Analyzing and Explaining Default Recovery Rates

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Abstract

This comprehensive report analyzes the impact of various assumptions about the association between aggregate default probabilities and the loss given default on bank loans and corporate bonds, and seeks to empirically explain this critical relationship. The analysis has important implications for the results of various value-at-risk credit risk models as well as the fundamental factors which influence fixed income portfolio models and strategies. Virtually all of the literature on credit risk management models and tools treat the important recovery rate variable as a function of historic average default recovery rates, conditioned perhaps on seniority and collateral factors, and in almost all cases as independent of expected or actual default rates. This report examines that assumption in the extant literature, in value-at-risk simulations based on several critical assumptions on the correlation between default and recovery rates, and in actual empirical tests. We specify and empirically test for a negative relationship between these two key inputs to credit loss estimates, and find that the result is indeed significantly negative with profound effects on value-at-risk and other measures.

We present the analysis and results in three distinct sections. The first section examines the literature of the last three decades of the various structural-form, closed-form and other credit risk and portfolio credit value-at-risk (VaR) models and the way they explicitly or implicitly treat the recovery rate variable. Our conclusion is that this critical variable has been treated in a rather simplistic and perhaps unrealistic manner. Section II presents simulation results under three different recovery rate scenarios and examines the

impact of these scenarios on the resulting risk measures. Specifically, we assume (i) that recovery is deterministic (fixed), (ii) that recovery rates are stochastic yet uncorrelated with the probability of default and (iii) recovery rates are stochastic and negatively correlated with default probabilities. Our results show a significant increase in both expected an unexpected losses in specification (iii) compared with either specification (i) or (ii), with virtually no difference between (i) and (ii).

Finally, in Section III, we empirically examine the recovery rates on corporate bond defaults, over the period 1982-2000. We attempt to explain recovery rates by specifying a rather straightforward statistical least squares regression model. The central thesis is that aggregate recovery rates are basically a function of supply and demand for the securities. Our econometric univariate and multivariate time series models explain a substantial proportion of the variance in bond recovery rates aggregated across all seniority and collateral levels. The models are also extremely accurate in forecasting 2001 recovery rates.

Our results have important implications for just about all portfolio credit risk models, for markets which depend on recovery rates as a key variable (e.g., securitizations, credit derivatives, etc.), for the current debate on the revised BIS guidelines for capital requirements on bank credit assets and for investors in corporate bonds of all credit qualities.

This report will be presented in three separate Sections along the lines as discussed above.

Section I

The Relationship Between Default Rates and Recovery Rates in Credit Risk Modeling: a Review of the Theoretical and Empirical Literature

I.1 Introduction

Credit risk affects virtually every financial contract. Therefore the measurement, pricing and management of credit risk has received much attention from practitioners, who have a strong interest in accurately pricing and managing this kind of risk, financial economists, who have much to learn from the way such risk is priced in the market, and bank supervisors, who need to design minimum capital requirements that correctly reflect the credit risk of banks' loan portfolios. Following the recent attempt of the Basel Committee on Banking Supervision to reform the capital adequacy framework by introducing risk-sensitive capital requirements, significant attention has been devoted to the subject of credit risk measurement by the international regulatory, academic and banking communities.

Three main variables affect the credit risk of a financial asset: (i) the probability of default (PD), (ii) the "loss given default" (LGD), which is equal to one minus the recovery rate in the event of default (RR), and (iii) the exposure at default (EAD). While there has been much work in the credit risk literature on the estimation of the first component (PD), much less attention has been dedicated to the estimation of RR and to the relationship between PD and RR. This is mainly the consequence of two related factors. First, credit pricing models and risk management applications tend to focus on the systematic risk components of credit risk, as these are the only ones that attract risk-premia. Second, credit risk models traditionally assumed RR to be independent of PD.

Evidence from many countries in recent years suggest that collateral values and

recovery rates can be volatile and, moreover, they tend to go down just when the number of defaults goes up in economic downturns (Schleifer and Vishny [1992], Altman [2001], Hamilton, Gupton and Berthault [2001]). Still, little quantitative analysis has appeared during the eighties and the nineties to assist bond portfolio managers and banks in setting sufficient collateral haircut[1], or in estimating RRs, or in pricing debt with stochastic collateral values[2]. Most credit risk models have indeed been based on static loss assumptions with, at best, a single average RR used for all secured loans and bonds and another single average used for all unsecured assets. These simplifying assumptions are particularly relevant given the high standard deviations and fat tails of the empirical distributions of RRs[3].

This traditional focus on default analysis has been partly reversed by the significant increase in the number of studies dedicated to the subject of RR estimation and the relationship between PD and RR that appeared during the last two years (Fridson, Garman and Okashima [2000], Gupton, Gates and Carty [2000], Jokivuolle and Peura [2000], Carey and Gordy [2001], Frye [2000a, 2000b and 2000c], Jarrow [2001]). This is partly the consequence of the parallel increase in default rates and decrease of recovery rates registered during the 1999-2001 period.

This literature review briefly summarizes the way credit risk models, which have developed during the last thirty years, treat RR and, more specifically, their relationship with the PD of an obligor. These models can be divided into two main categories: (a) credit pricing models, and (b) portfolio credit value-at-risk (VaR) models. Credit pricing models can in turn be divided into three main approaches: (i) "first generation" structural-form models, (ii) "second generation"

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[1] A collateral haircut is equivalent to a limit on the loan to value ratio, i.e., the maximum amount of loan that can be granted against a given amount of collateral in order to retain the risk of the loan at a desired level. Both recovery rates and collateral are dealt with in the Basel Committee proposals to reform the capital adequacy framework (Basel, 2001).

[2] A relevant exception being represented by Altman and Kishore (1996).

[3] See Van de Castle and Keisman (1999 and 2000) for empirical evidence on this point.

structural-form models, and (iii) reduced-form models. These three different approaches, together with their basic assumptions, advantages, drawbacks and empirical performance, are briefly outlined in sections 2, 3 and 4. Credit VaR models are then examined in section 5. Finally, the more recent studies explicitly modeling and empirically investigating the relationship between PD and RR are briefly analyzed in section 6. Section 7 concludes by summarizing the main results. A descriptive summary of these models can be found in Table I.1.

I.2 First generation structural-form models: the Merton approach

The first category of credit risk models are the ones based on the original framework developed by Merton (1974) using the principles of option pricing (Black and Scholes, 1973). In such a framework, the default process of a company is driven by the value of the company's assets and the risk of a firm's default is therefore explicitly linked to the variability in the firm's asset value. The basic intuition behind the Merton model is relatively simple: default occurs when the value of a firm's assets (the market value of the firm) is lower than that of its liabilities. The payment to the debtholders at the maturity of the debt is therefore the smaller of two quantities: the face value of the debt or the market value of the firm's assets. Assuming that the company's debt is entirely represented by a zero-coupon bond, if the value of the firm at maturity is greater than the face value of the bond, then the bondholder gets back the face value of the bond. However, if the value of the firm is less than the face value of the bond, the equityholders get nothing and the bondholder gets back the market value of the firm. The payoff at maturity to the bondholder is therefore equivalent to the face value of the bond minus a put option on the value of the firm, with a strike price equal to the face value of the bond and a maturity equal to the maturity of the bond. Following this basic intuition, Merton derived an explicit formula for default risky bonds which can be used both to estimate the PD of a firm and to estimate the yield differential between a risky bond and a default-free bond[4].

In addition to Merton (1974), first generation structural-form models include Black and Cox (1976), Geske (1977), and Vasicek (1984). Each of these models tries to refine the original Merton framework by removing one or more of the unrealistic assumptions. Black and Cox (1976) introduce the possibility of more complex capital structures, with subordinated debt; Geske (1977) introduces interest paying debt; Vasicek (1984) introduces the distinction between short and long term liabilities, which now represents a distinctive feature of the KMV model[5].

Under these models all the relevant credit risk elements, including default and recovery at default, are a function of the structural characteristics of the firm: asset volatility (business risk) and leverage (financial risk). The RR is therefore an endogenous variable, as the creditors' payoff is a function of the residual value of the defaulted company's assets. More precisely, under Merton's theoretical framework, PD and RR are inversely related (see Appendix I.A for a formal analysis of this relationship). If, for example, the firm's value increases, then its PD tends to decrease while the expected RR at default increases (ceteris paribus). On the other side, if the firm's debt increases, its PD increases while the expected RR at default decreases. Finally, if the firm's asset volatility increases, its PD increases while the expected RR at default decreases.

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[4] A formal analysis of the Merton (1974) model together with an examination of the relationship between the default probability and the recovery rate at default is presented in Appendix A.

[5] In the KMV model, default occurs when the firm's asset value goes below a threshold represented by the sum of the total amount of short term liabilities and half of the amount of long term liabilities. There is no theory, as far as we can tell, which specifies why the appropriate amount of debt includes 50% of long-term liabilities.

Although the line of research that followed the Merton approach has proven very useful in addressing the qualitatively important aspects of pricing credit risks, it has been less successful in practical applications [6]. This lack of success has been attributed to different reasons. First, under Merton's model the firm defaults only at maturity of the debt, a scenario that is at odds with reality. Second, for the model to be used in valuing default-risky debts of a firm with more than one class of debt in its capital structure (complex capital structures), the priority/seniority structures of various debts have to be specified. Also, this framework assumes that the absolute-priority rules are actually adhered to upon default in that debts are paid off in the order of their seniority. However, empirical evidence in Franks and Torous (1994) and others, indicates that the absolute-priority rules are often violated.

I.3 Second generation structural-form models

In response to such difficulties, an alternative approach has been developed which still adopts the original framework developed by Merton as far as the default process is concerned but, at the same time, removes one of the unrealistic assumptions of the Merton model, namely, that default can occur only at maturity of the debt when the firm's assets are no longer sufficient to cover debt obligations. Instead, it is assumes that default may occur any time between the issuance and maturity of the debt and that default is triggered when the value of the firm's assets reaches a lower threshold level[7]. These models include Kim, Ramaswamy and Sundaresan (1993), Hull

[6] The standard reference is Jones, Mason and Rosenfeld (1984), who find that, even for firms with very simple capital structures, a Merton-type model is unable to price investment-grade corporate bonds better than a naive model that assumes no risk of default.

[7] One of the earliest studies based on this framework is Black and Cox (1976). However, this is not included in the second generation models in terms of the treatment of the recovery rate.

and White (1995), Nielsen, Saà-Requejo, Santa Clara (1993), Longstaff and Schwartz (1995) and others.

Under these models the RR in the event of default is exogenous and independent from the firm's asset value. It is generally defined as a fixed ratio of the outstanding debt value and is therefore independent from the PD.

