

# **THE MARKET PRICE OF CREDIT RISK:**

## **An Empirical Analysis of Interest Rate Swap Spreads**

**Jun Liu\***  
**Francis A. Longstaff\***  
**Ravit E. Mandell\*\***

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\* The Anderson School at UCLA, Box 951481, Los Angeles, CA 90095-1481. \*\* Salomon Smith Barney. Corresponding author: Francis Longstaff, Phone: (310) 825-2218, Email Address: francis.longstaff@anderson.ucla.edu. We are grateful for helpful discussions with Don Chin, Qiang Dai, Peter Hirsch, Antti Ilmanen, Josh Mandell, Yoshihiro Mikami, Jun Pan, Monica Piazzesi, Walter Robinson, Pedro Santa-Clara, Janet Showers, Suresh Sundaresan, Rossen Valkanov, Toshiki Yotsuzuka, and seminar participants at the Risk Conferences on Credit Risk in London and New York, and Simplex Asset Management. All errors are our responsibility.

## ABSTRACT

This paper studies the market price of credit risk incorporated into one of the most important credit spreads in the financial markets: interest-rate swap spreads. Our approach consists of jointly modeling the swap and Treasury term structures using a four-factor affine credit framework and estimating the parameters by maximum likelihood. We solve for the implied special financing rate for Treasury bonds and find that the liquidity component of on-the-run bond prices can be very significant. We show that most of the variation in swap spreads is driven by changes in the liquidity of Treasury bonds rather than changes in default risk. We find that there are positive credit premia in swap spreads on average. These premia, however, vary significantly over time and were negative for much of the 1990s. Since the hedge-fund crisis of 1998, credit premia have become positive and are currently at historical highs.

## 1. INTRODUCTION

One of the most fundamental issues in finance is how the market compensates investors for bearing the credit risk inherent in securities issued by entities that may default on their obligations. Recent events such as the default by Russia on its ruble-denominated debt in August 1998 and the subsequent flight to quality which resulted in major losses at many hedge funds and investment banks demonstrate that changes in the willingness to bear credit risk can have dramatic effects on the financial markets.<sup>1</sup> Furthermore, these events indicate that variation in credit spreads may reflect both changes in perceived default risk and in the relative liquidity of bonds. Understanding the risk and return tradeoffs for these types of securities will become even more important in the future as the supply of U.S. Treasury securities available in the market decreases.

This paper studies the market price of credit risk incorporated into what is rapidly becoming one of the most important credit spreads in the financial markets: interest rate swap spreads. Since swap spreads represent the difference between swap rates and Treasury bond yields, they reflect the difference in the default risk of the financial sector quoting Libor rates and the U.S. Treasury. In addition, swap spreads may include a significant liquidity component if the relevant Treasury bond trades special in the repo market. Thus, swap spreads represent a near-ideal data set for examining how both default and liquidity risks influence security returns. The importance of swap spreads derives from the dramatic recent growth in the notional amount of interest rate swaps outstanding relative to the size of the Treasury bond market. For example, the total amount of Treasury debt outstanding at the end of 1999 was \$ 5.7 trillion. In contrast, the Bank for International Settlements (BIS) estimates that the total notional amount of interest rate swaps outstanding at the end of 1999 was \$ 43.9 trillion, or nearly eight times the amount of Treasury debt.

Since swap spreads are fundamentally credit spreads, our approach consists of jointly modeling the interest rate swap and Treasury term structures using the reduced-form credit framework of Duffie and Singleton (1997, 1999). To capture the rich dynamics of the swap and Treasury curves, we use a four-factor affine term structure model which allows the swap spread to include both default-risk and liquidity components and to be correlated with interest rates. In addition, our specification allows market prices of risk to vary over time to reflect the possibility that the willingness of investors to bear credit and liquidity risk may change. Using an approach motivated by Dai and Singleton (2000a), we estimate the parameters of the model by maximum likelihood. The data for the study consist of an extensive set of rates spanning nearly

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<sup>1</sup>Duffie, Peterson, and Singleton (2000) provide a in-depth analysis of how the market values of Russian bonds were affected by changes in default risk and liquidity.

the full history of the swap market. We show that both the swap and Treasury term structures are well described by the four-factor affine model. In particular, both curves can be fit simultaneously with a root mean squared error over the entire sample period of only about six basis points. Because of this, we are able to examine credit and liquidity effects at a higher resolution than in a number of previous studies.

A number of interesting results emerge from this analysis. First, we solve for the short-term riskless rate implied by Treasury bond prices. We find that the implied riskless rate can differ substantially from the Treasury-bill rate and is often much higher. This supports the widespread view on Wall Street that because of the extreme liquidity of Treasury bills, their yields tend to be downward-biased estimates of the true riskless rate. Since the implied short-term riskless rate can be interpreted as the special repo rate for the on-the-run Treasury bonds in the sample, we contrast them with repo rates for generic or general Treasury collateral.<sup>2</sup> We find that these implied special repo rates are slightly less than the general repo rates on average, implying that the prices of the on-the-run Treasury bonds in the sample include premia for their liquidity or specialness relative to off-the-run Treasury securities. By integrating the difference between the general and implied special repo rates, we obtain direct estimates of the size of the specialness component in on-the-run Treasury security prices. These specialness premia can be large in economic terms. For example, the specialness premium for the ten-year Treasury note can be as much as .66 percent of its notional amount, which translates into a nine basis point effect on its yield to maturity. The estimated specialness premia match closely those implied by a limited sample of market term special repo rates provided to us.

We then solve for the implied spread process representing the sum of the instantaneous default risk and liquidity components as in Duffie and Singleton (1997). We find that this spread varies significantly over time, but is nearly zero for an extended period during the mid to latter 1990s. Using the earlier results for the implied special repo rate, we decompose the spread into its default-risk and liquidity components. We show that the default-risk component is typically the largest component of the spread. The liquidity component, however, is much more volatile and can often exceed the size of the default-risk component. Thus, most of the variation in swap spreads is attributable to changes in the relative liquidity of swaps and Treasury bonds. This suggests that if there are credit premia incorporated into swap spreads, they should be interpreted as primarily liquidity premia. Furthermore, our results imply that the historically high swap spreads recently observed in the financial markets are largely due to an increase in the liquidity of Treasury securities rather than to a decline in the creditworthiness of the financial sector.

Finally, we examine the implications of the model for the market prices of interest-

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<sup>2</sup>In a recent paper, Duffie (1996) studies the causes and effects of special repo rates in the Treasury repo market.

rate and credit-related risk. Consistent with previous research, we find that there are significant time-varying term premia embedded in Treasury bond prices. The results suggest, however, that these term premia have decreased substantially during recent years. We also find that there are significant credit premia embedded in the swap curve. On average, these premia are positive, ranging from two basis points for a one-year horizon to 42 basis points for a ten-year horizon. These credit premia also display substantial time variation. Surprisingly, we find evidence that credit premia were significantly negative for an extended period during the 1990s. Since the hedge-fund crisis of 1998, however, credit premia have become positive and are currently at historically high levels. Taken together, these results suggest that there have been major changes over time in the expected returns from bearing the default and liquidity risk inherent in interest rate swaps.

This paper complements and extends the important recent paper by Duffie and Singleton (1997) who apply a reduced-form credit modeling approach to the swap curve and examine the properties of swap spreads. Our results support their finding that both default-risk and liquidity components are present in swap spreads and our approach allows us to estimate the size of the components. By modeling both the swap and Treasury curves simultaneously, however, we are also able to address the issue of how credit risk is priced in the market, which is the primary focus of this paper. Another important related paper is He (2000) who independently uses a multi-factor affine term structure framework similar to ours in modeling swap spreads. While He does not estimate the parameters of his model, our empirical results provide support for both swap spread modeling frameworks. Grinblatt (2001) models the swap spread as the annuitized value of an instantaneous convenience yield. If this convenience yield is interpreted as the liquidity component of the spread process, then our results can also be viewed as providing support for the implications of his model. Other related papers include Sun, Sundaresan, and Wang (1993) who study the extent to which counterparty credit risk affects market swap rates, and Collin-Dufresne and Solnik (2001) who focus on the spread between Libor corporate rates and swap rates.

The remainder of this paper is organized as follows. Section 2 explains the framework used to model the swap and Treasury term structures. Section 3 describes the data. Section 4 discusses the maximum likelihood estimation of the model. Section 5 focuses on the implications of the results for the liquidity of Treasury securities. Section 6 discusses the empirical results about the properties of swap spreads. Section 7 presents the results about the pricing of default and liquidity risk. Section 8 summarizes the results and makes concluding remarks.

## 2. MODELING SWAP SPREADS

To understand how the market prices credit risk over time, we need a framework for estimating expected returns implied by the swap and Treasury term structures. In

this section, we use the Duffie and Singleton (1997, 1999) credit modeling approach as the underlying framework in which to analyze the behavior of swap spreads. In particular, we jointly model the swap and Treasury term structures using a four-factor affine framework and estimate the parameters of the model by maximum likelihood.<sup>3</sup>

Recall that under standard no-arbitrage assumptions, the value  $D(T)$  of a riskless zero-coupon bond with maturity  $T$  can be expressed as

$$D(T) = E_Q \left[ \exp \left( - \int_0^T r_s \, ds \right) \right], \quad (1)$$

where  $r$  denotes the instantaneous riskless rate and the expectation is taken with respect to the risk-neutral measure  $Q$  rather than the objective measure  $P$ . In the Duffie and Singleton (1997, 1999) framework, default is modeled as the realization of a Poisson process with an intensity which may be time varying. Under some assumptions about the nature of recovery in the event of default, they demonstrate that the value of a risky zero-coupon bond  $C(T)$  can be expressed in the following form

$$C(T) = E_Q \left[ \exp \left( - \int_0^T r_s + \lambda_s \, ds \right) \right], \quad (2)$$

where  $\lambda$  is a credit-spread process.<sup>4</sup> They also show that this credit-spread process may be viewed as the product of the time-varying Poisson intensity and the recovery-rate process. Furthermore, they argue that the credit-spread process could also include a time-varying liquidity component which may be either positive or negative. In this paper, we simply refer to  $\lambda$  as the credit-spread process, keeping in mind, however, that  $\lambda$  may include both default-risk and liquidity components. Consequently, the term credit risk is used in a general sense throughout this paper, reflecting that variation in credit or swap spreads may be due to changes in either default risk or liquidity.

In applying the Duffie and Singleton (1997, 1999) credit model to swaps, we are implicitly making two assumptions. First, we assume that there is no counterparty credit risk. This is consistent with recent papers by Grinblatt (2001), Duffie and Singleton (1997), and He (2000) that argue that the effects of counterparty credit risk

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<sup>3</sup>Other examples of affine credit models include Duffie (1999, 2000), Duffie and Liu (2000), He (2000), Duffie, Petersen, and Singleton (2000) and Colin-Dufresne and Solnik (2001).

<sup>4</sup>In the Duffie and Singleton (1999) model, the recovery rate is linked to the value of the bond immediately prior to the default event. While this assumption has often been criticized, the fact that Libor is computed from a set of banks that may change over time if some banks experience a deterioration in their credit rating argues that this assumption may be more defensible when applied to the swap curve.

on market swap rates should be negligible because of the standard marking-to-market or posting-of-collateral and haircut requirements almost universally applied in swap markets.<sup>5</sup> Second, we make the relatively weak assumption that the credit risk inherent in the Libor rate (which determines the swap rate) can be modeled as the credit risk of a single defaultable entity. In actuality, the Libor rate is a composite of rates quoted by 16 banks and, as such, need not represent the credit risk of any particular bank.<sup>6</sup> In this sense, the credit risk implicit in the swap curve can be viewed essentially as the average credit risk of the most representative banks providing quotations for Eurodollar balances.<sup>7</sup>

To model the discount bond prices  $D(T)$  and  $C(T)$ , we next need to specify the dynamics of  $r$  and  $\lambda$ . In doing this, we parallel the approach used by Duffie and Kan (1996), Duffie and Singleton (1997), Dai and Singleton (2000a, 2000b) and others by assuming that the riskless rate is given as the sum of three affine state variables,

$$r = W + X + Y. \quad (3)$$

By allowing the riskless rate to be driven by three distinct state variables, the model is consistent with the empirical evidence of Litterman and Scheinkman (1991), Knez, Litterman, and Scheinkman (1994), Longstaff, Santa-Clara, and Schwartz (2000) and many others who find evidence of at least three factors in term structure dynamics.<sup>8</sup>

In modeling the dynamics of the spread  $\lambda$ , we assume that

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<sup>5</sup>Even in the absence of these requirements, the effects of counterparty credit risk for swaps between similar counterparties are very small relative to the size of the swap spread. For example, see Cooper and Mello (1991), Sun, Sundaresan, and Wang (1993), Bollier and Sorensen (1994), Longstaff and Schwartz (1995), Duffie and Huang (1996), and Minton (1997).

