Is Default Event Risk Priced in Corporate Bonds?

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Abstract

We identify and estimate the sources of risk that cause corporate bonds to earn an excess return over default-free bonds. In particular, we estimate the risk premium associated with a default event. Default is modelled using a jump process with stochastic intensity. For a large set of firms, we model the default intensity of each firm as a function of common and firm-specific factors. In the model, corporate bond excess returns can be due to risk premia on factors driving the intensities and due to a risk premium on the default jump risk. The model is estimated using data on corporate bond prices for 104 US firms and historical default rate data. We find significant risk premia on the factors that drive intensities. However, these risk premia cannot fully explain the size of corporate bond excess returns. Next, we estimate the size of the default jump risk premium, correcting for possible tax and liquidity effects. The estimates show that this event risk premium is a significant and economically important determinant of excess corporate bond returns.

JEL Codes: E43; G12; G13.

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1 Introduction

Given the extensive literature on risk premia in equity markets, relatively little is known about expected returns and risk premia in the corporate bond market. Recent empirical evidence by Elton et al. (2001) suggests that corporate bonds earn an expected excess return over default-free government bonds, even after correcting for the likelihood of default and tax differences. As shown by Elton et al. (2001), part of this expected excess return is due to the fact that changes in credit spreads (if no default occurs) are systematic, implying that the risk of these changes should be priced. The current empirical literature has, however, neglected the possibility that the risk associated with the default event itself is (also) priced. Typically, a default event causes a jump in bond prices and this jump risk may have a risk premium. Jarrow, Lando, and Yu (JLY, 2001) and Yu (2001) discuss the possible existence of a default jump risk premium, but do not estimate the size of this premium.

In this paper, we distinguish the risk of credit spread changes, if no default occurs, and the risk of the default event itself. We use credit spread data of many different firms and historical default rates to estimate the size of the default jump risk premium, along with the risk prices of credit spread changes. We show that, in order to fully explain the size of expected excess corporate bond returns, an economically and statistically significant default jump risk premium is necessary, on top of the risk premia that are due to the risk of credit spread changes.

By estimating the default jump risk premium, this paper essentially tests the assumptions underlying the conditional diversification hypothesis of JLY (2001). These authors prove that, if default jumps are conditionally independent across firms and if the economy contains an infinite number of bonds, default jump risk cannot be priced. Intuitively, in this case the default jump risk can be fully diversified. Our results indicate that default jumps are not conditionally independent across firms and/or that not enough corporate bonds are traded to fully diversify default jump risk. A particularly appealing explanation for the existence of a default jump risk premium is that investors take into account the possibility of a multiple defaults scenario (a ‘contagious defaults’ scenario).

The model that we use is specified according to the Duffie and Singleton (1999) framework. In these intensity-based models, firms can default at each instant with some probability. In case of a default event, there is a downward jump in the bond price that equals a loss rate times the bond price just before default. The product of the risk-neutral default intensity and the loss rate equals the instantaneous credit spread. Like Duffee (1999) and Elton et al. (2001), we assume a constant loss rate and allow the default intensity to vary stochastically over time. We model each firm's default intensity as a function of a low number of latent common factors and a latent firm-specific factor. This extends the analysis of Duffee (1999), who estimates a separate model for each firm. As in Duffee (1999), all factors follow square-root diffusion processes. We use a latent factor model, since Collin-Dufresne et al. (2001) show that observable financial and economic variables cannot explain the correlation of credit spread changes across firms. In line with empirical evidence provided by Longstaff and Schwartz (1995) and Duffee (1998), the model also allows for correlation between credit spreads and default-free interest rates, which are modelled by a two-factor affine model used by Duffie, Pedersen, and Singleton (2001). Finally, we model the relation between risk-neutral and actual default intensities. The ratio of the risk-neutral default intensity and the actual intensity defines the jump risk premium, which we assume to be constant over time.

In total, the model can generate expected excess corporate bond returns in four ways. First, through the dependence of credit spreads (or, equivalently, default intensities) on default-free term structure factors. Second, because the risk of common or systematic changes in credit spreads across firms is priced. Third, via a risk premium on firm-specific credit spread changes, and, fourth, due to a risk premium on the default jump.[1] Empirically, we find that all these terms contribute to the expected excess corporate bond return, except for the risk of firm-specific credit spread changes.

We use a data set of weekly US corporate bond prices for 592 bonds of 104 firms, from 1991 to 2000. All bonds in the data set are rated investment-grade. The estimation methodology consists of four steps. First, using data on Treasury bond yields, we estimate the two-factor model for the default-free term structure using Quasi Maximum Likelihood based on the Kalman filter. Second, we estimate the common factor processes that influence corporate bond spreads of all firms, again using Quasi Maximum Likelihood based on the Kalman filter. Third, the

[1] Yu (2001) also provides a decomposition of corporate bond returns, but does not estimate the size of the components.

residual bond pricing errors are used to estimate the firm-specific factor for each firm. In the final step, we use data on historical default rates to estimate the default jump risk premium.

The empirical results are as follows. We estimate a model with two common factors and a firm-specific factor for each firm. The common factors are statistically significant and reduce the corporate bond pricing errors. These factors have economically and statistically significant risk prices, while the risk associated with the firm-specific factors of our model is not priced. Thus, our results indicate that the market-wide spread risk, represented by movements in the common factors, is priced in the corporate bond prices, whereas the firm-specific risk is not. We also find a negative relation between credit spreads and the default-free term structure.

Next we show that, if we would not include a default jump risk premium in this model, the model largely overestimates observed default rates, and, therefore, underestimates expected excess corporate bond returns. Subsequently, we estimate the size of the default jump risk premium using historical default rate data, and find an economically and statistically large value for this parameter. For example, the default jump risk premium accounts for about 68% of the total expected excess return on a 10-year BBB rated corporate bond. If we correct for tax and liquidity differences between corporate and government bonds, the estimate for the risk premium remains economically important and, in most cases, statistically significant.

Our results on the default risk premium are somewhat different from the results on the test of 'conditional diversification' in JLY (2001), who use the estimates of the Duffee (1999) model. The main reason for these differences is that JLY (2001) do not use historically observed cumulative default rates to perform their test, but the cumulative default rates implied by a Markov model for rating migrations. The observed cumulative default rates are, however, much lower than these model-implied default rates. Using cumulative default probabilities that are based on the Markov migration model therefore leads to downward biased estimates of the default jump risk premium.

We end the paper with an application of our model to the pricing of a n^{th}-to-default swap. This application highlights the importance of a multiple defaults scenario. Incorporating such a scenario leads to a large change in the price for a credit default swap, relative to a model with independent default events. Finally, we note that another practical application of our model is that it allows financial institutions to extract actual default probabilities from corporate bond prices, which is useful for risk management purposes.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 describes the corporate bond data set. In Section 4, the estimation methodology for the factor model is outlined, and the estimation results for the factor model are presented. In Section 5, we discuss the estimation of the default jump risk premium and present the results, as well as corrections for tax and liquidity effects. In Section 6 we apply our model to price basket credit default swaps. Section 7 concludes.

2 A Model for Defaultable Bond Prices

2.1 Model Setup

The first part of the model describes default-free interest rates. We assume that US Treasury bonds cannot default. This part of the model is identical to the affine model for the default-free term structure of Duffie, Pedersen, and Singleton (DPS, 1999). The model implies the following process for the instantaneous default-free short rate r_t under the 'true' or 'actual' probability measure P

$$\begin{bmatrix} dr_t \\ dv_t \end{bmatrix} = \begin{bmatrix} k_{rr} & k_{rv} \\ 0 & k_{vv} \end{bmatrix} \begin{bmatrix} \theta_r - r_t \\ \theta_v - v_t \end{bmatrix} dt + \sqrt{v_t} \begin{bmatrix} \sigma_{rr} & \sigma_{rv} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} dW_{1,t} \\ dW_{2,t} \end{bmatrix} \tag{1}$$

This model allows for correlation between the factors r_t and v_t. Dai and Singleton (2000) argue that this is important to obtain an accurate fit of US government bond data. $W_{1,t}$ and $W_{2,t}$ are independent Brownian motions under the true probability measure P.

We model the risk premia in the government bond market in the same way as DPS: the Brownian motions $\hat{W}_{1,t}$ and $\hat{W}_{2,t}$ under a risk-neutral probability measure Q are related to the P-Brownian motions through $d\hat{W}_{1t} = dW_{1,t} + \lambda_r \sqrt{v_t} dt$ and $d\hat{W}_{2,t} = dW_{2,t} + \lambda_v \sqrt{v_t} dt$. This way, the model is still affine under a risk-neutral probability measure Q. This model leads to an exponential-affine pricing formula for bonds that are not subject to default risk

$$-\log(P(t,T))/(T-t) \;=\; A_r(T-t) \;+\; D_r(1,T-t)\,r_t \;+\; D_{rv}(1,T-t)\,v_t \tag{2}$$

where $P(t,T)$ is the time t price of a default-free discount bond maturing at T. The functions $A_r(.), D_r(.,.)$, and $D_{rv}(.,.)$ satisfy differential equations that can easily be solved numerically (Duffie and Kan (1996)). The first argument of the functions $D_r(.,.)$ and $D_{rv}(.,.)$ is a scale parameter that allows for scaling the short rate r_t with a multiplicative constant. This notation will be useful later. For default-free bonds this scale parameter simply equals one.

As in Duffie and Singleton (1999), Madan and Unal (1998), and Jarrow and Turnbull (1995), default is modelled as an unpredictable jump of a conditional Poisson process. The stochastic intensity of this jump process at time *t* under the true probability measure is denoted by $h_{j,t}^{P}$, for firm *j*, *j=1,..,N*, and, consequently, the actual default probability in the time interval $(t, t+dt)$ is equal to $h_{j,t}^{P}\,dt$ (for an infinitesimal time change *dt*). For now, we do not specify whether the default jumps of different firms are independent or not (conditional on the default intensity). We return to this issue later.

In case of a default event at time *t*, there is a downward jump in the bond price equal to $L_{j,t}$ times the market price of the bond just before the default event. Duffie and Singleton (1999) call this the Recovery of Market Value (RMV) assumption. In line with Duffee (1999) and Elton et al. (2001), we assume this loss rate to be constant. We use the same value of 56% for this loss rate as Duffee (1999). Below, we will see that, from corporate bond price data only, it is not possible to separately identify this loss rate and the default intensity.

Assuming the absence of arbitrage opportunities guarantees the existence of an equivalent martingale measure *Q*. As noted by JLY (2001), the intensity under this measure, which we denote $h_{j,t}^{Q}$, is related to the *P*-intensity through the risk premium parameter μ on the default jump

$$h_{j,t}^{Q} \;=\; \mu\, h_{j,t}^{P} \tag{3}$$

If the risk associated with default events is priced, the parameter μ will exceed 1. Although this risk premium parameter can be time-varying, we assume it to be constant for simplicity.

In this setup, Duffie and Singleton (1999) show that, conditional upon no default before time

t, the time t price $V_j(t,T)$ of a defaultable zero-coupon bond, issued by firm j and maturing at time T, is given by

$$V_j(t,T) \;=\; E_t^Q[\exp(-\int_t^T (r_s + h_{j,s}^Q L)\,ds)] \tag{4}$$

where E_t^Q denotes the Q-expectation conditional upon the information set at time t. Formula (4) shows that, given an appropriate model for the default-free rate r_t, it suffices to model the instantaneous spread, defined as $s_{j,t} \;=\; h_{j,t}^Q L$, to price defaultable bonds. Given our assumption that the loss rate L is constant, modelling the credit spreads is equivalent to modelling default intensities, and we use these two terms interchangeably in this paper.