For example, Longstaff and Schwartz (1995) argue that, by looking at the history of defaults and the recovery ratios for various classes of debt of comparable firms, one can form a reliable estimate of the RR. In their model, they allow for stochastic term structure of interest rates and for the correlation between defaults and interest rates. They find that this correlation between default risk and the interest rate has a significant effect on the properties of the credit spread[8]. This approach simplifies the first class of models by both exogenously specifying the cash flows to risky debt in the event of bankruptcy and simplifying the bankruptcy process. This occurs when the value of the firm's underlying assets hits some exogenously specified boundary.

Despite these improvements with respect to the original Merton framework, second generation structural-form models still suffer from three main drawbacks, which represent the main reasons behind their relatively poor empirical performance[9]. First, they still require estimates for the parameters of the firm's asset value, which is nonobservable. Indeed, unlike the stock price in the Black and Scholes formula for valuing equity options, the current market value of a firm is not easily observable. Second, structural-form models cannot incorporate credit-rating changes that occur quite frequently for default-risky corporate debts. As is well known, most corporate

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[8] Using Moody's corporate bond yield data, they find that credit spreads are negatively related to interest rates and that durations of risky bonds depend on the correlation with interest rates.

[9] See Eom, Helwege and Huang (2001) for an empirical analysis of structural-form models.

bonds undergo credit downgrades before they actually default. As a consequence, any credit risk model should take into account the uncertainty associated to credit rating changes as well as the uncertainty concerning default, i.e., mark-to-market models. Finally, most structural-form models assume that the value of the firm is continuous in time. As a result, the time of default can be predicted just before it happens and hence, as argued by Duffie and Lando (2000), there are no "sudden surprises".

I.4 Reduced-form models

The attempt to overcome the above mentioned shortcomings of structural-form models gave rise to reduced-form models. These include Litterman and Iben (1991), Madan and Unal (1995), Jarrow and Turnbull (1995), Jarrow, Lando and Turnbull (1997), Lando (1998), Duffie and Singleton (1999), and Duffie (1998). Unlike structural-form models, reduced-form models do not condition default on the value of the firm, and parameters related to the firm's value need not be estimated to implement them. In addition to that, **reduced-form models introduce separate, explicit assumptions on the dynamic of both PD and RR. These variables are modeled independently from the structural features of the firm, its asset volatility and leverage**. **Generally speaking, reduced-form models assume an exogenous RR that is independent from the PD.** More specifically, reduced-form models take as primitives the behavior of default-free interest rates, the RR of defaultable bonds at default, as well as a stochastic intensity process for default. At each instant there is some probability that a firm defaults on its obligations. Both this probability and the RR in the event of default may vary stochastically through time. The stochastic processes determine the price of credit risk. Although these processes are not formally linked to the firm's asset value, there is presumably some underlying relation, thus Duffie and Singleton (1999) describe these alternative approaches as reduced-form models.

Reduced-form models fundamentally differ from typical structural-form models in the degree of predictability of the default. A typical reduced-form model assumes that an exogenous random variable drives default and that the probability of default over any time interval is nonzero. Default occurs when the random variable undergoes a discrete shift in its level. These models treat defaults as unpredictable Poisson events. The time at which the discrete shift will occur cannot be foretold on the basis of information available today.

Reduced-form models are distinguished somewhat by the manner in which the RR is parameterized. For example, Jarrow and Turnbull (1995) assume that, at default, a bond would have a market value equal to an exogenously specified fraction of an otherwise equivalent default-free bond. Duffie and Singleton (1999) followed with a model that, when specified to exogenous fractional recovery of market value at default, allows for closed-form solutions for the term-structure of credit spreads. Their model also allows for a random RR that depends on the pre-default value of the bond. Other models assume that bonds of the same issuer, seniority, and face value have the same RR at default, regardless of remaining maturity. For example, Duffie (1998) assumes that, at default, the holder of a bond of given face value receives a fixed payment, irrespective of coupon level or maturity, and the same fraction of face value as any other bond of the same seniority. This allows him to use recovery parameters based on statistics provided by rating agencies such as Moody's (2000) or from Altman (2001) – see our discussion in Section III of this report. Jarrow, Lando and Turnbull (1997) also allow for different seniority debt for a particular firm to be incorporated via different RRs in the event of default. Both Lando (1998) and Jarrow, Lando and Turnbull (1997) use transition matrices (historical probabilities of credit rating changes) to price defaultable bonds.

Empirical evidence concerning reduced-form models is rather limited. Using the Duffie and Singleton (1999) framework, Duffee (1999) finds that these models have difficulty in explaining the observed term structure of credit spreads across firms of different qualities. In particular, such models have difficulty generating both relatively flat yield spreads when firms have low credit risk and steeper yield spreads when firms have higher credit risk.

A recent attempt to combine the advantages of structural-form models – a clear economic mechanism behind the default process - and the ones of reduced-form models – unpredictability of default - can be found in Zhou (2001). This is done by modeling the evolution of firm value as a jump-diffusion process. This model links RRs to the firm value at default so that the variation in RRs is endogenously generated and the correlation between RRs and credit ratings before default, reported in Altman (1989) and Gupton, Gates and Carty (2000), is justified.

I.5 Credit Value-at-Risk Models

During the second part of the nineties, both banks and consultants started developing credit risk models aimed at measuring the potential loss, with a predetermined confidence level, that a portfolio of credit exposures could suffer within a specified time horizon (generally one year). These vaue-at-risk (VaR) models include J.P. Morgan's CreditMetrics® (Gupton, Finger and Bhatia [1997]), Credit Risk Financial Products' CreditRisk^{+}® (1997), McKinsey's CreditPortfolioView® (Wilson [1997a, 1997b and 1998]), and KMV's CreditPortfolioManager®. These models can largely be seen as reduced-form models, where the RR is typically taken as an exogenous constant parameter or a stochastic variable independent from PD. Some of these models, such as CreditMetrics®, CreditPortfolioView® and CreditManager®, treat the RR in the event of default as a stochastic variable – generally modeled through a beta distribution - independent from the PD. Others, such as CreditRisk^{+}®, treat it as a constant parameter that must be specified as an input for each single credit exposure.

While a comprehensive analysis of these models goes beyond the aim of this literature review[10], it is important to highlight that all credit VaR models treat RR and PD as two independent variables.

I.6 Some recent contributions on the PD-RR relationship

During the last two years, new approaches explicitly modeling and empirically investigating the relationship between PD and RR have been developed. These models include Frye (2000a and 2000b), Jokivuolle and Peura (2000), Jarrow (2001) and, Carey and Gordy (2001). Our discussion in Section III of this report, based on the research of Altman & Brady (2002), provides, we believe, the clearest evidence of a strong negative correlation between PD and RR, at the macro level.

The model proposed by Frye (2000a and 2000b) draws from the conditional approach suggested by Finger (1999) and Gordy (2000). In these models, defaults are driven by a single systematic factor – the state of the economy - rather than by a multitude of correlation parameters. These models are based on the assumption that the same economic conditions that cause default to rise might cause RRs to decline, i.e. that the distribution of recovery is different in high-default time periods from low-default ones. In Frye's model, both PD and RR depend on the state of the systematic factor. The correlation between these two variables therefore derives from their mutual dependence on the systematic factor.

The intuition behind Frye's theoretical model is relatively simple: if a borrower defaults on a loan, a bank's recovery may depend on the value of the loan collateral. The value of the

[10] For a comprehensive analysis of these models, see Crouhy, Galai and Mark (2000), Gordy (2000) Saunders (1999) and Allen and Saunders (2002).

collateral, like the value of other assets, depends on economic conditions. If the economy experiences a recession, RRs may decrease just as default rates tend to increase. This gives rise to a negative correlation between default rates and RRs.

While the model originally developed by Frye (2000a) implied recovery from an equation that determines collateral, Frye (2000b) modeled recovery directly. This allowed him to empirically test his model using data on defaults and recoveries from the U.S. corporate bond data. More precisely, data from Moody's Default Risk Service database for the 1982-1997 period have been used for the empirical analysis[11]. Results show a strong negative correlation between default rates and RRs for corporate bonds. This evidence is consistent with the most recent U.S. bond market data, indicating a simultaneous increase in default rates and LGDs for both 1999 and 2000[12]. Frye's (2000b and 2000c) empirical analysis allows him to conclude that in a severe economic downturn, bond recoveries might decline 20-25 percentage points from their normal-year average. Loan recoveries may decline by a similar amount, but from a higher level.

Jarrow (2001) presents a new methodology for estimating RRs and PDs implicit in both debt and equity prices. As in Frye (2000a and 2000b), RRs and PDs are correlated and depend on the state of the macroeconomy. However, Jarrow's methodology explicitly incorporates equity prices in the estimation procedure, allowing the separate identification of RRs and PDs and the use of an expanded and relevant dataset. In addition to that, the methodology explicitly incorporates a

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[11] Data for the 1970-1981 period have been eliminated from the sample period because of the low number of default prices with which to construct recovery rate annual averages.

[12] Hamilton, Gupton and Berthault (2001) and Altman and Brady (2002) provide clear empirical evidence of this phenomenon.

liquidity premium in the estimation procedure, which is considered essential in light of the high variability in the yield spreads between risky debt and U.S. Treasury securities.

Using four different datasets, Carey and Gordy (2001) analyze LGD measures and their correlation with default rates. Their preliminary results contrasts with the findings of Frye (2000b): estimates of simple default rate-LGD correlation are close to zero. They also find that limiting the sample period to 1988-1998, estimated correlations are more in line with Frye's results (0.45 for senior debt and 0.8 for subordinated debt). The authors note that, during this short period, the correlation rises, not so much because LGDs are low during the low-default years 1993-1996, but rather because LGDs are relatively high during the high-default years 1990 and 1991. They therefore conclude that the basic intuition behind the Frye's model may not adequately characterize the relationship between default rates and LGDs. Indeed, a weak or asymmetric relationship suggests that default rates and LGDs may be influenced by different components of the economic cycle.

Using defaulted bonds' data for the sample period 1982-2000, which include the relatively high default period of 1999 and 2000, we will show empirical results that appear consistent with Frye's intuition: a negative correlation between default rates and RRs. However, we find that the single systematic risk factor – i.e. the performance of the economy - is less predictive than Frye's model would suggest. We show that a simple microeconomic interpretation based on supply and demand drives aggregate recovery rate values rather than a macroeconomic model based on the common dependence of the two variables (default rates and recovery rates) on the state of the economy. In high default years, supply of defaulted securities tends to exceed demand[13], thereby

[13] Demand mostly comes from niche investors called "vultures", who intentionally purchase bonds in default. These investors represent a relatively small and specialized segment of the fixed income market.

driving secondary market prices – and therefore RRs - down. This in turn negatively affects RR estimates, as these are generally measured as the bond price shortly after default. We devote all of Section III of this report to this analysis.