<sup>6</sup>The official Libor rate is determined by eliminating the highest and lowest four bank quotes and then averaging the remaining eight. Furthermore, the set of 16 banks whose quotes are included in determining Libor may change over time. Thus, the credit risk inherent in Libor may be ‘refreshed’ periodically as low credit banks are dropped from the sample and higher credit banks are added. The effects of this ‘refreshing’ phenomenon on the differences between Libor rates and swap rates are discussed in Colin-Dufresne and Solnik (2001).

<sup>7</sup>For discussions about the economic role that interest-rate swaps play in financial markets, see Bicksler and Chen (1986), Turnbull (1987), Smith, Smithson, and Wakeman (1988), (1988), Wall and Pringle (1989), Macfarlane, Ross, and Showers (1991), Sundaresan (1991), Litzenberger (1992), Sun, Sundaresan, and Wang (1993), Brown, Harlow, and Smith (1994), Minton (1997), Gupta and Subrahmanyam (2000), and Longstaff, Santa-Clara, and Schwartz (2000).

<sup>8</sup>Other examples of multi-factor affine term structure models include Cox, Ingersoll, and Ross (1985), Longstaff and Schwartz (1992), Chen and Scott (1993), Pearson and Sun (1993), Piazzesi (2000) and others.

$$\lambda = \gamma r + Z, \quad (4)$$

where  $\gamma$  is a constant which may be positive or negative, and  $Z$  is also an affine state variable. By allowing the spread  $\lambda$  to depend on the riskless rate  $r$ , the model captures the potential dependence of spreads on the term structure. For example, Longstaff and Schwartz (1995) and Duffee (1999) find evidence that credit spreads are negatively related to the level of interest rates, an empirical result consistent with theoretical models of credit spreads such as Merton (1974), Black and Cox (1976), and Longstaff and Schwartz. The state variable  $Z$  allows the risky term structure to be influenced by an additional credit and/or liquidity factor which does not affect the riskless term structure.

To close the model, we need to specify the dynamics of the four state variables driving  $r$  and  $\lambda$ . To focus more clearly on the intuition of how the market prices the risk of variation in swap spreads, we use a particularly tractable term structure specification: a four-factor Vasicek (1976) framework. Specifically, we assume that the dynamics of  $W$ ,  $X$ ,  $Y$ , and  $Z$  under the objective measure are given by

$$\begin{aligned} dW &= \beta_W (\alpha_W - W) dt + \sigma_W dB_W, \\ dX &= \beta_X (\alpha_X - X) dt + \sigma_X dB_X, \\ dY &= \beta_Y (\alpha_Y - Y) dt + \sigma_Y dB_Y, \\ dZ &= \beta_Z (\alpha_Z - Z) dt + \sigma_Z dB_Z, \end{aligned} \quad (5)$$

where the  $\alpha$ ,  $\beta$ , and  $\sigma$  terms are constants, and  $B_W$ ,  $B_X$ ,  $B_Y$ , and  $B_Z$  are independent Brownian motions. Since  $\lambda$  is linear in the state variables, this model implies that the spread could potentially be negative. There are several reasons why this assumption may be appropriate in this context. First, the process  $\lambda$  reflects the differential credit between the swap and Treasury curves. While swap spreads have been uniformly positive in the U.S., swap spreads have occasionally been negative in other currencies. Allowing  $\lambda$  to take on negative values enables the model to applied more generally. Secondly,  $\lambda$  also reflects potential differences in liquidity. Again, while the liquidity of Treasury bonds has historically been very high, the liquidity of the swaps market is growing rapidly while the total notional amount of Treasury debt is currently shrinking.

Since our primary objective is to study how the market compensates investors over time for bearing the credit risk, it is important to allow a general specification of the market prices of risk in this affine term structure framework. Accordingly, we assume that the dynamics of the state variables  $W$ ,  $X$ ,  $Y$ , and  $Z$  under the risk-neutral measure are given by



$$\begin{aligned}
dW &= \kappa_W (\mu_W - W) dt + \sigma_W dB_W, \\
dX &= \kappa_X (\mu_X - X) dt + \sigma_X dB_X, \\
dY &= \kappa_Y (\mu_Y - Y) dt + \sigma_Y dB_Y, \\
dZ &= \kappa_Z (\mu_Z - Z) dt + \sigma_Z dB_Z.
\end{aligned} \tag{6}$$

This specification allows both the long-term mean and mean-reversion parameters appearing in the drift of the state variable process to take different values under the risk-neutral measure. This approach differs from the traditional Vasicek (1976) formulation in that market prices of risk are allowed to be time varying, but is similar to the specification used in Liu (1999) and Dai and Singleton (2000b). Because the dynamics imply that the state variables are Gaussian, allowing both of the drift parameters to differ under the risk-neutral measure does not change the sets of measure zero, preserving the absolute continuity of the risk-neutral measure. Note that the volatility parameters are the same under both the objective and risk-neutral measures. Allowing for time-varying prices of risk is important since recent events such as the flight to quality during the latter part of 1998 suggest that the willingness of investors to bear default and liquidity risk may depend on market conditions.

Given the risk-neutral dynamics of the state variables, it is straightforward to obtain closed-form solutions for the prices of riskless zero-coupon bonds,

$$\begin{aligned}
D(T) &= \exp \left( A_W(T) + A_X(T) + A_Y(T) \right. \\
&\quad \left. - B_W(T)W - B_X(T)X - B_Y(T)Y \right),
\end{aligned} \tag{7}$$

where

$$A_i(T) = -\mu_i T + \mu_i B_i(T) + \frac{\sigma^2}{2\kappa_i^2} \left( T - 2B_i(T) + \frac{1}{2\kappa_i} (1 - e^{-2\kappa_i T}) \right),$$

$$B_i(T) = \frac{1}{\kappa_i} (1 - \exp(-\kappa_i T)),$$

$$i = W, X, Y, Z.$$

Similarly, the prices of risky zero-coupon bonds are given by

$$C(T) = \exp\left(A_W^*(T) + A_X^*(T) + A_Y^*(T) + A_Z(T) - B_W^*(T)W - B_X^*(T)X - B_Y^*(T)Y - B_Z(T)Z\right), \quad (8)$$

where

$$A_i^*(T) = -(1 + \gamma)\mu_i T + (1 + \gamma)\mu_i B_i(T) + \frac{(1 + \gamma)^2 \sigma^2}{2\kappa_i^2} \left( T - 2B_i(T) + \frac{1}{2\kappa_i} (1 - e^{-2\kappa_i T}) \right),$$

$$B_i^*(T) = \frac{1 + \gamma}{\kappa_i} (1 - \exp(-\kappa_i T)),$$

$$i = W, X, Y.$$

With these closed-form solutions, market bond prices can be inverted to solve directly for the unobservable state variables.

### 3. THE DATA

Given this framework for modeling the swap and Treasury term structures, the next step is to estimate the parameters of the model using historical market data. In doing this, we use one of the most extensive sets of U.S. swap data available, covering the period from January 1988 to June 2000. This period includes most of the active history of the U.S. swap market.

The swap data for the study consist of weekly (Friday) observations of the three-month Libor rate and midmarket constant maturity swap (CMS) rates for maturities of two, three, five and ten years. These maturities represent the most liquid and actively-traded maturities for swap contracts. All of these rates are based on end-of-trading-day quotes available in New York to insure comparability of the data. In estimating the parameters, we are careful to take into account daycount differences among the rates since Libor rates are quoted on an actual/360 basis while swap rates are semiannual bond equivalent yields. There are two sources for the swap data. The primary source is the Bloomberg system which uses quotations from a number of swap brokers. The data for Libor rates and for swap rates from the pre-1990 period are provided by Salomon Smith Barney Inc. As an independent check on the data, we also compare the rates with quotes obtained from Datastream; the two sources of data are generally very consistent.

The Treasury data consists of weekly (Friday) observations of the constant maturity Treasury (CMT) rates published by the Federal Reserve in the H-15 release for maturities of two, three, five, and ten years. These rates are based on the yields of on-the-run Treasury bonds of various maturities and reflects the Federal Reserve's estimate of what the par or coupon rate would be for these maturities. CMT rates are widely used in financial markets as indicators of Treasury rates for the most-actively-traded-bond maturities. Since CMT rates are based heavily on the most-recently-auctioned bonds for each maturity, CMT rates provide accurate estimates of yields for liquid on-the-run Treasury bonds. As such, these rates are more likely to reflect actual market prices than quotations for less-liquid off-the-run Treasury bonds. Note that since CMT rates are based on more-recently-issued bonds, however, they may incorporate the effects of any special repo financing that may be associated with these bonds. The possibility that these bonds may trade special in the repo market is taken into account explicitly in the estimation of the model. The sources of this data are the same as for swaps. Finally, data on three-month general collateral repo rates are provided by Salomon Smith Barney, who also provided us with a set of term special repo rates for June 30, 2000. Data for three-month Treasury bill rates are obtained from the Federal Reserve.

Table 1 presents summary statistics for the swap and Treasury data, as well as the corresponding swap spreads. In this paper, we define the swap spread to be the difference between the CMS rate and the corresponding-maturity CMT rate. Figure 1 plots the two-year, three-year, five-year, and ten-year swap spreads over the sample period. As shown, swap spreads average between 35 and 60 basis points during the sample period, with standard deviations on the order of 20 to 25 basis points. Thus, swap spreads have been fairly stable over time. The standard deviations of weekly changes in swap spreads are only on the order of five to seven basis points. Note, however, that there are weeks during which swap spreads narrow or widen by as much as 35 basis points. In general, swap spreads are less serially correlated than the interest rates. The first difference of swap spreads, however, displays significantly more negative serial correlation. This implies that there is a strong mean reverting component to swap spreads.

#### 4. ESTIMATING THE TERM STRUCTURE MODEL

In this section, we describe the empirical approach used in estimating the term structure model and report the maximum likelihood parameter estimates. The empirical approach closely parallels that of the recent papers by Duffie and Singleton (1997) and Dai and Singleton (2000a). This approach also draws on other papers in the empirical term structure literature such as Longstaff and Schwartz (1992), Chen and Scott (1993), Duffee (1999, 2000) and others.

In this four-factor model, the parameters of both the objective and risk-neutral

dynamics of the state variables need to be estimated. In addition, we need to solve for the values of each of the four state variables  $W$ ,  $X$ ,  $Y$ , and  $Z$  for each of the 650 weeks in the sample period. At each date, the information set consists of observations of four points along the Treasury curve and five points along the swap curve. Specifically, we use the CMT2, CMT3, CMT5 and CMT10 rates for the Treasury curve, and the three-month Libor, CMS2, CMS3, CMS5, and CMS10 rates for the swap curve. The use of this extensive time series of data makes possible accurate estimation of the parameters of the objective dynamics of the state variables. In addition, since the model involves only four state variables, using nine observations at each date provides us with significant additional cross-sectional pricing information from which the parameters of the risk-neutral dynamics can be more precisely identified.