Given the existing evidence that changes in credit spreads across firms contain systematic components (see Collin-Dufresne, Goldstein, and Martin (2001) and Elton et al. (2001)), we model the risk-neutral default intensities as a function of common and firm-specific latent factors. We use a latent factor model since Collin-Dufresne, Goldstein, and Martin (2001) show that financial and economic variables cannot explain the correlation structure of credit spreads across firms. In our model, the risk-neutral default intensity of firm j, $j=1,..,N$, is a function of K common factors $F_{i,t}$, $i=1,..,K$, and a firm-specific factor $G_{j,t}$, plus two terms that allow for correlation between spreads and default-free rates

$$s_{j,t} \;\equiv\; h_{j,t}^Q L \;=\; \alpha_j \;+\; \sum_{i=1}^{K} \gamma_{ij} F_{i,t} \;+\; G_{j,t} \;+\; \beta_{r,j} r_t \;+\; \beta_{v,j} v_t \tag{5}$$

where the K common factors $F_{i,t}$, i=1,..,K, follow independent square-root processes under the true probability measure P

$$dF_{i,t} \;=\; \kappa_i^F(\theta_i^F - F_{i,t})\,dt \;+\; \sigma_i^F \sqrt{F_{i,t}}\,dW_{i,t}^F, \qquad i=1,..,K \tag{6}$$

and where the N firm-specific factors $G_{j,t}$, j=1,..,N, also follow independent square-root processes under P

$$dG_{j,t} = \kappa_j^G(\theta_j^G - G_{j,t})\,dt + \sigma_j^G \sqrt{G_{j,t}}\,dW_{j,t}^G, \quad j=1,\dots,N \tag{7}$$

Here, the κ-parameters are mean-reversion parameters, the θ-parameters represent the unconditional factor means, and the σ-parameters can be interpreted as volatility parameters.[2] All Brownian motions are assumed to be independent from each other. The model implies that credit spreads of firm j are influenced by the common factors through the factor loadings γ_{ij}. To allow for correlation between spreads and default-free rates the instantaneous spread is influenced by the default-free factors through the parameters $\beta_{r,j}$ and $\beta_{v,j}$. Finally, the credit spreads of each firm are also determined by a firm-specific (or, idiosyncratic) factor. As in the default-free model, we assume the market price of factor risk to be proportional to the factor level; for example, for the common factors we have $d\hat{W}_{i,t}^F = dW_{i,t}^F + (\lambda_i^F/\sigma_i^F)\sqrt{F_{i,t}}\,dt$, where $\hat{W}_{i,t}^F$ is a Brownian motion under Q, so that the market price of factor risk is equal to $(\lambda_i^F/\sigma_i^F)\sqrt{F_{i,t}}$. For the firm-specific factors, a completely similar assumption for the risk adjustment is made.

Equations (4)-(7) imply that the corporate bond price $V_j(t,T)$ is given by the well-known exponential-affine function of all factors in the model (Duffie and Kan (1996)). Thus, the $(T\text{-}t)$-maturity zero-coupon credit spread $S_j(t,T)$ is an affine function of all factors

$$\begin{gathered} S_j(t,T) \equiv -\log(V_j(t,T))/(T-t) + \log(P(t,T))/(T-t) = \\ A_j(T-t) + \sum_{i=1}^{K} B_{i,j}(T-t)F_{i,t} + C_j(T-t)G_{j,t} + \\ (D_r(1+\beta_{r,j},T-t) - D_r(1,T-t))\,r_t + (D_{rv}(1+\beta_{r,j},T-t) - D_{rv}(1,T-t) + D_v(\beta_{j,v},T-t))\,v_t \end{gathered} \tag{8}$$

where the functions $A_j(.)$, $B_{i,j}(.)$, $C_j(.)$, $D_r(.,.)$, $D_v(.,.)$, and $D_{rv}(.,.)$ depend on the model parameters (see, for example, Pearson and Sun (1994) for explicit expressions for these loading functions in square-root models). The function $D_v(\beta_{v,j}, T-t)$ appears in (8) due to the separate dependence

[2] Not all parameters in the process in equation (6) are identified. In Appendix A we show that the identification problem can be solved by normalizing the means of the factors θ_i^F, $i=1,..,K$.

of the instantaneous spread on the volatility of the short rate v_t via the parameter $\beta_{v,j}$.

In practice, coupon-paying bonds are traded instead of zero-coupon bonds. The prices of these coupon bonds are simply the sum of the prices of the coupon payments and the notional payment. Finally, note that, if the number of common factors K is equal to zero, we obtain the purely firm-specific model that is similar to Duffee (1999).

2.2 Expected Bond Returns and Conditional Diversification

We start with default-free bond returns. Applying Ito's lemma to the bond price expression in (2) it follows that

$$E_t^P\left[\frac{dP(t,T)}{P(t,T)}\right] = r_t dt + \tilde{D}(1, T-t)\,dt \tag{9}$$

with

$$\tilde{D}(\beta, t, T) \equiv -(T-t)\begin{bmatrix} D_r(\beta, T-t) & D_{rv}(\beta, T-t)\end{bmatrix}\begin{bmatrix} \beta\sigma_{rr} & \beta\sigma_{rv} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \lambda_r \\ \lambda_v \end{bmatrix} v_t \tag{10}$$

For corporate bond returns, the expression is slightly more complicated, because one has to incorporate the influence of a default event on the expected return. Using results in Yu (2001), Appendix B derives the following expression for the instantaneous expected return on a corporate discount bond, in excess over a government bond with the same maturity

$$\begin{aligned}&\Big[-\sum_{i=1}^{K}(T-t)B_{i,j}(T-t)\lambda_i^F F_{i,t} - (T-t)C_j(T-t)\lambda_j^G G_{j,t} + \\ &(\tilde{D}(1+\beta_{r,j}, t, T) - \tilde{D}(1, t, T)) - (T-t)D_v(\beta_{v,j}, T-t)\lambda_v v_t + (\mu-1)h_{j,t}^P L\Big]\,dt\end{aligned} \tag{11}$$

Equation (11) illustrates that, in total, the model can generate expected excess corporate bond returns in four ways. First, because the risk of common or systematic changes in credit spreads (or, equivalently, default intensity changes) across firms is priced. Second, via a risk premium on firm-specific credit spread changes. Third, through the dependence of credit spread changes on default-free term structure variables r_t and v_t. The fourth component is the risk premium on the default jump. Expected bond returns are positively related to the default jump risk premium μ. If the parameter μ deviates from 1, default jump risk priced.

At this stage it is appropriate to discuss the ‘asymptotic equivalence’ results of JLY (2001). They prove that, if one assumes that (i) default events are modelled by a conditional Poisson process, (ii) the default intensities across firms depend on a set of state variables, (iii) default processes are independent, conditional on the path of default intensities, (iv) there are a countably infinite number of firms in the economy, then the default intensities under the true probability measure P and the equivalent martingale measure Q are approximately equal, so that there is no risk premium on the default jumps $(\mu \approx 1)$. JLY refer to this as a case of ‘diversifiable default risk’. Intuitively, given the conditional independence assumption, default jumps can be (approximately) diversified away and are, therefore, not priced.

By estimating the default jump risk premium, we can test the assumptions underlying the conditional diversification hypothesis. In particular, JLY mention two reasons why default jump risk could be priced. First, there may be a (small) possibility that a contagious default event takes place, in which some firms simultaneously default. In this case, the asymptotic equivalence result does not hold. Second, it may be that in practice not enough bonds are traded to justify the assumption of an infinite number of bonds. As noted by JLY, in a finite economy there can be perturbations to default risk premia, even if the default jumps are conditionally independent.

It is important to note that both explanations for the existence of a default jump risk premium can be in line with our model for default events. This is because we do not have to specify whether the default events are conditionally independent or not in order to price corporate bonds. In our model, default jumps can be independent, conditional upon the intensity process. As an alternative, the model can also allow for contagious default, if, for example, the common intensity factors $F_{i,t}$ each drive a Poisson jump that triggers default of several firms. In Section 6, we precisely describe how such a contagious default scenario can be incorporated in our model, and we assess the implications for credit derivative pricing.

3 Description of Data Set

3.1 Data

Data on US-dollar corporate bond prices are taken from the Bloomberg Corporate Bonds Database (BCBD), that contains mid-quotes for corporate bond prices. Besides these mid-quotes, the dataset contains for each bond the maturity date, the coupon size and coupon frequency, the (S&P) rating, the firm's industry sector, and the amount issued. Although the data are available on a daily basis, we use weekly observations (i.e., the observation on each Friday) to reduce the influence of possible measurement errors and stale prices. We collect data from February 22, 1991 until February 18, 2000.

To facilitate the comparison, we restrict ourselves to the set of 161 firms that is analyzed by Duffee (1999). Unfortunately, due to missing observations the BCBD does not contain (sufficient) data for all 161 firms. We only include a firm in our analysis if there are data on at least two corporate bonds for at least 100 weeks, which leaves us with 104 of the 161 firms. We only use bonds with constant, semiannual coupon payments, that do not contain any put or call options, or sinking fund provisions. As in Duffee (1999), observations on bond prices with remaining maturity less than one year are dropped. Also, for a given firm, we only include bonds that have maturities that are more than 6 months apart. If the maturity difference of two bonds is smaller than 6 months, we keep the most recently issued bond[3]. More than 80% of the remaining bonds is senior unsecured. We only include other bonds, such as subordinated bonds, if this bond has the same rating as the senior unsecured bonds. At the end of the sample period, all 104 firms are rated investment-grade: 2 AAA-rated firms, 13 AA-rated firms, 51 A-rated firms, and 31 BBB-rated firms. In the remainder of the paper, we will several times refer to the rating of a firm. Since our model does not explicitly account for rating migrations, we always use the rating at the end of the data period.

Table 1, that is similar to Table 1 of Duffee (1999), contains information on the bond data. For the median firm, on average 3 different bonds are used to estimate the model, and at 378 of

[3]This elimination of near-maturity bonds is slightly different from Duffee (1999). His elimination scheme implies that bonds with maturities of 3.9 and 4.1 years would both be included.

the 470 weeks in the data at least two bond prices are observed for this median firm. Also, there is considerable variation in the bond maturities.

Besides corporate bond price data, we also use Bloomberg data on the 6-month US Treasury bill, and the most recently issued US Treasury bonds, for the maturities 2, 3, 5, 7, 10, and 30 years. These bonds are typically more liquid than the off-the-run Treasury bonds, see Duffie (1996). In Section 5, we discuss the issue of liquidity differences between corporate and government bonds.

3.2 Analysis of Coupon Spreads

To further analyze the corporate bond price data, we provide summary statistics on the coupon spreads of the corporate bonds. We define the coupon spread as the difference between the yield-to-maturity on a given corporate coupon-paying bond, and the yield-to-maturity of a default-free bond (i.e., government bond) with the same coupon and maturity. The latter yield-to-maturity is not directly available in our data for all maturities and coupon sizes. We use the estimation results (discussed in Section 4) for the two-factor affine model in equation (1) to calculate the necessary yield-to-maturities of default-free coupon bonds and the coupon spreads.

There are some bond price observations in the data with coupon spreads that are very likely incorrect. Therefore, we eliminate observations for which the coupon spread is above 400 basis points or below -50 basis points, as well as observations that are related to a coupon spread movement of more than 100 basis points in one week. Also, we delete the ‘middle’ observation for observations for which the coupon spread moves more than 50 basis points in one week, and again more than 50 basis points in the opposite direction in the next week. This way, we eliminate 616 of the 140,389 bond price observations.

In Figure 1, we plot the average term structure of the coupon spreads, per rating category[4]. The figure shows that, on average, high-rated bonds have a low and slowly increasing spread term structure, whereas the lower-rated bonds have higher and more steeply increasing spread

[4]Of course, the coupon spreads of two bonds with different coupon sizes, that are otherwise identical, can be different. By averaging spreads of bonds within a rating category and maturity class, this effect is averaged out to a large extent.

term structures. In Figure 2, we plot the time-series behaviour of the coupon spreads, averaged within each rating category. The graph shows that from 1991 to 1998 especially spreads of lower-rated firms have declined, thereby decreasing the difference between spreads of firms with different ratings. Due to the Russia/LTCM crisis in the fall of 1998, spreads increased dramatically, and have remained high since. Figure 2 also shows that the difference between the spread term structures of the different rating categories has increased again since the crisis, and that there is considerable correlation between the spreads of the different rating categories, which motivates our common factor model.