A rather different approach is the one proposed by Jokivuolle and Peura (2000). The authors present a model for bank loans in which collateral value is correlated with the PD. They use the option pricing framework for modeling risky debt: the borrowing firm's total asset value determines the event of default. However, the firm's asset value does not determine the RR. Rather, the collateral value is in turn assumed to be the only stochastic element determining recovery. Because of this simplifying assumption, the model can be implemented using an exogenous PD, so that the firm asset value parameters need not be estimated. In this respect, the model combines features of both structural-form and reduced-form models.[14]

Jokivuolle and Peura's model takes into account the stochastic properties of the collateral value: its volatility, drift and correlation with the event of default. The theoretical results of this model show that the expected RR is a decreasing function of the collateral volatility and the correlation between collateral and the firm value driving default. A counterintuitive result is that the expected RR increases as PD increases.

However, this result is obtained assuming: (i) a positive correlation between a firm's asset value and collateral value, and that (ii) the value of collateral always equals the total face value of debt of the borrowing firm. A low PD implies that the firm's asset value has to strongly decline in the

[14] Because of this simplifying assumption the model can be implemented using an exogenous PD, so that the firm asset value parameters need not be estimated. In this respect, the model combines features of both structural –form and reduced-form models.

future before default can occur. Therefore, a positive correlation between asset value and collateral value implies that the latter is likely to be relatively low, too, in the case of default. For high PDs the firm asset value does not have to decline substantially before default can occur. Hence, the collateral value in default is on average also higher relative to its original value than in the case of low PD. This finding depends on two rather unrealistic assumptions, since the value of the assets chosen as collateral tends to be uncorrelated with the borrower's prospect and, moreover, not all loans are exactly 100% (fully) collateralized. Indeed, the above mentioned positive correlation between PD and RR vanishes for zero correlation between the collateral value and the firm value.

I.7 Concluding remarks

Table I.1 summarizes the way RR and its relationship with PD are dealt with in the different credit models described in the previous sections of this literature review.

TABLE I.1 APPROXIMATELY HERE

As clearly highlighted in Table I.1, while in the original Merton (1974) framework an inverse relationship between PD and RR exists, the credit risk models developed during the nineties treat these two variables as independent. The currently available and mostly used credit pricing and credit VaR models are indeed based on this independence assumption and treat RR either as a constant parameter or as a stochastic variable independent from PD. In the latter case, RR volatility is assumed to represent an idiosyncratic risk which can be eliminated through adequate portfolio diversification.

This assumption strongly contrasts with the growing empirical evidence showing a negative correlation between default and recovery rates (Frye [2000b and 2000c], Altman [2001], Carey and Gordy [2001], and Hamilton, Gupton and Berthault [2001]). This evidence indicates that recovery risk is a systematic risk component. As such, it should attract risk premia and should adequately be considered in credit risk management applications.

In the next section (Section II), we relax the assumption of independence between PD and RR and simulate the impact on value-at-risk models when these two variables are negatively correlated. In Section III, we carefully empirically estimate this critical dependence based on corporate bond market data over the last two decades.

Appendix I.A - The relationship between PD and RR in the Merton model

Merton-like default models provide us with a framework for deriving the expected recovery rate on a defaulted firm, as well as its default probability. While the latter was given much attention by subsequent research (see e.g. Crosbie, 1999), the former has been somewhat overlooked.

We therefore would like to briefly review the Merton model, emphasizing its implications for recovery rates, and showing how it can be used as a theoretical guideline to investigate the empirical link between default probabilities and severity.

In Merton-like models, the asset value of the firm follows a geometric Brownian motion:

$$dV_A = \mu V_A dt + \sigma_A V_A dz$$

where μ and σ_A are the firm's asset value drift and the volatility rate and *dz* are Wiener processes. This implies that the log of the asset value at a given future date *t*

$$\log V'_A = \log V_A + \left(\mu - \frac{\sigma_A^2}{2}\right)t + \sigma_A \sqrt{t}\varepsilon$$

follows a normal distribution with mean $\log V_A + \left(\mu - \frac{\sigma_A^2}{2}\right)t$ and variance $\sigma_A^2 t$. In turn, the asset value at time t will follow a lognormal distribution with mean $V_A e^{\mu t}$ and variance $V_A^2 e^{2\mu t}\left(e^{\sigma_A^2 t} - 1\right)$ (see e.g. Hull, 1997, for details).

Default happens if and only if, at time t, the value of the firm's assets, V'_A, is lower than its debt[15] X_t. That means that the firm's *probability of default*, *PD*, equals:

$$PD = p\left[V'_A < X_t\right] = p\left[\log V'_A < \log X_t\right] = p\left[\log V_A + \left(m - \frac{s_A^2}{2}\right)t + s_A\sqrt{t}e < \log X_t\right] =$$

$$= p\left[\frac{\log\frac{V_A}{X_t} + \left(m - \frac{s_A^2}{2}\right)t}{s_A\sqrt{t}} < -e\right] = \Phi\left(-\frac{\log\frac{V_A}{X_t} + \left(m - \frac{s_A^2}{2}\right)t}{s_A\sqrt{t}}\right) = \Phi\left(-d_2\right)$$

where $\Phi(.)$ is the normal c.d.f. and d_2 is similar to the quantity used in the standard Black-Scholes option pricing formula.

When default occurs, the recovery rate *RR* is given by the ratio of the asset value to the debt[16], V'_A/X_t. The expected recovery rate therefore is $E(V'_A/X_t)$, that is $E(V'_A)/X_t$. However, this is true only if $V'_A < X_t$, otherwise no default happens and no recovery can be observed. More formally, *the expected recovery rate, RR*, can then be defined as:

$$E\left(\frac{V'_A}{X_t}\middle| V'_A < X_t\right) = \frac{1}{X_t}E\left(V'_A\middle| V'_A < X_t\right)$$

that is, as $1/X_t$ times the mean of a truncated lognormal variable. This, in turn, is given by:

$$E\left(V'_A\middle| V'_A < X_t\right) = e^{m_* + \frac{s_*^2}{2}}\,\frac{\Phi\left(\frac{\log X_t - m_*}{s_*} - s_*\right)}{\Phi\left(\frac{\log X_t - m_*}{s_*}\right)}$$

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[15] Short-term debt due at time t can be used instead, since the inability to repay long-term debt does not, by itself, trigger insolvency.

[16] Assuming that bankruptcy costs are negligible.

(see Liu et al. [1997], for a formal proof), where $m_* = \log V_A + \left(m - \frac{s_A^2}{2} \right) t$ and $s_*^2 = s_A^2 t$ are the mean and variance of log V'_A.

Plugging these two quantities into the above equation gives the following result:

$$E\left(V_A' \middle| V_A' < X_t\right) = e^{\log V_A + mt} \frac{\Phi\left(- \frac{\log \frac{V_A}{X_t} + \left(m + \frac{s_A^2}{2} \right) t}{s_A \sqrt{t}} \right)}{\Phi\left(- \frac{\log \frac{V_A}{X_t} + \left(m - \frac{s_A^2}{2} \right) t}{s_A \sqrt{t}} \right)} = V_A e^{mt} \frac{\Phi(-d_1)}{\Phi(-d_2)} = E(V_A') \frac{\Phi(-d_1)}{\Phi(-d_2)}$$

where the meaning of d_1 and d_2 is similar as in the standard Black-Scholes formula.

The expected recovery rate therefore turns out to be:

$$RR = E\left(\frac{V_A'}{X_t} \middle| V_A' < X_t \right) = \frac{V_A}{X_t} e^{mt} \frac{\Phi(-d_1)}{\Phi(-d_2)} = E\left(\frac{V_A'}{X_t} \right) \frac{\Phi(-d_1)}{\Phi(-d_2)}$$

Figure II.1 shows a graphic representation of PD and RR. The left panel shows the normal distribution for $\log V_A'$, with PD given by the grey area on the left; the right panel shows the lognormal distribution for V'_A/X_t, the expected RR being the average of the values below 1, i.e., the mean of the values in the grey tail.

FIGURE I.1 APPROXIMATELY HERE

Given the expressions for PD and RR derived above, we can make sensitivity analyses on the link between those two variables. Figures I.2-4 consider the case of a firm with debt (X_t) worth 80, total assets (V_A) of 100, an annual asset volatility of 20% and an expected return on assets of 5%. This base case will be indicated by dotted vertical lines in the graphs; each time, one of the three main variables (X_t, V_A and s_A) will be shocked (both halved and doubled) to see how PD and RR change.

FIGURE I.2 APPROXIMATELY HERE

First, in Figure I.2, we see that an increase in debt makes default more likely, while reducing the recovery rate on the defaulted loan (this could happen when a firm has to face an unexpected liability, e.g. because of legal claims due to polluting factories, oil leaks and so on); the opposite happens in Figure I.3, when the initial value of the firm's assets is revised upwards (e.g., for a pharmaceutical concern announcing a new treatment for some lethal disease), the PD shrinks and the RR grows higher.

FIGURE I.3 APPROXIMATELY HERE

Finally, in Figure I.4, we see what happens when asset volatility increases. This could be the case of the telecommunications industry over the last two years: as the demand for e-commerce and Internet services has slowed down, the value of the investments made in broadband lines and UMTS licences has become more uncertain. From Figure I.4 we see that, in such instances, an increase in asset volatility – even leaving leverage unchanged – brings about higher default probabilities and lower recovery rates.