We focus first on how the four values of the state variables are determined. Similar to Chen and Scott (1993), Duffie and Singleton (1997), Dai and Singleton (2000a) and others, we solve for the values of  $W$ ,  $X$ ,  $Y$ , and  $Z$  by assuming that specific rates are observed without error each week. In particular, we assume that the Libor, CMS10, CMT2, and CMT10 rates are observed without error. These four rates represent the shortest and longest maturity rates along both curves and are typically among the most-liquid maturities quoted, and hence, the most likely to be observed with a minimum of error. Note that Libor is given simply from the expression for a risky zero-coupon bond,

$$\text{Libor} = \frac{a}{360} \left[ \frac{1}{C(1/4)} - 1 \right], \quad (9)$$

where  $a$  is the actual number of days during the next three months. Since CMT and CMS rates represent par rates, they are also easily expressed as explicit function of the values of riskless and risky zero coupon bonds,

$$CMT_T = 2 \left[ \frac{1 - D(T)}{\sum_{i=1}^{2T} D(i/2)} \right], \quad (10)$$

$$CMS_T = 2 \left[ \frac{1 - C(T)}{\sum_{i=1}^{2T} C(i/2)} \right]. \quad (11)$$

Given a parameter vector, we can then invert the closed-form expressions for these four rates to solve for the corresponding four values of the state variables using a standard nonlinear optimization technique. While this process is straightforward, it is computationally very intensive since the inversion must be repeated for every trial value of the parameter vector utilized by the numerical search algorithm in maximizing the likelihood function.

For each of the four state variables, the parameters to be estimated are the  $\alpha$ ,  $\beta$ , and  $\sigma$  terms for the objective dynamics, and the  $\mu$  and  $\kappa$  terms defining the drift of the risk-neutral dynamics. Along with the parameter  $\gamma$  in equation (4), this gives a total of 21 parameters to estimate. In actuality, however, not all parameters can be identified from the data. For example, Dai and Singleton (2000a) show that only a linear combination of the  $\mu$  terms can be identified from zero-coupon bond prices in a Gaussian framework. To avoid identification problems, we impose the constraint  $\mu_X = \mu_Y = 0$ . It is easily shown that this constraint has no effect on the properties of the estimated values of  $r$  and  $\lambda$ . With this constraint, the total number of model parameters to be estimated is 19.

To define the log likelihood function, let  $S'_t = [W_t, X_t, Y_t, Z_t]$ . Furthermore, let  $R_{1,t}$  be the vector of the four rates assumed to be observed without error at time  $t$ , and let  $R_{2,t}$  be the vector of the remaining five observed rates. Using the closed-form solution, we can solve for  $S_t$  from  $R_{1,t}$

$$S_t = h(R_{1,t}, \Theta), \quad (12)$$

where  $\Theta$  is the parameter vector. The conditional log likelihood function for  $S_{t+\Delta t}$  is

$$f_{t+\Delta t}(S) = -\frac{1}{2}(S_{t+\Delta t} - M_t)' \Sigma_S^{-1} (S_{t+\Delta t} - M_t) - \frac{1}{2} \ln |\Sigma_S|, \quad (13)$$

where

$$M_t = \begin{bmatrix} e^{-\beta_W \Delta t} W_t + \alpha_W (1 - e^{-\beta_W \Delta t}) \\ e^{-\beta_X \Delta t} X_t + \alpha_X (1 - e^{-\beta_X \Delta t}) \\ e^{-\beta_Y \Delta t} Y_t + \alpha_Y (1 - e^{-\beta_Y \Delta t}) \\ e^{-\beta_Z \Delta t} Z_t + \alpha_Z (1 - e^{-\beta_Z \Delta t}) \end{bmatrix},$$

and where the covariance matrix  $\Sigma_S$  is diagonal with diagonal elements

$$\begin{bmatrix} \frac{\sigma_W^2}{2\beta_W} (1 - e^{-2\beta_W \Delta t}) \\ \frac{\sigma_X^2}{2\beta_X} (1 - e^{-2\beta_X \Delta t}) \\ \frac{\sigma_Y^2}{2\beta_Y} (1 - e^{-2\beta_Y \Delta t}) \\ \frac{\sigma_Z^2}{2\beta_Z} (1 - e^{-2\beta_Z \Delta t}) \end{bmatrix}.$$

Let  $\epsilon_{t+\Delta t}$  denote the vector of differences between the observed value of  $R_{2,t+\Delta t}$  and the value implied by the model.<sup>9</sup> The log likelihood function for  $\epsilon_{t+\Delta t}$  is given by

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<sup>9</sup>We assume that the  $\epsilon$  terms are independent. In actuality, the  $\epsilon$  terms could be correlated. As is shown later, however, the variances of the  $\epsilon$  terms are very small

$$g_{t+\Delta t}(\epsilon) = -\frac{1}{2}\epsilon'_{t+\Delta t} \Sigma_{\epsilon}^{-1} \epsilon_{t+\Delta t} - \frac{1}{2} \ln |\Sigma_{\epsilon}|, \quad (14)$$

where  $\Sigma_{\epsilon}$  is a diagonal matrix with diagonal elements  $\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2$ , and  $\sigma_5^2$ . Since  $S_{t+\Delta t}$  and  $\epsilon_{t+\Delta t}$  are assumed to be independent, the log likelihood function for  $[S_{t+\Delta t}, \epsilon_{t+\Delta t}]'$  is simply

$$f_{t+\Delta t}(S) + g_{t+\Delta t}(\epsilon). \quad (15)$$

The final step in specifying the likelihood function consists of changing variables from the vector  $[S_t, \epsilon_t]'$  of state variables and error terms to the vector  $[R_{1,t}, R_{2,t}]'$  of rates actually observed. It is easily shown that the determinant of the Jacobian matrix is given by  $|J| = \left| \frac{\partial h(R_1)}{\partial R'_1} \right|$ . This then implies that the log likelihood function for the data is

$$\sum_t (f_{t+\Delta t}(S(R_1)) + g_{t+\Delta t}(\epsilon(R_1, R_2)) + |J|). \quad (16)$$

With the introduction of the five parameters for the variance of the  $\epsilon$  terms,  $\sigma_i^2, i = 1, 2, \dots, 5$ , the log likelihood function now depends explicitly on 24 parameters.

From this log likelihood function, we now solve directly for the maximum likelihood parameter estimates using a standard nonlinear optimization algorithm. In doing this, we initiate the algorithm at a wide variety of starting values to insure that the global maximum is achieved. Furthermore, we check the results using an alternative genetic algorithm which has the property of being less susceptible to finding local minima. These diagnostic checks confirm that the algorithm converges to the global maximum and that the parameter estimates are robust to perturbations of the starting values.

Table 2 reports the maximum likelihood parameter estimates and their asymptotic standard errors. As shown, there are clear differences between the objective and risk-neutral parameters. These differences have major implications for the dynamics of the key variables  $r$  and  $\lambda$  which we consider in the next two sections. The differences themselves reflect the market prices of risk for the state variables and also have important implications for the expected returns from bearing credit and liquidity risk. We note that the  $\beta$  and  $\kappa$  parameters are all estimated to be positive; the estimation procedure does not constrain these parameters to be positive.

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and the assumption of independence is unlikely to have much effect on the estimated model parameters.

One key result that emerges from the maximum likelihood estimation is that the four-factor model fits the data extremely well. At the maximum likelihood parameter estimates, the RMSE taken over all of the data points in the sample is only 6.08 basis points. The fact that the model captures the market rates within a few basis points is particularly impressive when one considers that we are estimating the parameters of both the swap and Treasury term structures simultaneously. In addition, the magnitude of the pricing errors is less than those reported in previous studies.<sup>10</sup> This suggests that the use of four factors results in an improved description of the term structure. Furthermore, these results imply that the joint estimation of both term structures may provide more accurate estimates than when the two term structures are estimated individually.

## 5. THE IMPLIED FINANCING RATE

The instantaneous riskless rate  $r$  plays a central role in many continuous-time term structure models. In addition to being the shortest-maturity rate,  $r$  can also be viewed as the cost of borrowing on short-term riskless loans such as those fully secured by riskless Treasury bond collateral. Traditionally, the cost of riskless borrowing is often interpreted as the Treasury-bill rate since this is the rate at which the U.S. Treasury can borrow short-term funds. Among practitioners, however, the Treasury-bill rate is generally viewed as a contaminated measure of the true riskless rate. The reason for this is the widespread belief that the extreme liquidity of Treasury bills makes them worth slightly more than the present value of their cash flows, and hence, that Treasury-bill rates are downward biased estimates of the true cost of riskless borrowing. In a recent paper, Longstaff (2000) suggests considering general Treasury collateral repo rates as an alternative measure of the riskless rate. The rationale for this measure is that repo loans that are overcollateralized by default-free Treasury bonds are essentially riskless short-term loans. Because repo loans are financial contracts rather than securities, however, they are not as affected by liquidity effects as actual Treasury securities.<sup>11</sup>

An important advantage of our approach is that we can solve for the value of  $r$  endogenously and then contrast it with market rates. This allows us to examine directly whether the implied value of  $r$  obtained from longer-term Treasury bonds more closely resembles Treasury-bill rates or repo rates. In this model, the implied rate  $r$  has the interpretation as the cost of carrying a position in the longer-term Treasury bonds defining the CMT rates. If these longer-term bonds do not have special liquidity value, then  $r$  should represent the riskless interest rate for the market. On the other hand, if

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<sup>10</sup>For example, see Duffie and Singleton (1997) and Duffie (2000).

<sup>11</sup>For a discussion of special and general collateral repo rates, see Duffie (1996) and Longstaff (2000).

longer-term Treasury bonds have liquidity value, then the estimated value of  $r$  takes on the interpretation of a special repo rate in the sense of Duffie (1996). Specifically, since  $r$  is implied from the cross section of CMT rates,  $r$  represents the average or typical short-term special repo rate for the on-the-run bonds in the sample.<sup>12</sup> In this case,  $r$  may then be less than the true riskless rate. Thus,  $r$  should represent a lower bound on the true riskless rate. To reflect the role that  $r$  plays in this model, we designate  $r$  the implied financing rate.

From equation (3),  $r$  is the sum of the values of the state variables  $W$ ,  $X$ , and  $Y$ . To make estimates of the implied finance rate comparable with the three-month general collateral repo and Treasury-bill rates in the sample, we redefine the implied finance rate slightly to be the yield implied by a three-month zero coupon bond  $D(1/4)$ . Using the maximum likelihood parameter values, the values of the state variables are implied from the data as described previously. Given the values of the state variables,  $D(1/4)$  is obtained directly from the closed-form expression in equation (3). Table 3 reports summary statistics for the three-month general collateral repo rates, implied finance rates, and Treasury-bill rates along with the spreads between these rates. Figure 2 graphs the difference between the implied finance rate and the Treasury-bill rate, and difference between the repo rate and the implied finance rate.

As shown, the implied finance rate typically lies between the general collateral repo rate and the Treasury-bill rate. On average, the implied finance rate is 5.9 basis points below the repo rate, but 31.9 basis points above the Treasury-bill rate. The median implied finance rate is 1.7 basis points below the median repo rate, but 30.8 basis points above the median Treasury-bill rate. Since the implied finance rate can be viewed as a lower bound on the actual riskless rate, these results strongly suggest that the general collateral repo rate is closer to the actual riskless rate than the Treasury-bill rate. In particular, the difference between the repo rate and the implied finance rate is typically positive, but there is an extended period when the implied finance rate essentially equals the repo rate. Thus, the general collateral repo rate appears to roughly represent an upper bound on the implied finance rate.