4 Estimation of Factor Model for Risk-Neutral Intensities

Our model setup is such that we can estimate it in two parts. First, in this section, we describe the estimation methodology and results for the model for the risk-neutral intensity $h_{j,t}^{Q}$. Given these estimation results, we describe the estimation of the default jump risk premium μ in Section 5.

4.1 Estimation Methodology

Similar to Chen and Scott (1995), De Jong (2000), Duan and Simonato (1999), and Duffee (1999), we use Quasi Maximum Likelihood (QML) based on the Kalman Filter to estimate our model for the risk-neutral intensity in equations (4)-(7). As in Duffee (1999) we directly use the yield-to-maturities of coupon-paying Treasury and corporate bonds to estimate the parameters. This avoids using an ad-hoc smoothing method to calculate zero-coupon interest rates.

In principle, a joint estimation of all parameters in the model is most efficient. However, since the number of parameters is large, we choose to perform estimation in three steps. First, we estimate the two-factor affine model in equation (1) for the default-free term structure using the Treasury bill rate and the Treasury bond yields. We use Quasi Maximum Likelihood (QML) based on the Kalman Filter to estimate the parameters. It is assumed that each interest rate is observed with an i.i.d. measurement error, that is uncorrelated with measurement errors of other interest rates. Besides parameter estimates, the Kalman Filter estimation also gives estimates for the factor values (r_t, v_t) at all dates[5]. We refer to the Appendix C for further details on Kalman Filter QML estimation.

This estimation method is *Quasi* Maximum Likelihood because the conditional distribution of (r_t, v_t) conditional upon (r_{t-1}, v_{t-1}) is not normal, but the conditional means and variances are known explicitly (after transforming these state variables, see Appendix C). Moreover, the conditional variance in this distribution depends on the unknown values (r_{t-1}, v_{t-1}), which makes the QML estimator based on the Kalman Filter strictly speaking inconsistent. Simulation experiments by Duan and Simonato (1999) and De Jong (2000) show that the induced biases are very small. Consistent parameter estimates can be obtained by using the Efficient Method of Moments (EMM, Gallant and Tauchen (1996)), combined with the semi-nonparametric (SNP) method of Gallant and Tauchen (1992). Dai and Singleton (2000) use this method to estimate affine term structure models. Duffee and Stanton (2000) compare EMM/SNP estimation of affine term structure models with QML estimation using the Kalman filter. They document considerable small-sample biases for the EMM/SNP method, and conclude that 'for reasonable sample sizes, the results strongly support the choice of the Kalman filter'.

The second step of our estimation procedure involves the estimation of the parameters that determine the processes of the common factors $F_{i,t}$ and the relation with default-free interest rates, which is determined by $\beta_{r,j}$ and $\beta_{v,j}$. Given the parameter estimates for the default-free term structure model, we use again QML based on the Kalman Filter, which also gives us estimates for the factor values $F_{i,t}$. In Appendix C we explain how we deal with missing observations. For this estimation, we assume that bond yields of all firms are measured with error, and that the yield measurement errors are all independent from each other and i.i.d. distributed with the same variance. The constant terms α_j in equation (5) are also estimated in this step.

To reduce the number of parameters to be estimated in the second step, we restrict the firm-specific parameters $\alpha_j, \beta_{r,j}, \beta_{v,j}, \gamma_{1,j}, .., \gamma_{K,j}$ to be constant across firms that have the same credit

[5]In this paper, we always use so-called unsmoothed estimates for the factor values at time t, which means that only information up to time t is used to estimate the factor values.

rating.[6] This restriction significantly reduces the number of parameters.[7] Our model does not contain rating migrations: if the rating of a firm changes, the factor loading of firm does not change. For each firm we use the rating at the end of the sample period to determine which factor loading applies to this firm for the entire sample period. We assume that this rating is good proxy for the creditworthiness of the firm throughout the sample. As shown below, we indeed find that the factor loadings based on the end-of-sample ratings are systematically different, which is evidence that our classification indeed picks up differences in creditworthiness.

The third step is the estimation of the parameters of the firm-specific factors $G_{j,t}$, j=1,..,104. Given the parameter estimates obtained in the first two steps, we estimate the parameters in the firm-specific factor process for each firm j separately, using again QML based on the Kalman Filter. Again, we assume for each firm that yields are measured with error, where the measurement errors have constant variance and are independent across yields and over time.

Finally, we repeat the second step and third step of this estimation procedure, where in the second step we now replace the i.i.d. measurement error assumption by the structure implied by the estimated firm-specific factor processes and the measurement error structure that was assumed in this third step. This way, we explicitly incorporate the presence of firm-specific factors when estimating the common factor processes. We have analyzed whether applying another iteration leads to important changes in the parameter estimates, but this is not the case.

If the common factor values were known and exogenous, we could first regress the spreads on the common factors, and in a second step use the Kalman filter on the regression residuals to estimate the firm-specific factors. In our case, the common factors are estimated from spread data of all 104 firms, so that the common factor is approximately exogenous to a given firm. Our multi-step estimation strategy can thus be motivated by the fact that the influence of a single firm on the common factor is negligible.

For comparison, we also estimate the model proposed by Duffee (1999), that only contains firm-specific factors. In this case, step two is skipped in the estimation procedure and the parameters α_j, $\beta_{r,j}$, and $\beta_{v,j}$ are estimated along with the parameters of the firm-specific factor.

[6] Since there are only two AAA rated firms, we treat these two firms as AA rated firms.

[7] We have also estimated these firm-specific parameters without this restriction. The results for the common factors and the price of the default jump risk are similar to the results presented in this paper.

In each estimation step we calculate standard errors and t-ratios for the parameter estimates (correcting for heteroskedasticity using White (1982)), assuming that the parameters that are estimated in previous steps are estimated without error. In principle, it is possible to calculate the standard errors taking into account the previous steps, for instance, by means of bootstrapping, but this is excessively time-consuming.

For all square-root processes in the model, we estimate the parameters given the Feller condition on the parameters. For example, for the common factors, we impose the restriction $2 k_i^F \theta_i^F > (\sigma_i^F)^2, \ i = 1,...,K.$

4.2 Estimation Results

In Tables 2 and 3, we give summary statistics on the estimation results for the default-free part of our model.[8] Table 2 shows, amongst others, that the estimates for the interest rate risk premium parameter is negative, which implies that government bonds earn a positive excess return over the risk-free short rate. Table 3 shows a reasonable fit on the Treasury yields. Since all corporate bonds that are analyzed have maturities larger than 1 year, the pricing errors for the 6-month T-bill are not a great concern.

Before estimating the model with common factors, we replicate the analysis of Duffee (1999) by estimating a model with firm-specific factors only. This model is obtained by setting *K*, the number of common factors, equal to zero in equation (5). The estimation results are given in Table 4. We report quantiles of the distribution of parameter estimates across firms. These quantiles give an indication of the accuracy that would result if parameters were assumed to be constant across firms. The differences with Duffee (1999) are due to a smaller set of firms, and a different data period and data frequency. Qualitatively, however, the estimates are quite similar. In particular, as in Duffee (1999), we find a positive estimate for the mean-reversion parameter (under the real probability measure) κ_j^H, whereas the estimate for the mean-reversion parameter under the risk-neutral measure $\kappa_j^H + \lambda_j^H$ is negative for most firms. This implies that,

[8]As in DPS, to simplify estimation, we first set the mean of the short rate θ_r equal to the mean of the 6-month T-bill rate (which equals 5.20%), and estimate the other parameters. Next, we calculate the average of the fitted instantaneous short rate, which equals 4.79%. In a second step, we set the mean of the short rate θ_r equal to this average, and re-estimate the other parameters. The average fitted instantaneous short rate hardly changes.

as spreads increase, the spread term structure is more steeply increasing, which is a feature that is present in the data. We also find that the market prices of spread risk λ_j^H are negative for almost all firms. Finally, we find that the average of the cross-firm correlations of weekly changes in the firm-specific factors is 0.318, which shows the positive relation between spread movements across firms.

Next, we estimate models with common factors. Using a 'Likelihood Ratio' test[9] we end up with a model with two common factors. The estimates, reported in Table 5, show that the market prices of risk of both factors are negative and jointly statistically significant, indicating that corporate bond investors demand an excess return over the default-free bond returns, to be compensated for the risk associated with common spread movements.

For the first factor, the estimates for the factor loadings $\gamma_{1,j}$ are positive for all three rating categories (AAA/AA, A, and BBB). Thus, this factor causes movements in credit spreads in the same direction for all firms. This first factor has very slow mean-reversion under the real probability measure, and is trending under the risk-neutral measure. This implies that the loading function $B_{1,j}(T\text{-}t)$ in equation (8) is increasing with maturity $(T\text{-}t)$. Other results, which are not presented here, show that this first common factor has the highest values during and after the Russia/LTCM crisis, to account for the higher and steeper spread term structures during this period.

For the second factor, the estimates for the factor loadings $\gamma_{2,j}$ are again positive for all three rating categories. This factor also represents movements in credit spreads of almost all firms in the same direction. The factor has somewhat stronger mean-reversion under the true probability measure than the first factor, and is still mean-reverting under the risk-neutral measure. Therefore, the loading function $B_{2,j}(T\text{-}t)$ is decreasing in maturity, so that the second factor mostly influences short-maturity spreads.

For both factors, lower-rated firms are more sensitive to the common factors than high-rated firms. The explanation for this result is twofold. First, spread term structures are more steeply increasing for lower ratings, and, second, spreads are more volatile for lower-rated firms. Indeed, a higher value for $\gamma_{i,j}$ both implies steeper spread term structures (especially for the first factor)

[9]For this Likelihood Ratio test, we neglect the non-normality of the factor changes and use a normal approximation for these changes.

and more volatile spreads. The first effect is shown in Figure 3, where we plot the average term structures of zero-spreads, as implied by the two-factor model.

In line with results by Longstaff and Schwartz (1995), and Duffee (1998, 1999), there is a negative correlation between spreads and default-free rates for firms in all three rating categories. However, the explained variation is small, and the estimates are mostly insignificant. For example, an increase in the default-free short rate of 100 basis points implies on average a decrease in the instantaneous spread of 4.5 basis points for a BBB rated firm, ceteris paribus.

Other results, which are not presented here, show that changes in the common intensity factors are negatively related to equity returns, and positively to changes in equity volatility. This is in line with results by Longstaff and Schwartz (1995), Kwan (1996), Duffee (1999), and Collin-Dufresne, Goldstein, and Martin (2001).

In Table 6 we present the estimation results for the parameters in the firm-specific factor processes. Most strikingly, the market price of the risk associated with movements in the firm-specific factors is close to zero for the median firm. This is in contrast with the results for the model without common factors in Table 4, where we found large market prices of risk for the firm-specific factors. Thus, after correcting for market-wide spread risk by including two common factors, the remaining firm-specific movements in spreads are hardly priced. This is in line with most equity pricing models, where idiosyncratic or firm-specific movements in stock prices are not priced, and only the covariances of the stock return with the common factors determine expected returns.

To verify that the firm-specific factors are really firm-specific, we calculate the cross-firm correlations of weekly changes in these firm-specific factors. The average of these cross-firm correlations is 0.087, which is much lower than 0.318, the average cross-firm correlation that was found for the model without common factors.

4.3 Corporate Bond Pricing Errors

Next, we analyze how well the common factor model fits the observed coupon bond prices, by comparing the observed yield-to-maturity of the coupon bonds with the model-implied yield-to-maturity. We distinguish three models: (i) a model with only firm-specific factors, (ii) a model with two common factors only and no firm-specific factors, and (iii) the model with both two common factors and firm-specific factors. Model (ii) is included to show the relative importance of the common and firm-specific factors.

