sides to obtain the following identity

$$\begin{aligned}
& P_D \kappa [\log \mu - \log D(t)] D(t) + \frac{1}{2} P_{DD} \sigma_V^2 D(t)^2 + P_r [\zeta - \beta r(t)] + \frac{1}{2} P_{rr} \eta^2 \\
& - P_{Dr} \rho_{Vr} \sigma_V \eta D(t) + P_t - P_D \mu_B D(t) + P_r \mu_B \frac{1}{C(m)} + P_D \mu_V D(t) \\
& - r(t) P(D, r, T - t) + P_{Dr}(t) D(t) - P_r r(t) \frac{1}{C(m)} - P_D r(t) D(t) = 0. \quad (25)
\end{aligned}$$

The partial differential equation in the Lemma is then obtained by reorganizing and by making the following substitutions: $\lambda \equiv \mu_B - \mu_V$, $\sigma^2 = \sigma_V^2$ and $\rho = -\rho_{Vr}$. Q.E.D.

C Proof of Proposition 1

The Feynman-Kac probabilistic solution to the partial differential equation in the Lemma subject to its boundary condition is given by (Karatzas and Shreve's (1991) theorem 4.2)

$$P(D, r, T) = E_0^Q \left[\exp \left(- \int_0^T r dt \right) (1 - \tilde{w} 1_{\tau \leq T}) \right], \quad (26)$$

where $E_0^Q[\cdot]$ denotes an expectation with respect to the risk-neutral probability measure Q taken at the current time 0. The existence of Q is implied by the no-arbitrage condition and the completeness and perfectness of the capital markets. This equation can be expressed in the following more convenient form which allows separation of the expectations (see, for example, the separation theorem in Longstaff (1990))

$$P(D, r, T) = E_0^Q \left[\exp \left(- \int_0^T r dt \right) \right] E_0^Q [1 - \tilde{w} 1_{\tau \leq T}], \quad (27)$$

where the risk-neutral process for the debt ratio is adjusted to reflect its correlation with the default-free interest rate in the following way

$$dD(t) = \{ \kappa [\log \mu - \log D(t)] - \lambda - \rho \sigma \eta C(T - t) \} D(t) dt + \sigma D(t) dW^Q(t). \quad (28)$$

Since the stochastic recovery rate \tilde{w} is independent of the default-free interest rate r , no adjustment needs to be made to its distribution. The solution to the first expectation is from Vasicek (1977) and given by $A(r, T)$ in Equation (9). The independence of the recovery rate from the debt ratio can be used to obtain the following form for Equation (27)

$$P(D, r, T) = A(r, T) \left[1 - E_0^Q(\tilde{w}) E_0^Q(1_{\tau \leq T}) \right], \quad (29)$$

where $E_0^Q(\tilde{w}) = E_0(\tilde{w}) = w$ since by assumption the risk of recovery is not priced in the capital markets. The expectation of the indicator function $1_{\tau \leq T}$ equals the first-passage-

time probability of the adjusted risk-neutral process of D to the boundary K in time T . Therefore, the solution to the partial differential equation can be written as follows

$$P(D, r, T) = A(r, T) [1 - w Q(D, T)], \quad (30)$$

where the first-passage-time probability is denoted by $Q(D, T)$.

The subsequent parts of the proof show that Equations (10) to (14) give the relevant first-passage-time probability. This probability is obtained by considering the isomorphic problem of the first-passage-time of the logarithm of the debt ratio to the logarithm of the default boundary. The following dynamics of $\log D$ are implied by Itô's Lemma

$$d \log D(t) = \{\kappa[\log \mu - \log D(t)] - \sigma^2/2 - \lambda - \rho\sigma\eta C(T - t)\} dt + \sigma dW^Q(t). \quad (31)$$

This is an Ornstein-Uhlenbeck process with a time-dependent drift. Let $q(\log K, t \mid \log D(0), 0)$ denote the first-passage-time density function for the logarithmic process to attain the boundary $\log K$ at time $t \in (0, T]$ conditional upon its value of $\log D < \log K$ at time 0. This density function is defined implicitly by the following integral equation (Fortet (1943))

$$f(\log D(t), t \mid \log D(0), 0) = \int_0^t q(\log K, s \mid \log D(0), 0) f(\log D(t), t \mid \log K, s) ds, \quad (32)$$

where $f(\cdot, t \mid \log D(0), 0)$ is the free transition density of $\log D$ at time t (that is, its density in the absence of the default boundary) conditional upon its value at time 0. Integration of both sides of the equation with respect to $\log D(t)$ between $\log K$ and a natural upper boundary and evaluation of the integrals using Fubini's theorem yields

$$1 - F(\log K, t \mid \log D(0), 0) = \int_0^t q(\log K, s \mid \log D(0), 0) \times [1 - F(\log K, t \mid \log K, s)] ds, \quad (33)$$

where $F(\cdot, t \mid \log D(0), 0)$ is the free transition distribution of $\log D$ at time t conditional upon its value at time 0. This transition distribution can be obtained from the following solution to the stochastic differential equation (31) (Karatzas and Shreve pp. 360)

$$\begin{aligned} \log D(t) = & e^{-\kappa t} \log D(0) + \int_0^t e^{-\kappa(t-s)} [\kappa \log \mu - \sigma^2/2 - \lambda - \rho \sigma \eta C(T-s)] ds \\ & + \sigma \int_0^t e^{-\kappa(t-s)} dW_s^Q. \end{aligned} \quad (34)$$

The conditional mean of the free transition distribution of $\log D$ is given by the sum of the first two terms of the equation. The second term contains an ordinary Riemann integral. The solution to the Itô stochastic integral in the third term gives the conditional variance of the Ornstein-Uhlenbeck process (Karatzas and Shreve pp. 200). This process is conditionally normally distributed. By exploiting the symmetry of the normal distribution, Equation (33) can be written as follows

$$\begin{aligned} N \left(\frac{M(\log D, t, T) - \log K}{\sqrt{V(t)}} \right) = & \int_0^t q(\log K, s \mid \log D(0), 0) \times \\ & N \left(\frac{M(\log K, t, T) - M(\log K, s, T)}{\sqrt{V(t) - V(s)}} \right) ds, \end{aligned} \quad (35)$$

where $M(X, t, T)$ and $V(t)$ denote the conditional mean and variance of the logarithmic process, which are presented in Equations (13) and (14), respectively, and $N(\cdot)$ denotes the standard normal distribution function.