The spread between the implied finance rate and the Treasury-bill rate can be

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<sup>12</sup>Note that we are using a slightly broader interpretation of the special repo rate since special repo rates are typically associated with a specific Treasury bond. An advantage of this approach, however, is that by using a common special short-term repo rate  $r$  for all on-the-run bonds, we can solve for the implied specialness premia for individual bonds by integrating the difference between the short-term special and general collateral repo rates over the appropriate horizon. This is equivalent to solving for the implied longer-term special repo rates for individual Treasury bonds. Because these specialness premia or implied term special repo rates are based on the dynamics of  $r$ , they reflect not only the current liquidity of the bonds, but also the possibility of future increases in their liquidity. In contrast, term special repo rates quoted in the market may be for horizons that are too short to fully capture the effect of specialness on the value of a bond throughout its life.



interpreted as a measure of the relative liquidity of Treasury bills and on-the-run Treasury bonds. As shown, Treasury bills appear to be much more liquid than Treasury bonds during much of the sample period. During the 1990-93 period, however, the liquidity of Treasury bonds and bills appears to converge. During this period, the spread between the repo rate and the implied finance rate also converges to near zero. This suggests that there is little liquidity component in the prices of either Treasury bonds or bills during this period. After the hedge fund crisis of 1998, the implied finance rate actually dips below the Treasury-bill rate, which suggests that longer-term on-the-run Treasury bonds may actually have become more liquid than Treasury bills. This may be related to the fact that the U.S. Treasury no longer auctions one-year Treasury bills on a regular basis.

If we equate the general collateral repo rate with the riskless rate, then the difference between the repo rate and the implied finance rate has the simple interpretation of the average implied specialness of the on-the-run Treasury bonds used to compute CMT rates. Figure 2 shows that this implied specialness varies significantly over time. During the first part of the sample period, the implied specialness is as high as 45 basis points, suggesting that the prices of Treasury bonds have a large liquidity component. During the 1995-1998 period, the implied specialness of the Treasury bonds essentially disappears and the implied finance rate closely mirrors the general collateral repo rate. After the hedge-fund crisis of 1998, however, the implied specialness of the bonds increases dramatically, reaching a high of 78 basis points near the end of the sample period.

To quantify the size of the liquidity or specialness component in the prices of on-the-run Treasury bonds, we do the following. First, we denote the implied specialness (general collateral repo rate - the implied finance rate) by  $I_t$ , and assume that  $I_t$  follows a standard Ornstein-Uhlenbeck process. Estimating the parameters of this process by maximum likelihood gives the following dynamic specification for  $I_t$ ,

$$dI = 7.14896 ( .000636 - I ) dt + .00789 dB_I, \quad (17)$$

where  $B_I$  is a standard Brownian motion. For a zero-coupon Treasury bond with maturity  $T$ , the present value benefit or specialness premium from being able to borrow at the special repo rate rather than the general repo rate equals

$$D(T) - D(T) E \left[ \exp \left( - \int_0^T I_t dt \right) \right], \quad (18)$$

under the assumptions that  $B_I$  is independent of the other Brownian motions in the term structure model and that the market price of  $I$  risk is zero. Evaluating this expectation gives the following expression for the liquidity premium,

$$D(T) (1 - A_I(T) \exp(-B_I(T)I)), \quad (19)$$

where  $A_I$  and  $B_I$  are defined as in equation (7) with the corresponding parameter estimates in equation (17) substituted in for  $\kappa$ ,  $\mu$ , and  $\sigma$ . Based on this approach, Table 4 provides estimates of the size of the liquidity or specialness premia in the prices of the Treasury bonds. Figure 3 graphs the estimated premia during the sample period.

As shown, the value of liquidity or specialness premium in the prices of on-the-run Treasury bonds can be substantial. For the two-year Treasury note, the premium ranges from about 10 cents to 20 cents during the sample period per \$100 notional amount, which translates roughly to a 4 to 9 basis point effect on the yield. Thus, there is significant time variation in the value of the premium. For the ten-year Treasury note, the premium was typically in excess of 50 cents per \$100 notional or roughly 6.7 basis points in terms of yield to maturity. During the latter portion of the sample period, the premium was as high as 66 cents, or 8.8 basis points of yield. These results indicate that the value of liquidity can represent an important time-varying component of the value of a Treasury bond. These estimates of the liquidity premia in bond prices are generally consistent with those reported by Amihud and Mendelson (1991), Boudoukh and Whitelaw (1993), Kamara (1994), Longstaff (1995), Jordan and Jordan (1997) and others.

As an additional diagnostic for the estimated specialness premia, we also use a set of term special repo rates provided to us by Salomon Smith Barney. This data set reports the longest term special repo rates for individual Treasury bonds available in the market as of June 30, 2000, along with the general collateral repo rate for the same term. The implied premium per \$100 value of the bond is given by simply taking the difference between the general collateral and special repo rates and multiplying by the term of the repo measured in years. This makes clear that the value of the specialness premium can be viewed as the interest savings an investor who finances his purchase of the bond would receive by being able to finance at the special repo rate rather than the general collateral repo rate. Table 5 reports the special and general collateral rates for the bonds with maturities of ten years or less along with the implied specialness premia. The two-year, five-year, and ten-year on-the-run bonds are denoted by an asterisk.

As shown in Table 5, a number of Treasury bonds trade special in the repo market. For many of these bonds, the difference between the term special and general collateral repo rates is small, and the implied specialness premium is likewise small. For the on-the-run bonds, however, the value of the specialness premia is substantial. In particular, the specialness premia for the two-year, five-year, and ten-year on-the-run bonds given in Table 5 are 8.0 cents, 50.5 cents, and 64.3 cents respectively.<sup>13</sup>

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<sup>13</sup>There is no on-the-run three-year bond on June 30, 2000 because the Treasury

This agrees well with the average implied specialness premia reported in Table 4.<sup>14</sup>

## 6. THE SPREAD PROCESS

The spread process  $\lambda$  plays a particularly important role in the Duffie and Singleton (1997) credit modeling framework. Recall that in this framework, the spread  $\lambda$  may consist of both default-risk and liquidity components. Since the Libor rate is fitted exactly in the maximum likelihood estimation, the implied spread  $\lambda$  can be thought of as the difference between the Libor rate and the implied finance rate. From equations (3) and (4),  $\lambda$  is a function of all four state variables. Table 2 reports that the maximum likelihood estimate of the parameter  $\gamma$  is -.07017, which implies that there is a strong negative relation between the level of  $\lambda$  and the level of the riskless rate  $r$ . This relation is consistent with the negative relation between rates and spreads implied by a number of fundamental models of credit spreads including Merton (1974), Black and Cox (1976), and Longstaff and Schwartz (1995).

Figure 4 graphs the time series of  $\lambda$  for the sample period. As illustrated, the spread  $\lambda$  varies significantly over time. For example, at the beginning of the sample period, the spread is on the order of 80 basis points. During the latter 1990s, however, the spread decreases significantly and at one point becomes nearly zero. The period during which the spread is nearly zero coincides with the period during which there is little apparent liquidity component in Treasury bonds as measured by the implied specialness estimate. This suggests that this period may represent a time when the market viewed both the liquidity of the swap market as identical to that of Treasury bonds and the probability that banks quoting Libor rates could default as essentially zero. Although the model allows the spread to become negative, only two observations are actually (slightly) negative.

The maximum likelihood parameter estimates provide a complete specification of both the objective and risk-neutral dynamics of the state variables. From these dynamics, it is straightforward to solve for the objective and risk-neutral dynamics of the spread. Perhaps the best way to illustrate the differences between the objective and risk-neutral dynamics of the spread is by contrasting its expected value under the two measures. Figure 5 graphs the expected value of  $\lambda$  five years in the future conditional on the current value of the state variables. As shown, the expected value of the spread under the objective measure is nearly constant. The reason for this is that under the objective measure, all of the state variables display significant mean reversion. For example, the objective mean-reversion parameter for the  $Z$  component

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recently stopped auctioning three-year bonds.

<sup>14</sup>Conversations with market practitioners indicate that there are occasionally periods during which the specialness premia for some on-the-run Treasury issues implied by market term special repo rates are on the order of twice those shown in Table 5.

of the spread is .878, implying that shocks in  $Z$  dissipate within several years. In contrast, the risk-neutral mean-reversion parameter for the  $Z$  component of the spread is .002, implying that  $Z$  is essentially a random walk under the risk-neutral measure. Similar results hold for the state variable  $Y$ . Because these state variables display little mean reversion under the risk-neutral measure, the expected value of the spread five years in the future is virtually the same as its current value, and the expected value of the spread is much more variable under the risk-neutral measure. An important implication of this is that the market acts as if shocks to credit spreads were permanent when pricing securities, despite the fact that actual spreads are rapidly mean reverting.

In theory, the spread  $\lambda$  could include both default-risk and liquidity components. This raises the question of whether the separate components can be identified. From equation (4), the spread  $\lambda$  consists of a component that is proportion to  $r$ , and an orthogonal component  $Z$ . Clearly, however, there is no way in which these two components could be mapped directly into the default-risk and liquidity components. For example, one could argue that default risk for banks quoting Libor could be related to the level of  $r$ . Alternatively, one could just as easily argue that default risk may be orthogonal to  $r$ .

An alternative and more promising approach to identifying the default-risk and liquidity components of the spread is suggested by the results in the previous section. Observe that the spread  $\lambda$  can be decomposed into the difference between Libor and the general collateral repo rate, and the difference between the general collateral repo rate and the implied finance rate. This first component can be given the interpretation as a pure default-risk spread since it measures the difference between Libor and generic Treasury bonds which are not on special, and presumably, have little or no liquidity component to their value. The second component is directly a measure of the specialness of the bonds used in our sample and can be viewed as a pure liquidity spread since the spread between on-the-run Treasury bonds and generic or general collateral Treasury bonds should not include a default-risk component.

Adopting this approach to decomposing the spread  $\lambda$  into default-risk and liquidity components, Table 6 reports summary statistics for the spread and its components. As can be seen, the liquidity component is typically much smaller than the default-risk component. In particular, the mean of the liquidity component is .059 which is only 15.7 percent of the average value of the spread. On the other hand, however, the liquidity component is significantly more volatile than the credit component.

While the liquidity component is smaller than the default-risk component on average, the liquidity component may become larger. To illustrate this, Table 6 also reports summary statistics for the ratio of the liquidity component to the default-risk component.<sup>15</sup> Figure 6 graphs the default-risk and liquidity components as well as the

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<sup>15</sup>We compute this ratio rather than the liquidity to spread ratio because the spread can become negative.

the ratio of the liquidity to default-risk components. As shown, the liquidity component is typically fairly small relative to the default-risk component. Occasionally, however, the liquidity component can represent a much larger portion of the total spread than the default-risk component. This is particularly true after the hedge-fund crisis of 1998 where the liquidity component is often twice or three times as large as default-risk component.

Although the liquidity component tends to be a smaller part of the total credit spread, it is important to recognize that it is responsible for a disproportionately large portion of the total variation in the credit spread. To see this, Figure 7 presents scatterdiagrams of the credit spread against its default-risk and liquidity components. As illustrated, the correlation between the spread and the liquidity component is much higher than the correlation between the spread and the default-risk component. In particular, the correlation of the spread with the default-risk component is .346 while the correlation of the spread with the liquidity component is .786. Furthermore, the correlation of weekly changes in the spread with changes in the default-risk component is .064 while the correlation of weekly changes in the spread with changes in the liquidity component is .418. Taken together, these results suggest that most of the variation in spreads is driven by liquidity rather than changes in the probability of default. An immediate implication of this is that if there are any credit premia in market prices, these credit premia are more likely compensation for liquidity risk rather than default risk.

## 7. THE MARKET PRICE OF CREDIT RISK

The primary objective of this paper is to examine how the market prices the credit risk in interest rate swaps. To this end, we focus on the premia that are incorporated into the expected returns of bonds implied by the estimated term structure model. These premia are given directly from the differences between the objective and risk-neutral parameters of the model.