FIGURE I.4 APPROXIMATELY HERE

Table I.1 – The Treatment of LGD and Default Rates within Different Credit Risk Models

	MAIN MODELS & RELATED EMPIRICAL STUDIES	TREATMENT OF LGD	RELATIONSHIP BETWEEN RR AND PD
Credit Pricing Models			
First generation structural-form models	Merton (1974), Black and Cox (1976), Geske (1977), Vasicek (1984), Crouhy and Galai (1994), Mason and Rosenfeld (1984).	PD and RR are a function of the structural characteristics of the firm. RR is therefore an endogenous variable.	PD and RR are inversely related (see Appendix I.A).
Second generation structural-form models	Kim, Ramaswamy e Sundaresan (1993), Nielsen, Saà-Requejo, Santa Clara (1993), Hull and White (1995), Longstaff and Schwartz (1995).	RR is exogenous and independent from the firm's asset value.	RR is generally defined as a fixed ratio of the outstanding debt value and is therefore independent from PD.
Reduced-form models	Litterman and Iben (1991), Madan and Unal (1995), Jarrow and Turnbull (1995), Jarrow, Lando and Turnbull (1997), Lando (1998), Duffie and Singleton (1999), Duffie (1998) and Duffee (1999).	Reduced-form models assume an exogenous RR that is either a constant or a stochastic variable independent from PD.	Reduced-form models introduce separate assumptions on the dynamic of PD and RR, which are modeled independently from the structural features of the firm.
Single systematic factor models	Frye (2000a and 2000b), Jarrow (2001), Carey and Gordy (2001), Altman and Brady (2002).	Both PD and RR are stochastic variables which depend on a common systematic risk factor (the state of the economy).	PD and RR are negatively correlated. In the "macroeconomic approach" this derives from the common dependence on one single systematic factor. In the "microeconomic approach" it derives from the supply and demand of defaulted securities.
Credit Value at Risk Models			
CreditMetrics®	Gupton, Finger and Bhatia (1997).	Stochastic variable (beta distr.)	RR independent from PD
CreditPortfolioView®	Wilson (1997a and 1997b).	Stochastic variable	RR independent from PD
CreditRisk+®	Credit Suisse Financial Products (1997).	Constant	RR independent from PD
KMV CreditManager®	McQuown (1997), Crosbie (1999).	Stochastic variable	RR independent from PD

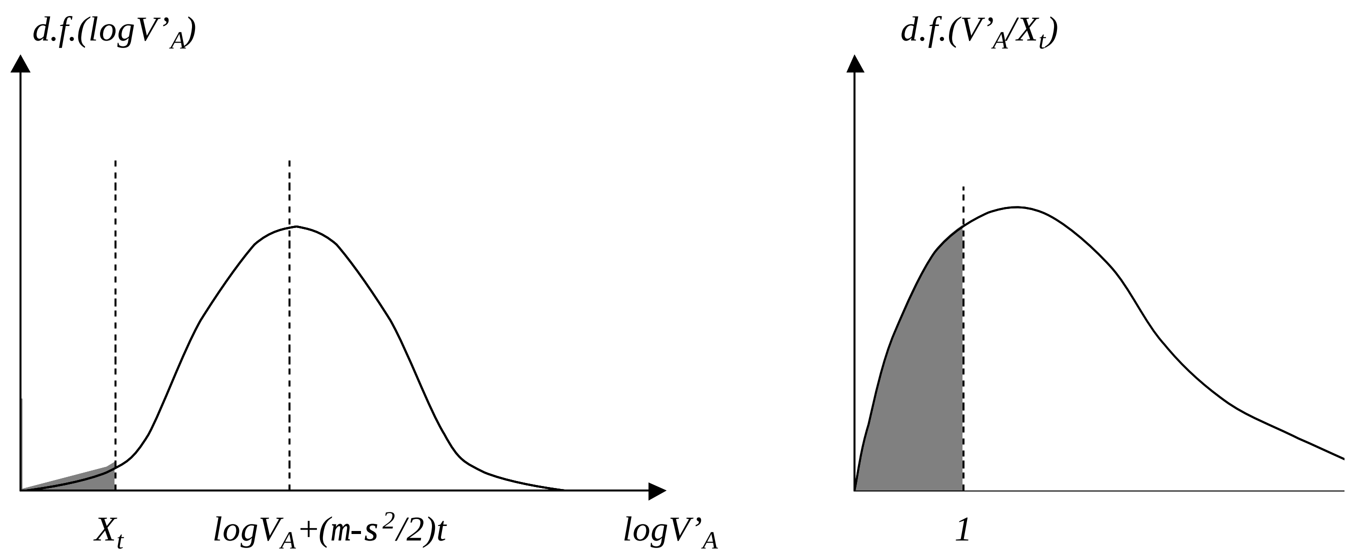

Figure I.1: Graphic Representation of PD and RR

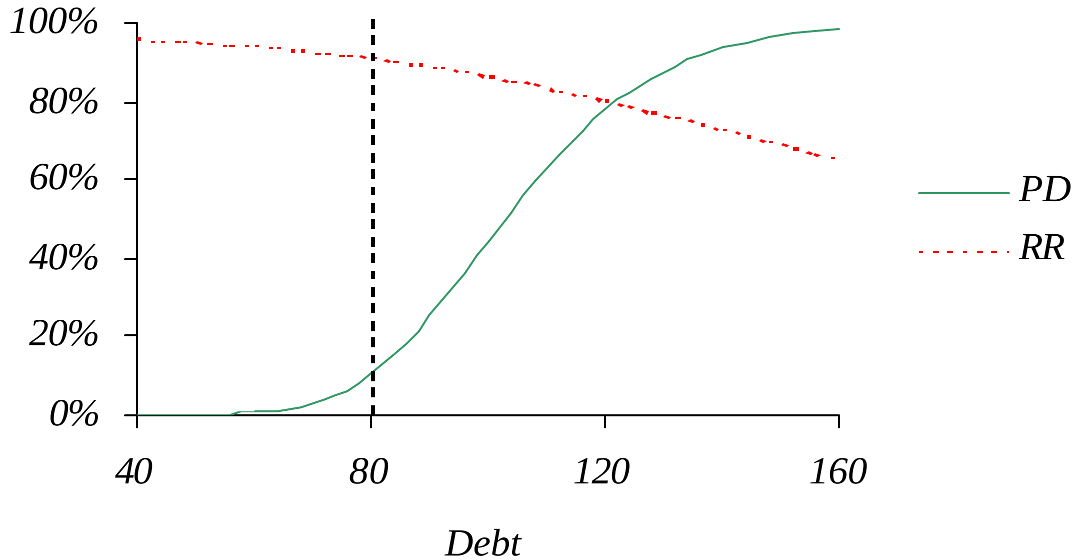

Figure I 2: the effect of debt value on PD and RR

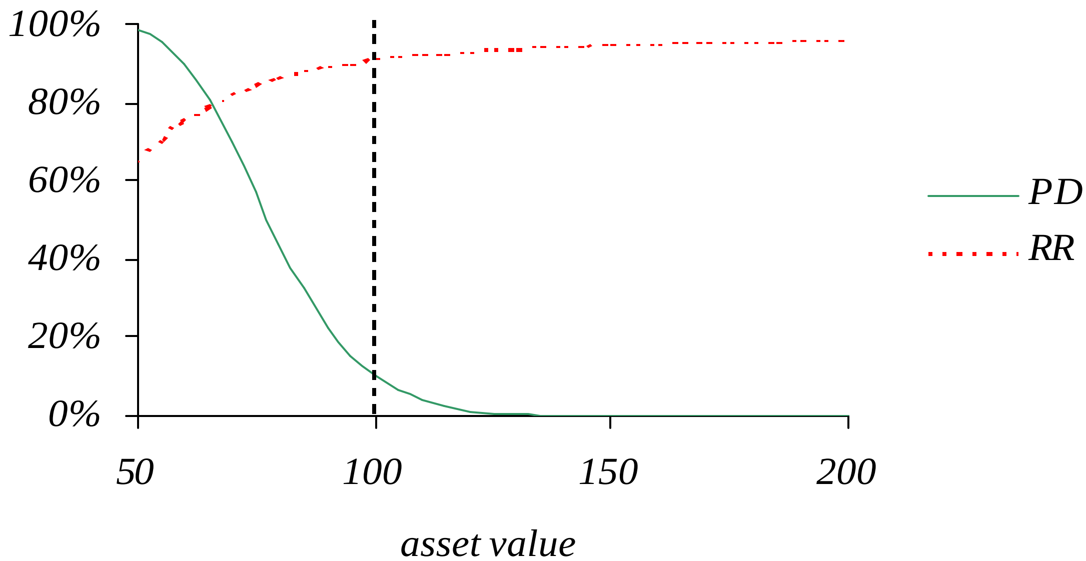

Figure I 3: the effect of asset value on PD and RR

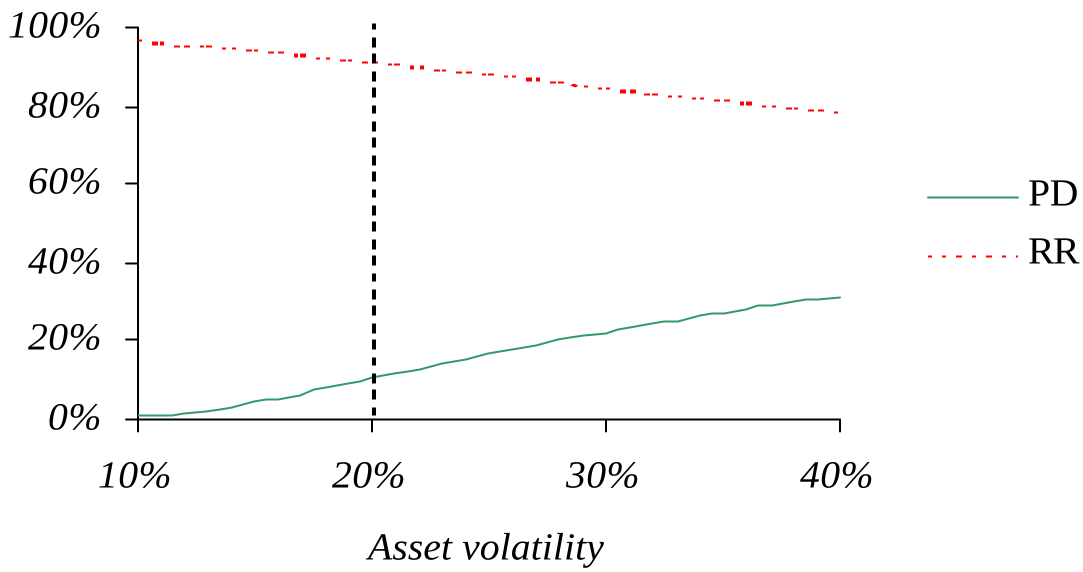

Figure I.4: the effect of asset volatility on PD and RR

Section II

The Effects of the Probability of Default-Loss Given Default Correlation on Credit Risk Measures: Simulation Results

II.1 Overview of the issues

This report is dedicated to an analysis of the empirical correlation between default and recovery risk. Before we proceed to consider the results derived from US bond market, this section focuses on what effects such a correlation, if found, *would imply* for the risk measures derived from the most commonly found and used credit risk VaR models. For example, as discussed earlier in the literature review and shown in Figure II.1, the basic version of the Creditrisk+® model treats recovery as a deterministic component; in other words, a credit exposure of 100 dollars with an estimated recovery rate after default of 30% is dealt with the same as an exposure of 70 dollars with a fixed loss given default (LGD) of 100%. The Creditmetrics® model allows for individual LGDs to be stochastic (the actual recovery rate on a defaulted loan is drawn from a beta distribution, through a Montecarlo simulation); however, the recovery rate is drawn independently of default probabilities, and an increase in default risk leaves the distribution of recovery rates unchanged.