The remaining parts of the proof show how Equations (10) to (12) follow from Equation (35). To see this, first divide the time period $[0, T]$ into n equal subperiods. Then discretize the above integral equation to obtain the following finite sum

$$N \left(\frac{M(\log D, iT/n, T) - \log K}{\sqrt{V(iT/n)}} \right) = \sum_{j=1}^i q_j N \left(\frac{M(\log K, iT/n, T) - M(\log K, jT/n, T)}{\sqrt{V(iT/n) - V(jT/n)}} \right), \quad (36)$$

where $i \in [1, 2, \dots, n]$ and $q_j \equiv q(\log K, jT/n \mid \log D(0), 0)$. These equations can be solved

recursively for the q_i terms by first solving q_1 . Their sum that is presented in Equation (10) provides an approximation to the value of $Q(D, T)$. As n approaches infinity, the approximation converges to the exact value of the first-passage-time probability. Q.E.D.

References

- Anderson, Ronald W. and Suresh Sundaresan, 1996, Design and valuation of debt contracts, *Review of Financial Studies* 9, 37-68.
- Auerbach, Alan J., 1985, Real determinants of corporate leverage, in Benjamin M. Friedman ed.: *Corporate Capital Structures in the United States* (University of Chicago Press, Chicago, I.L.).
- Black, Fisher and Myron Scholes, 1973, The pricing of options and corporate liabilities, *Journal of Political Economy* 81, 637-54.
- Black, Fisher and John C. Cox, 1976, Valuing corporate securities: Some effects of bond indenture provisions, *Journal of Finance* 31, 351-367.
- Bookstaber, Richard and David P. Jacob, 1986, The composite hedge: Controlling the credit risk of high-yield bonds, *Financial Analysts Journal* 42, 25-36.
- Brennan, Michael J. and Eduardo S. Schwartz, 1984, Optimal financial policy and firm valuation, *Journal of Finance* 39, 593-607.
- Briys, Eric and Francois de Varenne, 1997, Valuing risky fixed rate debt: An extension, *Journal of Financial and Quantitative Analysis* 32, 239-248.
- Carty, Lea V. and Dana Lieberman, 1998, Historical default rates of corporate bond issuers, 1920-1996, in Satyajit Das ed.: *Credit Derivatives* (John Wiley, Singapore).
- Crosbie, Peter, 1998, Credit valuation, in Satyajit Das ed.: *Credit Derivatives* (John Wiley, Singapore).
- Chance, Don. M., 1990, Default risk and the duration of zero coupon bonds, *Journal of Finance* 45, 265-274.
- Dickey, David A. and Wayne A. Fuller, 1979, Distributions of the estimators for autoregressive time series with a unit root, *Journal of the American Statistical Association* 74, 427-431.
- Duffee, Gregory R., 1998a, Estimating the price of default risk, *Review of Financial Studies*, forthcoming.
- Duffee, Gregory R., 1998b, The relation between treasury yields and corporate bond yield spreads, *Journal of Finance*, forthcoming.
- Duffie, Darrell and Kenneth J. Singleton, 1999, Modeling term structures of defaultable bonds, *Review of Financial Studies*, forthcoming.
- Fama, Eugene F., 1986, Term premiums and default premiums in money markets, *Journal of Financial Economics* 17, 175-196.
- Fan, Hua and Suresh Sundaresan, 1998, Debt valuation, renegotiations and optimal dividend policy, Working Paper, Columbia University.
- Fischer Edwin O., Heinkel Robert and Josef Zechner, 1989, Dynamic capital structure choice: Theory and tests, *Journal of Finance* 44, 19-40.

- Fons, Jerome S., 1994, Using default rates to model the term structure of credit risk, *Financial Analysts Journal* September-October, 25-32.
- Fortet, R., 1943, Les fonctions aléatoires du type de Markoff associées à certaines équations linéaires aux dérivées partielles du type parabolique, *Journal de Mathématiques Pures et Appliquées* 22, 177-243.
- Franks, Julian R. and Walter N. Torous, 1994, A comparison of financial restructuring in distressed exchanges and Chapter 11 reorganizations, *Journal of Financial Economics* 35, 349-370.
- Goldstein, Robert, Nengjiu Ju and Hayne Leland, 1998, An EBIT-based Model of Dynamic Capital Structure, Working Paper, Ohio State University.
- Grives, Robin, 1986, Hedging corporate bond portfolios, *Journal of Portfolio Management* 12, 23-25.
- Helwege, Jean and Christopher M. Turner, 1999, The slope of the credit yield curve for speculative-grade issuers, *Journal of Finance*, forthcoming.
- Ilmanen, Antti, Donald McGuire and Arthur Warga, 1994, The value of duration as a risk measure for corporate debt, *Journal of Fixed Income* June, 132-148.
- Izvorski, Ivailo, 1996, Strategic and liquidity default, Working Paper, Yale University.
- Jalilvand, Abolhassan and Robert S. Harris, 1984, Corporate behavior in adjusting to capital structure and dividend targets: An econometric study, *Journal of Finance* 39, 127-145.
- Jones, Philip E., Scott P. Mason and Eric Rosenfeld, 1984, Contingent claims analysis of corporate capital structures: An empirical investigation, *Journal of Finance* 39, 611-625.
- Johnson, Ramon E., 1967, Term structures of corporate bond yields as a function of risk of default, *Journal of Finance* 22, 313-345.
- Karatzas, Ioannis and Steven E. Shreve, 1991, *Brownian motion and stochastic calculus*, 2nd ed., (Springer-Verlag, New York, N.Y.)
- Kim, In Joon, Krishna Ramaswamy and Suresh Sundaresan, 1993, Does default risk in coupons affect the valuation of corporate bonds?: A contingent claims model, *Financial Management* Autumn, 117-131.
- Kwan, Simon H., 1996, Firm-specific information and the correlation between individual stocks and bonds, *Journal of Financial Economics* 40, 63-80.
- Leland, Hayne E., 1995, Corporate debt value, bond covenants and optimal capital structure, *Journal of Finance* 49, 1213-1242.
- Leland, Hayne E., 1998, Agency costs, risk management and capital structure, *Journal of Finance* 53, 1213-1243.
- Leland, Hayne E. and Klaus B. Toft, 1996, Optimal capital structure, endogenous bankruptcy and the term structure of credit spreads, *Journal of Finance* 51, 987-1019.
- Longstaff, Francis A., 1990, The valuation of options on yields, *Journal of Financial Economics* 26, 97-122.