To provide some perspective for these results, however, it is useful to also examine the implications of the model for the term premia in Treasury bond prices. Applying Ito's Lemma to the closed-form expression for the value of a riskless zero-coupon bond  $D(T)$  given in equation (7) results in the following expression for its instantaneous expected return

$$\begin{aligned} & r + ((\beta_W - \kappa_W)W + \mu_W\kappa_W - \alpha_W\beta_W))B_W(T) \\ & + ((\beta_X - \kappa_X)X + \mu_X\kappa_X - \alpha_X\beta_X))B_X(T) \\ & + ((\beta_Y - \kappa_Y)Y + \mu_Y\kappa_Y - \alpha_Y\beta_Y))B_Y(T). \end{aligned} \tag{20}$$

The first term in this expression is the riskless rate and the sum of the remaining terms

is the instantaneous term premium for the bond. This term premium is time varying since it depends explicitly on the state variables. To solve for the unconditional term premium, we take the expectation over the objective measure of the state variables which gives

$$\kappa_W(\mu_W - \alpha_W)B_W(T) + \kappa_X(\mu_X - \alpha_X)B_X(T) + \kappa_Y(\mu_Y - \alpha_Y)B_Y(T). \quad (21)$$

Now applying Ito's Lemma to the closed-form expression for the price of the risky zero-coupon bond  $C(T)$  given in equation (8) leads to the following expression for the instantaneous expected return

$$\begin{aligned} r + \lambda + ((\beta_W - \kappa_W)W + \mu_W\kappa_W - \alpha_W\beta_W))B_W^*(T) \\ + ((\beta_X - \kappa_X)X + \mu_X\kappa_X - \alpha_X\beta_X))B_X^*(T) \\ + ((\beta_Y - \kappa_Y)Y + \mu_Y\kappa_Y - \alpha_Y\beta_Y))B_Y^*(T) \\ + ((\beta_Z - \kappa_Z)Z + \mu_Z\kappa_Z - \alpha_Z\beta_Z))B_Z(T). \end{aligned} \quad (22)$$

The sum of the first two terms  $r + \lambda$  in this expression represents the instantaneous risky rate. The sum of the remaining four terms can be interpreted as the combined term premium and credit premium. To identify the credit premium separately, we subtract the term premium for a zero-coupon riskless bond with the same maturity from the combined term and credit premium in the risky bond. Thus, the credit premium equals the difference between the expected return of a risky zero-coupon bond (minus  $r + \lambda$ ) and the expected return on a riskless zero-coupon bond (minus  $r$ ) with the same maturity. As before, the credit premium is time varying through its dependence on the state variables. Taking the expectation with respect to the objective measure for the state variables and subtracting the expression for the unconditional term premium gives the following expression for the unconditional credit premium

$$\begin{aligned} \gamma\kappa_W(\mu_W - \alpha_W)B_W(T) + \gamma\kappa_X(\mu_X - \alpha_X)B_X(T) \\ + \gamma\kappa_Y(\mu_Y - \alpha_Y)B_Y(T) + \kappa_Z(\mu_Z - \alpha_Z)B_Z(T). \end{aligned} \quad (23)$$

Focusing first on the unconditional premia, Table 7 reports the unconditional term premia for riskless zero-coupon bonds with maturities ranging from one to ten years. Table 7 also reports the unconditional credit premia for risky zero-coupon bonds with the same maturities. These unconditional premia are also graphed in Figure 8. As shown, the mean term premia are positive and monotonically increasing functions of time to maturity. Mean term premia range from about 64 basis points for a one-year horizon to about 237 basis points for a ten-year horizon. These estimates of unconditional term premia are similar to those reported by Fama (1984), Fama and Bliss (1987), and others.

Table 7 also shows that unconditional credit premia are positive and increasing functions of maturity. The mean credit premium for a one-year horizon is only about 2 basis points. Thus, there is very little compensation on average for bearing short-term credit risk. At longer horizons, however, the mean credit premium is much larger. For example, the mean credit premium for a ten-year horizon is 45 basis points. The convex shape of the unconditional credit premium curve indicates that investors require sharply higher credit premia as the maturity of the bond increases. This pattern contrasts with that observed for the unconditional term premia.

To give some sense of the time variation in term and credit premia, Figure 9 graphs these premia for a one-year maturity zero-coupon bond. As illustrated, the term premium displays a significant amount of variation. The term premium is usually positive, but has generally tended downward throughout the sample period. During the latter part of the sample period, the estimated term premium is occasionally as negative as -150 basis points. The gradual decline in term premia throughout the sample period is consistent with the recent tendency of the Treasury term structure towards flatter shapes.

The time series of the credit premium displays a number of surprising features. Recall that the average credit premium for a one-year horizon is only about two basis points. Figure 9 shows that while the average is small, the actual credit premium varies significantly over time and is often large in absolute terms. Most surprisingly, the credit premium is significantly negative for nearly one half of the sample period. During this timeframe, the credit premium appears relatively stable at a level of about -20 basis points. The credit premium first becomes negative in approximately 1992 and remains negative until the summer of 1998. This is about when the Russian government defaulted on a large issue of its ruble-denominated debt. The credit premium ranges from a high of about 59 basis points to a low of about -35 basis points. Despite the variation, however, the credit premium appears to be much more predictable and less volatile than the term premium.

## 8. CONCLUSION

This paper examines how the market prices the credit and liquidity risk inherent in interest rate swaps relative to Treasury bonds. A number of key results emerge from this analysis. First, we find that on-the-run Treasury bonds have a significant liquidity component to their value. This liquidity component can be as much as .66 percent of the notional amount of a ten-year Treasury bond. The value of this liquidity component varies significantly over time. Second, we find that most of the variation in swap spreads is due to changes in the liquidity of Treasury bonds rather than to changes in the default risk associated with the swap curve. This implies that swap spread risk premia built into swap rates should be viewed primarily as compensation for liquidity risk. Finally, we find that the market prices the credit risk of swaps.

The market price of credit and risk, however, varies over time and was significantly negative for much of the 1990s.

There are a number of possible extensions to this research. For example, the approach of solving for the implied financing rate could be applied to the term structures for corporate bond issuers and then used to identify the liquidity components of their spreads.<sup>16</sup> One major puzzle is why the credit premia implicit in swap spreads was so negative during the 1990s, and only became positive again after the hedge-fund crisis of Fall 1998. Certainly, these results are difficult to reconcile with a view of the market in which investors are aware of the historical variability in swap spreads and where expected returns compensate investors for their exposure to risk. A possible resolution of this puzzle may be that most of the credit risk reflected in swap spreads may actually represent the liquidity risk of Treasury bonds. From this perspective, Treasury bonds may be subject to a unique risk which does not affect pure contracts such as swaps, and may be priced accordingly in the market. Clearly, further research is necessary to resolve this issue.

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<sup>16</sup>Huang and Huang (2000) focus on the estimation of the liquidity components in corporate bond prices.



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Table 1

**Summary Statistics for the Data.** This table reports the indicated summary statistics for both the level and first difference of the indicated data series. The term  $\rho$  represents the first-order serial correlation coefficient. The data consist of 650 weekly observations from January 1988 to June 2000. Libor denotes the three-month Libor rate, CMS denotes the swap rate for the indicated maturity, CMT denotes the constant maturity Treasury rate for the indicated maturity, and SS denotes the swap spread for the indicated maturity where the swap spread is defined as the difference between the corresponding CMS and CMT rates.

	Levels						First Differences					
	Mean	Standard Deviation	Minimum	Median	Maximum	$\rho$	Mean	Standard Deviation	Minimum	Median	Maximum	$\rho$
Libor	5.984	1.733	3.188	5.719	10.560	.998	-.001	.109	-.505	.000	.563	.090
CMS2	6.599	1.558	3.840	6.255	10.750	.995	-.002	.154	-.540	.000	.560	-.012
CMS3	6.841	1.475	4.230	6.453	10.560	.994	-.003	.151	-.510	.000	.540	-.041
CMS5	7.155	1.371	4.800	6.840	10.330	.994	-.003	.145	-.460	.000	.480	-.057
CMS10	7.529	1.286	5.170	7.245	10.130	.995	-.004	.134	-.420	.000	.530	-.069
CMT2	6.224	1.434	3.710	5.965	9.840	.995	-.002	.143	-.440	.000	.470	-.008
CMT3	6.404	1.361	4.040	6.165	9.760	.994	-.003	.144	-.450	.000	.480	-.012
CMT5	6.670	1.260	4.040	6.430	9.650	.993	-.004	.143	-.460	-.010	.450	-.039
CMT10	6.965	1.214	4.310	6.730	9.490	.994	-.005	.132	-.380	.000	.460	-.056
SS2	.375	.191	.040	.363	.940	.932	.000	.071	-.340	.000	.350	-.396
SS3	.436	.200	.100	.418	.900	.949	.000	.064	-.300	.000	.340	-.403
SS5	.485	.226	.120	.435	1.043	.965	.000	.059	-.290	.000	.270	-.362
SS10	.565	.235	.155	.503	1.343	.977	.000	.050	-.230	.000	.220	-.340

Table 2

**Maximum Likelihood Estimates of the Model Parameters.** This table reports the maximum likelihood estimates of the parameters of the four-factor term structure model along with their asymptotic standard errors. The  $\alpha$  and  $\beta$  terms define the drift of the objective dynamics of the state variables, while the  $\mu$  and  $\kappa$  terms define the drift of the risk-neutral dynamics of the state variables. The  $\sigma$  terms denote the instantaneous volatilities of the state variables and  $\gamma$  denotes the sensitivity of the credit spread to the riskless rate. The terms  $\sigma_i, i = 1, 2, \dots, 5$  denote the standard deviations of the difference between model and observed values of the CMS2, CMS3, CMS5, CMT3, and CMT5 rates respectively. The asymptotic standard errors are based on the inverse of the information matrix computed from the Hessian matrix for the log likelihood function.

Parameter	Value	Std. Error
$\alpha_W$	.11363	.00686
$\alpha_X$	-.01657	.03966
$\alpha_Y$	-.04158	.01694
$\alpha_Z$	.00763	.00127
$\mu_W$	.11391	.00686
$\mu_X$	—	—
$\mu_Y$	—	—
$\mu_Z$	.33941	.46019
$\beta_W$	2.56874	.60167
$\beta_X$	.24633	.23571
$\beta_Y$	.35665	.22794
$\beta_Z$	.87755	.39172
$\kappa_W$	5.25031	.62448
$\kappa_X$	.38516	.00907
$\kappa_Y$	.01912	.00197
$\kappa_Z$	.00186	.00262
$\sigma_W$	.02658	.00110
$\sigma_X$	.01623	.00049
$\sigma_Y$	.01186	.00045
$\sigma_Z$	.00359	.00008
$\gamma$	-.07017	.00506
$\sigma_1$	.00115	.00007
$\sigma_2$	.00088	.00006
$\sigma_3$	.00068	.00003
$\sigma_4$	.00042	.00001
$\sigma_5$	.00060	.00002

**Table 3**

**Summary Statistics for the Three-Month General Collateral Repo Rate, the Implied Three-Month Finance Rate, and the Three-Month Treasury-Bill Rate.** This table reports the indicated summary statistics for the level and spreads of the data series. The term  $\rho$  represents the first-order serial correlation coefficient. The data consist of 650 weekly observations from January 1988 to June 2000.

	Mean	Standard Deviation	Minimum	Median	Maximum	$\rho$
General Collateral Rate	5.668	1.682	2.900	5.480	10.150	.998
Implied Finance Rate	5.609	1.672	2.894	5.484	10.274	.997
Treasury-Bill Rate	5.289	1.491	2.690	5.080	9.903	.998
General Collateral Minus Implied Finance Rate	.059	.205	-.620	.017	.782	.857
Implied Finance Minus Treasury-Bill Rate	.319	.315	-.442	.308	1.534	.933
General Collateral Minus Treasury-Bill Rate	.378	.265	-.200	.330	1.410	.931

Table 4

**Summary Statistics for the Liquidity or Specialness Premia in On-The-Run Treasury Bond Prices.** This table reports the indicated summary statistics for the estimated liquidity or specialness premium incorporated into the values of the Treasury bonds. The premium is estimated from the spread between the general collateral repo rate and the implied finance rate. The premia are reported in units of dollars per \$100 dollar notional amount. The data consist of 650 weekly observations from January 1988 to June 2000.

