EMPIRICAL EVIDENCE ON VOLATILITY ESTIMATORS

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ABSTRACT

Are historical volatilities better then implied volatilities in estimeting future (also known as actual or realised) volatilities? Which method of measuring historical or implied volatility is best? In this paper we discuss the methodology for calculating these approaches to volatility, carry out empirical tests on each estimator, as well as on their interrelations.

In order to test the "quality" of the estimators, comparisons among historical, implied and future volatilities were used for a full range of estimators. This identifies some of the criticisms for each estimator. The differences found among different estimators are statistically significant and should became fully noted by users of volatilities in the pricing and trading “volatility dependent securities” such as options. Moreover we observed some empirical evidence of the so-called "smile effect" that explains why implied volatility estimators that embody the moneyness effect show lower errors in predicting future volatilities. We also found some empirical evidence for the increase of the smile effect with the approach of the maturity. We also found that the selection of a specific estimator can lead to biased conclusions when studying the forecast ability of implied volatilities. Finally the exercise price effect seems to be asymmetrically dependent on stock price changes.

JEL: N2 - Financial Markets and Institutions
Key Words: Volatility estimators; implied volatility; historical volatility; efficiency.

Empirical Evidence on Volatility Estimators

1. INTRODUCTION

Volatility is usually viewed from three different perspectives: historical, implied or future (also known as actual or realised). Different methods have been suggested to measure the three types of volatility. When studying the forecasting of future volatility, it is common to use a single implied or historical volatility estimator. However, given that several estimators for historical and for implied volatility are suggested in the literature, we tested if significant differences are found when using different estimators. We first review the literature on implied volatility and historical volatility estimators. Then we describe our data sources and the methodology. Finally we present the empirical conclusions.

2. LITERATURE REVIEW ON VOLATILITY ESTIMATORS

2.1. Implied Volatility Estimators

The implied volatility of an option is the volatility that used in an option valuation model equates the theoretical value and the market value. If option pricing models are valid, implied volatilities express the market expectation about future volatility. The main reason for using implied volatility is the assumption that the market as a whole "*may know some things about the future volatility in the stock that we don't know*", Black [1975]. However, when implied volatility is computed, we find as many estimates as the number of series traded on a specific stock. Also there are complications introduced by three factors: (1) non-simultaneous data in the options and stock market, (2) improper model specification if the market does not price options which conform to the chosen model and (3) market inefficiencies, that allow for profitable arbitrage opportunities. There have been several attempts to develop methods that could embody in a single figure the information content of the implied volatilities “observed” in several series and to estimate the volatility of a specific stock.

We found five different estimating methodologies, with variations within each methodology. The first group of authors suggests estimators based on the calculation of weighted averages (called "**simple weighted estimators**"). Within this category there are suggestions to use: 1) the implied volatility of a single option series as representative of all of the series (Gemmill [1986] or Sheikh [1993]); 2) a simple average estimator (Trippi [1977]); 3) a derivative weighted estimator (Latane and Rendleman [1976]); 4) an elasticity weighted estimator (Chiras and Manaster [1978]); 5) a trading volume weighted estimator (Clewlow and Xu [1993]); and 6) an open interest weighted estimator (Duque [1994]) .

A second group uses a minimum **least squares approach** for estimating implied volatility. Suggestions can be found for: 1) a non-weighted estimator of implied volatility (Whaley [1982]); 2) a non-weighted estimator for two implied variables - the implied volatility and the implied underlying security price - (Manaster and Rendleman [1982]); 3) an elasticity weighted estimator (Beckers [1981]); and 4) a trading volume weighted estimator (Day and Lewis [1988]).

A third group of **estimators that account for the "smile" effect** were suggested by MacBeth and Merville [1979] and by Finucane [1989a] and [1989b].

In a fourth category, Brenner and Galai [1984] suggested a **time average estimator** in order to improve the forecasting power of the implied volatility estimator.

Finally, Brenner and Galai [1986] advocated **flexible methods**, arguing that the quality of the estimators is time dependent.

2.2. Historical Volatility Estimators

One of the frequently used methods to estimate past volatility is the "**classical**" estimator (so-called because it is traditionally used). It requires only closing prices and can be defined as the standard deviation of the daily price returns for a period of time. It is not clear how long should be the time of sampling, so this issue remains open to empirical research. However, it is common to use the past 20 or 50 days period (see Gemmill [1993]). Its simplicity is said to be its great advantage and also its disadvantage, since it ignores other commonly available information.

Parkinson [1980], has suggested another method that is said to be "far superior to the traditional method". It incorporates the intraday information of **high and low prices**. This estimator embodies more information than the previous one and it is claimed to ensure the same accuracy with about 80% less data and with an efficiency of 5.2 when the classical estimator is considered as the benchmark (with an efficiency of 1)[1].

Another method that has been frequently used in the literature is the Garman and Klass [1980] extreme-value estimator. Garman and Klass have suggested several different estimators in their article with different degrees of efficiency, but the preferred estimator with the highest efficiency score, if the information is available, uses the **opening-high-low-closing prices**. This is said to be 8.4 times more efficient than the classical estimator. It not only incorporates the close to close information but also combines the Parkinson measure.

One of the shortcomings of the Garman and Klass [1980] estimator is that it was developed assuming that the underlying asset follows a continuous Brownian motion process. However, stock prices are observable only at discrete time moments, creating a possible source of bias. In an empirical study, Beckers [1983] reinforces this idea that non-continuous prices will bias downward the extreme value estimators. Marsh and Rosenfeld [1986], Edwards [1988], Wiggins [1991] and [1992] also emphasise that non-trading activity will affect the efficiency of these estimators. In fact, only by pure chance, the extreme observable high and low prices are also the highest and lowest

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[1] Efficiency of an estimator is defined as the variance of a benchmark estimator divided by the variance of that particular estimator,

$$\text{Efficiency (Estimator)} = \frac{\text{VAR (Benchmark)}}{\text{VAR (Estimator)}}$$

continuous prices. It is common for the low price to be higher than the "true" lowest price and the high price to be lower than the "true" highest price. This criticism assumes that prices are continuously formed but only some are registered on the stock exchange. Closely related to this topic is the frequency of stock price observation. If there are few transactions during the trading day, then the high and low prices are likely to be less close to the "true" high and low prices than if there are many transactions. This issue was also raised by Garman and Klass, who suggested an adjustment for their estimator, dividing the figures found by some constants based on trading frequency. A lower number of bargains on a stock will result in a higher adjustment.

Beckers [1983] suggests that instead of a fixed weighted average of the classical close-to-close estimator and the high-low estimator as proposed by Garman and Klass [1980], the volatility estimator should allow for **frequent changes in the weights**. First, he regresses the values of the estimators for a period of time (*Classical = f(high-low)*). Then, he uses the constant and the slope coefficients from the regression equation to estimate the value of his suggested estimator. He claims that this estimator has outperformed the classical, the Parkinson [1980] and the Garman and Klass [1980] estimators.

Butler and Schachter [1986] proposed also a volatility estimator for the Black-Scholes model. They claim to present the **minimum variance unbiased estimator** (MVU) that should be used in order to generate unbiased estimates of the Black and Scholes [1973] option pricing model. However they claim that the variance rate of the volatility estimate does not seem to be constant. The longer the time series of observations used, the more biased the variance estimate may be so that using more data over a longer period involves a trade-off between reducing the variance of the variance estimate and increasing its bias.

Some years later Barnaud [1990] presented another method that can be seen as a **simplified opening-high-low-closing prices estimator** purposed by Garman and Klass [1980]. He developed another intraday estimator that requires less data than the Garman and Klass [1980] estimator, but incorporates more information than the classical or the Parkinson [1980] estimator. That estimator has an efficiency ratio around 4. As expected, it requires only 25% of the data needed for the classical estimator, but 20% more than the Parkinson [1980] to achieve the same accuracy.