In Table 7 we present results on the fit. All three models give a reasonable fit on the bond yields for most firms. The two-factor model with firm-specific factors gives a mean absolute yield error of 9.11 basis points for the median firm, which is in the same order of magnitude as the fit on the Treasury bonds. This model has the best fit, which makes sense, since this model nests the two other models. To compare the pricing results with Duffee (1999), we also report the root mean squared yield errors (RMSE). These numbers are slightly larger than in Duffee (1999): he reports a median RMSE of 9.83 basis points, while we find for our dataset a median RMSE equal to 13.49 basis points for the purely firm-specific model, and 12.89 basis points for the common and firm-specific factor model. One explanation for this difference could be the fact that our data period includes the Russia/LTCM crisis, as it turns out that the yield errors are largest in this period for all models. Table 7 also shows that the model with only two common factors (and no firm-specific factors) already gives a reasonable fit of the coupon bond yields, although this model has much less parameters than the purely firm-specific model.

5 Estimation of the Default Jump Risk Premium

5.1 Estimation Methodology

Given that we have estimated the process for the risk-neutral default intensity, we describe in this section how we estimate the risk premium of the default jump μ. The estimation procedure is based on the following result for the actual probability $p_{j,t}(n, \mu)$ that a firm defaults within the next n years, conditional upon that no default has occurred yet

$$p_{j,t}(n, \mu) = 1 - E_t^P[\exp(-\int_t^{t+n} h_{j,s}^P ds)] = 1 - E_t^P[\exp(-\int_t^{t+n} \frac{h_{j,s}^Q}{\mu} ds)] \qquad (12)$$

Given the affine process for $h_{j,t}^{Q}$, this probability is an explicit function of the risk premium μ. A higher value for μ leads to lower model-implied default probabilities under the actual probability measure. The expectation in (12) can explicitly be calculated because the process for $h_{j,t}^{Q}$ is affine. Notice that, because we assumed constant parameters for firms that have the same rating at the end of the sample, the model-implied default probabilities are the same for these firms. We therefore denote the default probability of a, say, A-rated firm by $p_{A,t}(n,\mu)$.

The probabilities in (12) depend on the current factor values. To obtain the average over time of the model-implied cumulative default probabilities, we calculate (12) using the factor values at each week in the sample and take the time series average.[10] We denote the resulting probabilities $p_{Rating}(n,\mu)$. These cumulative default probabilities can be easily transformed into yearly conditional default probabilities. The latter probability is defined as the probability of default event in the next year, conditional upon having survived for n years, and equals

$$q_{Rating}(n,\mu) \equiv 1 - \frac{1 - p_{Rating}(n+1,\mu)}{1 - p_{Rating}(n,\mu)} \tag{13}$$

This model-implied conditional default probability is a decreasing function of μ. For example, if the risk-neutral default intensity is constant at h_{Rating}^{Q}, it is easy to see that $q_{Rating}(n,\mu) = 1 - \exp(-h_{Rating}^{Q}/\mu)$.

By confronting (13) with actual default rates, μ can be estimated. Both Moody's and Standard & Poor's provide average cumulative default rates per rating category. These default rates are averages of cumulative default rates of cohorts of firms that are formed each year. Given that our data on credit spreads start in 1991 and end in 2000, one would ideally use default rates on the cohort that starts in 1991 up to the cohort starting in 2000. However, since defaults do not occur frequently, one needs a relatively long period to reliably estimate default probabilities. For example, in the 1991-2000 period, default rates are low relative to the seventies and eighties. Using the 1991-2000 period would lead to very high estimates for the risk premium μ. Moreover, if one would use default rate data for the 1991-2000 period, cumulative default

[10]As opposed to using this time series average, we could have used the unconditional expectation of (12) for estimation of μ. Since the average fitted factor values are mostly lower than their unconditional mean, this would most likely lead to higher estimates for μ.

rates for more than a 10-year period cannot be used, while the long-maturity bonds provide information on these long-term default rates. Therefore, we use a longer data period to obtain historical default rates. Of course, this has the disadvantage that the credit spread data and the default rate data are not entirely based on the same sample period. From an econometric viewpoint the use of two partly overlapping data periods is not a problem, since our estimation is based on unconditional moment conditions (see below) and the assumption of stationarity.

For the estimation of μ we both use S&P and Moody's data. The S&P default rate data are based on cohorts starting in 1981 up to 2000.[11] For comparison, we also perform an estimation based on Moody's data, which are based on the 1970-2000 period.[12] We use cumulative default rates from 1 year up to 15 years. We do not use longer horizons for two reasons. First, Standard & Poor's does not provide default rate data for longer horizons, and, second, Table 1 shows that for the median firm the maximum maturity of the fitted bonds is 15.0 years. For both data sets we convert the cumulative default rates into yearly conditional default rates $q_{Rating}^{Data}(n)$. Next, we estimate μ by minimizing the sum of squared differences between the model-implied and observed conditional default probabilities

$$\min_{\mu}\left[\sum_{Rating=AA,A,B}\ \sum_{n=0}^{14}\ (q_{Rating}(n,\mu)\ -\ q_{Rating}^{Data}(n))^2\right] \tag{14}$$

over μ. In Appendix D we describe the procedure for calculation of the standard error of the estimate for μ.

5.2 Estimation Results

In Figures 4a-c we illustrate what the influence of the risk premium μ is on default probabilities. These figures plot yearly conditional default probabilities for different rating categories. In all graphs, the upper line ('Q-Prob') depicts the risk-neutral default probabilities under the risk-neutral probability measure. These are calculated in the same way as in equations (12) and

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[11] See Standard & Poor's special report (2001).

[12] See Moody's special comment (2001). Here we assume that the Moody's and S&P ratings are the same.

(13), taking μ equal to one and using the risk-neutral measure Q instead of the actual probability measure P for calculating the expectation. The line 'P-prob, mu=1' shows what the model implies for actual default probabilities, assuming that there is no risk-premium on the default jump (μ=1). The difference between these actual default probabilities and the risk-neutral default probabilities is completely caused by the risk premia on the factors that drive the default intensities. This is because the presence of risk premia on these factors implies that the expectation of the path of the default intensity under *Q* differs from the expectation under *P*.

Figures 4a-c also contain the empirical yearly conditional default rates based on S&P data from 1981-2000. Clearly, these default rates are well below the default probabilities implied by a model with no risk premium on the default jump. In other words, allowing for risk premia on the intensity factors only is not enough to match the default probabilities that are observed in the data: it leads to overestimation of default probabilities.

Next, we estimate the risk premium on the default jump μ using equation (14). Table 8 shows that this gives an estimate of 5.83 in case of the S&P data and 5.55 based on the Moody's data. Both estimates are statistically significantly different from 1. In Figures 4a-c it is shown that including a risk premium on the default jump gives a much better description of observed default rates (as shown by the lines 'P-prob: mu*').

The size of the estimate for μ is also economically significant. In effect, investors multiply the actual default probability with a factor of almost 6 when pricing corporate bonds (using the risk-neutral probability measure). The importance of the risk premium μ is also highlighted when looking at the decomposition of excess corporate bond returns proposed in equation (11), as shown in Figure 5a. This figure shows that, for BBB bonds, the default jump risk premium causes an excess return of 0.62% per year. For a 10-year bond, this is about 68% of the total excess corporate bond return. Of the other components of the corporate bond excess return, the risk premia on the common factor risk and the relation with default-free rates are most influential. Since credit spreads are negatively related to default-free rates, they provide a partial hedge against interest rate risk, which lowers expected corporate bond returns. As noted in Section 4, the risk premium associated with firm-specific credit spread movements is small.

Similar results are obtained for the other rating categories.[13]

JLY (2001) use the estimates of Duffee (1999) model to compare the model-implied default probabilities, with $\mu = 1$, with default probabilities that are implied by a Markov migration model with a one-year Moody's migration probability matrix. It turns out that the historically estimated cumulative default rates, as used in this paper, are much lower than the cumulative default rates implied by this Markov model for rating migrations. This is evidence against the assumption that rating migrations are independent over time, as is assumed in the Markov migration model. It also implies that using the Markov migration model leads to a downward bias in the estimate for μ. Furthermore, JLY (2001) find that the shape of the conditional default probabilities, as implied by the intensity model, is much flatter than the shape of the conditional default probabilities implied by the Markov migration model. Figures 4a-c show that, if one uses the cumulative default rates that are directly observed in the data, these shape differences disappear to a large extent.

5.3 Tax and Liquidity Effects

As documented by Elton et al. (2001), there are tax differences between corporate and government bonds: coupons on corporate bonds are subject to state taxes, while government bond coupons are not. This may partially explain the size of credit spreads and corporate bond returns. In Appendix E we show how this tax effect can be incorporated in the intensity-based pricing model. Elton et al. (2001) estimate the effective tax rate to be 4.875%. Using this tax rate, we re-estimate the intensity-based pricing model and the default jump risk premium for the tax-corrected bond prices. In Table 8 we report the resulting estimates for the risk premium μ. As expected, including the tax effect leads to a lower estimate for μ, since part of the observed size of credit spreads is now due to a tax effect instead of a risk-neutral default intensity. A lower risk-neutral default intensity implies that a lower value for μ is necessary to match the observed default probabilities. This is illustrated in Figures 4a-c, where we depict the tax-corrected default

[13] We have also estimated the default jump risk premium using Duffee's model with no common factors to model intensities. In this case, the estimate for the default jump risk premium is of similar size as reported in Table 8. The advantage of our common factor model over Duffee's model is that it provides a decomposition of the total risk premium on the risk of intensity changes into common and firm-specific risk.

probabilities if there is no default jump risk premium ('P-prob: mu=1, tax correction'). Still, even if we correct for taxes, ignoring a risk premium on the default jump leads to overestimation of default probabilities. Thus, in case of a tax correction the estimates for μ are still well above 1, as shown in Table 8.

Next, we allow for the presence of a liquidity component in credit spreads. Chakravarty and Sarkar (1999) show that bid-ask spreads of US corporate bonds are typically larger than of government bonds. Part of the spread between corporate and government bond prices may therefore be a liquidity premium. Following Duffie and Singleton (1999), to account for liquidity differences a liquidity spread v can be included to obtain a liquidity-adjusted instantaneous credit spread $s_{j,t} = v + h_{j,t}^{Q}L$. For simplicity we assume this spread v to be constant over time and across firms and re-estimate the model. Because the liquidity spread is constant, we do not have re-estimate the entire corporate bond pricing model, since the model for $h_{j,t}^{Q}L$ contains a constant term that can be adjusted for the presence of v. We then estimate v along with the risk premium μ, by minimizing (14) over both v and μ.

Table 8 gives the resulting parameter estimates. If no tax correction is applied, the estimate for the liquidity spread equals 32 basis points (S&P data) or 41 basis points (Moody's data). If we correct for a tax effect, these estimates go down to 17 or 18 basis points, respectively. In all cases, the estimate for the risk premium μ is larger than 1. This estimate remains statistically significantly different from 1 in case of the S&P data, but not for the Moody's data.

In Figure 6b, we again provide a decomposition of excess corporate bond returns for BBB rated firms, now including the tax correction and the estimated liquidity spread. The three most important components of the corporate bond excess return are the tax effect, the risk premia on common intensity factors, and the risk premium on the default jump. Similar results are obtained for firms with other ratings.

JLY (2001) correct for the presence of taxes and liquidity differences by calculating credit spreads against the AAA yield curve. By assuming a liquidity spread that is constant across firms, we obtain a similar effect, since in this case the risk premium μ is essentially estimated from the differences across firms with different ratings.

6 Pricing a Basket Credit Derivative

An interesting application of our multi-firm model is to look at the pricing of basket credit derivatives. For these basket credit derivatives, whose payoffs depend on default events of several firms, a joint model for default events of several firms is necessary. We illustrate this by pricing a particular basket credit derivative: the n^{th}-to-default swap.

The most common examples of the n^{th}-to-default swap are the first-to-default swap (n=1) and the second-to-default swap (n=2). In case of a first-to-default swap, the buyer of the swap periodically pays a fixed amount to the seller, up to a maturity date. In return, the swap-seller pays an amount to the buyer the first time that a bond in a given portfolio of bonds of different issuers defaults, if this default occurs before the maturity date. The size of this payment is typically the loss in market value on the defaulting bond. In case of further defaults no other payments are made. In case of the second-to-default swap, only the loss on the second bond that goes into default is compensated; the buyer is not compensated for the first defaulting bond. The n^{th}-to-default swap is defined in a similar way. These instruments can be used by institutions to hedge the default risk of a portfolio of corporate bonds.