	Mean	Standard Deviation	Minimum	Median	Maximum	$\rho$
Two-Year Treasury Bond	.1256	.0285	.0312	.1200	.2260	.8571
Three-Year Treasury Bond	.1870	.0282	.0932	.1814	.2871	.8570
Five-Year Treasury Bond	.3032	.0277	.2097	.2987	.4047	.8608
Ten-Year Treasury Bond	.5364	.0355	.4356	.5331	.6555	.9308



Table 5

**Special and General Collateral Term Repo Rates and the Implied Specialness Premia for Treasury Bonds as of June 30, 2000.** This table reports the longest quoted term special repo rates for the indicated Treasury bonds along with the corresponding term general collateral repo rate. The implied specialness is computed by multiplying the difference between the two rates by the term of the repo rates measured in years and represents the specialness premium per \$100 value. The on-the-run issues are denoted by an asterisk.

Coupon	Maturity	Longest Repo Term in Days	General Collateral Term Repo Rate	Special Term Repo Rate	Difference	Implied Specialness
5.875	30-Nov-01	92	.06461	.06450	.00011	.003
6.625	31-May-02	78	.06424	.06050	.00374	.080
6.375*	30-Jun-02	32	.06346	.05550	.00796	.070
5.250	15-Aug-03	92	.06461	.06400	.00061	.016
4.250	15-Nov-03	286	.06714	.06650	.00064	.050
4.750	15-Feb-04	92	.06461	.06350	.00111	.028
6.000	15-Aug-04	92	.06461	.06400	.00061	.016
5.875	15-Nov-04	92	.06461	.06300	.00161	.041
6.750*	15-May-05	358	.06765	.06250	.00515	.505
6.500	15-May-05	93	.06462	.06350	.00112	.029
5.625	15-Feb-06	93	.06462	.06400	.00062	.016
6.875	15-May-06	93	.06462	.06400	.00062	.016
7.000	15-Jul-06	93	.06462	.06350	.00112	.029
6.500	15-Oct-06	93	.06462	.06400	.00062	.016
6.250	15-Feb-07	92	.06461	.06400	.00061	.015
6.625	15-May-07	93	.06462	.06400	.00062	.016
6.125	15-Aug-07	93	.06462	.06400	.00062	.016
5.500	15-Feb-08	93	.06462	.06400	.00062	.016
5.625	15-May-08	93	.06462	.06400	.00062	.016
4.750	15-Nov-08	183	.06592	.06500	.00092	.046
5.500	15-May-09	259	.06687	.06600	.00087	.062
6.000	15-Aug-09	298	.06724	.06600	.00124	.101
6.250*	15-Feb-10	354	.06763	.06100	.00663	.643

**Table 6**

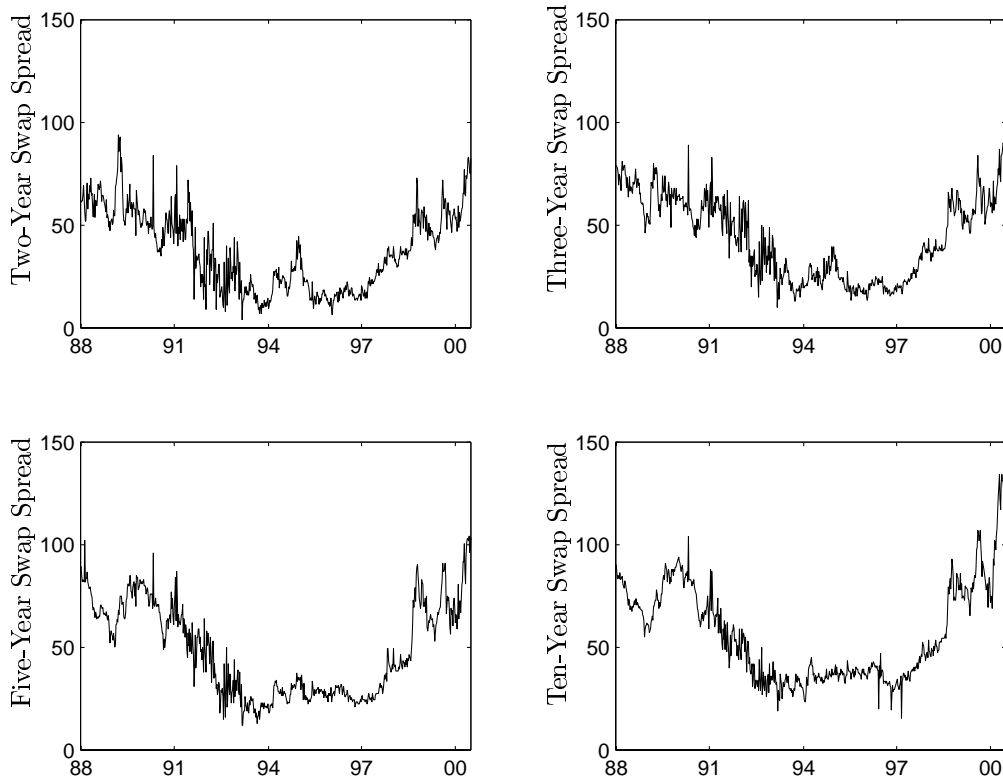
**Summary Statistics for the Credit-Spread Process and its Default-Risk and Liquidity Components.** This table reports the indicated summary statistics for the spread process  $\lambda$  and its default-risk and liquidity components. The default-risk component is defined as the difference between three-month Libor and the general collateral repo rate. The liquidity component is defined as the difference between the general collateral repo rate and the implied financing rate. The data consist of 650 weekly observations from January 1988 to June 2000.

	Mean	Standard Deviation	Minimum	Median	Maximum	$\rho$
Credit-Spread Process	.375	.208	-.058	.333	1.086	.968
Default-Risk Component	.316	.135	.055	.280	1.063	.728
Liquidity Component	.059	.205	-.620	.017	.782	.857
Liquidity/Default-Risk Ratio	.293	.850	-1.214	.050	6.638	.731

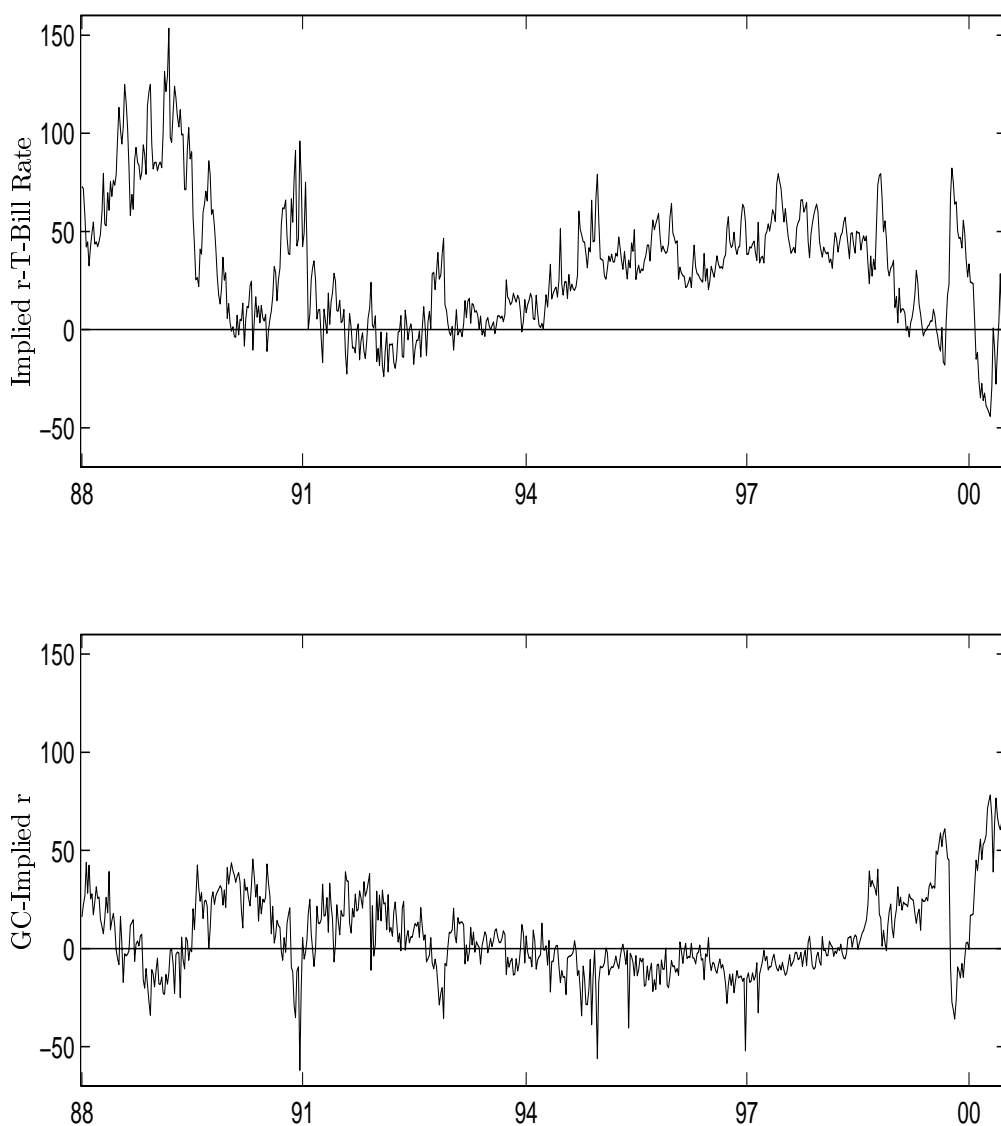
**Table 7**

**Unconditional Term and Credit Premia.** This table reports unconditional values of the term premia in zero-coupon Treasury bond prices implied by the model. Also reported are the unconditional values of the credit premia in zero-coupon risky bonds implied by the model.

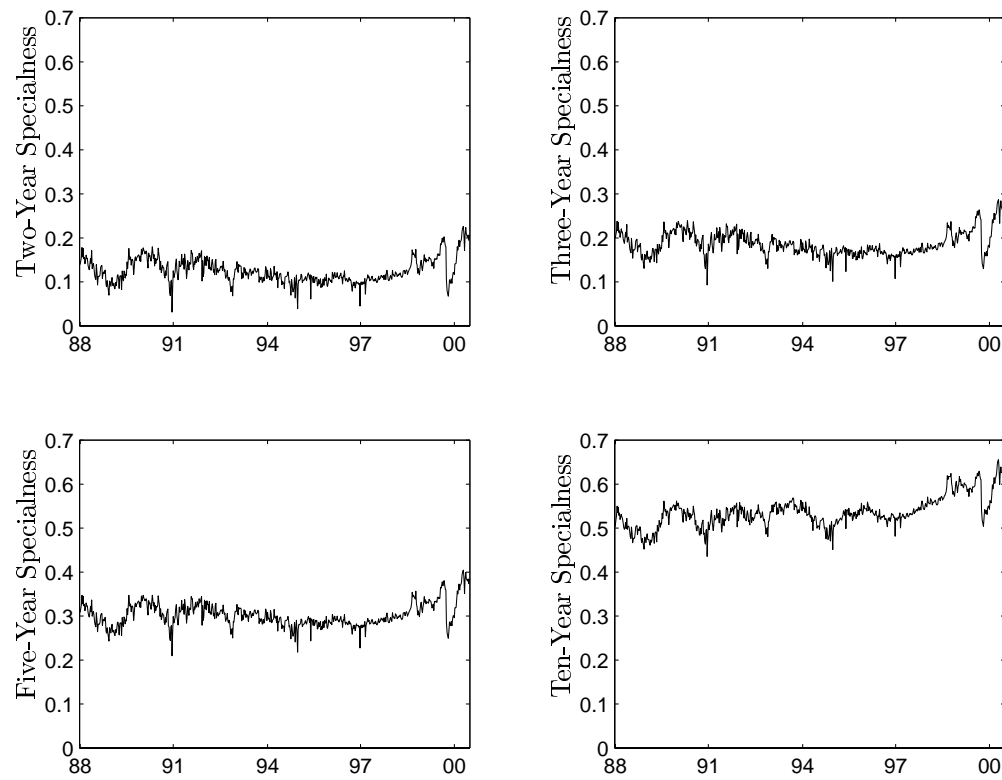
	Maturity in Years									
	1	2	3	4	5	6	7	8	9	10
Term Premium	.636	1.074	1.295	1.636	1.822	1.971	2.094	2.199	2.291	2.373
Credit Premium	.017	.048	.087	.131	.180	.230	.283	.336	.391	.446