FIGURE II.1 APPROXIMATELY HERE

Following this introduction, we will run Montecarlo experiments on a sample portfolio and compare the risk measures obtained under three different approaches (see Figure II.1). Recovery rates will be alternatively treated as:

a. deterministic (like in the Creditrisk+ approach);

b. stochastic, yet uncorrelated with the probabilities of default (PDs - like in the Creditmetrics framework);

c. stochastic, and partially correlated with default risk (as might happen in real life).

By doing so, we will be able to assess whether the computations of risk are different among the three approaches. In other words, if we will eventually find that default and recovery rates are significantly and negatively correlated, as we suspect, then our simulations will show by how much the first and second approaches underestimate risk, compared to the third one.

The results obtained depend on the actual portfolio considered in the simulation. Since we will use a large portfolio (with a high number of assets of different credit quality), however, we believe that the final outcome will be general enough to apply to a wide array of real-life situations.

II.2 Experimental setup: the sample and the process leading to default

Figure II.2 presents the benchmark portfolio used in our experiment. It includes 250 loans, generating a total exposure of 7.5 million Euros belonging to seven different rating grades. Individual exposures are shown on the x-axis, while the y-axis reports the PD levels associated with the rating classes[17] (ranging from 0.5% to 5%). As can be seen, the array of borrowers included in the benchmark portfolio looks widely diversified, as regards both credit quality and size; it should therefore be general enough to represent real-life loan portfolios.

FIGURE II.2 APPROXIMATELY HERE

[17] Note that these are long-term PDs that are going to be revised upwards or downwards in the short term because of both macroeconomic and idiosyncratic factors (see below).

Figure II.3 summarizes our simulation procedure. Our simulation engine draws heavily on the Creditrisk+ approach, as described in Credit Suisse Financial Products (1997). Note however that we are not going to follow the Creditrisk+ model as far as the computation of expected losses and risk measures is concerned, but will keep the simulation framework as flexible as possible to accommodate the three different treatments of recovery risk outlined in the section's overview.

As in all Montecarlo experiments, a large number of scenarios (100,000) will be drawn from a simulation engine, and the empirical distribution of such scenarios will then be used as a proxy for the theoretical distribution of losses (computing its expected value, standard deviation and some percentile-based risk measures).

FIGURE II.3 APPROXIMATELY HERE

Every scenario will be based on the following logic: in the short run, the default probability of each obligor can be seen as the product of two components: the long-term PD of the borrower (i.e., the value reported on the y-axis in Figure II.2) and a short-term shock, due both to macroeconomic and individual factors. Individual characteristics may be based, for example, on the obligor's industry, its size and the age of loan/bond facility. In symbols:

$$PD_{short} = PD_{long} \cdot Shock$$

This approach accounts for the fact that firms with different ratings tend to have, on average, different default rates, and that, nevertheless, their actual PDs might fluctuate over time according to the state of the economy and the firms' cash flow and profit cycles.

More specifically, the short-term shock can be thought of as the weighted sum of two random components, both drawn from independent gamma distributions with mean equal to one[18]: x_1 represents a background factor that is common to all the borrowers in the portfolio (the risk of an economic downturn affecting all bank customers), while x_2 is different for every obligor, and represents idiosyncratic risk:

$$Shock = w_1 x_1 + w_2 x_2$$

Note that, according to this framework, a recession would bring about a very high value for x_1 which, after being combined with the individual components (the x_2s), would significantly increase the short term PDs of most borrowers, bringing them above their average long-term values. This would make the bank's portfolio more vulnerable to default risk, since the actual number of defaults experienced over the following year would be higher. Indeed, if we were simulating all rating changes, the number of downgrades vs. no change or upgrades would increase as well. This is related to the "procyclicality" effect that may be an important issue inherent in any rating-based capital requirement standards.

The weights w_1 and w_2, through which the macroeconomic and individual shocks are combined, must be set carefully, since they play a very important role in the final results. If too much emphasis is attributed to systemic risk x_1, then the short-term PDs of all borrowers would mechanically respond to the macroeconomic cycle, and defaults would take place in thick clusters (increasing the variance of bank losses, i.e., the risk that must be faced by bank shareholders and regulators). Conversely, if a significant weight is given to the idiosyncratic risk

[18] In this way, the expected short-term PD will be the long-term value associated with each rating class.

x_2, then the defaults by different borrowers would be entirely uncorrelated and the stream of bank losses over time could appear quite smooth (since individual risks could be diversified away).

In order to keep things as simple and transparent as possible, we will use a simple fifty-fifty weighting scheme in our simulation. Note that – although it represents an arbitrary choice - this is not dramatically different from the 33%-66% scheme underlying the new regulatory framework proposed by the Basel committee in its January 2001 document[19].

We now return to Figure II.3, to see how this logic was implemented in our simulation. For each scenario:

1. A value for the background factor x_1 is drawn from a gamma distribution[20]; this value, which is common to all borrowers in the portfolio, is combined with an idiosyncratic noise term (x_2, also taken from a gamma), which is different for every obligor.

---

[19] In the January 2001 Basel document, default occurs because of changes in a firm's asset value; these, in turn, follow a standard normal distribution which combines a macro factor (with a weight of about .45) and an idiosyncratic term (with a weight of .89); hence the 33%-66% proportion quoted in the text. However, as noted by many observers who discussed the Basel proposals, the idiosyncratic component should probably be given more importance for small borrowers, while the systematic component should be more relevant for large firms, the credit quality of which tend to depend more heavily on the overall economic cycle. This remark sounds quite correct, yet using different weights for each borrower, depending on her size, would have made our simulation longer and less transparent. Therefore, we decided to stick to the simplest rule, the "fifty-fifty" weighting.

[20] We use gamma distributions because they are highly skewed to the right, accounting for the fact that default probabilities tend to stay low most of the time, but can increase dramatically in some (rare) extreme scenarios.

~~1.~~2. The combination of x_1 and x_2 is used to shock the long term values of the obligors' PDs in order to obtain the short term probabilities that will be used in the following steps. Note that when x_1 is low, most PDs will be revised downwards (as it happens when a healthy economy makes default risk smaller for most borrowers); on the other hand, when x_1 is high, default probabilities will be adjusted according to a more risky economic environment (as shown in our example in Figure II.3).

1.3. Based on the adjusted PDs, the computer draws which borrowers will actually default in this scenario. A loan with a 10% PD is more likely to default than one with a 2% PD. However, due to the random error, the latter might go bust while the former survives. This step of the simulation provides us with a list of defaulted borrowers.

~~1.~~4. For each defaulted loan in the list, the amount of losses is computed. This step can be performed in three different ways, depending on the assumptions concerning LGD outlined in Figure II.1. More details will be given in the following section.

~~1.~~5. The loss amount generated by this scenario is filed, and a new scenario is started.

Before moving to the results, the next section explains in more detail how the LGD computations were carried out.

II.3 Experimental setup: the computation of LGDs

The Montecarlo simulation described in the previous section was repeated three times, changing the way in which LGDs were handled. Following Figure II.1, we tested three different approaches:

a) First, LGD is deterministic. In this case we simply multiply the exposure of each defaulted asset by an "average" loss given default. To keep things simple, we use a 30% LGD for all borrowers, which is the mean of the beta distribution utilized in approach #2.

~~a)~~b) Secondly, LGD is stochastic but uncorrelated with default probabilities. In this case, LGD is separately drawn for each borrower from a beta distribution limited between 10% and 50%, with mean 30% and with a variance such that 5/9 of all values are bound between 20% and 40%.

~~a)~~c) Finally, LGD is stochastic and correlated with default probabilities. In this case, we are still using the same beta distribution as above, but we impose a perfect rank correlation between the LGD and the background factor x_1[21]. For example, when the background factor x_1 takes a very high value (thereby signalling that the economy is facing a recession), the LGDs increase up to 50%; on the other hand, when the economy improves, LGDs can become as low as 10%.[22].

---

[21] In other words, for every possible value x_1^* of the background factor x_1, such that $p(x_1^* < x_1) = P$, we choose the LGD as the *P*th percentile of its (beta) distribution.

[22] Note that this might look as a very "mild" way of implementing correlations between PDs and LGDs, since recoveries are still thought to be independent of the long-term rating of a borrower (the one that expresses her unconditional probability of default, i.e. PD_{long}). Actually, one might impose that LGDs depend directly on individual PDs, not on a macroeconomic background factor like x_1, and this would be somewhat more consistent with our empirical results on US bonds (see below) where ratings, not macroeconomic variables, emerge as the main driver behind changes in recovery rates. This would imply that the "unconditional", expected value of the LGD for each borrower, instead of being uniformly fixed at 30%, would be scaled up or down according to her rating; any differences among the three approaches quoted in the text would still arise *only from unexpected changes* in the LGDs, and any increase in expected and unexpected loss - when moving from approaches 1 and 2 to approach 3 - would remain the same as those shown in the next section.

II.4 Main Results of the Simulation

Table II.1 shows the main outcomes of our simulation exercise. The first three columns of data show loss and risk indicators obtained under the three approaches discussed in the previous paragraph. The last column quantifies the increase in those indicators when we move from the "quiet" world where no recovery risk is present to the more "dangerous" one where default and recovery risk tend to move together. All VaR measures (regardless of the confidence interval chosen), as well as the standard deviation, look considerably underestimated when recovery risk is overlooked.

We find that not only unexpected losses (i.e. the standard error and percentiles), but also expected losses tend to increase dramatically when shifting from column "a" to "c". This looks especially important since expected losses are generally thought to be computed correctly by multiplying the (long term) PD by the expected LGD. The numbers in Table II.1 suggest that such a straightforward practice might not be correct and seriously understate the actual loss. This finding will be scrutinized more carefully and further analysed by means of a simple numeric example, in the following section.

TABLE II.1 APPROXIMATELY HERE

Another noteworthy result is that no significant differences arise when we move from column (a) to (b): in other words, when recovery rates are considered stochastic, but independent on each other, the law of large numbers ensures that all uncorrelated risks can be effectively disposed of.

A portfolio of 250 loans already looks large enough to exploit this diversification effect, since its risk measures are not significantly different from the deterministic case. In other words, it is not uncertainty in recovery rates, but positive correlation, that brings about an increase in credit risk. Among all possible kinds of correlation, the link between recovery and default looks as the most significant and possibly dangerous one, since it increases both unexpected and expected losses. Moreover, the percent error found when moving from (a), or (b), to (c) is approximately the same (about 30%) for all risk and loss measures (expected and unexpected losses, percentile-based indices).