According to Rogers and Satchell [1991], the Garman and Klass [1980] estimator seems to present two major drawbacks: (i) the estimator will be biased in the case of a non-zero drift rate of the stock return over time and (ii) the empirical observations of stock prices are not continuous as it is assumed in the Brownian motion process. While the first drawback seems to have no effect since the estimator "*works just as well for non-zero [drift rate]*", Rogers and Satchell [1991], the second has some consequences. Garman and Klass suggested the use of a given set of values to adjust the figures found when historical volatilities are calculated. However, Rogers and Satchell [1991] tried to embody the frequency of price observations in the model in order to overcome the drawback. They claim that the **corrected estimator** outperforms the uncorrected, in a study based on simulated data.

Kunitomo [1992] returned to the same subject of the **non-zero drift rate** of the stochastic process, and tried to develop an estimator based on the Parkinson [1980] **high-low estimator**. Absolute high or low prices, apparently the extreme observations, can be transformed into non-extreme, once the drift rate observed during the time period is deducted. As a consequence, other observations can become the "true" highs and lows. This estimator is said to be 10 times more efficient when compared with the classical volatility estimator. However it does not include the additional information that is embodied in the Garman and Klass [1980] estimator.

Finally Gemmill [1993] suggests improvements in the classical estimator using **weighted data**, as a practical way to compute historical volatilities. The rational behind these weighted estimates is based on the belief that investors tend to give more importance to recent than to long dated past information.

A quite different approach to the subject was **the maximum likelihood estimator** for volatility suggested by Ball and Torous [1984]. Their method involves the maximization of the logarithm of the likelihood function of volatility. This requires the use of numerical procedures in order to find the estimator for volatility. Starting from the Garman and Klass [1980] estimator, which is inserted into the logarithm of the likelihood function, and following the Newton-Raphson method, it is possible to converge to the searched value. This estimator is empirically presented as being on average 8.1 times more efficient than the classical. It is also important to underline that the efficiency of this estimator seems to be dependent on the sample size used in the estimation. However, given the enormous complexity of this method as well as the required computing skills and time, and given the small or nil increase in efficiency when compared with Garman and Klass [1980], it seems unlikely to be adopted.

Another completely different source of bias is related to **time and sample size**. This subject was addressed by Burghardt and Lane [1990] who state: "*Because an option has a specific time horizon - its time to expiration - any comparison of its implied volatility with a fixed-period historical volatility is inappropriate*". Burghardt and Lane [1990] suggest that when historical volatility needs to be calculated for option pricing purposes, **the time horizon for sampling should be equal to the time to maturity of the option**. Instead of a single value for the volatility of a stock, as many estimates as the number of option maturities still "alive" should be calculated. Other studies also address the problem and present different if not opposed conclusions. Butler and Schachter [1986] conclude that when historical data is used for forecasting purposes, the bias found in the estimator increases with the length of the sampling period. But, Beckers [1981] notes that information based on short periods of time can present high volatility that embody other phenomena such as overreaction corrections.

Figure 1 gives a general idea about how the recent developments have been integrated. Table 1 summarizes the literature on historical volatilities.

2.3. Future Volatility Estimators

Future volatility is the observed volatility of the stock price from the current date until the option maturity. When future volatility is used to address the topic of forecasting ability, it is usually computed according to one single estimator (see among others Latané and Rendleman [1976], Chiras and Manaster [1978], Beckers [1981], Park and Sears [1985], Heaton [1986], Gemmill [1986], Scott and Tucker [1989], Wilson and

Fung [1990], Randolph, Rubin and Cross [1990], Canina and Figlewski [1991], Barone and Cuoco [1991], Fung and Hsieh [1991]). But all the estimators used to estimate historical volatility can be applied. If differences are found among historical estimators, similar differences will be found when computing future volatility estimators. The strong empirical weakness for future volatility occurs when it is calculated for very short periods of time. This becomes particularly significant when these figures are compared with implied or historical volatilities.

3. DATA

The data was collected from Datastream and consists of a set of 30,612 call option quotations written on 9 liquid stocks (Amstrad, British Airways, British Gas, British Petroleum, British Telecom, Forte, General Electric, Hanson and Rolls-Royce) traded on the London Stock Exchange. The option prices were middle bid and ask quotations at the close of the London Traded Options Market (now the LIFFE Equity Options Market) from April 1990 to December 1991.

Share prices (closing, opening, high and low) to calculate historical as well as future volatility were also collected from Datastream, while simultaneous share prices with the closing price quotations of the options market were collected from the Daily Official List of the London Stock Exchange.

Interest Rates (LIMEAN) were obtained from Datastream, and dividends both from Datastream and Microview.

4. METHODOLOGY

4.1. Implied Volatilities

We have selected eight different estimators of implied volatility that were previously reviewed: Trippi [1977] (TRIPPI), Latané and Rendleman [1976] (LATANE), Chiras and Manaster [1978] (CHIRAS), Duque [1994] (OISIMPL), Whaley [1982] (WHALEY), Beckers [1981] (BECKERS), Day and Lewis [1988] (DAY_LEWI), MacBeth and Merville [1979] (MCBTH_B0) and Finucane [1989a] [1989b] (FINUCANE).

Our concept of moneyness is slightly different than MacBeth and Merville's. While their moneyness ratio of at-the-money options is equal to zero, ours is equal to one. Therefore we deducted one unit from our moneyness ratio to calculate the MacBeth and Merville [1979] estimator:

$$Mnss_{MacBeth} = \frac{S - NPV(Div) - X\, e^{-r(T-t)}}{Xe^{-r(T-t)}} \qquad \textbf{(1)}$$

$$\mathrm{Mnss} = \mathrm{Mnss}_{\mathrm{MacBeth}} + 1 = \frac{S\text{-}NPV(Div)}{Xe^{-r(T-t)}} \qquad \textbf{(2)}$$

Another adjustment had to be done when calculating the Finucane [1989] estimator. While he takes R=X/S, we assumed R=1/Mnss where Mnss is defined as equation (2):

$$\mathrm{R} = \frac{Xe^{-r(T-t)}}{S\text{-}NPV(Div)} \qquad \textbf{(3)}$$

In order to compare Finucane's estimator with others, we have calculated the estimated at-the-money Finucane implied volatility (FINUC_AT). Finucane assumes that at-the-money options are correctly priced. But the moneyness degree of an at-the-money option can diverge slightly from one. However, it is possible to estimate what is the correspondent implied volatility for a moneyness ratio of one. As he assumes a linear relation between moneyness ratio and implied volatility, we have calculated the moneyness premium from two option series: an at-the-money and a deep in-the-money option. Taking σimp_L as the implied volatility of a deep in-the-money option, σimp_A as the implied volatility of an at-the-money option, R_L and R_A as the correspondent moneyness ratios of the deep in-the-money and the at-the-money options, the moneyness premium in terms of volatility, of any option with moneyness ratio $R \leq R_A$ is given by

$$\text{Mnss Premium} = \frac{\sigma\ imp_L\ -\ \sigma\ imp_A}{R_L\ -\ R_A}\ R \qquad \textbf{(4)}$$

Therefore, the implied volatility for an at-the-money option with R=1 becomes equal to the first term of the right hand side of equation (4).

4.2. Historical Volatilities

We have calculated almost all of the historical estimators reviewed in the literature. The classical estimator was computed using both 20 and 50 past trading days (respectively CLASIC20 and CLASIC50). When the classical weighted estimator suggested by Gemmill [1993] was computed (GEMMILL), we have used the last 20 trading days weighted exponentially.