Clearly, the price of the n^{th}-to-default swap depends on the joint distribution of default events of the different firms in the portfolio. For example, the price of the first-to-default swap is sensitive to the (risk-neutral) probability that the minimum of the default times of the different firms is smaller than the maturity date. In this case, a positive correlation between the default times of different firms gives a lower price for the first-to-default swap than zero or negative correlation between these default times.

In Section 2 we noted that in the intensity-model of equations (4)-(7) it is not necessary to specify whether the default jumps are independent or not. In the previous section, we have provided evidence that there is a risk premium on the jump risk associated with a default event. As indicated in Section 2, one explanation for this risk premium is the existence of a contagious default scenario. Therefore, we consider in this section two models that have the same processes for the default intensities, but have a different specification of the dependence of default jumps.

In the first model default jumps are assumed to be conditionally independent. In this case, all unconditional correlation between the default times of different firms is generated by correlation between the intensity processes of the different firms. In the second model, we assume that the common intensity factors each drive a Poisson jump that triggers default of several firms with some probability, depending on the firm's factor loading. More precisely, we define a modified intensity factor $\tilde{F}_{i,t} \equiv (\gamma_{AA} + \gamma_A + \gamma_{BBB}) F_{i,t}$, where $\gamma_{Rating(j)}$ refers to the factor loading of firm *j*. The sensitivity of firm *j* to this modified factor equals $\gamma_{Rating(j)} / (\gamma_{AA} + \gamma_A + \gamma_{BBB})$, which is by construction between 0 and 1. We assume that the intensity factor $\tilde{F}_{i,t}$ drives a Poisson jump process. If there is a jump in this common process, firm *j* will default with probability $\gamma_{Rating(j)} / (\gamma_{AA} + \gamma_A + \gamma_{BBB})$. Next to this contagious default event, firms can also default due to a jump in the process driven by the firm-specific default intensity $G_{j,t}$.[14] Naturally, these default jumps are assumed to be independent across firms. The total default intensity for firm *j* is then given by equation (5). For simplicity, we neglect the correlation between the default intensity and default-free interest rates in both models (i.e., we set $\beta_{r,j} = \beta_{v,j} = 0$).

We focus on pricing a n^{th}-to-default swap with 3 years maturity. The bond portfolio is an equally weighted portfolio of 15-year maturity coupon bonds. The portfolio includes bonds of 30 different firms, with 10 firms per rating category (AA, A, BBB). The coupon of each bond is chosen such that the bond trades at par at the initial date. At the time of the n^{th} default event, the payoff is equal to the loss in market value on the defaulting bond, in line with the Recovery of Market Value assumption. As before, we use a constant loss rate of 56%.

We use simulation to price the instrument. The factor processes under the risk-neutral measure Q in equations (5)-(7) are discretized using the Euler discretization, from which we can simulate movements in instantaneous interest rates and intensities over time. At each time-step, we simulate a Poisson jump for each firm (and the common Poisson jumps), using a simulation methodology outlined by Duffie and Singleton (1998). We calculate the discounted payoff of the n^{th}-to-default swap in each simulation step. The resulting price could thereafter be converted to a series of periodic payments.

In Figure 6, the resulting prices are plotted, for *n*=1,..,20. The independent-default model gives the highest prices for first-, second-, and third-to-default swaps, whereas the contagious default model gives the highest prices for the other n^{th}-to-default swaps. By construction, the contagious default model generates higher correlation between default times of different firms

[14] The constant α_j is assumed to be part of the firm-specific intensity.

than the independent default model. Therefore, for the latter model the probability that a large number of firms defaults before maturity is smaller, leading to lower prices for the n^{th}-to-default swap if *n* is large. In contrast, if *n* is small, higher correlation between default times lowers the price of the default swap, which explains why the contagious default model gives the lowest prices for small *n*.

Summarizing, Figure 6 shows that prices of n^{th}-to-default swaps are very sensitive to assumptions on the dependence of default jumps. Although we have provided some evidence in this paper that a contagious default scenario is of importance in pricing corporate bonds, more work is needed to determine what degree of default dependence gives a realistic description of reality. Clearly, this is important for pricing basket credit derivatives.

7 Concluding Remarks

In this paper, we have looked at different sources of risk in the corporate bond market. The main contribution of this paper is that we estimate the size of the risk premium associated with the jump in prices in case of a default event. This risk premium turns out to explain a significant part of corporate bond returns, even when tax and liquidity effects are included. This is evidence that default jump risk can not be fully diversified. There are, at least, two explanations for this imperfect diversification. The first is that not enough bonds are traded to obtain full diversification of independent default jumps. Second, it may be that default jumps are not (conditionally) independent. This could be the case if there is a possibility of multiple defaults at the same point in time ('contagious default scenario').

There are several extensions to the analysis in this paper. First, while in this paper we have focussed on estimating the size of the default jump risk premium, it would be interesting to see whether utility-based models with standard preferences can explain the size of this risk premium (see Karoui and Martellini (2001)). Another extension would be to include high-yield bonds in the analysis. Finally, Duffie, Pedersen, and Singleton (2000) and Keswani (1999) study the pricing of defaultable sovereign debt with reduced-form default models. The model in this paper can be used to analyze the joint behaviour of spreads of sovereign debt of many countries.

Appendix A: Parameter Identification

From Duffee (1999) and DPS (2000) it follows that all parameters related to the default-free and firm-specific factors can be identified. In this appendix, we analyze which parameters in the common factor processes can be identified. In equation (5) it is shown that the contribution of the common factor i to the instantaneous spread of firm j is given by $\bar{F}_{i,j,t} = \gamma_{i,j} F_{i,t}$. The process of $\bar{F}_{i,j,t}$ under the risk-neutral measure Q is given by

$$d\bar{F}_{i,j,t} = (\gamma_{i,j}\kappa_i^F\theta_i^F - (\kappa_i^F + \lambda_i^F)\bar{F}_{i,j,t})dt + \sqrt{\gamma_{i,j}}\sigma_i^F\sqrt{\bar{F}_{i,j,t}}d\hat{W}_{i,t}^F, \quad i=1,..,K \tag{A.1}$$

The process under the true probability measure is, heuristically speaking, obtained by removing λ_i^F from equation (A.1) and replacing the Q-Brownian motion $\hat{W}_{i,t}^F$ with a P-Brownian motion $W_{i,t}^F$. Besides the constant α_j, the identifiable parameters are the parameters in the processes under P and Q, which are thus given by the four reduced-form parameters $(\gamma_{i,j}\kappa_i^F\theta_i^F,\ \kappa_i^F,\ \kappa_i^F + \lambda_i^F,\ \sqrt{\gamma_{i,j}}\sigma_i^F)$. These four reduced-form parameters are a function of five structural parameters. For another firm k the identifiable parameters are $(\gamma_{i,k}\kappa_i^F\theta_i^F,\ \kappa_i^F,\ \kappa_i^F + \lambda_i^F,\ \sqrt{\gamma_{i,k}}\sigma_i^F)$. It then follows that, besides κ_i^F and λ_i^F, it is not possible to recover the remaining structural parameters from the reduced form parameter estimates, and that normalizing θ_i^F, i=1,..,K, solves this identification problem (other normalizations are also possible). We normalize θ_i^F to 50 basis points.

Appendix B: Expected Corporate Bond Returns

In this appendix, we derive the expression for the instantaneous expected excess return on a corporate bond. The derivation is based on Yu (2001).

First, we neglect for a moment the possibility of a default event. In this case, we can apply Ito's lemma (without a jump correction) to equation (4), using the fact that the model implies a standard exponential-affine pricing relation for corporate bonds. This gives for the expected return in case of no default

$$E_t[\frac{dV_j(t,T)}{V_j(t,T)}|no\ default] = [r_t + s_{j,t} - \sum_{i=1}^{K} (T-t)B_{i,j}(T-t)\lambda_i^F F_{i,t} - (T-t)C_j(T-t)\lambda_j^G G_{j,t} + \tilde{D}(1+\beta_j,t,T) - (T-t)D_v(\beta_{v,j},T-t)\lambda_v v_t]\,dt \tag{B.1}$$

with $\tilde{D}(\beta_j,t,T)$ defined in equation (10).

Of course, equation (B.1) neglects the loss in case of default events. In a small time interval $(t,t+\Delta t)$ the default probability under the true probability measure P approximately equals $h_{j,t}^P \Delta t$ and the loss in case of default equals $L \cdot V_j(t-,T)$. Then, the expected return over the next time interval approximately equals (normalized by the time interval)

$$\frac{(1-h_{j,t}^P \Delta t)E_t[\frac{\Delta V_j(t,T)}{V_j(t,T)}|no\ default] + (h_{j,t}^P \Delta t)(-L)}{\Delta t} \tag{B.2}$$

This expression becomes exact if we let $\Delta t \to 0$, which gives us the instantaneous expected return

$$r_t + s_{j,t} - \sum_{i=1}^{K} (T-t)B_{i,j}(T-t)\lambda_i^F F_{i,t} - (T-t)C_j(T-t)\lambda_j^G G_{j,t} + \tilde{D}(1+\beta_j,t,T) - (T-t)D_v(\beta_{v,j},T-t)\lambda_v v_t - h_{j,t}^P L \tag{B.3}$$

The excess return over a default-free bond (with the same maturity) is obtained by subtracting the expected return in (9) from the expression in (B.3). Using that $s_{j,t} = h_{j,t}^Q L$ and the risk premium assumption in (3), we obtain equation (11).

Appendix C: Kalman Filter Setup

In this appendix, we briefly describe the general setup for the extended Kalman filter estimation of affine bond pricing models. This setup applies both to the estimation of the default-free term structure model and to the estimation of the common factor intensity model. We therefore use a general notation that is unrelated to the notation in the main text. Let F_t be a K-dimensional vector that satisfies the following process under the actual probability measure

$$dF_t = \Lambda(\theta - F_t)dt + \Sigma(\alpha + \mathrm{B}'F_t)_d^{1/2} dW_t \tag{C.1}$$

Here $\Lambda, \Sigma,$ and B are K by K matrices, θ and α are K-dimensional vectors, and W_t is a K-dimensional vector of independent Brownian motions. The term $(\alpha + \mathrm{B}'F_t)_d^{1/2}$ is defined as a diagonal matrix with as diagonal elements the square-root of the elements of the vector $(\alpha + \mathrm{B}'F_t)$.

Without loss of generality we can assume the matrix Λ to be diagonal, $\Lambda = diag(\kappa_1,...,\kappa_K)$. In other words, one can transform each affine model into a model with diagonal Λ, by rotating the factors. This applies in particular to the default-free term structure model in equation (1).

De Jong (2000) derives the conditional expectations and (co)variances for the factors, which we repeat for convenience

$$\begin{gathered} E[F_{i,t+h}|F_t] = \theta_i + e^{-\kappa_i h}(F_{i,t} - \theta_i) \\ Q_{ij}(F_t) \equiv Cov[F_{i,t+h}, F_{j,t+h}|F_t] = \\ \frac{1 - e^{-(\kappa_i + \kappa_j)h}}{\kappa_i + \kappa_j}(a_{ij} + b_{ij}'\theta) + \sum_{k=1}^{K} \frac{e^{-\kappa_k h} - e^{-(\kappa_i + \kappa_j)h}}{\kappa_i + \kappa_j - \kappa_k} b_{ij,k}(F_{k,t} - \theta_k) \end{gathered} \tag{C.2}$$

where $[\Sigma diag(\alpha + \mathrm{B}'x)\Sigma']_{ij} = a_{ij} + b_{ij}'x$. This gives us the following transition equation for the state-space model

$$F_{t+h} = \theta + e^{-\Lambda h}(F_t - \theta) + \eta_{t+h}, \quad V_t(\eta_{t+h}) = Q(F_t) \tag{C.3}$$

The second part of a state-space model is the measurement equation. Similar to Duffee (1999), we have yields on coupon-paying bonds as observations, both in case of the default-free model and in case of the intensity model. Let Y_t denote a N-dimensional vector with the observed yields at time t. In case of the default-free model, this vector contains all Treasury bond yields, while for the intensity model this vector contains all observed corporate bond yields. The measurement equation for Y_t is then given by

$$Y_{t+h} = z(F_{t+h}) + \varepsilon_{t+h}, \quad V_t(\varepsilon_{t+h}) = H_t \tag{C.4}$$

Here $z(F_{t+h})$ is a function that relates the yields to the factors and ε_t is a zero-expectation measurement error that is uncorrelated with F_{t+h}. The function $z(F_{t+h})$ would be affine if zero-coupon yields are used. Since we use coupon-yields, this function is nonlinear. As in Duffee (1999), in order to linearize the model, we use a Taylor approximation of this function around the one-period forecast of F_{t+h}.