II.5. The Effect of PD/LGD Correlation on Expected Losses

According to a widely accepted practice, expected losses can be computed directly by multiplying PD by the expected LGD. Since expected LGD is the conditional mean of loss given that a default has occurred, this is just an application of the standard formula for unconditional means:

$$E(L) = E(L|D) \cdot PD + E(L|\neg D) \cdot (1 - PD) = E(LGD) \cdot PD + 0 \cdot (1 - PD) = E(LGD) \cdot PD$$

However, such a result holds when the PD is known, and the uncertainty on future losses arises only from the fact that default might actually take place or not. In real life, though, PD itself is uncertain in the short term (although its long term value might be known, as in our simulation exercise). In this case, the simple relationship above does not hold any more. Consider, e.g., the case in which the long-term PD of a borrower is 6%, but this value grows to 10% in a recession ($PD|R$=10%) and shrinks to 2% when the economy is in expansion ($PD|E$=2%). To keep things simple, suppose that both scenarios (expansion and recession) are equally likely on an *ex ante* basis. Finally, suppose that the expected LGD is 50%, yet this value moves up to 70% ($E(LGD|R)$) in recession years and decreases to 30% ($E(LGD|E)$) during expansion years.

The expected loss – computed as an unconditional mean over all possible scenarios – will be:

$$E(L) = \frac{1}{2} E(LGD|E) \cdot PD|E + \frac{1}{2} E(LGD|R) \cdot PD|R = \frac{1}{2} 70\% \cdot 10\% + \frac{1}{2} 30\% \cdot 2\% = 3.8\%$$

while the simple product between expected LGD and unconditional PD would of course equal 3% and would underestimate the true value by 80 basis points (i.e. by 21% in relative terms).

This suggests that the standard way of computing expected losses, although it is clearly appropriate each time that the PD of a loan is fixed and certain, might be far from accurate when default and recovery risk respond – to some extent – to the same background factors.

In other words, *should PD and LGD be driven by some common causes*, then not only the risk measures based on standard errors and percentiles (i.e., the unexpected losses usually covered with bank capital), but *even the amount of "normal" losses to be expected on a given loan* (and to be shielded through charge-offs and reserves) *could be seriously underestimated* by most credit risk models. In our opinion, this reinforces the theoretical interest for the empirical tests to be presented in the following Section of this report.

Table II.1

Main results of the LGD simulation

	LGD modelled according to approach			
	(a)	*(b)*	*(c)*	*% error**
Expected Loss	463	458	598	29.4%
Standard error	982	978	1,272	29.5%
95% VaR	1,899	1,880	2,449	28.9%
99% VaR	3,835	3,851	4,972	29.6%
99.5% VaR	3,591	3,579	4,653	29.6%
99.9% VaR	3,738	3,774	4,887	30.7%

** computed as [(c) – (a)] / (a)*

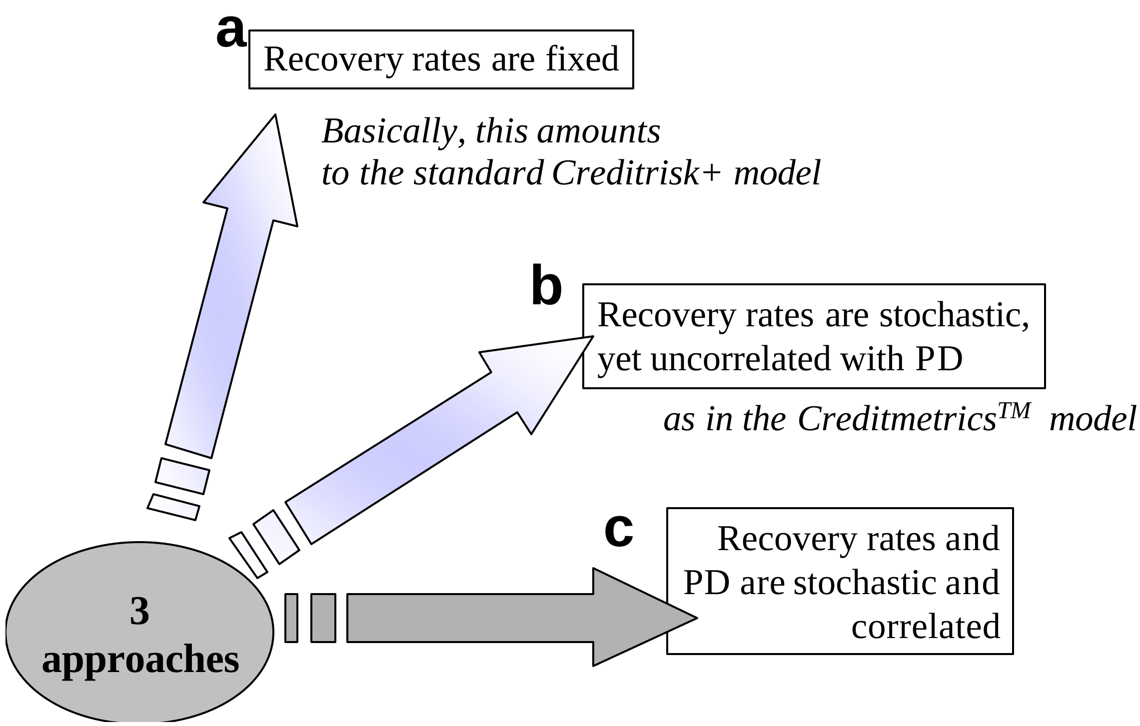

Figure II.1: three different approaches to modeling recovery rates

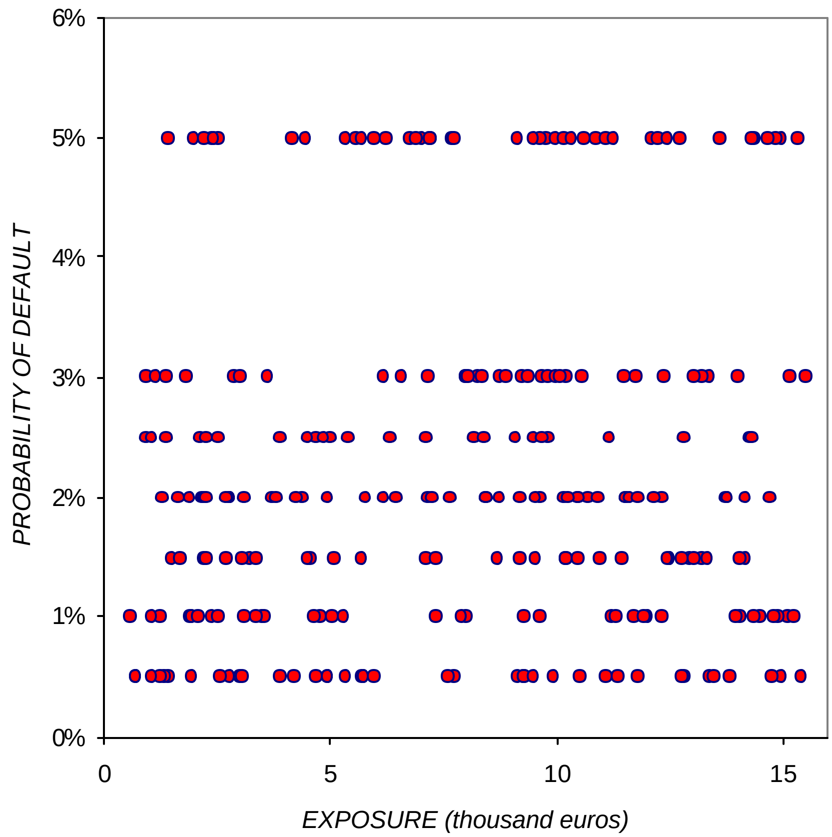

Figure II.2: PD and exposure of the 250 loans included in the benchmark portfolio

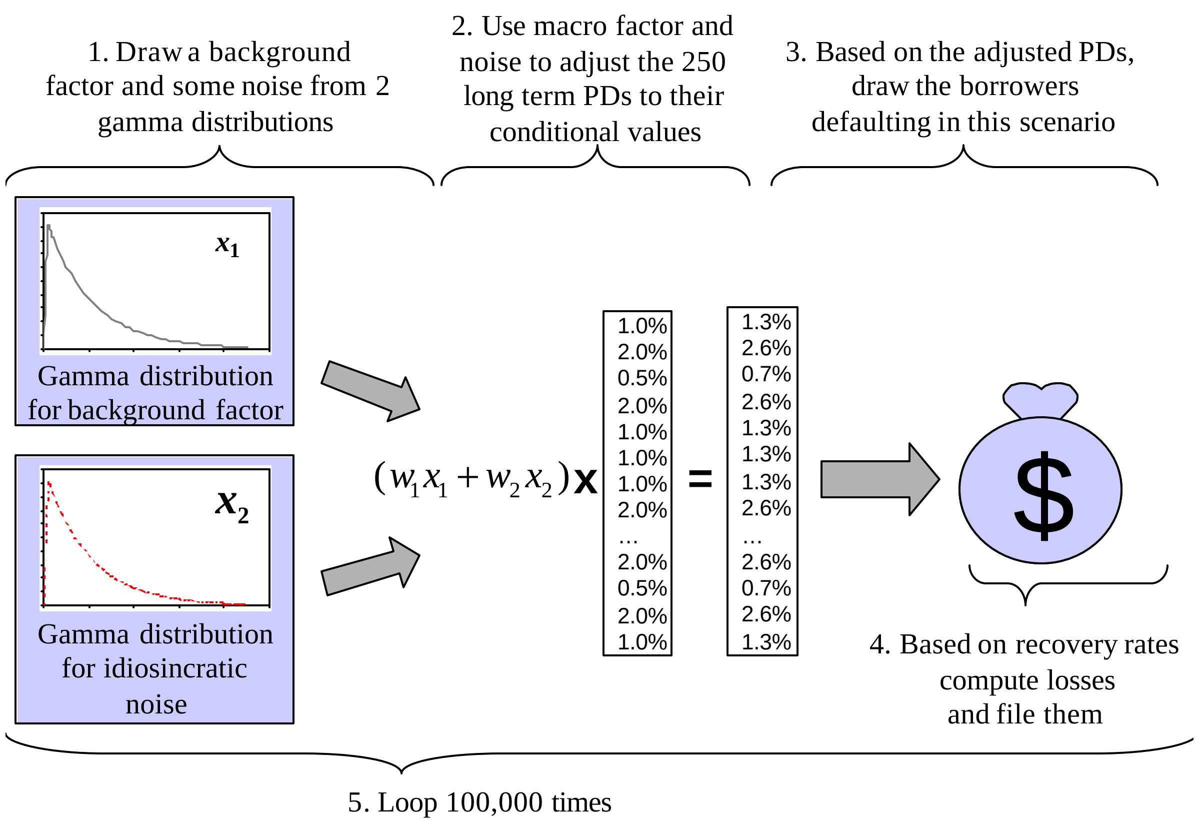

Figure II.3: the simulation engine used in our experiment

Section III

Explaining Aggregate Recovery Rates on Corporate Bond Defaults: Empirical Results

III.1 Introduction and Purpose

Until just a few years ago, credit risk management models and techniques have focused on the probability of default component in the calculation of loss-given-default (LGD) estimates for individual and portfolio counterparties. Virtually all of the literature treated the second important component, the recovery rate after default, as a function of average recovery rates on either bonds or bank loans and, in almost all cases, as independent of expected or actual default rates.