The same number of trading days was used to estimate the Parkinson [1980] estimator (PRKNS20). The Kunitomo [1992] estimator was calculated using moving periods of 20 days (KUNITOMO). A drift rate was calculated for each period and the data within the period was adjusted according to the observed drift rate.

The Garman and Klass [1980] estimator was also computed using the last 20 trading days (GK20). It was considered that the ratio of non-trading hours was 2/3, since the London Stock Exchange was trading from 8:30 to 16:30 (8 hours per working day). The results were adjusted according to the number of bargains observed per trading day according to Table 1 in Garman and Klass [1980]. The same number of days was used to estimate the Barnaud [1990] estimator (BARNAUD). To calculate Beckers [1983] estimator (BECKER) we have considered moving periods of 60 trading days. For each day, the last 60 days were considered. Next we regressed the classical estimator (as dependent) with the Parkinson's estimator (as independent variable). The final Beckers' estimator was calculated using the estimated parameters.[2]

[2] We have also calculated the Rogers and Satchell [1991] estimator. However, we found a very high number of unacceptable values. As we could not find the reason for such phenomenon, we have excluded this methodology.

In the database, for each stock and for each day, there are three maturity dates available. Therefore, according to Burghardt and Lane [1990] we have computed three historical volatilities, using the classical estimator. For each calculation the number of observations equals the time-to-maturity measured in terms of days.

All the estimators were annualized multiplying the results by $\sqrt{253}$ which is the square root of number of trading days in 1990 and 1991.

4.3. Future Volatilities

We have used only two different historical estimators to compute future volatilities: the "classical" estimator and the open-high-low-close estimator suggested by Garman and Klass [80].

4.4. Hypotheses

We can summarize the usual concern with volatility with four questions:

- How are the estimators related?
- How strongly are the implied estimators related to past information?
- How strong is the forecasting power of each method?
- How dependent are the estimators and their forecasting power on other variables?

We have addressed and tested several hypotheses that can help to clarify some of the questions raised above.

Hypothesis I - Are historical volatility estimators significantly different from each other?

We first summarize some descriptive statistics on the different estimators for historical, implied and future volatility (Table 2). When the classic estimator using 20 trading days is compared to the same estimator using 50 trading days, significant differences are found in terms of dispersion and shape of the distributions, but not in terms of means.

Differences are noticeable when all the methods are compared particularly GK20, KUNITOMO and BARNAUD. The systematic differences between the GK20 and the classical estimators confirms the findings of Chu and Bubnys [1990], who claimed that the Garman and Klass [1980] estimator was almost always above the classical estimator results. Consequently, we tried to test if the differences found among the means were statistically significant. We used two different tests: a t-test and a Wilcoxon signed-ranks test. In both cases we found statistically significant differences between all pairs compared, at a confidence level of 95%. The Garman and Klass [1980] estimator presents a higher sum of the absolute mean differences (0.5829), meaning that, on average, this estimator presents estimates that tend to differ strongly from all the others.

Subsequently, we tried to regress all the historical estimators with each other. Once again the results seem to evidence differences. In particular the multiple R statistics seem to show a close relation among extreme value estimators (GK20, PARKSN20, BECKER) and a close relation among classic estimators (CLASIC20, GEMMILL). However, some peculiar results appear. For instance KUNITOMO, which is intended to be an improved Parkinson [1980] estimator, appears to be closer to classical (CLASIC20, GEMMILL) than to extreme value estimators. Nevertheless, in all these regressions the Durbin-Watson statistic shows a very strong positive autocorrelation in the residuals that reduces the power of the test.

From the previous tests it seems possible to reject the null hypothesis of Hypothesis I, since all them show differences among historical estimates of volatility that are statistically significant.

Hypothesis II - Are implied volatility estimators significantly different from each other?

Next, we ran the same range of statistical analysis and tests on implied volatility estimators that are also presented in Table 2. Contrary to the previous set, implied volatility estimates seem to be far more homogeneous. A first look suggests that there are no significant differences between the implied volatility estimators. However, both the t-test and the Wilcoxon signed-ranks test indicate that for a confidence level of 95%

all the pairs of means are significantly different. We also regressed the estimators against each other. For all the cases, the correlation is positive and very high and in almost the cases we can reject the null hypothesis of $b_1=0$, since they are quite close to 1 as expected from the observation of Table 2. It is also noticeable that no autocorrelation seems to be present in all the regressions with a confidence level of 99%.

The differences found among implied volatility estimators do not lead to any conclusion on their "quality". If we calculate for every estimator the value of the implied volatility of the underlying asset for each maturity j (σimp_j), and having calculated the implied volatility for the option ij on the i[th] exercise price (σimp_{ij}), then we can estimate both the absolute error (AE) and the square error (SE) implied volatilities.

$$AE_{ij} = \left| \sigma imp_{ij} - \sigma imp_j \right| \quad \textbf{(5)}$$

$$SE_{ij} = (\sigma imp_{ij} - \sigma imp_j)^2 \quad \textbf{(6)}$$

Both measures indicate how far the individual option implied volatility (σimp_{ij}) is from the unique ("true") estimated implied volatility for maturity j using a specific estimator (σimp_j). Then taking N as the total sample size, it is possible to calculate the mean absolute error (MAE) and the mean square error (MSE) for each method M:

$$MAE_M = \frac{\sum_{i=1}^{N} | \sigma imp_{ij} - \sigma imp_j |}{N} \quad \textbf{(7)}$$

$$MSE_M = \frac{\sum_{i=1}^{N} (\sigma imp_{ij} - \sigma imp_j)^2}{N} \quad \textbf{(8)}$$

Table 3 presents the results. We split the estimators according to their characteristics that we found when reviewing the literature (Weighted Averages, Least Squares and Exercise Price Effect). Differences seem to be quite significant for methods that incorporate the exercise price effect (MACBETH and FINUCANE). These two estimators show significantly smaller mean absolute errors and mean squared errors than others (see column "mean - Group" where the means are averaged). Moreover, methods based on the least squares method present higher mean absolute errors and mean squared errors. We therefore conclude that when implied volatility estimators are used as input in option pricing models, those which incorporate the exercise price effect tend to present less errors than others. We also do not see any advantage in the use of sophisticated methods that employ the least squares technique requiring higher computational time, since the errors found are on average higher than in the simple weighted schemes. Remember that these methods were built in order to minimize conditions similar to equation (6). However, we observed that, on average, other simpler methods using weighted averages tend to be more effective. In Table 3 we defined the average effectiveness of a group of estimators as

$$\text{Effectiveness}_{\text{Group i}} = \frac{1}{\dfrac{\text{Mean Group i}}{\text{Mean Group Ex Price Effect}}} \qquad \textbf{(9)}$$

The effectiveness of the group of estimators[3] that are called Ex. Price Effect is assumed to be 100%. Using both the mean absolute error and the mean square error we arrive at the same conclusion that the group of estimator, called Least Squares estimators present on average a lower degree of effectiveness.

Concluding, although implied volatility estimators seem to be more homogeneous than historical volatility estimators, there is some evidence that differences exist. These differences seem to be significant when groups were created based on their methodology. The group of estimators that take into account the exercise price effect ("smile effect") is the one that presents lower errors. Simple weighted methods seem to present smaller errors than more sophisticated methods such as those based on the least squares approach. Therefore, it is possible to reject the null hypothesis corresponding to Hypothesis II since the differences found among estimators are statistically significant.

Hypothesis III - Are future volatility estimators significantly different from each other?

[3] This is not the same as the effectiveness of an estimator presented above.