In case of the default-free model, we assume that the covariance matrix of the measurement error ε_t is a diagonal matrix that is constant over time, $H_t = diag(\sigma^2_{\varepsilon,1},...,\sigma^2_{\varepsilon,N})$. As explained in the main text, we estimate the common factor model for the intensity process twice. In the first estimation, we assume $H_t = \sigma^2_{\varepsilon} I_N$. After this first estimation the firm-specific factors are estimated. This estimation of the firm-specific factor processes is based the residuals ε_t from the common factor model. For each firm, we formulate a state-space model for these residuals in the same way as outlined above. Given the estimation results for the firm-specific factors, we re-estimate the common factor model, now using a block-diagonal structure for H_t, where each block relates to all corporate bond yields of a single firm. Measurement errors of yields across firms are assumed to be uncorrelated. For the covariance matrix of the measurement errors of the yields of a single firm we use the conditional covariance matrix of yield errors $V_t(\varepsilon_{t+h})$ implied by the estimates for the firm-specific factor process of this firm.

An important issue when estimating the common factors in the intensity model is the presence of missing observations. At each week, not all corporate bond yields are observed. Still, we always observe some corporate yields in the data each week. Therefore, to construct the likelihood contribution for each week, we use all available observations for that week. For the estimation of the firm-specific factors the treatment of missing observations is different because in this case there are weeks with no observations at all for a single firm. Therefore, as in Duffee (1999), we let the length of the time interval h vary over time in this case.

In all cases, we assume that all factors follow stationary processes under the true probability measure, so that we can use the unconditional expectation and (co)variances of the factors to initiate the Kalman filter. We refer to De Jong (2000) for all equations in the Kalman filter recursion.

Appendix D: Calculation of Standard Error of Default Jump Risk Premium

To obtain a standard error on the estimate for μ, we ignore, as before, estimation error for the parameters estimated in all previous steps. We thus treat the model-implied probability $q_{Rating}(n, \mu)$ as a deterministic function. Next, we calculate the variances of the observed conditional default rates $q^{Data}_{Rating}(n)$, by assuming for simplicity that in each year defaults are independently generated from a binomial distribution with probability $q^{Data}_{Rating}(n)$. S&P and Moody's report the number of firms per cohort (and per rating category), and we sum these over all cohorts that are relevant for the particular default probability. For example, for the default probability in the first year, $q^{Data}_{Rating}(0)$, we take the sum over all cohorts, while for the 14-year conditional default probability $q^{Data}_{Rating}(14)$, we only sum over the cohorts starting in 1981 up to 1986 in case of the S&P data, and over the cohorts starting in 1971 up to 1986 in case of the Moody's data. Together with the assumption of the binomial distribution and the associated observed default rate, the variance of this default probability estimate can be determined as $q^{Data}_{Rating}(n)\,(1 - q^{Data}_{Rating}(n)) \,/\, N_{Rating}$, where N_{Rating} is the total number of firms in all yearly cohorts of a given rating category.

A more difficult issue is the correlation between the observed default rates. Clearly, there is a large overlap for the conditional default rates $q^{Data}_{Rating}(n)$ across the conditioning period n, within the same rating category. For example, a large fraction of the firms in the AA-cohort that starts in, say, 1982, will also be present in the AA-cohort that starts in 1983. To be conservative, we assume perfect correlation between the estimated default rates for different n within each rating category. We assume zero correlation between the estimated default rates across rating categories. Given these assumptions, we can calculate the full covariance matrix W of the default rates $q_{Rating}^{Data}(n)$ across rating categories and conditioning periods n. Since our estimation method is the first step of the Generalized Method of Moments (Hansen (1982)), the asymptotic variance of the estimator for μ is given by

$$\left(\frac{\partial q(\mu)'}{\partial \mu}\frac{\partial q(\mu)}{\partial \mu}\right)^{-1}\frac{\partial q(\mu)'}{\partial \mu}\,W\,\frac{\partial q(\mu)}{\partial \mu}\left(\frac{\partial q(\mu)'}{\partial \mu}\frac{\partial q(\mu)}{\partial \mu}\right)^{-1}\Big|_{\mu=\hat{\mu}} \tag{D.1}$$

where W is the estimated covariance matrix of the (S&P or Moody's) default rates and the vector $q(\mu)$ is defined as $(q_{AA}(0,\mu),...,q_{AA}(14,\mu),q_A(0,\mu),...,q_A(14,\mu),q_{BBB}(0,\mu),...,q_{BBB}(14,\mu))'$.

Appendix E: Tax Correction on Corporate Bond Prices

As indicated by Elton et al. (2001), coupon payments on corporate bonds are subject to state taxes, whereas government bond coupons are not. In this appendix, we work out the correction for this tax effect in the intensity-based framework. As before, we assume recovery of market value at the default event date.

Denote t_s the state tax rate and t_g the federal tax rate. As shown by Elton et al. (2001), the effective tax rate is $t_s(1-t_g)$, which we denote τ. This tax rate changes the corporate bond price in two ways. First, it changes the net coupon payment in case no default occurs. If we denote the coupon size C, the net coupon payment is $(1-\tau)C$. Second, there is a tax recovery on the default loss if the firm defaults, which changes the loss rate to $(1-\tau)L$. We assume that this loss rate applies to the net coupon payments. This leads to the following valuation equation for a corporate bond that has n coupon payments and associated payment dates $T_1,...,T_n$

$$\begin{aligned} V_j(t,T,n) \;=\; & (1-\tau)C\sum_{i=1}^{n} E_t^Q\Big[\exp\Big(-\int_t^{T_i}(r_s+h_{j,s}^Q(1-\tau)L)\,ds\Big)\Big] \;+ \\ & E_t^Q\Big[\exp\Big(-\int_t^{T_n}(r_s+h_{j,s}^Q(1-\tau)L)\,ds\Big)\Big] \end{aligned} \tag{E.1}$$

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Table 1. Summary Statistics Corporate Bond Data.

Summary statistics on weekly observations for corporate bond prices from February 22, 1991 until February 18, 2000, for 592 bonds of 104 firms. The row 'Weeks of data' contains the number of weeks for which at least two bond prices are observed for a given firm. 'Mean number of fitted bonds' contains the mean number of bonds fitted per week, conditional upon two bond prices observed at this week.

	Across 104 firms		
Firm-level statistic	Minimum	Median	Maximum
Weeks of data	100	378	470
Mean number of fitted bonds per week	2.0	3.0	8.0
Mean years to maturity of fitted bonds	2.6	7.7	22.9
Minimum years to maturity of fitted bonds	1.0	1.0	11.8
Maximum years to maturity of fitted bonds	4.7	15.0	50.0
Mean coupon of fitted bonds	5.8	7.6	10.1

Table 2. Kalman Filter Estimates of Two-Factor Affine Model for Default-Free Rates.

Using QML based on the Kalman Filter, the two-factor affine model in equation (1) is estimated using weekly data on the 6-month T-bill rate and Treasury bond yields with maturities of 2, 3, 5, 10 and 30 years. It is assumed that all interest rates are observed with i.i.d. measurement errors independent across instruments. Standard errors are corrected for heteroskedasticity using White (1982).

Parameter	Estimate	Standard Error
κ_{vv}	0.0330	0.0444
κ_{vr}	-0.0002	0.0011
κ_{rr}	0.4018	0.0712
θ_v	19.149	20.461
θ_r	0.0479	-
σ_{vr}	0.0013	0.0010
σ_{rr}	0.0020	0.0012
λ_v	-0.0298	0.0799
λ_r	-0.1163	0.0432

Table 3. Fit of Two-Factor Affine Model on Treasury Instruments.

The table reports the fit of the two-factor affine model, estimated using QML based on the Kalman filter, on the 6-month T-bill rate and Treasury bond yields (the yield-to-maturities of the coupon-paying bonds). Data are weekly from February 1991 until February 2000.

Bond Maturity	Mean Error	Mean Absolute Error	Root Mean Square Error
6 months	-4.54 bp	17.45 bp	21.13 bp
2 years	-3.07 bp	5.54 bp	7.18 bp
3 years	-1.03 bp	7.51 bp	9.15 bp
5 years	-3.45 bp	6.20 bp	7.77 bp
10 years	2.14 bp	5.47 bp	6.72 bp
30 years	2.18 bp	10.46 bp	13.54 bp

Table 4. Parameter Estimates Firm-Specific Models.

The table reports parameter estimates for the model with firm specific factors only, that is given in equations (4)-(7) with *K* equal to zero. Estimates are obtained using QML based on the extended Kalman Filter (see Appendix C).

	First Quartile Firm	Median Firm	Third Quartile Firm
Estimate δ_j	3.34 bp	5.43 bp	23.94 bp
Estimate κ_j^G	0.009	0.036	0.103
Estimate λ_j^G	-0.185	-0.106	-0.065
Estimate σ_j^G	0.022	0.030	0.044
Estimate θ_j^G	42.89 bp	58.59 bp	73.42 bp
Estimate $\beta_{r,j}^G$	-0.121	-0.0929	-0.0537
Estimate $\beta_{v,j}^G$	2.53e-5	-3.89e-5	-4.32e-5
Average Fitted Instantaneous Spread	37.68 bp	52.30 bp	65.09 bp

Table 5. Kalman Filter QML Estimates for Two Common Credit Factors.

Using QML based on the Kalman Filter, the model with two common factors in equations (4)-(7) is estimated. Appendix C contains details on the Kalman filter estimation. Standard errors are calculated using the White (1982) heteroskedasticity-consistent covariance matrix. The parameter θ_i^F is normalized to 50 basis points for both factors.

Factor i	κ_i^F	λ_i^F	σ_i^F
Factor 1	0.030 (0.082)	-0.072 (0.024)	0.016 (0.007)
Factor 2	0.490 (0.165)	-0.163 (0.116)	0.046 (0.011)
	AAA/AA Rating	A Rating	BBB Rating
α	15.18 bp (11.15 bp)	22.86 bp (13.54 bp)	22.43 bp (11.68 bp)
$\gamma_{1,Rating}$	1.675 (0.487)	1.971 (0.333)	3.196 (0.565)
$\gamma_{2,Rating}$	0.219 (0.073)	0.400 (0.052)	0.553 (0.094)
$\beta_{r,Rating}$	-0.047 (0.003)	-0.046 (0.004)	-0.045 (0.004)
$\beta_{v,Rating}$	4.54e-5 (2.33e-5)	-5.28e-5 (2.83e-5)	-6.37e-5 (5.94e-5)
Average Fitted Instantaneous Spread	38.95 bp	58.28 bp	74.87 bp

Table 6. Firm-Specific Factor Parameter Estimates for Common Factor Model.

The table reports quartiles of parameter estimates for the firm specific factors in the model with two common factors in equations (4)-(7). Estimates are obtained using QML based on the Kalman Filter (see Appendix C).

	First Quartile	Median	Third Quartile
Estimate κ_j^G	0.004	0.017	0.073
Estimate λ_j^G	-0.063	-0.008	0.032
Estimate σ_j^G	0.007	0.013	0.021
Estimate θ_j^G	16.01 bp	36.72 bp	83.12 bp

Table 7. Yield Errors for Corporate Bonds.

For each bond in the dataset, the yield error is defined as the difference between the model-implied yield-to-maturity and the observed yield-to-maturity. For three models results are given: the model with firm-specific factors only, the model with two common credit factors only, and the model with both two common credit factors and firm-specific factors. For each firm, the average of the absolute value of these yield errors and the root mean squared yield error (RMSE) is calculated. The table contains summary statistics on these average absolute yield errors and the RMSE for all firms.