These credit value at risk models, for example **CreditMetrics®** (Gupton, Finger and Bhatia, 1997), focused on the probability of default as systematic risk determined usually based on the rating of the counterparty, but left the LGD specification as independent of default probabilities. Following on this model, as well as those from CreditRisk+® (1997) and CreditPortfolioView® (1997), Basel II (1999 and 2001) essentially neglects the correlation between loss given default and the default rate. LGD is empirically specified based on average recovery rate statistics or somewhat arbitrary risk weights These specifications are not consistent with economic intuition and have not gone unnoticed by theorists[23] or by proponents of Basel II (e.g., Carty and Gordy 2001) and have motivated more in-depth studies -- one of which is our empirical investigation.

The average loss experience on credit assets is now well documented in studies by the various rating agencies (Moody's, S&P, and Fitch) as well as by academics. See Appendix III.A for a summary of these average recovery rate results for bonds, stratified by seniority, as well as for bank loans. The latter asset class can be further stratified by capital structure and collateral type. Altman and Kishore (1996) breakdown the results by industry affiliation and original bond

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[23] For example, Jarrow and Turnbull (1995) and Duffie and Singleton (1999) conceptually include fractional recovery values in their models.

rating, as well as the seniority of the issue. While quite informative, they say nothing about the recovery vs. default correlation. The purpose of this Section is to empirically test this relationship with actual default data from the U.S. corporate bond market over the last two decades.

As Frye (2000a), Altman (2001), and others point out, there is strong intuition suggesting that default and recovery rates are correlated. Models that use a type of Merton (1974) approach, suggesting that the simple relationship between asset values and liability obligations will determine the default probability and the time when the default will occur, imply that recovery rates should be extremely high if the asset values are still very close to liability levels when defaults occur. And, if these asset values are independent of the aggregate default level, then default probabilities and recovery rates will be relatively independent. On the other hand, if high default levels imply a systematic decline in asset values, then the intuition of non-independence should manifest. Indeed, Frye (2000a) posits that a single systematic factor, e.g., GDP, industry performance, etc., can and does significantly impact recovery rates on corporate bonds. On the other hand, Carey and Gordy (2001) find contrasting evidence when they analyze similar relationships for the 1970-98 period, as contrasted to Frye's shorter 1988-98 sample period. They conclude that the low default and high recovery results of 1993-96 influence Frye's results but that preliminary results for their longer period does not show any correlation whatsoever. Our multivariate results for the period 1982-2000, which includes the relatively high default years of 1999 and 2000, seem to be consistent with Frye's intuition. We find, however, that the asymptotic single risk factor or systematic risk component that Frye posits, is less predictive than one would assume. Still, Frye's objective to better align required regulatory capital with loss given default precision is our goal as well, and we do find a highly significant relationship between default rates and recoveries, especially in our multivariate results.[24]

We will concentrate on average annual recovery rates but not on the factors that contribute to understanding and explaining recovery rates on individual firm and issue defaults. Van de Castle and Keisman (1999) indicate that factors like capital structure, as well as collateral and seniority, are important determinants of recovery rates and Madan and Unal (2001) propose a model for estimating risk-neutral expected recovery rate distributions - - not empirically observable rates. The latter can be particularly useful in determining prices on credit derivative instruments, such as credit default swaps.

III.2 Explaining Recovery Rates

This third Section of our study attempts to explain the important recovery rate variable by specifying rather straightforward statistical models. The central thesis of these models is that aggregate recovery rates on corporate bond defaults are basically a function of supply and demand for the securities of defaulting companies. Moreover, the performance of the economy, in general, although negatively correlated with default rates, has only secondary importance. We do recognize the systematic relationship between economic performance measures and expected default rates but find that macroeconomic forces and their changes are less important in explaining default recovery rates. Since the demand for distressed and defaulted bonds and bank loans has been rather stable over the last decade or so and is extremely difficult, in fact, to estimate precisely, we will concentrate more on the supply side.

---

[24] Some of our preliminary results in Altman (2001) show a highly significant univariate relationship between aggregate default and recovery rates.

We measure aggregate annual recovery rates by the weighted average recovery of all corporate bond defaults, primarily in the United States, over the periods 1982-2000 and also for the shorter period, 1987-2000. The weights are based on the market value of defaulting debt issues of publicly traded corporate bonds. The sample includes annual averages from about 1000 bonds for which we were able to get reliable quotes on the price of these securities just after default. We utilize the database constructed and maintained by the NYU Salomon Center, under the direction of one of the authors. Our models are both univariate, one dependent and one independent (explanatory) variable, and multivariate, least squares regressions. We are able to explain up to 60% of the variation of average annual recovery rates with our univariate models and as much as 90% of the variation with our multivariate models.

The rest of this Section will proceed as follows. We begin our analysis by describing the independent variables used to explain the annual variation in recovery rates. These include supply-side aggregate variables that are specific to the market for corporate bonds, as well as macroeconomic factors. We also present some background data on the demand for distressed and defaulted securities for the period 1990-2001. Next, we describe the results of the univariate analysis in which we utilize these variables to help explain the variation in annual aggregate recovery rates. We then describe our basic multivariate model and attempt to enhance this model by including macroeconomic factors in addition to the bond market factors. Finally, we use our best model to predict recovery rates for 2001 and conclude with a discussion of procyclicality in debt markets.

III.2.1 – Explanatory Variables

We proceed by listing several variables we reasoned could be correlated with aggregate recovery rates. Table III.1 summarizes the effects we expect changes in these variables to have on recovery rates. We use two different time frames in our various analyses, 1982-2000 and

1987-2000, because the defaulted bond index return (***BIR***) has only been calculated since 1987. We go no earlier than 1982 because there are so few default observations before 1982. The exact definitions of the variables we use are:

BRR = The weighted average recovery rates on defaulted bonds calculated annual and its logarithm (***BLRR***). Weights are based on the market value of defaulting issues. Prices of defaulted bonds are based on the closing levels on or as close to the default date as possible.

BDR = The weighted average default rate on bonds in the high yield bond market and its logarithm (***BLDR)***. Weights are based on the face value of all high yield bonds outstanding each year[25].

BDRC = One Year Change in ***BDR***.

BOA = This is the total amount of high yield bonds outstanding for a particular year (measured at mid-year in trillions of dollars) and represents the potential supply of defaulted securities. We used this variable for both sample time periods. Since the size of the high yield market has grown in most years over the sample period, the ***BOA*** variable is picking up a time-series trend as well as representing a potential supply factor.

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[25] We did not include a variable that measures the distressed, but not defaulted, proportion of the high yield market since we do not know of a time series measure that goes back to 1987. We define distressed issues as yielding more than 1000 basis points over the risk-free 10-year Treasury Bond Rate. We did utilize the average yield spread in the market and found it was highly correlated (0.67) to the subsequent one year's default rate and hence did not add value (see discussion below). The high yield bond yield spread, however, can be quite helpful in forecasting the following year's BDR, a critical variable in our model.

BDA = We also examined the more directly related bond defaulted amount as an alternative for ***BOA*** (also measured in trillions of dollars).

BIR = This is the one year return on the Altman-NYU Salomon Center Index of Defaulted Bonds. The index is a monthly indicator of the market weighted average performance of a sample of defaulted publicly traded bonds. We have calculated this Index since 1987; more details can be found in Altman (1991) and Altman and Cyrus (2001). This is a measure of the price changes of existing defaulted issues as well as the "entry value" of new defaults and, as such, is impacted by supply and demand conditions in this "niche" market.[26]

GDP = The annual GDP growth rate.

GDPC = The change in the annual GDP growth rate from the previous year.

GDPI = Takes the value of 1 when GDP growth was less than 1.5% and 0 when GDP growth was greater than 1.5%.

SR = The annual return on the S&P 500 stock index.

SRC = The change in the annual return on the S&P 500 stock index from the previous year.

[26] We are aware of the fact that the average recovery rate on newly defaulted bond issues could influence the level of the defaulted bond index and vice-versa. The vast majority of issues in the index, however, are usually comprised of bonds that have defaulted in prior periods. And, as we will see, while this variable is significant on an univariate basis and does improve the overall explanatory power of the model, it is not an important contributor. We could only introduce this variable in the 1987-2000 regression.

Table III.1 summarizes the variables we use in our analysis and their expected effect on aggregate recovery rates.

TABLE III.1 APPROXIMATELY HERE

The Basic Explanatory Variable: Default Rates - It is clear that the supply of defaulted bonds is most vividly depicted by the aggregate amount of defaults and the rate of default. In the following section we will specify a univariate model, which correlates the concurrent corporate bond default rate with weighted average recovery rates.

FIGURE III.1 APPROXIMATELY HERE

Figure III.1 shows statistics on the traditional method for calculating default rates in the high yield bond market as the proportion of the market that has defaulted each year since 1971 through the third quarter of 2001. Since virtually all public defaults most immediately migrate to default from the non-investment grade or "junk" bond segment of the market, we use that market as our population base. The default rate is the par value of defaulting bonds divided by the total amount outstanding, measured at face values. We observe that this annual rate has varied between close to zero percent (0.16% in 1981) to as much as over ten percent in 1990 and 1991. The weighted (by amount outstanding each year of the total market) average annual default rate is 3.5%, with a standard error of about 2.6%. This annual default rate will be our primary explanatory variable and it will be our main supply side estimate of the defaulted bond market. We will also assess the influence of the absolute dollar amount of defaults.

FIGURE III.2 APPROXIMATELY HERE

Figure III.2 shows the same default rate data from 1978-2000 as well as the weighted annual recovery rates (our dependent variable) and the default loss rate (last column). The average annual recovery is 41.2% and the weighted average annual loss rate to investors is 2.45%[27]. Figure III.3 breaks down the annual and average recovery rates by seniority, over the same 1978-2000 period. Note the expected decreasing average and median recovery rates as the seniority level drops, except for the senior subordinate and subordinate classes which have the same median levels. Appendix III.A shows similar results based on studies from several of the rating agencies. Altman and Eberhart (1994) show recovery rates, by seniority, where the recovery date is at the time of emergence from bankruptcy reorganization and not the default date.