Subsequently we addressed similar questions for future volatilities. Once again different methods resulted in different outcomes. The means for the two methods (Table 2) were significantly different, but positively and highly correlated. However, the results are similar to those found in the study of historical estimators. The Garman and Klass [1980] estimator results in an average estimation that is higher than the average estimation when the classic estimator is used. Remember that future volatility estimators used were the classic and the Garman and Klass [1980] calculated each day for the time remaining to expiration. Nevertheless, the Durbin-Watson test shows a positive autocorrelation in the residuals. Given the significance of the differences found among future estimators it is also possible to reject the null hypothesis corresponding to Hypothesis III.

Hypothesis IV - Are the implied volatility estimates based on past information?

Having analyzed the relationships among each type of estimator we now study the relationships between them. Firstly, we were concerned with the relationship between implied and historical volatility. Are implied volatilities dependent on historical information? We regressed all the implied volatility estimators with all the historical volatility estimators for all the sample in the form of:

$$\sigma\, imp_{ij} = \beta_0 + \beta_1\, \sigma\, hist_{ik} + \varepsilon_i \qquad \textbf{(10)}$$

where σimp_{ij} is the implied volatility of option i using method j and $\sigma hist_{ik}$ is the historical volatility of the underlying asset of option i using the estimator k. The results show that estimators seem to be quite strongly and positively correlated. There do not seem to be significant differences in the way the different implied volatility estimators reflect historical volatility information. Hence, historical information is equally absorbed by implied volatility estimators. However, when historical estimators are observed, differences become amplified. When multiple-R is averaged by historical volatility estimator, some methods such as CLASIC50 or BURG_LAN are systematically more highly correlated with implied estimators than others (the lowest correlations are BARNAUD and GK20). This seems to be favourable to those that suggest the use of a longer period of observations to estimate historical volatilities. It also supports the suggestion that calculations of volatilities based on past information for option pricing purposes, should be done with the same time period observation as the time to expiration of the option. However the results can be biased by the sampling period of time, and Durbin-Watson statistics show a positive autocorrelation in the residuals.

In order to overcome this problem, we have used the Cochrane-Orcutt procedure as suggested by Montgomery and Peck [1992][4]. However, for computational reasons we had to restrict the sample size[5]. We have selected randomly a subsample of 3000 cases. As expected the explanatory power of the model decreases but the conclusion still holds. There appear to be no significant differences among implied volatility estimators, while some historical estimators seem to be systematically more correlated with the implied estimates (CLASIC50 still has the highest score).

As a result of the tests ran, we are able to reject the null hypothesis that states that no relation exists between implied and historical estimators. It seems that implied volatility estimates are partly based on historical information independently of the estimator in use to calculate both the implied and the historical volatility. However, some historical estimators seem to be more closely related with implied figures, especially the CLASIC50 and BURG_LAN estimators.

Hypothesis V - Are implied volatility estimates good predictors of future volatilities?

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[4] Although it is possible to use other methods to overcome serial autocorrelation, some tend to converge to Cochrane - Orcutt results when large samples are used (see Greene [1993] for proof). Cochrane and Orcutt [1949] has the additional advantage of easy calculation in statistical packages like SPSS.

[5] SPSS requires a reduction in the sample in order to compute the Cochrane-Orcutt procedure.

Next, we calculated the predictive power of implied volatilities on future volatility. Theoretically, implied volatilities should represent the market expectations of the future. An objective of this study is to discuss if any implied volatility estimator systematically under or outperforms others. In general, future volatility estimators were highly correlated with implied volatility estimators. Comparing the results, we found that implied volatility estimators seem to be more correlated with the future volatilities than with past information. Once again, different implied volatility estimators do not show significant differences in general terms. We also found that the future volatility estimates calculated according to the classical estimator show a higher degree of correlation with implied estimates than future volatility estimates computed according to the Garman and Klass [1980] estimator.

But once again the regression test presents problems of autocorrelation. We tried to overcome the problem using the Cochrane-Orcutt procedure. As a result some estimators seem to clearly outperform others, namely FINUC_AT and BECKERS. However, the most important issue is related to the differences between FUTCLASIC and FUTGK. These differences (already noticed with the entire sample and enlarged now with the Cochrane-Orcutt procedure) show possible weaknesses in studies that compare implied with past and future information, especially when conclusions are drawn on the predictive power of volatilities. If the forecasting ability of implied volatility is measured by its comparison with future volatility, then the result seems dependent on the future volatility estimator as well as on the implied volatility estimator. Please note that the highest multiple-R occurs using the BECKERS estimator when the FUTCLASIC is used (0.4968), while FINUC_AT results in the highest multiple-R when the FUTGK is used (0.2598). This conclusion is clearly opposed to Scott and Tucker [1989] who claim that forecasting quality does not depend on the chosen estimator, while it supports Clewlow and Xu [1993] who state the opposite.

Therefore, it is possible to reject the hypothesis that no correlation seems to exist between implied and future volatility. Even though the relation between future and implied volatility seems to be stronger than the relation between implied and historical, this observation does not hold after the Cochrane-Orcutt procedure. We also noticed that the predictive power of implied volatility estimates is dependent on the method used to estimate both the future and the implied figures.

Hypothesis VI - Are historical volatility estimates good predictors of future volatilities?

Next we evaluate historical volatility as a predictor of future volatility. The results do not seem to evidence profound differences but in general terms they are lower than previous comparisons using implied volatilities. Although a similar problem of autocorrelation is noticeable and the same procedure was adopted to overcome it, the conclusions do not change. This comparison shows how the sampling period influences the conclusions. For the period under analysis, it is clear that the classical historical estimator based on the past 50 observations (CLASIC50) outperformed the same historical estimator based on the last 20 observations (CLASIC20). The Multiple R is higher for CLASIC50 than for CLASIC20, b_1 is closer to 1 for CLASIC50 and b_0 is closer to 0 for CLASIC50. All this holds even after the Cochrane-Orcutt procedure. Such a conclusion is opposite to that stated by Butler and Schachter [1986] who believe that longer time series results in a higher bias in forecasting. However, if it is true that longer time series reduce the capacity of changing volatility parameters, this may be the most appropriate model when markets are relatively quiet and may also be the most appropriate estimator for mean-reverting processes. Longer time series are slower to react to short term and sometimes misleading information (see Beckers [1981], and follow closely a mean-reverting stochastic process.

Previous studies such as Black [1975], Beckers [1981], Park and Sears [1985], Heaton [1986], Fung and Hsieh [1991] or Barone and Cuoco [1991] support the idea that implied volatility is a good predictor for future volatility. Even stronger, Chiras and Manaster [1978], Gemmill [1986], Latané and Rendleman [1976], Heaton [1986], Scott and Tucker [1989] and Fung and Hsieh [1991] state the superior predictive ability of implied volatility over historical volatility when forecasting future volatility. In contrast, Canina and Figlewski [1991] conclude that their statistical tests show "*little or no correlation between implied volatilities and subsequent realized volatility at all. Moreover, implied volatility does not fully impound the information contained in the* *historical estimate*". Even Gemmill [1986] has concluded that implied volatility estimators have only marginally outperformed historical volatility estimators. Randolph, Rubin and Cross [1990] state that "*implied volatility estimators do not appear to be a useful predictor of upcoming changes in volatility*". Finally Wilson and Fung [1990] concluded that the relation between historical and implied volatility is variable according to different products.