	Avg. Absolute Yield Error per Firm: Across 104 firms		
	First Quartile	Median	Third Quartile
Firm-Specific Factors Only	6.68 bp	9.73 bp	13.45 bp
Common Factors Only	10.43 bp	14.63 bp	17.90 bp
Common and Firm-Specific Factors	5.97 bp	9.11 bp	13.14 bp
	Root Mean Squared Yield Error per Firm: Across 104 firms		
	First Quartile	Median	Third Quartile
Firm-Specific Factors Only	9.43 bp	13.49 bp	17.89 bp
Common Factors Only	12.34 bp	16.76 bp	18.55 bp
Common and Firm-Specific Factors	9.11 bp	12.89 bp	17.18 bp

Table 8. Estimates of Default Event Risk Premium.

The table contains estimates and standard errors for the default event risk premium μ, obtained using S&P data (upper panel) and Moody's data (lower panel). Estimation is performed as described in Section 5.1. The table also gives results for tax-corrected bond prices and estimates of the liquidity spread ν (see Section 5.3). The calculation of standard errors is described in Appendix D.

S&P Data: 1981-2000			
Tax Correction	Liquidity Correction	Risk premium μ	Liquidity spread ν
No	No	5.83 (0.79)	-
No	Yes	3.02 (0.82)	32.48 bp (9.12 bp)
Yes	No	3.57 (0.56)	-
Yes	Yes	2.94 (0.93)	16.82 bp (11.01 bp)
Moody's Data: 1970-2000			
Tax Correction	Liquidity Correction	Risk premium μ	Liquidity spread ν
No	No	5.55 (0.71)	-
No	Yes	1.86 (0.77)	41.45 bp (8.28 bp)
Yes	No	2.98 (0.50)	-
Yes	Yes	1.81 (0.90)	18.28 bp (8.84 bp)

Figure 1. Average Term Structures of Coupon-Spreads. Coupon spreads of 592 bonds of 104 firms are averaged over time, and within each rating category and maturity bucket. The graphs depicts these averages.

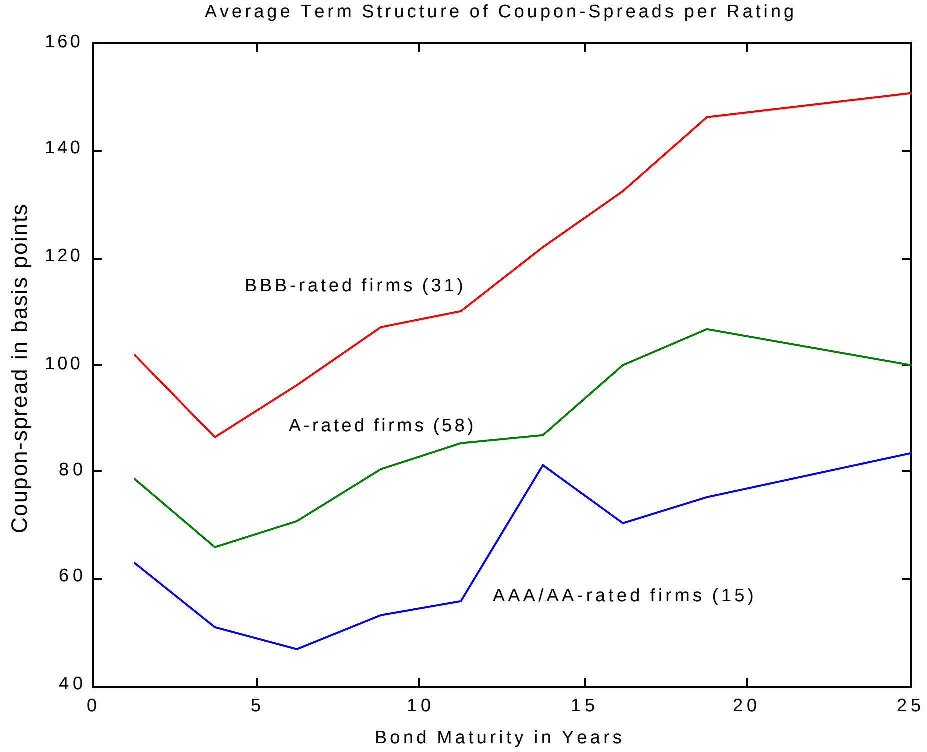


Figure 2. Time Series of Coupon spreads. At each week, coupon spreads of 592 bonds of 104 firms are averaged within each rating category. The graph depicts the resulting time series.

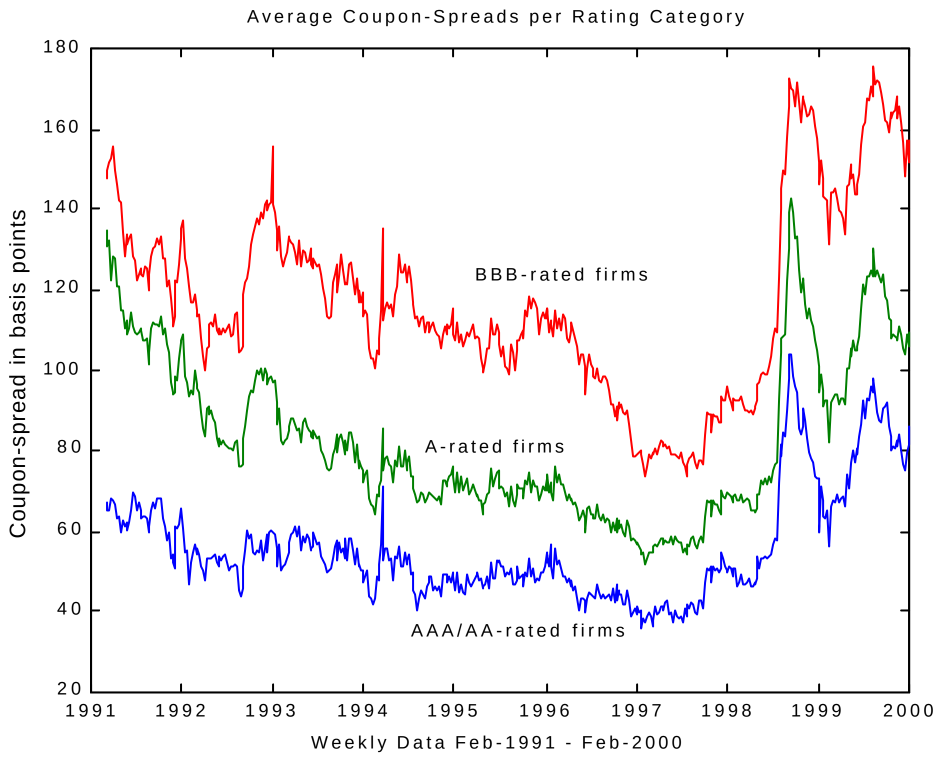
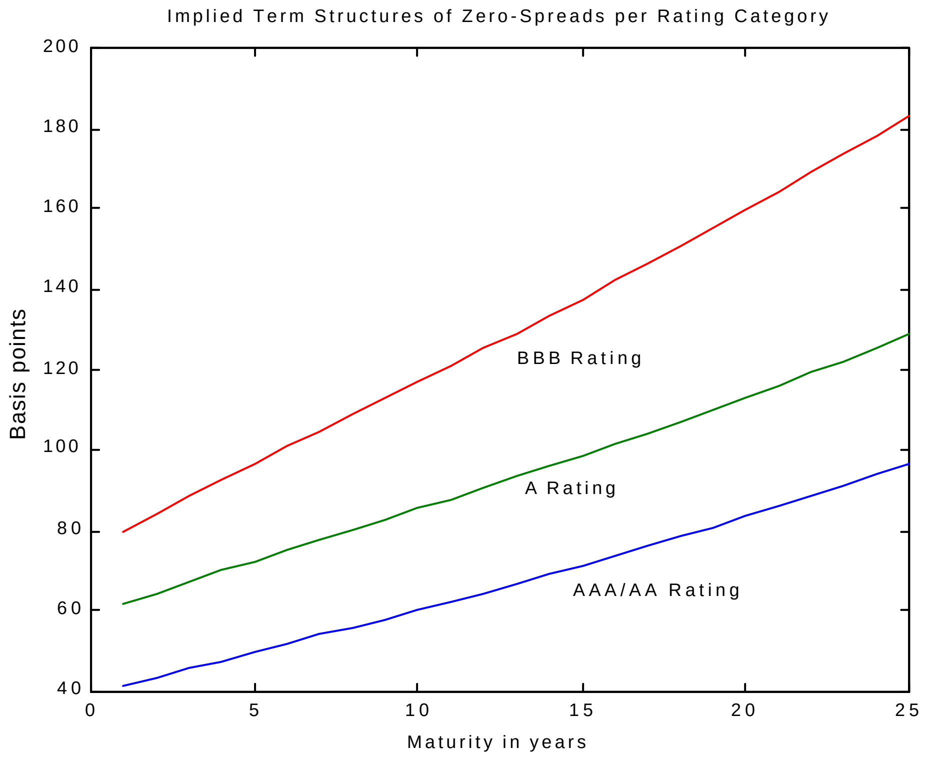


Figure 3. Model-Implied Term Structures of Zero-Coupon Spreads. At the average firm parameter estimates within each rating category and the average of the estimated factor values, the term structures of zero-coupon spreads implied by the common factor model are graphed.



Figures 4a-c. Conditional Default Probabilities. Each figure contains yearly conditional default probabilities, for AA rated firms (Figure 4a), for A rated firms (Figure 4b), and for BBB rated firms (Figure 4c). The line 'S&P Data' gives the historically estimated default probabilities, obtained from S&P data. The other lines all are model-implied default probabilities:

- 'Q-prob' refers to the risk-neutral probabilities.
- 'P-prob: mu=1' refers to the actual probabilities in case μ=1.
- 'P-prob: mu=1, tax correction' refers to the actual probabilities in case μ=1, with tax-corrected bond prices.
- 'P-prob: mu*' refers to the actual probabilities at the estimated value for μ (5.83).
- 'P-prob: mu*', tax correction' refers to the actual probabilities at the estimated value for μ (3.57), with tax-corrected bond prices.

The yearly conditional default probability gives the probability of default in the next year, given that no default has occurred before.