- Longstaff, Francis A. and Eduardo S. Schwartz, 1993, Valuing risky debt: A new approach, Working Paper, University of California, Los Angeles.
- Longstaff, Francis A. and Eduardo S. Schwartz, 1995a, A simple approach to valuing risky fixed and floating rate debt, *Journal of Finance* 50, 789-819.
- Longstaff, Francis A. and Eduardo S. Schwartz, 1995b, Valuing credit derivatives, *Journal of Fixed Income* 5, 6-12.
- Madan, Dilip B. and Haluk Unal, 1998, A two-factor hazard-rate model for pricing risky debt in a complex capital structure, Working Paper, University of Maryland, College Park.
- Marsh, Paul, 1982, The choice between equity and debt: An empirical study, *Journal of Finance* 37, 121-144.
- Mason, Scott P. and Sudipto Bhattacharya, 1981, Risky debt, jump processes and safety covenants, *Journal of Financial Economics* 9, 281-307.
- Merton, Robert C., 1974, On the pricing of corporate debt: The risk structure of interest rates, *Journal of Finance* 29, 449-470.
- Miller, Merton H. and Franco Modigliani, 1958, The cost of capital, corporation finance and the theory of investment, *American Economic Review* 48, 261-297.
- Morris, Charles, Robert Neal and Douglas Rolph, 1998, Credit spreads and interest rate risk: A cointegration approach, Working Paper, Federal Reserve Bank of Kansas City, Indiana University - Purdue University, Indianapolis and University of Washington, Seattle.
- Nawalkha, Sanjay K., 1996, A contingent claims analysis of the interest rate risk characteristics of corporate liabilities, *Journal of Banking and Finance* 20, 227-245.
- Newey, Whitney K. and Kenneth D. West, 1987, A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix, *Econometrica* 55, 703-708.
- Nielsen, Lars, Jesus Saá-Requejo and Pedro Santa-Clara, 1993, Default risk and interest rate risk: The term structure of default spreads, Working Paper, INSEAD.
- Nobile, A.G., Ricciardi L.M. and L. Sacerdote, 1985, A note on first-passage time and some related problems, *Journal of Applied Probability* 22, 346-359.
- Opler, Tim and Sheridan Titman, 1995, The debt-equity choice, Working Paper, Ohio State University and Boston College.
- Ramaswami, Murali, 1991, Hedging the equity risk of high-yield bonds, *Financial Analysts Journal* 47, 41-50.
- Ramaswamy, Krishna and Suresh Sundaresan, 1986, The valuation of floating-rate instruments: Theory and evidence, *Journal of Financial Economics* 17, 251-272.
- Sarig, Oded and Arthur Warga, 1989, Some empirical estimates of the risk structure of interest rates, *Journal of Finance* 44, 1351-1360.
- Shimko, David, Naohiko Tejima and Donald van Deventer, 1993, The pricing of risky debt when interest rates are stochastic, *Journal of Fixed Income* 3, 58-65.

- Shyam-Sunder, Lakshmi and Stewart C. Myers, 1999, Testing static tradeoff against pecking order models of capital structure, *Journal of Financial Economics* 51, 219-244.
- Skinner, Frank S., 1998, Hedging bonds subject to credit risk, *Journal of Banking and Finance* 22, 321-345.
- Smith, Clifford W. and Jerold B. Warner, 1979, On financial contracting: An analysis of bond covenants, *Journal of Financial Economics* 7, 167-211.
- Standard and Poor's, 1995, *Standard and Poor's CreditWeek* October 9, 54.
- Stevens, Jerry L, John. M. Clinebell and Douglas R. Kahl, 1998, Time series estimation of the bond default risk premium, *Quarterly Review of Economics and Finance*, forthcoming.
- Taggart, Robert, 1977, A model of financing decisions, *Journal of Finance* 32, 1467-1484.
- Taurén, Miikka, 1999, A comparison of bond pricing models in the pricing of credit risk, Working Paper, Indiana University, Bloomington.
- Titman, Sheridan and Walter Torous, 1989, Valuing commercial mortgages: An empirical investigation of the contingent-claims approach to pricing risky debt, *Journal of Finance* 44, 345-373.
- Vasicek, Oldrich, 1977, An equilibrium characterization of the term structure, *Journal of Financial Economics* 5, 177-188.
- Zhou, Chunsheng, 1997, A jump-diffusion approach to modeling credit risk and valuing defaultable securities, Working Paper, Board of Governors of the Federal Reserve System.

Table I
Descriptive Statistics of the Credit Spreads on
Boise Cascade Bonds in 1987–1994

Means, standard deviations, skewnesses, excess kurtoses and autocorrelations of the monthly credit spreads and their changes (in basis points) for Boise Cascade bonds in the period January 1987 to December 1994. The bonds are: 11.875% January 1993, 8.375% August 1994, 9.625% July 1998 and 9.900% March 2000. The credit spreads denoted by S_t are computed as differences between the yields of the bonds and those of the U.S. Treasury with similar maturities. The bond yields are obtained from bid prices quoted by traders of Salomon Brothers. Autocorrelation coefficients of order j are denoted by ρ_j . The Ljung-Box Q-statistic that is calculated for 5 autocorrelations is χ^2 distributed with 5 degrees of freedom. The number of observations is 269.

	Mean	Standard Deviation	Skew- ness	Excess Kurtosis	Autocorrelations					Ljung-Box Q-statistic
					ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	
S_t	188.00	95.41	1.41	2.59	0.92	0.85	0.77	0.68	0.61	820**
$S_t - S_{t-1}$	3.11	23.54	1.16	5.71	0.12	0.07	-0.01	-0.04	-0.18	14.7**

** indicates statistical significance at the 1% level.

* indicates statistical significance at the 5% level.

Table II
Comparison of the Models with the Credit Spreads of
Boise Cascade in the period 1987–1994

The credit spreads are from the period January 1987 to December 1994 for the following Boise Cascade bonds: 11.875% January 1993, 8.375% August 1994, 9.625% July 1998 and 9.900% March 2000. The parameters are: the value of the firm's assets V_t is the sum of the book value of debt B_t , preferred stock and the market value of equity, the debt ratio $D_t = B_t/V_t$, the default boundary $K = 81\%$ for models (1) and (2) and $K'_t = B_t/.81$ for models (3) and (4), the debt ratio volatility $\sigma^2 = 0.0328$, the risk premium $\lambda = -3\%$, the default-free interest rate r_t is the monthly aggregate yield on long-term Treasury bonds, the correlations $\rho = \rho_{Vr} = 0$ and the interest rate process parameters are: $\beta = 1.00$ and $\eta^2 = 0.001$. RMSE stands for root mean-squared error. The number of observations is 269.