How do our results compare with others? To summarize all the tests, we calculated the averages for b_0's, b_1's and multiple-R statistics for all the comparisons previously done. The results are summarized in Table 4. At this stage we are particularly concerned with the shadowed areas of Table 4. When we first observed the results before the Cochrane-Orcutt procedure, we concluded that the weakest relation was between historical and future volatility (the lowest multiple-R). This means that historical volatility is a weaker instrument for forecasting future volatility than implied volatility. Implied volatility has higher values in terms of all the statistics calculated: R-square, b_0 and b_1. However as the Durbin-Watson statistics show signs of positive autocorrelation, we have run the Cochrane-Orcutt procedure in order to overcome the problem. When we concentrate on the right side of Table 4 (after the Cochrane-Orcutt procedure), observing the outcomes of that procedure, some interesting effects happen. First, in terms of multiple-R future volatilities still seem to be better predicted by implied volatilities than by historical volatilities. However, as Kritzman [1991] states and Canina and Figlewski [1991] show, "*a high R-square in the equation (5)* [the regression equation of implied volatility on future volatility] *is not proof that a forecast is rational*". These authors claim that earlier studies did not apply the methodology correctly since these related tests have used R-square statistics to conclude what should be done with b_0 and b_1. "*Rather, they compared the R-square statistics and concluded that since R-square tended to be higher for the implied volatility equations than for historical volatility, implied volatility was informationally superior. [...] That comparing correlations or R-square's is an inadequate test of the joint hypothesis...*" - Canina and Figlewski [1991]. "*The regression equation with implied volatility as the independent variable may have a higher R-square, but the slope of the regression line may be significantly greater or less than one, or the intercept may be significantly greater or less than zero. In these cases,* *the forecasts would be biased, although the R-square alone would not reveal this*" (Kritzman [1991]).

In our results, when b_0 and b_1 are considered, historical volatility becomes a better predictor of future volatility than implied volatility. Therefore we would conclude that considering the statistical correction for autocorrelation, implied volatility seems to be quite strongly associated with historical volatility and that this seems to be a "better predictor" of future volatility than the former (see Figure 3).

Hypothesis VII - Are implied volatility with historical volatility estimates good predictors of future volatilities?

To test whether implied and historical volatility simultaneously considered improve the predictability of future volatility, we regressed historical and implied volatilities on future volatility. To study if there was any effect derived from the estimator in use we have combined all possible estimators of implied and historical volatility and regressed them on a single future volatility estimator that we selected (FUTCLASIC).

A first look at the results show that we have improved the multiple-R statistics when the results are compared with the corresponding multiple-R statistics obtained from simple regressions. However, such an improvement in multiple-R statistics is a trivial result. When t-tests on the parameters b_1 and b_2 are considered we notice that in general, both variables contribute to explain the dependent variable (future volatility estimates). The left side of Table 5 summarises the results. It is possible to observe that on average, implied volatility is a more important forecaster of future volatility than historical volatility ($E[b_1] > E[b_2]$). As both variables are expressed in similar units and ranges, we can compare the figures found for b_1 and b_2 directly. We are aware that this analysis suffers from problems of multicollinearity and autocorrelation. To overcome this problem we ran the Cochrane-Orcutt procedure after selecting a sample of 3000 cases. Once again we tested all possible implied and historical volatility estimators to test whether the results are estimator dependent. The results are summarized in the right side of Table 5. The multiple-R statistic was reduced significantly. But the most interesting results are the changing of importance of the independent variables and the estimator dependency of the results. We noticed that after this statistical correction, historical volatility is on average the most important variable in forecasting future volatility ($E[b_1] < E[b_2]$). However, it is important to emphasise that the relative importance of implied and historical volatility in predicting future volatility seems to be dependent of the estimator in use. For instance, if we consider the following regressions:

$$\text{FUTCLASIC} = \beta_0 + \beta_1 \text{ MCBTH}_{\text{B0}} + \beta_2 \text{ CLASIC50} + \varepsilon \qquad \textbf{(11)}$$

where β_1=0.2008 and β_2=0.5262 and

$$\text{FUTCLASIC} = \beta_0 + \beta_1 \text{ WHALEY} + \beta_2 \text{ GK20} + \varepsilon \qquad \textbf{(12)}$$

where β_1=0.6431 and β_2=0.0676, the conclusions are completely different!

Concluding, the results of the tests seem to show that future volatility is better predicted by combining implied and historical volatility. On average, after correcting for autocorrelation, historical volatility seems to be more important than implied volatility in predicting future volatility. However, these conclusions are completely dependent on the estimators used to estimate both implied and historical volatility.

We saw that implied volatility estimators have significant differences when MSE and MAE are estimated. This can indicate option mispricing. But to what extent are these differences stable or dependent on other variables? We tried two additional tests relating firstly MAE of each implied volatility estimator with time-to-maturity and secondly with the underlying stock price return.

Hypothesis VIII - Are the implied volatility estimators dependent on time-to-maturity?

In Duque [1994] it was concluded that as maturity approaches the number of out range options tends to increase[6]. It was stated that the "smile" effect tends to be more

[6]Out of range options are defined as those options with extreme volatilities, very

pronounced as maturity approaches. These two reasons are probably important to explain why some of the estimators tend to produce more errors than others when computing implied volatilities as maturity approaches. It is expected that implied volatility estimators that account for the "smile" show fewer errors than methods based on simple averages.

To test this hypothesis, we regressed the absolute errors calculated by equation (4) with time-to-maturity. The results show that the volatility estimators that differ significantly from the others are the Finucane [1989a] and [1989b] and the MacBeth and Merville [1979] estimator (see also Figure 2). They confirm that for all estimators the slope of the regression equation is negative and significantly different from zero, meaning that as the life of options declines, the errors tend to increase. But these tests present once again autocorrelation in the residuals. Therefore we have repeated the Cochrane-Orcutt procedure but the conclusions did not change. The results tend to support the rejection of the null hypothesis stating that no relation exists between time-to-maturity and errors obtained by the implied volatility estimators. Once again the Finucane [1989a] and [1989b] estimator is the estimator that seems to evidence less sensitivity to the approach of maturity of the options. It takes into account the so-called "smile" effect and therefore the adjustment in the estimator is observed along time. Other estimators that do not take into account that effect become increasingly biased once the effect is emphasized, that is, when time-to-maturity decreases. Hence, the smile effect seems to be more pronounced when maturity approaches and therefore the "quality" of the implied volatility estimators tends to decrease (assuming that the absolute error of the estimator as defined by equation (5) can be an indicator of "quality").

Hypothesis IX - Are the implied volatility estimators dependent on time-to-maturity and stock price changes?

The last hypothesis we tested was based on a multiple regression where implied volatility is regressed against historical volatility, time-to-maturity and the underlying stock price return.

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high or very low.

We saw previously that implied volatility is positively correlated with historical volatility. We also documented a tendency for increases in implied volatility figures when maturity approaches.

According to Geske [1979] and Christie [1982], volatility is negatively correlated with stock price returns. This should result from the fact that an increase of the stock price should reduce the degree of financial leverage and therefore reduce stock price volatility[7].

We tested whether these three variables are important in explaining implied volatility and whether they support previous conclusions and hypotheses when combined simultaneously.

The regression equation used was:

$$\sigma imp_i = \beta_0 + \beta_1 \sigma hist_i + \beta_2 (T-t) + \beta_3 R_i \qquad \textbf{(13)}$$

where σimp_i is the at-the-money implied volatility of stock i calculated according to the Finucane [1989a] and [1989b] estimator, $\sigma hist_i$ is the classic estimator of historical volatility based on the last 20 observations, T-t is the time-to-maturity and R_i is the daily return of the underlying stock i. We selected a sample of 2000 cases for computational reasons and once again we used the Cochrane-Orcutt procedure to overcome the problem of autocorrelation. The results are presented in Table 6 and confirm the hypothesised theory that implied volatility is positively related to historical volatility as we have concluded previously, but negatively correlated with time-to-maturity and with the underlying stock price return. As maturity approaches, volatility tends to increase. Implied volatility also seems to be negatively correlated with the underlying stock price return supporting previous research.