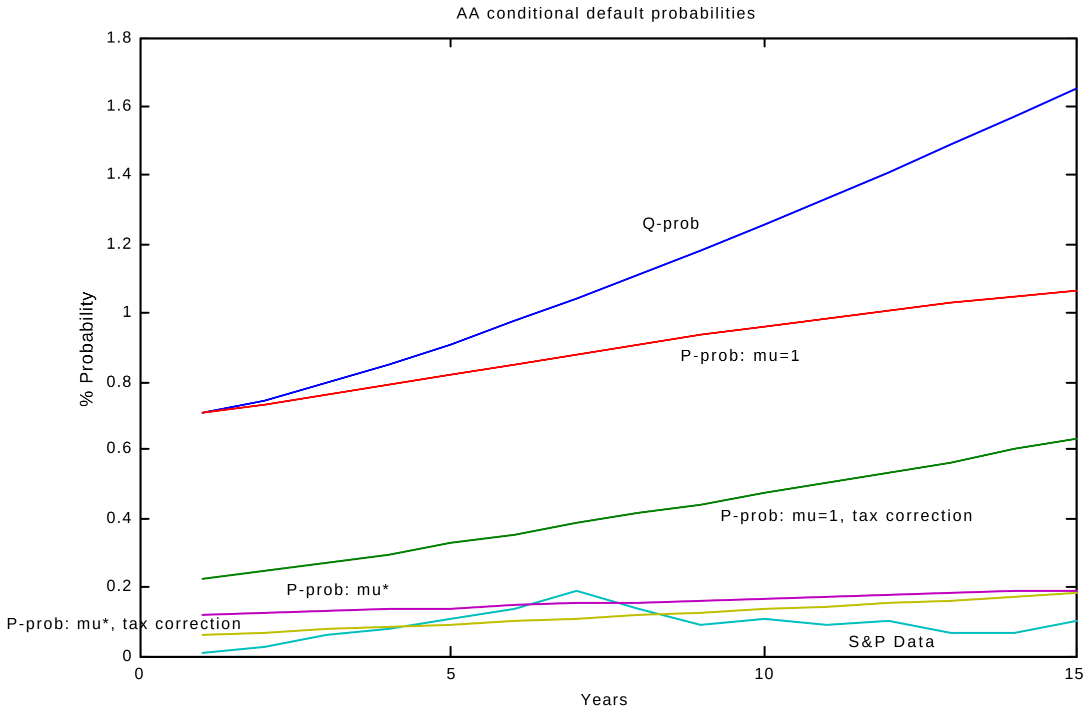


Figure 4b

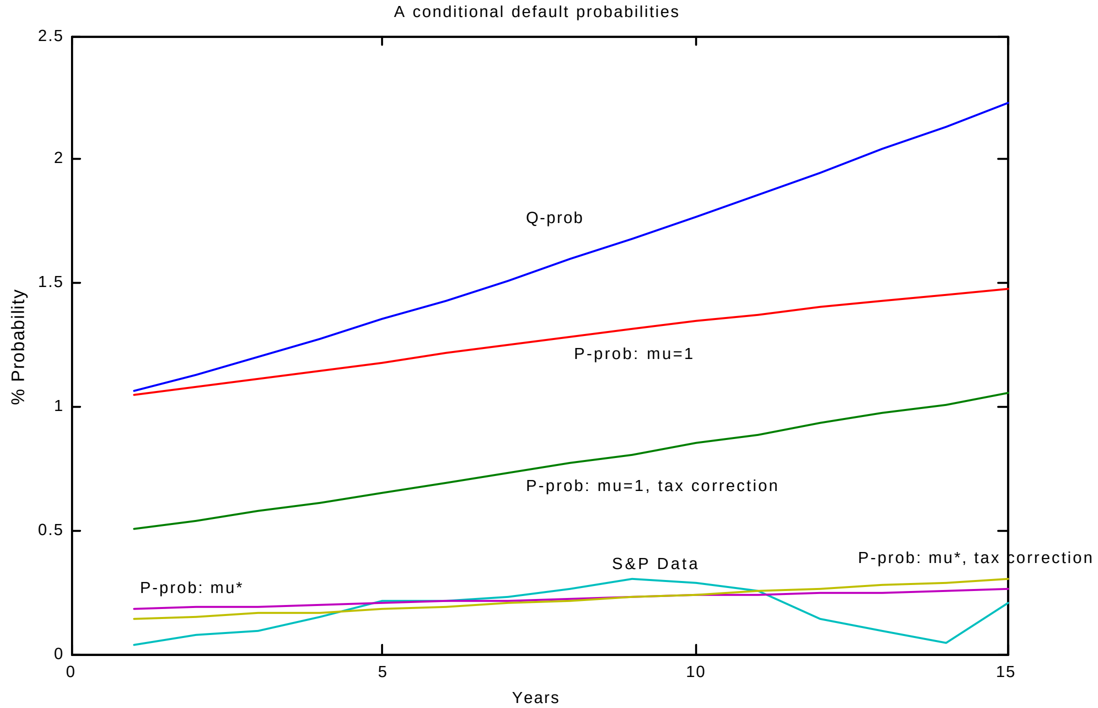
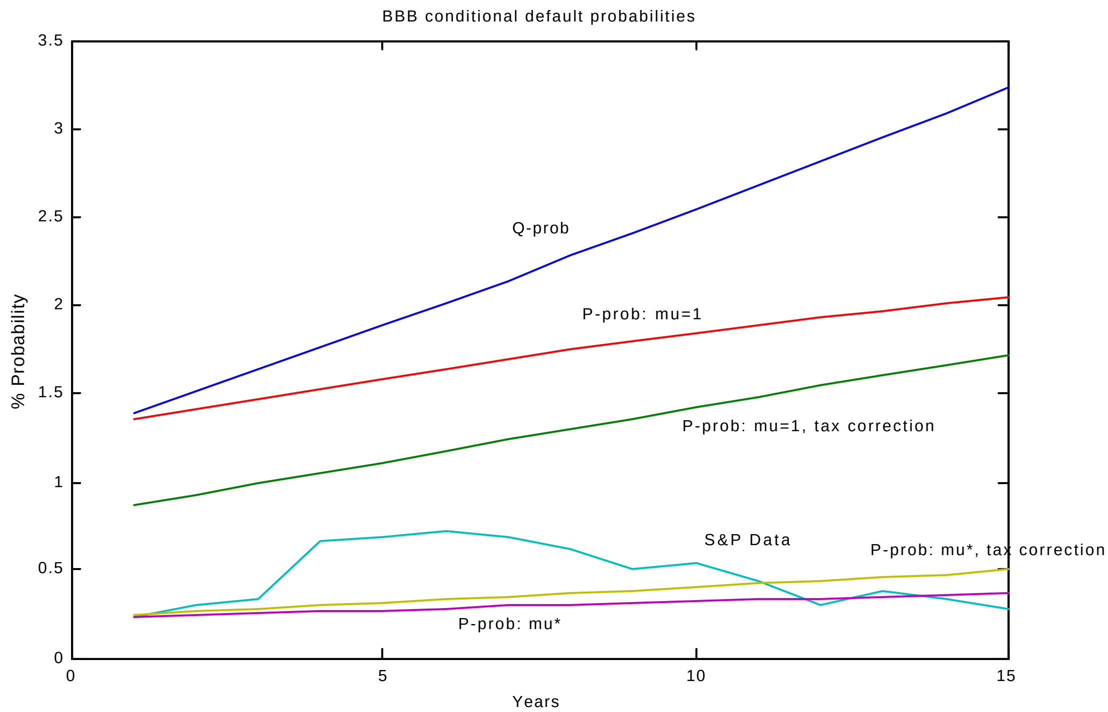
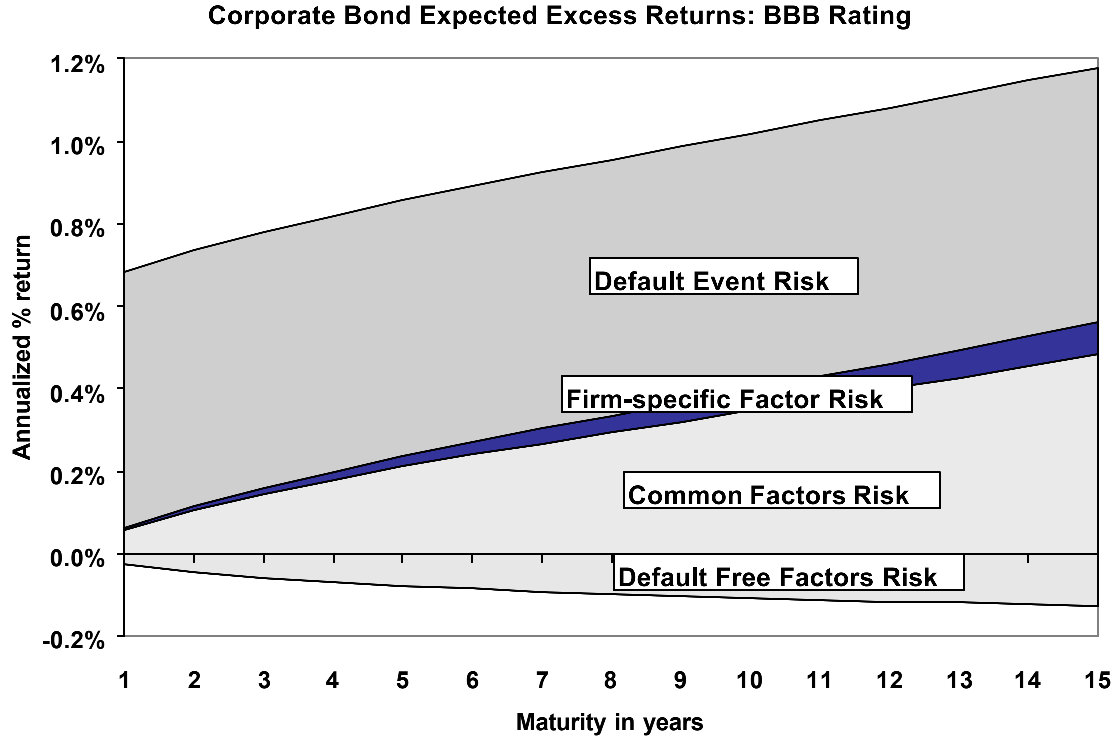


Figure 4c



Figures 5a-b. Corporate Bond Expected Excess Returns. For the median BBB rated firm, both figures provide the annualized unconditional expected return on a zero-coupon corporate bond, for different maturities, in excess of the return on a government bond with the same maturity. The expected return is decomposed four risk premia, as discussed in Section 2.2. In Figure 6b, tax and liquidity effects are included (see Section 5.3).



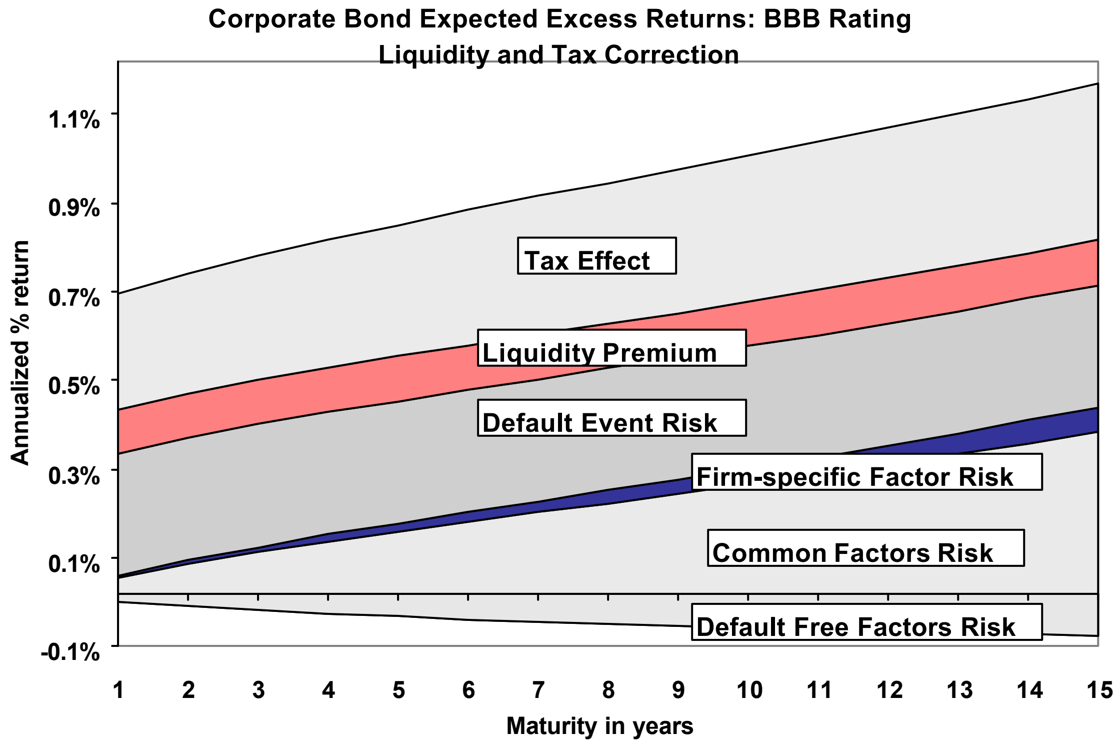


Figure 6. Prices of nth-to-default Swaps. The graph depicts prices of nth-to-default swaps for n=1,...,20. The swap has a maturity of 3 years, and is based on an equally weighted portfolio of bonds of 30 firms (10 AA rated, 10 A rated, and 10 BBB rated) with 15 years maturity. Two models are used to calculate prices: a model with conditionally independent defaults, and a model with dependent defaults (see Section 6). Prices are given as a percentage of the notional amount of a single bond.