Panel A: Credit Spread Levels					
	Mean	Standard Deviation	Min	Max	RMSE
Boise Cascade Bonds	188.00	95.41	53.60	575.90	
Current Model					
(1) $\mu = 32.7\%$ and $\kappa = 0.15$	81.96	64.08	0.00	368.55	139.98
(2) $\mu = 47.0\%$ and $\kappa = 0.20$	188.03	96.20	50.14	568.53	95.28
Longstaff-Schwartz Model					
(3) $\sigma_V^2 = 0.0328$	108.83	99.30	4.93	511.86	122.44
(4) $\sigma_V^2 = 0.0450$	189.37	155.18	20.67	886.12	137.69
Panel B: Credit Spread Changes					
	Mean	Standard Deviation	Min	Max	RMSE
Boise Cascade Bonds	3.11	23.54	-73.20	89.50	
Current Model					
(1) $\mu = 32.7\%$ and $\kappa = 0.15$	1.13	34.99	-331.61	197.12	42.52
(2) $\mu = 47.0\%$ and $\kappa = 0.20$	2.33	38.74	-198.31	266.07	44.77
Longstaff-Schwartz Model					
(3) $\sigma_V^2 = 0.0328$	2.75	38.55	-183.08	289.00	43.86
(4) $\sigma_V^2 = 0.0450$	4.81	52.42	-226.34	423.46	56.46

Table III
Regressions of the Model Errors on Term-to-Maturity,
the Debt Ratio and the Default-Free Interest Rate

The differences between the credit spreads implied by the models and the actual credit spreads on the bonds of Boise Cascade in the period 1987–1994 are regressed on a constant, the term-to-maturity of the bond $T - t$ (in years), the debt ratio D_t (in percent) and the default-free interest rate r_t (in basis points). The OLS regression allows for heteroskedascity and autocorrelation in the residuals using the Newey-West (1987) correction with 10 lags. R^2 denotes the coefficient of determination. DW refers to the Durbin-Watson statistic. The t -values are given in parentheses. The number of observations is 269.

Panel A: Current Model						
	Constant	$T - t$	D	r	R^2	DW
(1) $\mu = 32.7\%$ and $\kappa = 0.15$	-883.55** (6.98)	-3.07 (1.56)	351.43** (2.97)	0.76** (6.46)	0.51	0.60
(2) $\mu = 47.0\%$ and $\kappa = 0.20$	-868.71** (6.21)	-3.94 (1.90)	690.69** (5.52)	0.66** (5.30)	0.50	0.64
Panel B: Longstaff-Schwartz Model						
	Constant	$T - t$	D	r	R^2	DW
(3) $\sigma^2 = 0.0328$	-763.80** (5.74)	-6.82** (3.31)	610.23** (4.87)	0.52** (4.40)	0.48	0.62
(4) $\sigma^2 = 0.0450$	-817.13** (4.93)	-13.47** (4.64)	1112.36** (6.39)	0.43** (3.09)	0.59	0.63

** indicates statistical significance at the 1% level in a two-tailed test.

* indicates statistical significance at the 5% level in a two-tailed test.

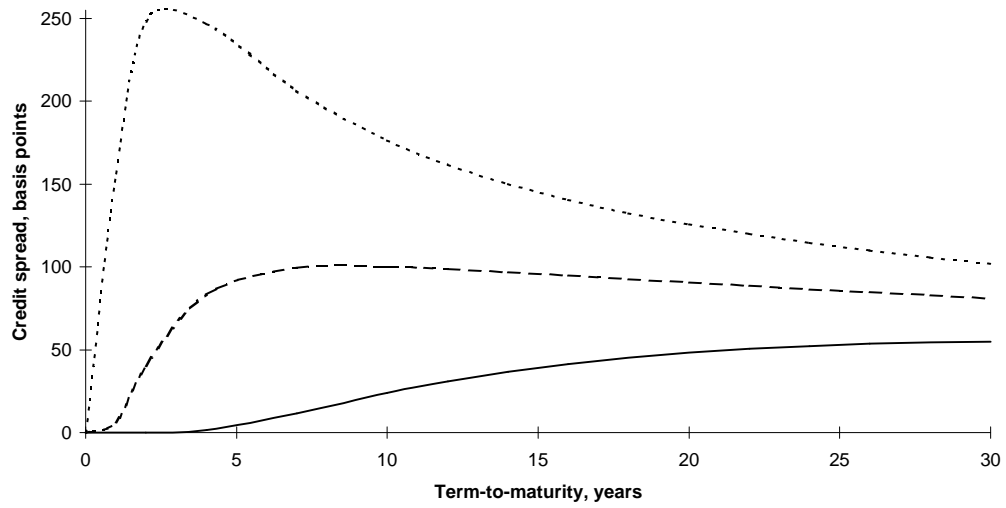


Figure 1. The credit spread as a function of term-to-maturity with debt ratios D of 12% (solid line), 33% (long dashed line) and 46% (short dashed line).

The other parameters are: the target debt ratio $\mu = 33\%$, the speed of debt ratio adjustment $\kappa = 0.15$, the volatility of the debt ratio $\sigma_2 = 0.09$, the debt ratio risk premium $\lambda = -3\%$, the default boundary $K = 81\%$, the recovery rate $1-w = 42\%$, the correlation between the debt ratio and the interest rate $\rho = 0.25$ and the interest rate process parameters $\beta = 1.00$ and $\eta_2 = 0.001$.