[7] However other authors support that this relation can be null (see Hull and White [1987]).

5. CONCLUSION

In this paper we observed different types of volatility estimators: historical, implied and future. We have studied their interrelations and conclude as follows.

(i) Differences among historical estimates of volatility exist and are statistically significant. In particular estimates obtained by Garman and Klass [1980] estimator are systematically higher than estimates presented by the remaining estimators.

(ii) Although implied volatility estimators seem to be more homogeneous than historical volatility estimators, there is evidence that differences exist. These differences seem to be significant when three groups were created based on their methodology (Weighted Averages, Least Squares and Exercise Price Effect). The group of estimators that take into account the exercise price effect ("smile effect") present lower errors. Simple weighted methods seem to present smaller errors than more sophisticated methods such as those based on the least squares approach.

(iii) When future volatility estimators are compared, we found differences that are statistically significant. Once again the Garman and Klass [1980] estimator seems to result in estimates that are higher than the alternative estimator (classic estimator).

(iv) We reject the hypothesis that no relation exists between implied and historical estimators. It seems that implied volatility estimates are partly based on historical information independently of the estimator used to calculate both the implied and the historical volatility. However, some historical estimators seem to be more closely related to implied figures especially the CLASIC50 and BURG_LAN estimators.

(v) We also reject the hypothesis that no correlation seems to exist between implied and future volatility. But it seems that the predictive power of implied volatility estimates is dependent on the method used to estimate both the future and the implied figures.

(vi) Considering the statistical correction for autocorrelation (Cochrane-Orcutt procedure), implied volatility seems to be quite strongly influenced by historical volatility and this seems to be a "better predictor" for future volatility than the former.

(vii) Future volatility is best predicted by combining implied and historical volatility. On average, after correcting for autocorrelation, historical volatility seems to be more significant than implied volatility in predicting / explaining future volatility.

However, these conclusions are dependent on the estimators used to calculate both implied and historical volatility.

(viii) Finally, the results confirm the hypothesised theory that implied volatility is positively related to historical volatility as we have concluded previously, but negatively correlated with time-to-maturity and with the underlying stock price return.

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TABLE 1

"Efficiency" of Estimators

Type	Author	Efficiency	Observation
Maximum Likelihood Estimator	Ball Torous [84]	8,1	Different estimator based on the Likelihood Function
Extreme Value Estimators	Parkinson [80]	5,2	Create a new estimator based on high-low prices
	Kunitomo [92]	10,0	Improves Parkinson's estimator for non-zero drift rate
	Garman and Klass [80]	8,4	Combines the Classic and the Parkinson's estimator
	Beckers [83]		Allows non-stationarity of weights in Garman-Klass' estimator
	Barnaud [90]	4,0	Simplifies the Garman-Klass' estimator when Opening prices are not available
	Rogers and Satchell [91]		Improves the Garman-Klass' estimator for non-continuous trading
Close-to-Close Estimators	Classic	1,0	
	Gemmill [93]		Improves the Classical estimator with different weights
Time and Sample Filters	Burghardt and Lane [90]		Concerned with the time of sampling for option pricing