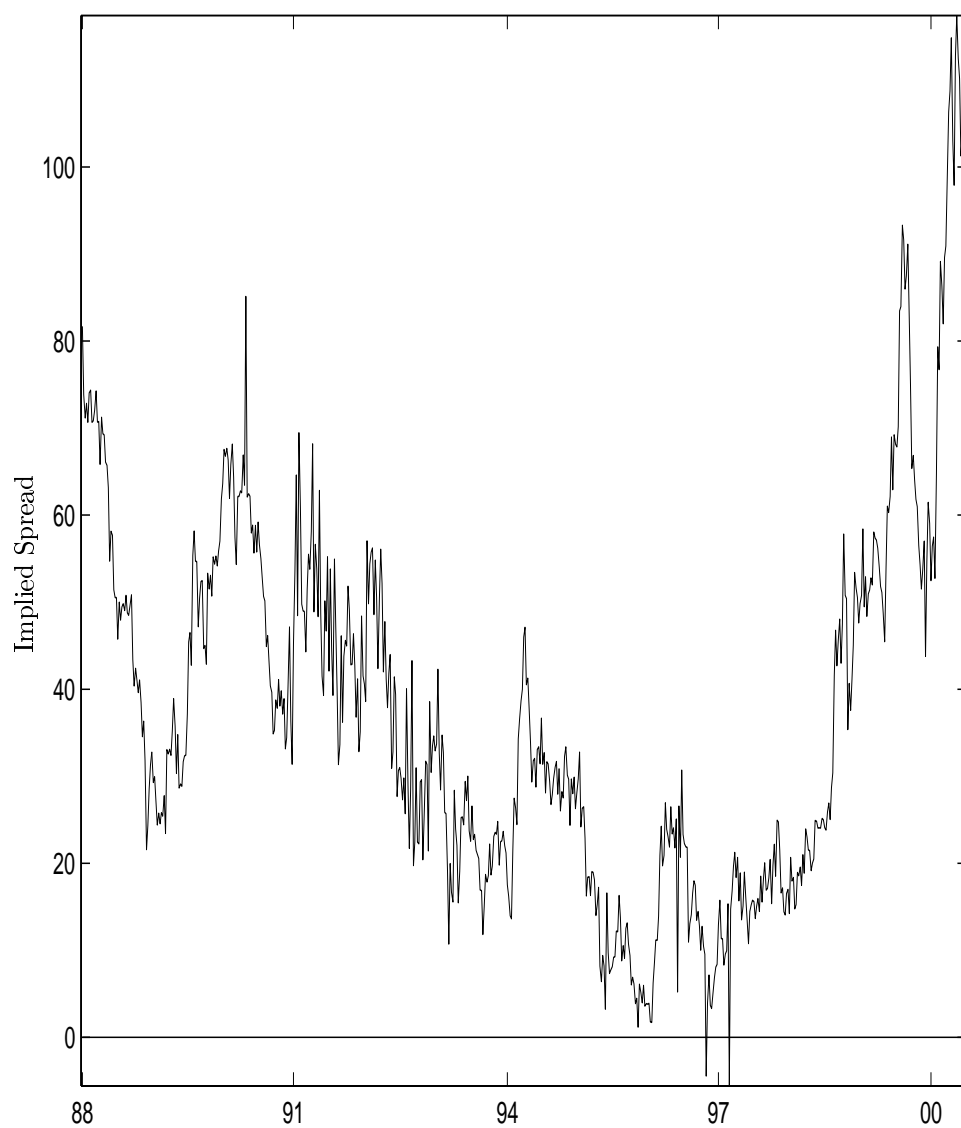
**Figure 1. Swap Spreads.** Weekly time series of swap spreads measured in basis points. The sample period is from January 1988 to June 2000.



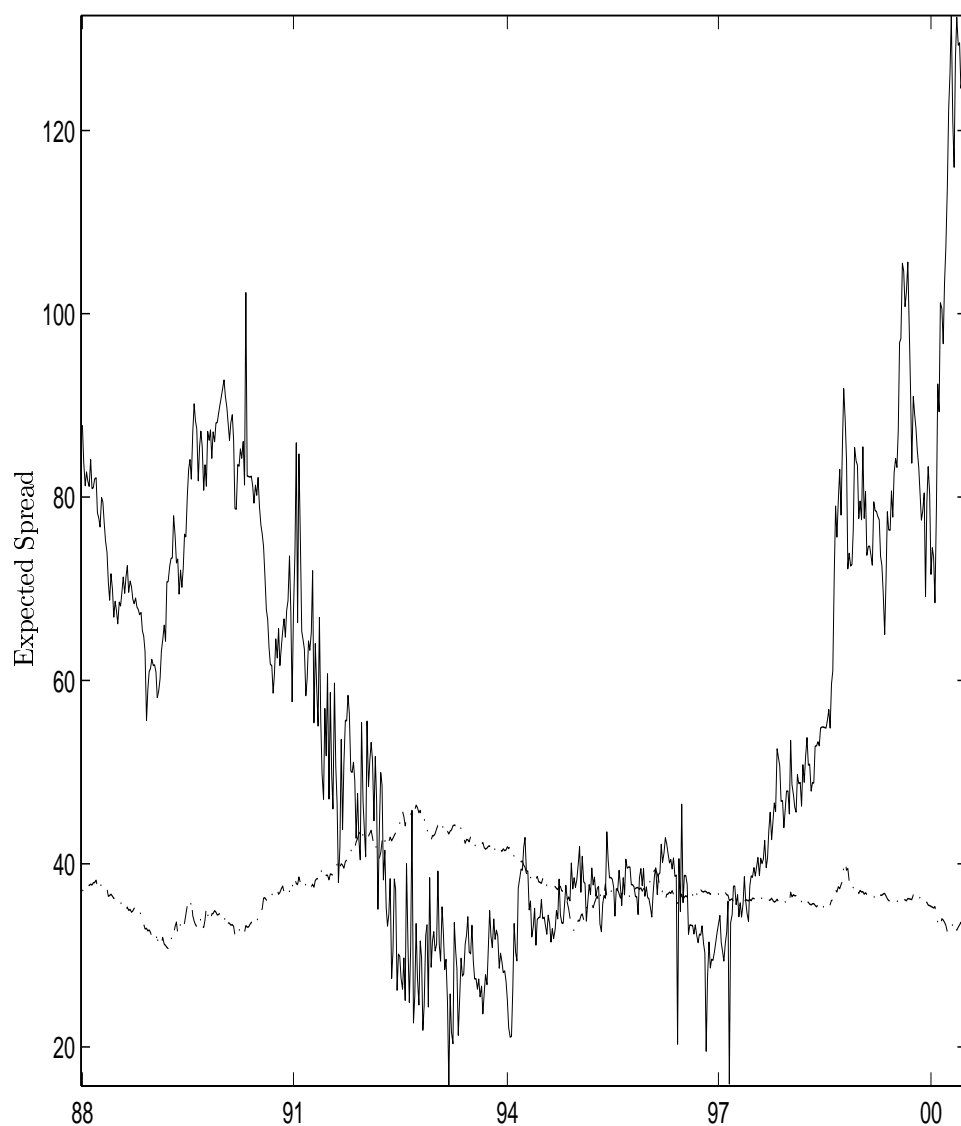
**Figure 2. Implied Finance Rate Spreads.** Weekly time series of the spread between the implied finance rate  $r$  and the Treasury-bill rate, and of the spread between the general collateral repo rate  $GC$  and the implied finance rate. Spreads are measured in basis points. The sample period is from January 1988 to June 2000.



**Figure 3. Specialness Premia.** Weekly time series of the estimated specialness premia in Treasury-bond prices. The specialness premia are measured in units of dollars per \$100 notional amount. The sample period is from January 1988 to June 2000.

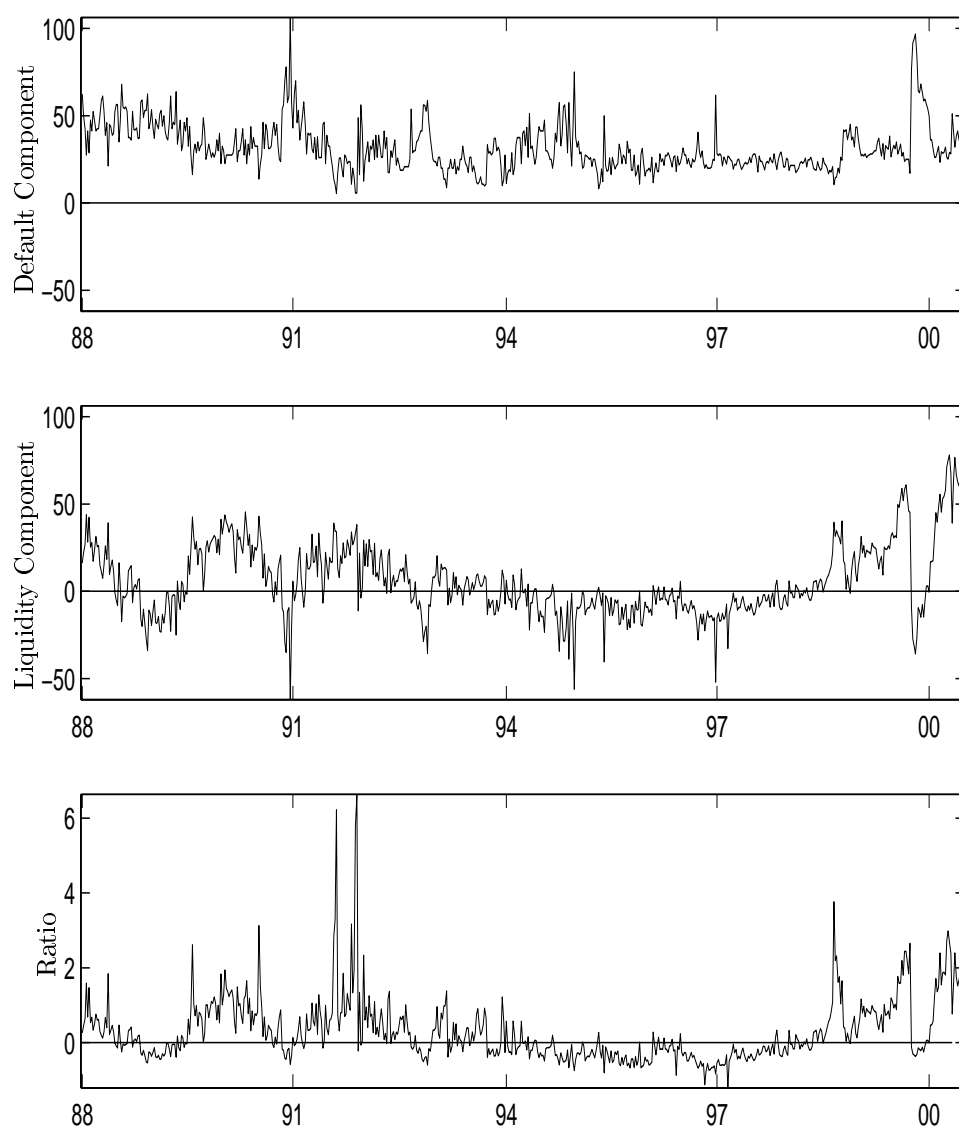


**Figure 4. The Implied Credit Spread.** The weekly time series of the implied credit spread. The credit spread is measured in basis points. The sample period is from January 1988 to June 2000.