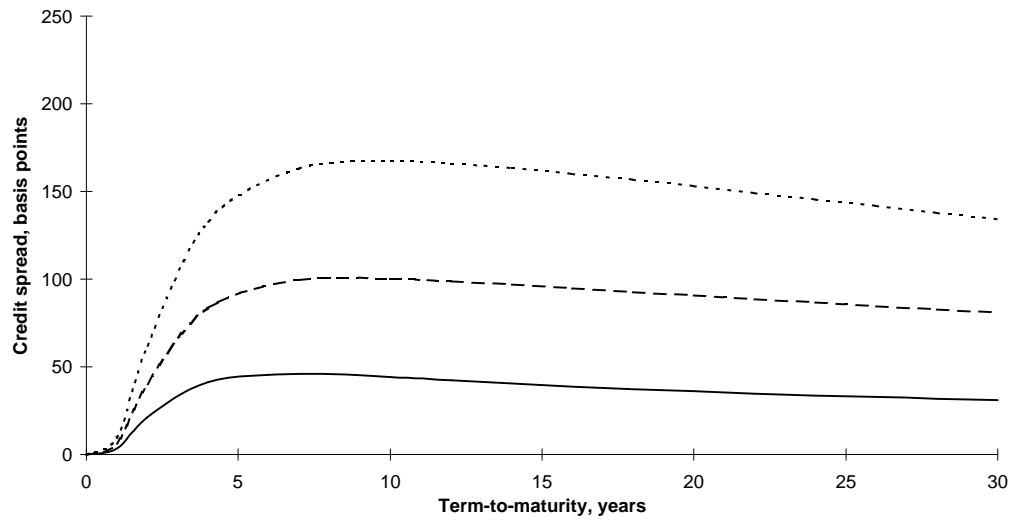


Figure 2. The credit spread as a function of term-to-maturity with target debt ratios μ of 22% (solid line), 33% (long dashed line) and 43% (short dashed line).

The other parameters are: the debt ratio $D = 33\%$, the speed of debt ratio adjustment $\kappa = 0.15$, the volatility of the debt ratio $\sigma_2 = 0.09$, the debt ratio risk premium $\lambda = -3\%$, the default boundary $K = 81\%$, the recovery rate $1-w = 42\%$, the correlation between the debt ratio and the interest rate $\rho = 0.25$ and the interest rate process parameters $\beta = 1.00$ and $\eta_2 = 0.001$.

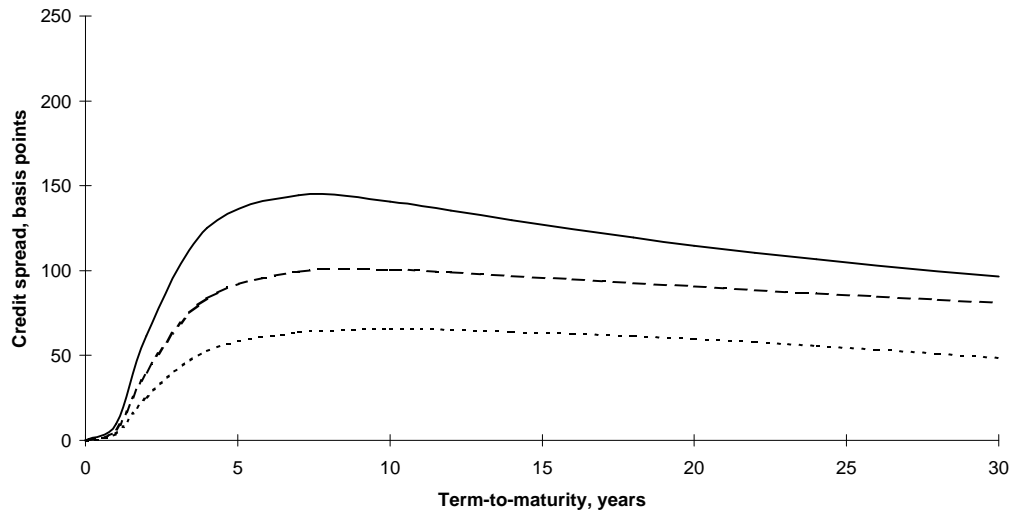


Figure 3. The credit spread as a function of term-to-maturity with debt ratio adjustment speeds κ of 0.05 (solid line), 0.15 (long dashed line) and 0.25 (short dashed line).

The other parameters are: the debt ratio $D = 33\%$, target debt ratio $\mu = 33\%$, the volatility of the debt ratio $\sigma_2 = 0.09$, the debt ratio risk premium $\lambda = -3\%$, the default boundary $K = 81\%$, the recovery rate $1-w = 42\%$, the correlation between the debt ratio and the interest rate $\rho = 0.25$ and the interest rate process parameters $\beta = 1.00$ and $\eta_2 = 0.001$.

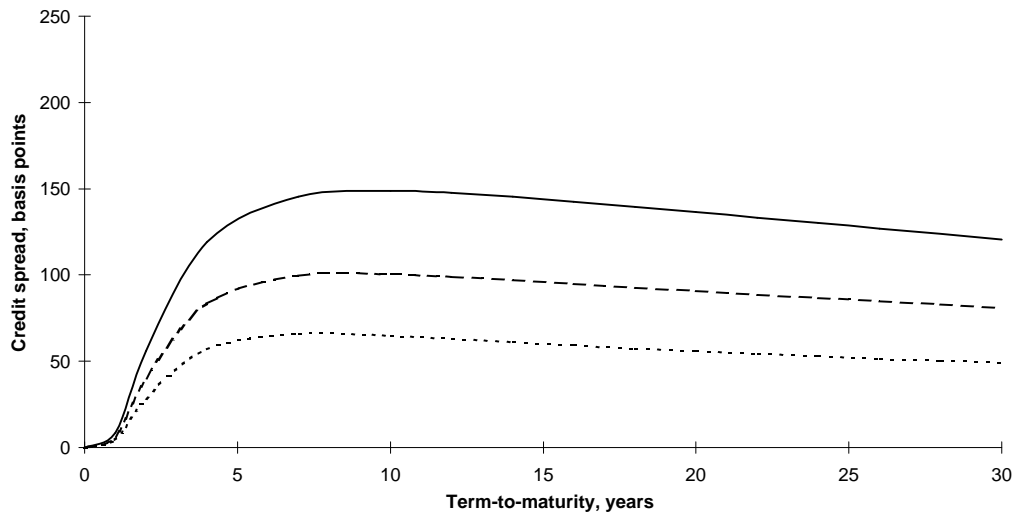


Figure 4. The credit spread as a function of term-to-maturity with debt ratio risk premia λ of -6% (solid line), -3% (long dashed line) and 0% (short dashed line).

The other parameters are: the debt ratio $D = 33\%$, target debt ratio $\mu = 33\%$, the debt ratio adjustment speed $\kappa = 0.15$, the volatility of the debt ratio $\sigma_2 = 0.09$, the default boundary $K = 81\%$, the recovery rate $1-w = 42\%$, the correlation between the debt ratio and the interest rate $\rho = 0.25$ and the interest rate process parameters $\beta = 1.00$ and $\eta_2 = 0.001$.