FIGURE 1

Evolution of Historical Volatility Estimators

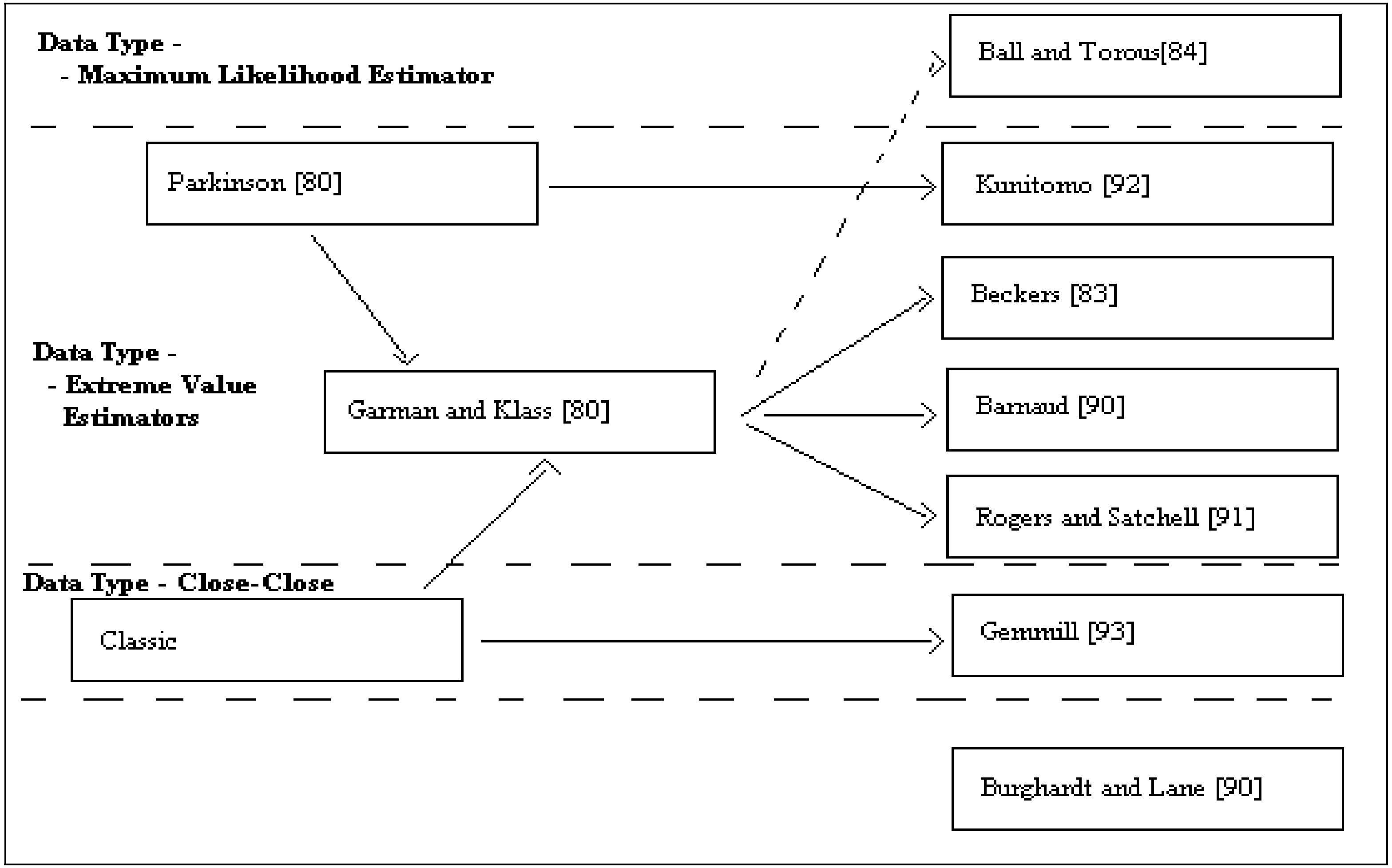

TABLE 2

HISTORICAL, IMPLIED AND FUTURE VOLATILITY ESTIMATORS

STATISTICS

		Mean	*Std Dev*	*Kurtosis*	*Skewness*	*Range*	*Maximum*	*Minimum*
	CLASIC20	0,262	0,118	8,242	2,439	0,862	0,964	0,102
	CLASIC50	0,266	0,103	3,603	1,825	0,578	0,708	0,130
	BURG_LAN	0,265	0,096	7,725	1,917	1,626	1,626	0,000
	PRKSN20	0,206	0,084	6,382	2,220	0,535	0,631	0,096
HISTORICAL	**GK20**	0,310	0,127	5,429	2,070	0,774	0,904	0,129
	BECKER	0,265	0,119	10,329	2,706	0,929	1,031	0,102
	KUNITOMO	0,193	0,103	9,825	2,662	0,815	0,878	0,063
	GEMMILL	0,254	0,121	12,306	2,807	1,190	1,274	0,083
	BARNAUD	0,181	0,082	8,653	2,359	0,662	0,662	0,000
	Average	**0,245**	**0,106**	--	--	--	--	--
	TRIPPI	0,283	0,116	6,584	2,236	0,918	1,049	0,131
	LATANE	0,277	0,110	7,731	2,381	0,918	1,049	0,131
	CHIRAS	0,286	0,120	7,584	2,441	0,918	1,049	0,131
	OISIMPL	0,277	0,114	8,484	2,479	0,934	1,058	0,124
IMPLIED	**MCBTH_B0**	0,274	0,107	7,160	2,307	0,903	1,038	0,135
	WHALEY	0,273	0,107	8,208	2,420	0,918	1,049	0,131
	BECKERS	0,272	0,106	8,370	2,444	0,918	1,049	0,131
	DAY_LEWI	0,273	0,109	9,222	2,547	0,941	1,064	0,123
	FINUC_AT	0,263	0,096	8,826	2,457	0,903	1,025	0,123
	Average	**0,275**	**0,109**					
FUTURE	**FUTCLSIC**	0,268	0,109	11,997	2,542	1,820	1,821	0,002
	FUTGK	0,313	0,110	5,765	1,822	1,414	1,461	0,047
	Average	**0,291**	**0,110**					

TABLE 3

ABSOLUTE AND SQUARED ERROR STATISTICS

		Mean	*Mean - Group*	*Std Dev*	*Kurtosis*	*Skewness*	*Range*	*Maximum*	*Minimum*
WEIGHTED AVERAGES	AE_TRIPP	0,0282		0,0367	17,593	3,397	0,4873	0,4873	0,0000
	AE_LATAN	0,0280		0,0425	23,576	4,021	0,5897	0,5897	0,0000
	AE_CHIRS	0,0297		0,0433	20,880	3,667	0,6187	0,6187	0,0000
	AE_OISMP	0,0293	0,0288	0,0436	24,496	3,985	0,6822	0,6822	0,0000
LEAST SQUARES	AE_WHLEY	0,0294		0,0480	27,924	4,387	0,7105	0,7105	0,0000
	AE_BCKRS	0,0298		0,0490	26,391	4,280	0,7218	0,7218	0,0000
	AE_DAYLW	0,0308	0,0300	0,0497	24,558	4,061	0,7403	0,7403	0,0000
EX. PRICE EFFECT	AE_MCBTH	0,0153		0,0215	23,525	3,800	0,3153	0,3153	0,0000
	AE_FINUC	0,0096	0,0124	0,0242	40,198	4,954	0,4346	0,4346	0,0000
Effectiveness									
WEIGHTED AVERAGES			43,24%						
LEAST SQUARES			41,43%						
EX. PRICE EFFECT			100,00%						
WEIGHTED AVERAGES	SE_TRIPP	0,0021		0,0077	179,289	10,670	0,2375	0,2375	0,0000
	SE_LATAN	0,0026		0,0111	183,344	11,251	0,3477	0,3477	0,0000
	SE_CHIRS	0,0028		0,0111	251,357	12,574	0,3828	0,3828	0,0000
	SE_OISMP	0,0028	0,0026	0,0118	291,791	13,423	0,4654	0,4654	0,0000
LEAST SQUARES	SE_WHLEY	0,0032		0,0149	211,597	12,189	0,5048	0,5048	0,0000
	SE_BCKRS	0,0033		0,0152	202,227	11,848	0,5209	0,5209	0,0000
	SE_DAYLW	0,0034	0,0033	0,0153	233,662	12,398	0,5480	0,5480	0,0000
EX. PRICE EFFECT	SE_MCBTH	0,0007		0,0029	269,504	13,428	0,0994	0,0994	0,0000
	SE_FINUC	0,0007	0,0007	0,0042	711,051	21,533	0,1889	0,1889	0,0000
Effectiveness									
WEIGHTED AVERAGES			26,79%						
LEAST SQUARES			20,83%						
EX. PRICE EFFECT			100,00%						

FIGURE 2

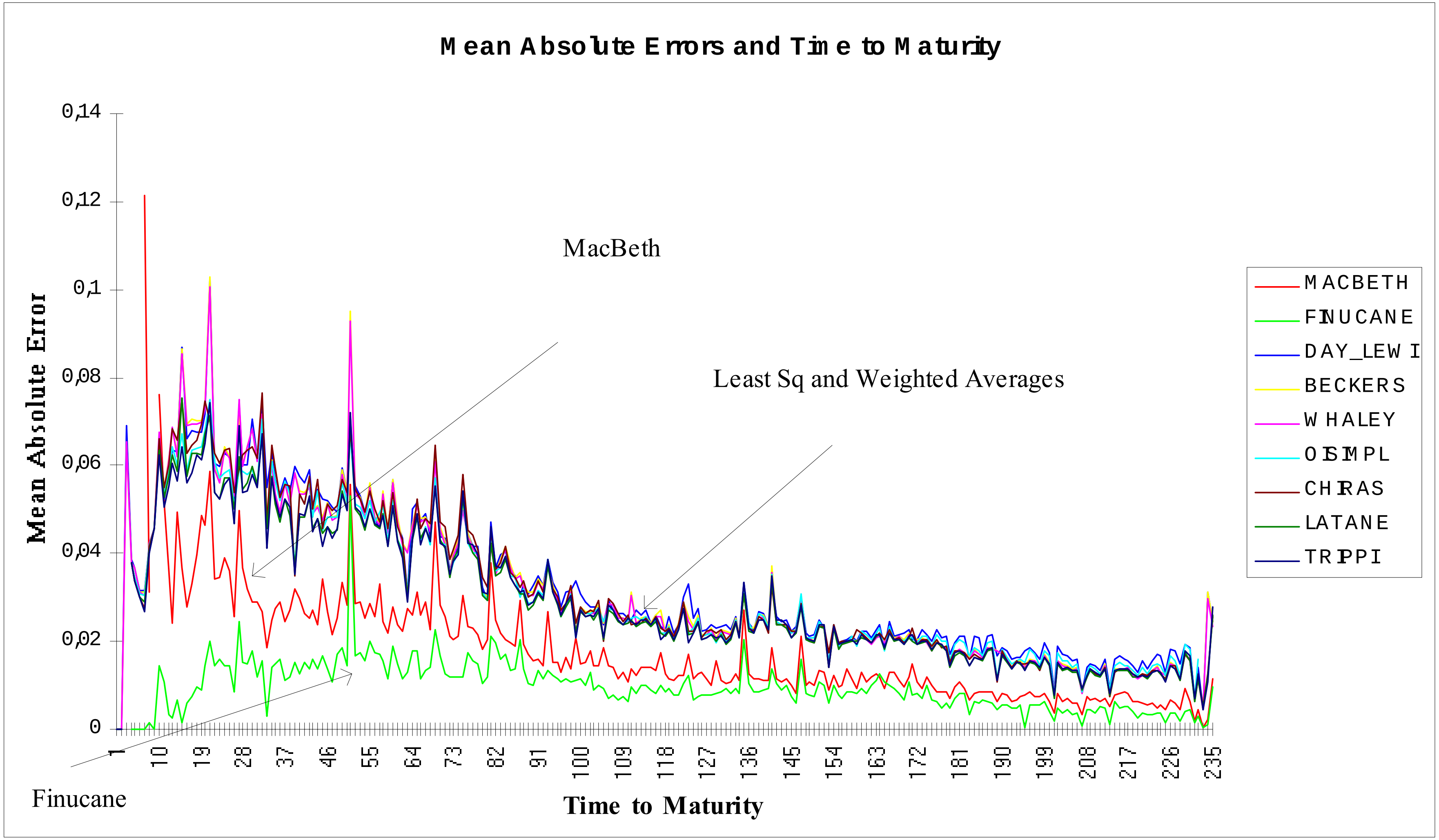
Mean Absolute Errors and Time to Maturity
0,14
0,12
0,1
0,08
0,06
0,04
0,02
0
Mean Absolute Error
MacBeth
Least Sq and Weighted Averages
Finucane
Time to Maturity
1
10
19
28
37
46
55
64
73
82
91
100
109
118
127
136
145
154
163
172
181
190
199
208
217
226
235
MACBETH
FINUCANE
DAY_LEWI
BECKERS
WHALEY
OISIMPL
CHIRAS
LATANE
TRIPPI