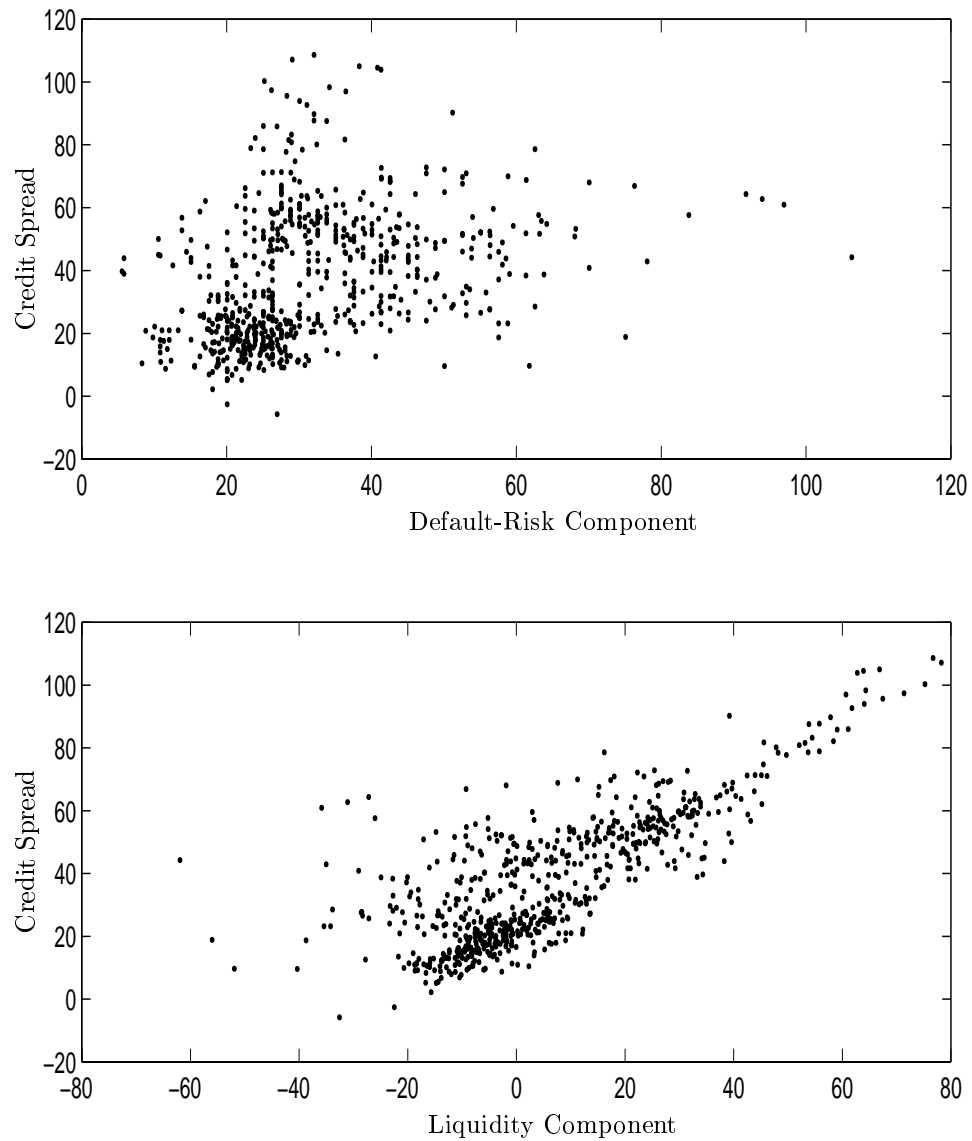


**Figure 5. Expected Credit Spreads.** Weekly time series of the expected value of the credit spread five years in the future, conditional on the current values of the state variables. The dashed-dotted line is the expectation under the objective measure. The solid line is the expectation under the risk-neutral measure. Expected spreads are measured in basis points. The sample period is from January 1988 to June 2000.

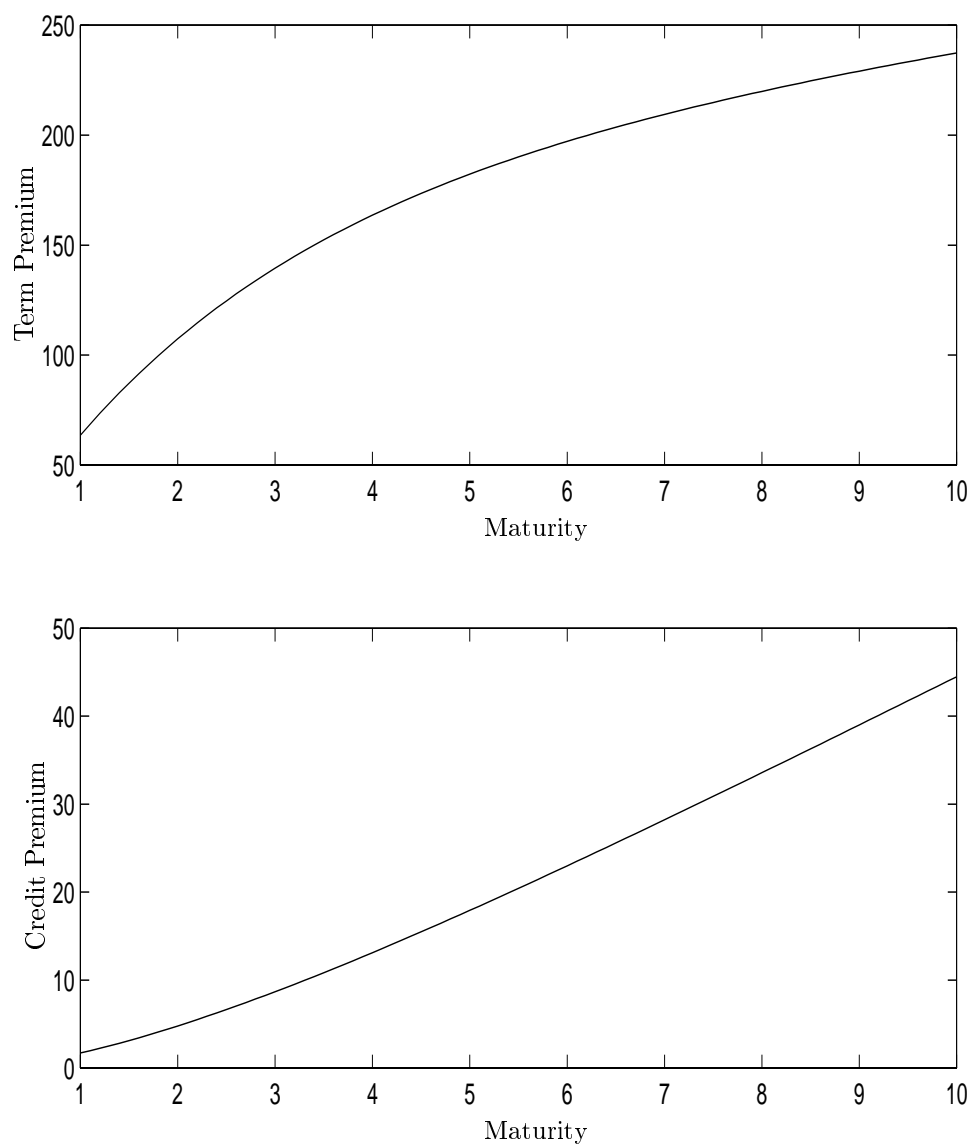




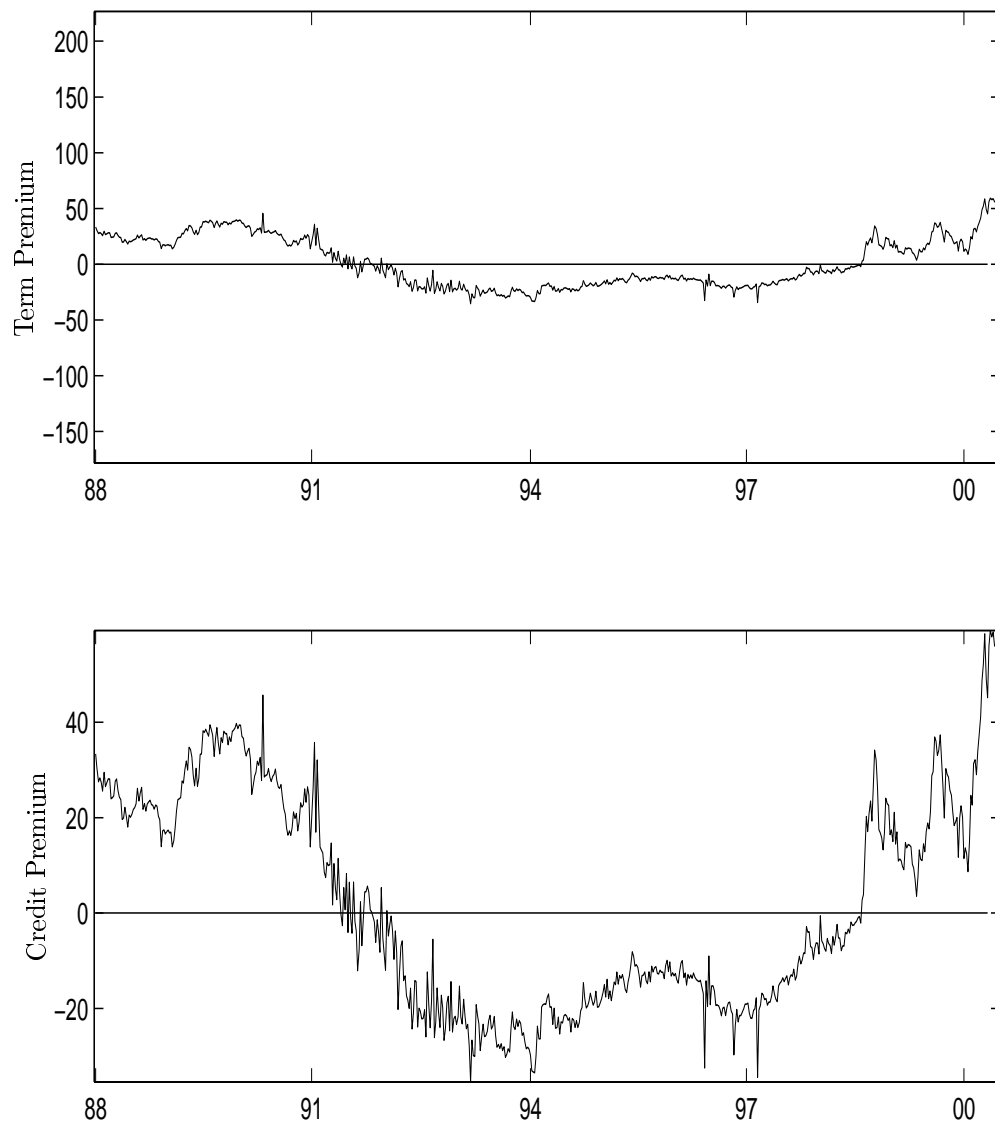
**Figure 6. Components of the Implied Credit Spread.** Weekly time series of the components of the implied credit spread. The top graph shows the default-risk component. The middle graph shows the liquidity component. The bottom graph shows the ratio of the liquidity component to the default-risk component. The default-risk and liquidity components are measured in basis points. The sample period is from January 1988 to June 2000.



**Figure 7. Scatterdiagrams of the Credit Spread and its Components.** The top graph plots the implied credit spread against the default-risk component. The bottom graph plots the implied credit spread against the liquidity component. The credit spread and the default-risk and liquidity components are measured in basis points.



**Figure 8. Unconditional Premiums.** The top graph plots the unconditional term premium of a riskless zero-coupon bond against the maturity of the bond in years. The bottom graph plots the unconditional credit premium of a risky zero-coupon bond against the maturity of the bond in years. Premiums are measured in basis points.



**Figure 9. Conditional Premiums.** The top graph plots the conditional term premium for a riskless one-year zero-coupon bond. The bottom graph plots the conditional credit premium for a risky one-year zero-coupon bond. Premiums are measured in basis points. The sample period is from January 1988 to June 2000.