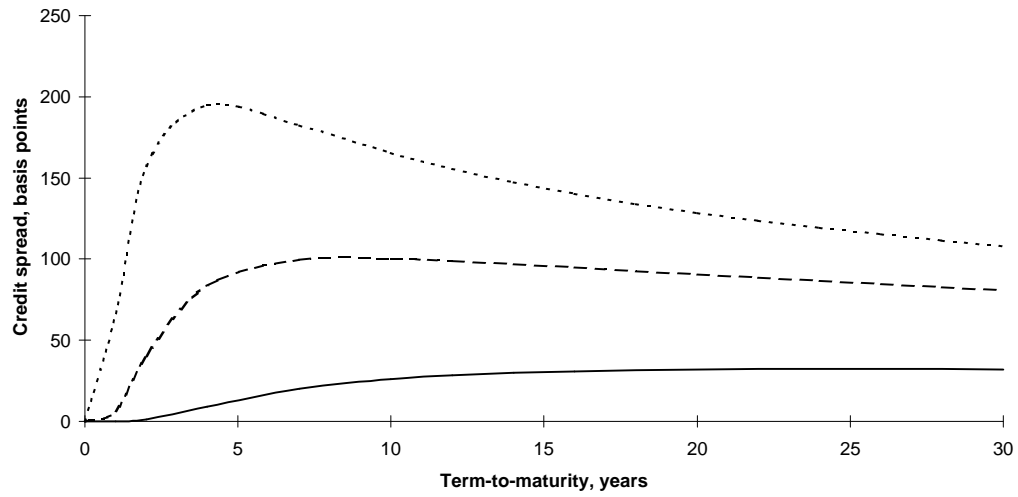


Figure 5. The credit spread as a function of term-to-maturity with debt ratio volatilities σ^2 of 0.04 (solid line), 0.09 (long dashed line) and 0.16 (short dashed line).

The other parameters are: the debt ratio $D = 33\%$, target debt ratio $\mu = 33\%$, the speed of debt ratio adjustment $\kappa = 0.15$, the debt ratio risk premium $\lambda = -3\%$, the default boundary $K = 81\%$, the recovery rate $1 - w = 42\%$, the correlation between the debt ratio and the interest rate $\rho = 0.25$ and the interest rate process parameters $\beta = 1.00$ and $\eta^2 = 0.001$.

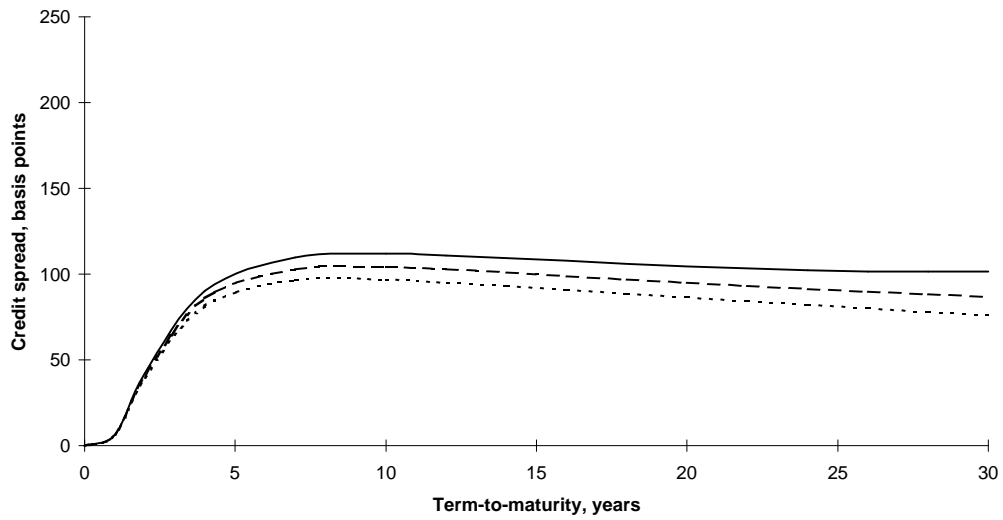


Figure 6. The credit spread as a function of term-to-maturity with correlations of the debt ratio with the interest rate ρ of -0.50 (solid line), 0 (long dashed line) and 0.50 (short dashed line). The other parameters are: the debt ratio $D = 33\%$, target debt ratio $\mu = 33\%$, the speed of debt ratio adjustment $\kappa = 0.15$, the volatility of the debt ratio $\sigma_2 = 0.09$, the debt ratio risk premium $\lambda = -3\%$, the default boundary $K = 81\%$, the recovery rate $1-w = 42\%$ and the interest rate process parameters $\beta = 1.00$ and $\eta_2 = 0.001$.

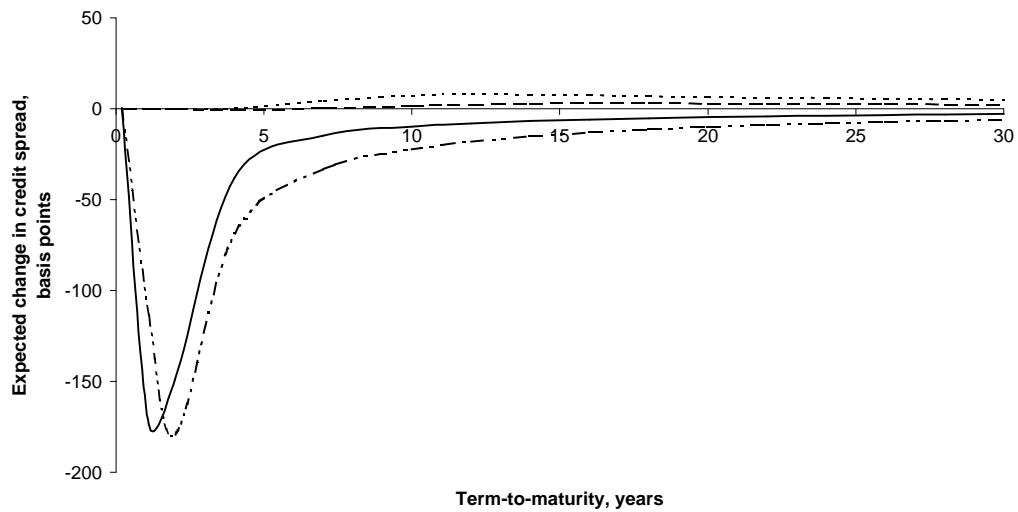


Figure 7. The expected change in the term structure of credit spreads in one year with debt ratios of 12% (short and long dashed lines for $\kappa = 0.15$ and 0.25 , respectively) and 46% (medium dashed and solid lines for κ , respectively).