TABLE 4

Regression Matrix of Estimator Regressions

Average B0
Average B1
Average R Square
Average Durbin-Watson

DEPENDENT VARIABLE	*INDEPENDENT VARIABLE* - Before Cochrane-Orcutt Procedure -			- After Cochrane-Orcutt Procedure -		
	HISTORICAL	IMPLIED	FUTURE	HISTORICAL	IMPLIED	FUTURE
HISTORICAL						
(B0)	0,0403					
(B1)	0,8538					
(Mult-R)	**0,8323**					
IMPLIED						
(B0)	0,0966	0,0074		0,1789		
(B1)	0,7247	0,9741		0,3729		
(Mult-R)	**0,6951**	**0,9719**		**0,2595**		
FUTURE						
(B0)	0,1354	0,0871	0,0215	0,2290	0,2480	
(B1)	0,6396	0,7548	0,9267	0,2507	0,1668	
(Mult-R)	**0,6031**	**0,7498**	**0,9267**	**0,1463**	**0,2203**	

FIGURE 3

Estimator Effectiveness Adjustments

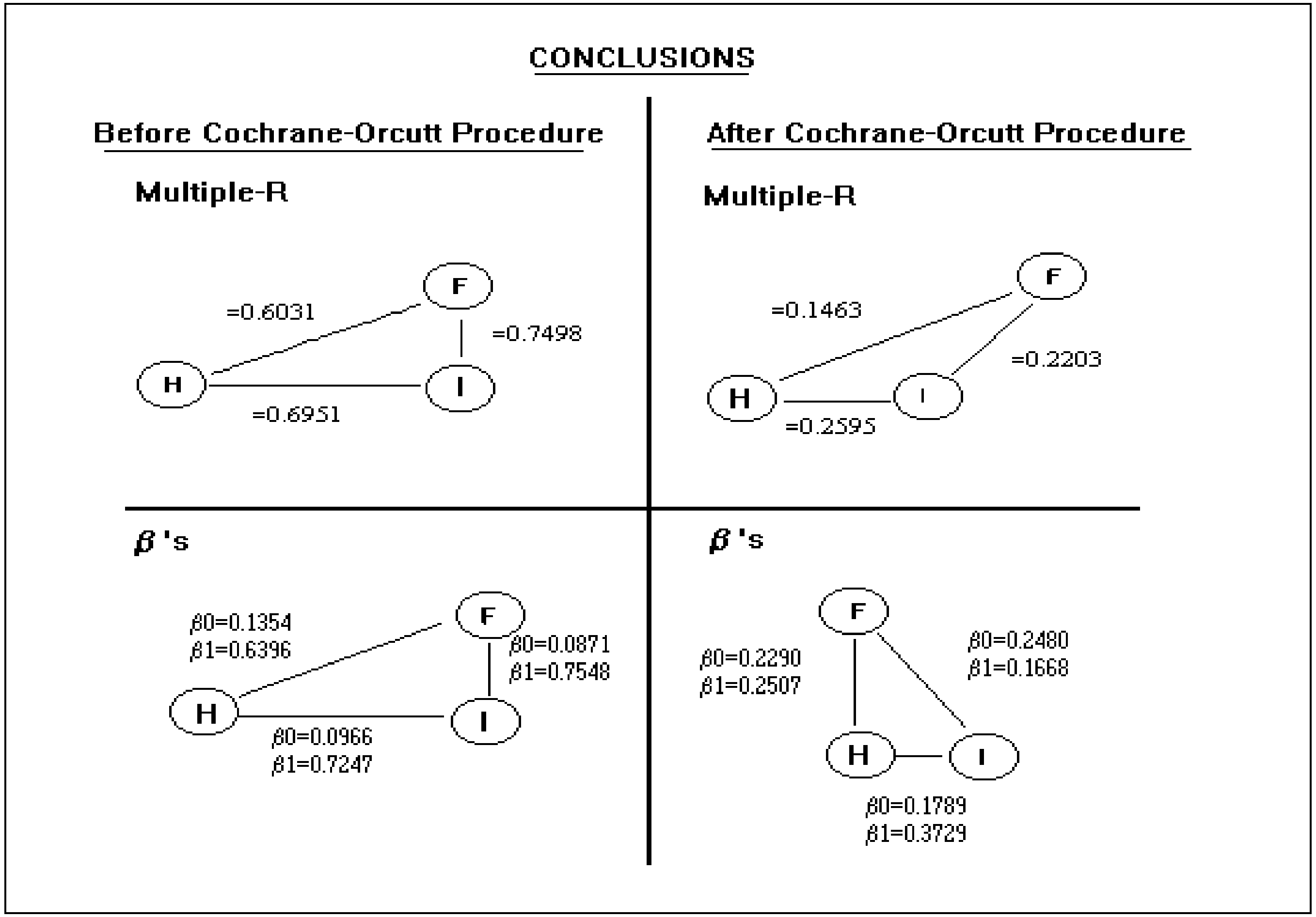

TABLE 5

Overall Statistics of the Parameters Obtained from the Regressions

	Before Cochrane-Orcutt Procedure				After Cochrane-Orcutt Procedure			
	Mult-R	*β0*	*β1*	*β2*	*Mult-R*	*β0*	*β1*	*β2*
Average	0,7977	0,0429	0,7413	0,0995	0,3264	0,1736	0,1553	0,2172
Stdev	0,0169	0,0088	0,0780	0,0386	0,1302	0,0751	0,2959	0,0592
Max	0,8141	0,0565	0,8250	0,1584	0,5346	0,2885	0,5175	0,3114
Min	0,7712	0,0337	0,6168	0,0524	0,1538	0,0826	-0,2777	0,1337

$$\sigma\ fut_i = \beta_0 + \beta_1\ \sigma\, imp_i + \beta_2\ \sigma\, hist_i + \varepsilon$$

TABLE 6

Adjustments to Finucane Estimator Regression

	Before Cochrane-Orcutt Procedure	*After Cochrane-Orcutt Procedure*
Multiple-R	0,700 221	0,341 455
b0	0,138 581	0,201 255
(t-value)	(26,74)	(22,53)
b1	0,567 372	0,288 994
(t-value)	(42,80)	(12,33)
b2	-0,000 161	-0,000 094
(t-value)	(-7,10)	(-9,19)
b3	-0,227 213	-0,077 610
(t-value)	(-2,90)	(-2,13)
DW	0,383 539	2,702 776
Sample Size	2.000	2.000

FINUC_AT = f (CLASIC20, T-t, STOCK RETURN)