The other parameters are: the target debt ratio $\mu = 33\%$, the speed of debt ratio adjustment $\kappa = 0.15$, the volatility of the debt ratio $\sigma_2 = 0.09$, the debt ratio risk premium $\lambda = -3\%$, the default boundary $K = 81\%$, the recovery rate $1 - w = 42\%$, the correlation between the debt ratio and the interest rate $\rho = 0.25$ and the interest rate process parameters $\beta = 1.00$ and $\eta_2 = 0.001$.

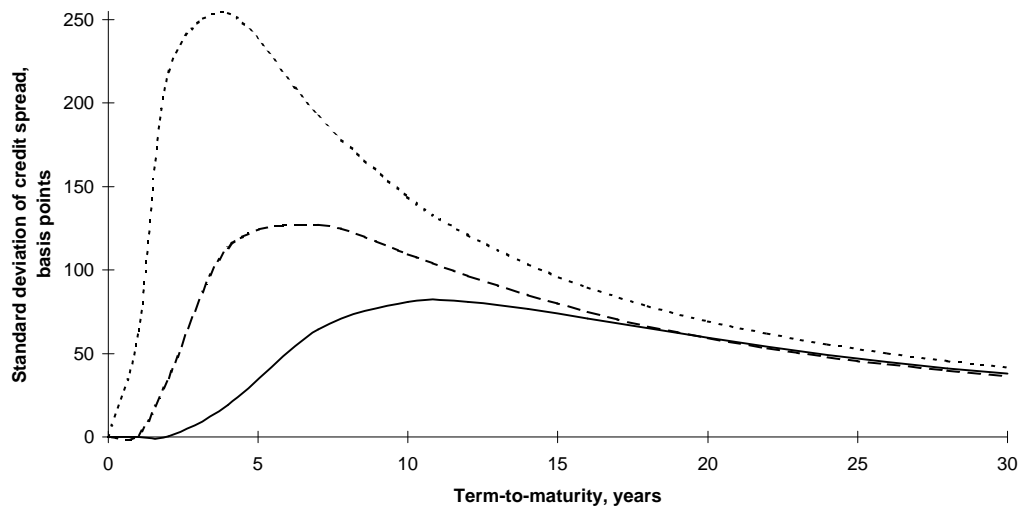


Figure 8. The standard deviation of the credit spreads as a function of term-to-maturity with debt ratios of 12% (solid line), 23% (long dashed line) and 33% (short dashed line). The other parameters are: the target debt ratio $\mu = 33\%$, the speed of debt ratio adjustment $\kappa = 0.15$, the volatility of the debt ratio $\sigma^2 = 0.09$, the debt ratio risk premium $\lambda = -3\%$, the default boundary $K = 81\%$, the recovery rate $1-w = 42\%$, the correlation between the debt ratio and the interest rate $\rho = 0.25$ and the interest rate process parameters $\beta = 1.00$ and $\eta^2 = 0.001$.

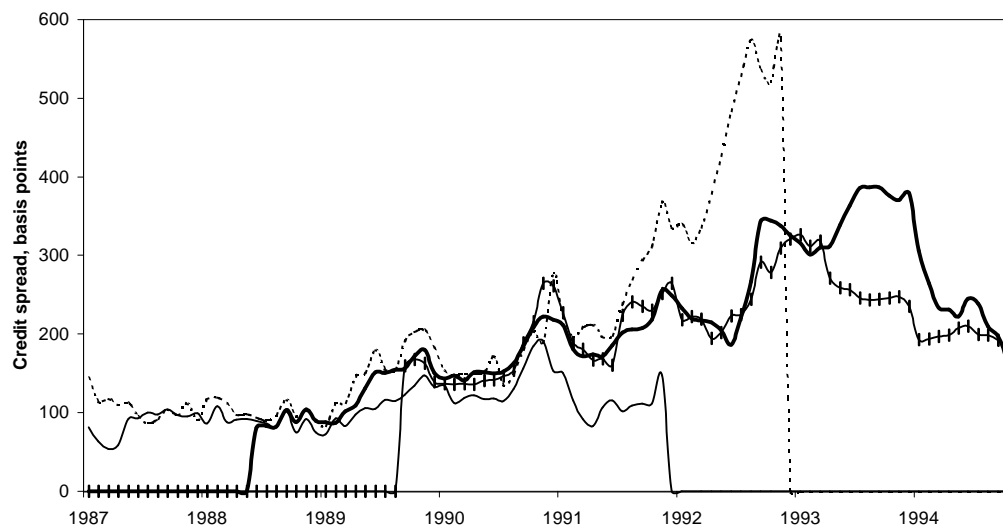


Figure 9. Monthly credit spreads on bonds of Boise Cascade in the period January 1987 to December 1994. The bonds are: 11.875% January 1993 (thin solid line), 8.375% August 1994 (short dashed line), 9.625% July 1998 (thick solid line) and 9.900% March 2000 (line with diamonds).

The credit spreads are calculated as the difference between the yield-to-maturity of the bond and that of a U.S. Treasury bond with a similar maturity. The yields are calculated for the last day of the month from the bid quotes of the bond traders of Salomon Smith Barney, Inc. The bonds are all straight senior and unsecured debt and the 1994 and 1998 issues are callable at 100% of the face value.

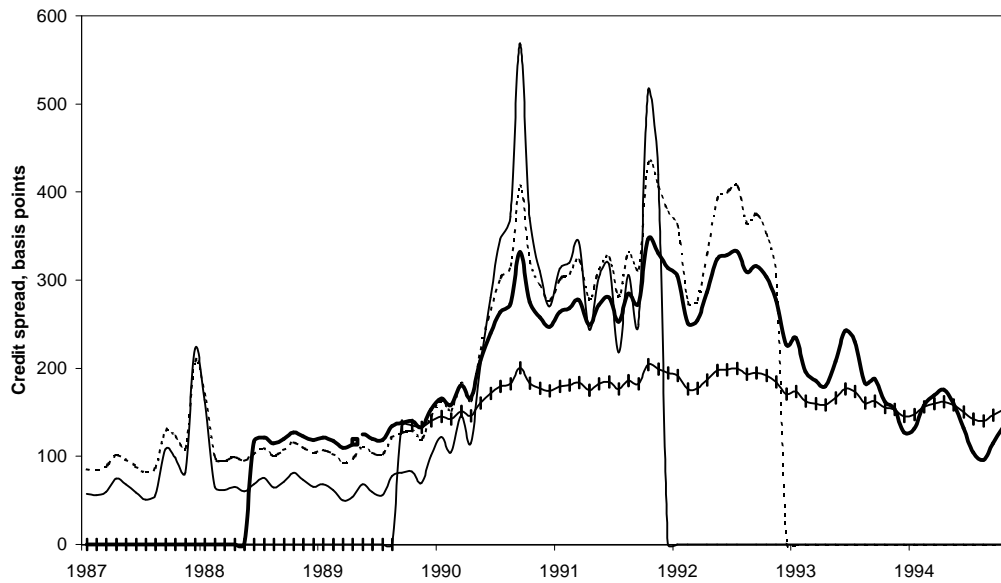


Figure 10. Monthly credit spreads predicted by the corporate bond pricing model for bonds of Boise Cascade in the period January 1987 to December 1994. The bonds are: 11.875% January 1993 (thin solid line), 8.375% August 1994 (short dashed line), 9.625% July 1998 (thick solid line) and 9.900% March 2000 (line with diamonds).

The model parameters are as follows. The debt ratio D is calculated monthly as the ratio of the firm's book value of debt to the sum of the book value of debt and the market value of equity, the target debt ratio $\mu = 47\%$, the speed of debt ratio adjustment $\kappa = 0.079$, the volatility of the debt ratio $\sigma^2 = 0.0328$, the debt ratio risk premium $\lambda = -3\%$, the default boundary $K = 81\%$, the recovery rate $1 - w = 48\%$, the correlation between the debt ratio and the interest rate $\rho = 0$ and the interest rate process parameters $\beta = 1.00$ and $\eta^2 = 0.001$.

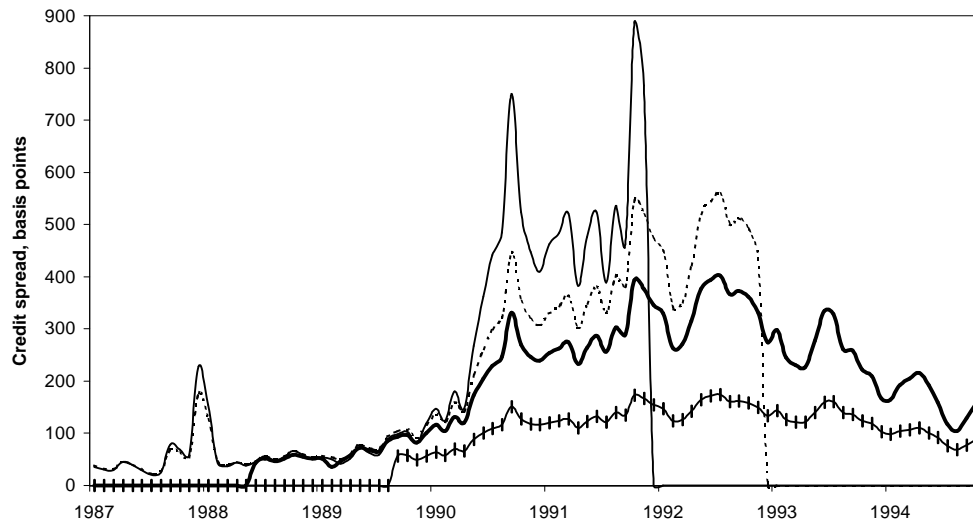


Figure 11. Monthly credit spreads predicted by the Longstaff and Schwartz (1995a) model for bonds of Boise Cascade in the period January 1987 to December 1994. The bonds are: 11.875% January 1993 (thin solid line), 8.375% August 1994 (short dashed line), 9.625% July 1998 (thick solid line) and 9.900% March 2000 (line with diamonds).

The model parameters are as follows. The value of the firm's assets V is calculated monthly as the sum of the book value of debt and the market value of equity, the volatility of the firm's asset value $\sigma^2 = 0.0328$, the default boundary $K' = 81\%$ is the most recent book value of debt divided by 0.81, the recovery rate $1-w = 48\%$, the default-free interest rate r is the monthly average yield on long-term U.S. Treasury bonds, the correlation between the firm's asset value and the interest rate $\rho = 0$ and the interest rate process parameters $\beta = 1.00$ and $\eta^2 = 0.001$.

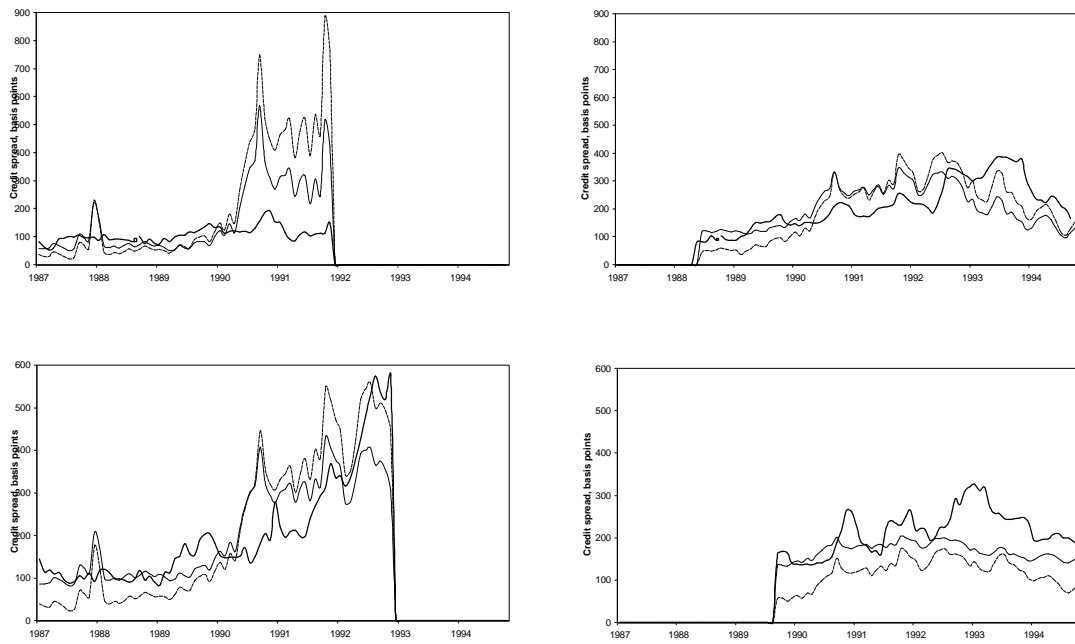


Figure 12. Monthly credit spreads for each Boise Cascade bond (thick solid line) and their values predicted by the corporate bond pricing model (thin solid line) and the Longstaff and Schwartz (1995a) model (short dashed line) in the period January 1987 to December 1994.