FIGURE III.3 APPROXIMATELY HERE

The Demand and Supply of Distressed Securities - The principal purchasers of defaulted securities, primarily bonds and bank loans, are niche investors called distressed asset or alternative investment managers - also called "vultures." These investors are now an established segment of the fixed-income market and include private partnerships, open and closed-end mutual funds, university endowment pension funds, fund-of-funds and, periodically, hedge fund investors who move in and out of the market as opportunities present themselves. Most of these asset pools hire specialist managers. Prior to 1990, there was little or no analytic interest in these investors, indeed in the distressed debt market, except for the occasional anecdotal evidence of performance in such securities. Bankrupt railroads investments following the depression of the 1930's or the REIT or energy industry crises of the 1970's and 1980's are examples. Since the

[27] The loss rate is impacted by the lost coupon at default as well as the more important lost principal.

total assets under management that is dedicated to the distressed securities market is, as we will show, relatively small, there was little effort to measure its flow of funds and performance.

Altman (1991) was the first to attempt an analysis of the size and performance of the distressed debt market and estimated, based on a fairly inclusive survey, that the amount of funds under management by these so-called vultures was at least $7.0 billion and if you include those investors who did not respond to the survey and non-dedicated investors, the total was probably in the $10-12 billion range. Cambridge Associates (2001) estimated that the amount of distressed assets under management in 1991 was $6.3 billion. Cambridge also estimated that distressed investors had about $20 billion under active management in the 1997-2000 period. More recent market activity in 2001 indicated several new investment pools and large increase in assets for established distressed debt investors to take advantage of opportunities in the booming supply (see below) of distressed securities. This has probably swelled the market to about $30-35 billion, according to market practitioners and our own estimates. If you add in the potential investor attention from other types of hedge funds, the amount of funds concerned with distressed and defaulted securities is somewhat greater.

On the supply side, the last decade has seen the amounts of distressed and defaulted public and private bonds and bank loans grow dramatically in 1990-1991 to as much as $300 billion (face value) and $200 billion (market value), then recede to much lower levels in the 1993-1998 period and grow enormously again in 2000-2001 to the unprecedented levels of $650 billion (face value) and almost $400 billion market value. These estimates can be found in Figure III.4

and are based on calculations in Altman and Cyrus (2001) from periodic, not continuous, market calculations and estimates.[28]

FIGURE III.4 APPROXIMATELY HERE

On a relative scale, the ratio of supply to demand of distressed and defaulted securities was something like ten to one in both 1990-1991 and also in 2000-2001. Dollarwise, of course, the amount of supply side money dwarfed the demand in both periods. And, as we will show, the price levels of new defaulting securities was relatively very low in both periods - - at the start of the 1990's and again at the start of the 2000 decade.

A Note on Recovery Rate Data - As indicated above, our recovery rate variable (BRR) is based on the market prices of defaulted bonds from a point in time as close to the default date as possible. In reality, this precise date pricing was only possible in the last ten years, or so, since market maker quotes were not available for the NYU Salomon Center database prior to 1990 and all prior date prices were acquired from secondary sources, primarily the S&P Bond Guides. Those latter prices were based on end-of-month closing bid prices only. We feel that more exact pricing is a virtue since we are trying to capture supply and demand dynamics which may impact prices negatively if some bondholders decide to sell their defaulted securities as fast as possible.

[28] Defaulted bonds and bank loans are relatively easy to define and are carefully documented by the rating agencies and others. Distressed securities are defined here as bonds selling at least 1000 basis points over comparable maturity Treasury Bonds (we use the 10-year T-Bond rate as our benchmark). Privately owned securities, primarily bank loans, are estimated as 1.5-1.8 x the level of publicly owned distressed and defaulted securities based on studies of a large sample of bankrupt companies (Altman and Cyrus, 2001).

In reality, we do not believe this is an important factor since many investors will have sold their holdings prior to default or are more deliberate in their "dumping" of defaulting issues.

III.2.2 – Univariate Models

We begin the discussion of our results with the univariate relationships between recovery rates and the explanatory variables described in the previous section. Figure III.5 and Figure III.6 display the results of the univariate regressions carried out using these variables.

FIGURE III.5 APPROXIMATELY HERE

These univariate regressions, and the multivariate regressions discussed in the following section, were calculated using both the recovery rate (***BRR***) and the natural log (***BLRR***) of the recovery rate as the dependent variables. The use of ***BLRR***, for the most part, resulted in better fits than ***BRR***, as judged by higher R-squared values, but both linear (***BRR***) and exponential (***BLRR***) results are displayed in Figure III.5 and Figure III.6, as signified by an "x" in the corresponding row.

FIGURE III.6 APPROXIMATELY HERE

We first examine the simple relationship between bond recovery rates and bond default rates for the period 1982-2000 (there simply are too few default observations in the 1978-1981 period). Figure III.7 shows several regressions between the two fundamental variables and we find that one can explain about 45% of the variation in the annual recovery rate with the level of default rates (this is the linear model, also shown in Figure III.6, regression 1) and as much as 60%, or more, with the quadratic and power[29] relationships (the power model is found in Figure III.6, regression 4). Hence, our basic thesis that the rate of default is a massive indicator of the likely average recovery rate amongst corporate bonds appears to be substantiated.

The other univariate results show the correct sign for each coefficient, but not all of the relationships are significant. ***BDRC*** is highly negatively correlated with recovery rates, as shown by the very significant t-ratios, although the t-ratios and R-squared values are not as significant as those for ***BLDR***. ***BOA*** and ***BDA*** are, as expected, both negatively correlated with recovery rates with ***BDA*** being more highly negatively correlated than ***BOA*** on a univariate basis.

Macroeconomic variables did not explain as much of the variation in recovery rates as the corporate bond market variables explained. In the 1987-2000 period (Figure III.5), none of the macroeconomic variables produced a significant t-ratio. In the 1982-2000 period, only the linear ***GDPC*** regression (regression 13) and the both ***GDPI*** regressions (regressions 15 and 16) resulted in t-ratios that are significant at the 10% level. This seems to confirm our argument that macroeconomic conditions have only a secondary effect on recovery rates.

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[29] The power relationship can be written using the following equivalent equations:

$$\boldsymbol{BLRR} = b_0 + b_1 \times \boldsymbol{BLDR}$$

$$ln(\boldsymbol{BRR}) = b_0 + b_1 \times ln(\boldsymbol{BDR})$$

$$\boldsymbol{BRR} = exp[b_0 + b_1 \times ln(\boldsymbol{BDR})]$$

$$\boldsymbol{BRR} = exp[b_0] \times exp[b_1 \times ln(\boldsymbol{BDR})]$$

$$\boldsymbol{BRR} = exp[b_0] \times \boldsymbol{BDR}^{b1}$$

and, hence, the name "power model."

III.2.3 – Multivariate Models

We now specify models to explain recovery rates that are somewhat more complex by including several additional variables to the important default rate measure. The basic structure of our most successful models is

(1) $\boldsymbol{BRR} = f(\boldsymbol{BDR}, \boldsymbol{BDRC}, \boldsymbol{BOA} \text{ or } \boldsymbol{BDA}, \boldsymbol{BIR})$

The actual model with the highest explanatory power and lowest "error" rates is called the power model[30] and utilized natural logarithms (*ln*) of both the Recovery Rate (***BLRR***) and the Default Rate (***BLDR***).

In addition, we tried the variables that represented macroeconomic measures, including Gross Domestic Product Growth (***GDP***) and the return on the Standard & Poor's 500 Stock Index (***SR***), as well as a yield spread variable (***Spread***) time variable.

The Multivariate Results We have constructed two simple regression structures in order to explain recovery rate results and to predict 2001 rates. One set is for the longer 1982-2000 period and the other is for the 1987-2000 period. Both sets involve linear and log-linear structures for the two key variables – recovery rates (dependent) and default rates (explanatory) with the log-

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[30] Like its univariate cousin, the multivariate power model can be written using several equivalent expressions

$$\boldsymbol{BLRR} = b_0 + b_1\times\boldsymbol{BLDR} + b_2\times\boldsymbol{BDRC} + b_3\times\boldsymbol{BIR} + b_4\times\boldsymbol{BOA}$$

$$ln(\boldsymbol{BLRR}) = b_0 + b_1\times ln(\boldsymbol{BDR}) + b_2\times\boldsymbol{BDRC} + b_3\times\boldsymbol{BIR} + b_4\times\boldsymbol{BOA}$$

$$\boldsymbol{BRR} = exp[b_0 + b_1\times ln(\boldsymbol{BDR}) + b_2\times\boldsymbol{BDRC} + b_3\times\boldsymbol{BIR} + b_4\times\boldsymbol{BOA}]$$

$$\boldsymbol{BRR} = exp[b_0] \times exp[b_1\times ln(\boldsymbol{BDR})] \times exp[b_2\times\boldsymbol{BDRC} + b_3\times\boldsymbol{BIR} + b_4\times\boldsymbol{BOA}]$$

$$\boldsymbol{BRR} = exp[b_0] \times \boldsymbol{BDR}^{b1} \times exp[b_2\times\boldsymbol{BDRC} + b_3\times\boldsymbol{BIR} + b_4\times\boldsymbol{BOA}]$$

and takes its name from BDR being raised to the power of its coefficient.

linear relationships somewhat more significant. These results appear in regressions 1 through 4 in Figure III.8 and Figure III.9.

FIGURE III.8 APPROXIMATELY HERE

We observe that the various models explain between 84% and 91% of the variation in recovery rates (unadjusted R-squared) and between 76% and 87% (adjusted R-squared) of the dependent variable. These results are for the shorter 1987-2000 period, with slightly lower, but still very meaningful, results (77% to 87% unadjusted and 73% to 84% adjusted) for the longer time frame (1982-2000). Hence, we are quite optimistic that the variable set, while probably not optimal, can be used to explain and predict recovery rates in the corporate defaulted bond market.

FIGURE III.9 APPROXIMATELY HERE

The Results for 1987-2000 - Our results for each structure are presented in subsequent figures in four panels for our basic model and two panels for the other models. Again, the basic structure of the first set of models is

(2) ***BRR*** = *f(**BDR**, **BDRC**, **BOA**, **BIR**)*

The four variants of the basic model:

1) Use the **actual** rates for Recovery (***BRR***) and Default (***BDR***) rates as well as the return on the Defaulted Debt Index (***BIR***) and the amount of high yield bonds outstanding (***BOA***), or the amount of defaulted bonds in any year (***BDA***). This is the "linear" model.

2) Use the **natural log** (*ln*) for the ***BLRR*** – Recovery Rate dependent variable – and the rest of the variables the same as in (a). This is the "exponential" model, and

3) Use the natural log for the primary independent variable (***BLDR***) and the absolute percentage for Recovery Rates (no logs). This is the "logarithmic" model.

4) Use the natural logs (*ln*) for both the primary dependent and independent variables. The other two variables remain unchanged. This is the "power" model.

Figure III.8 regressions 1-4 present our results. Note that most, but not all, of the variables are quite significant based on their t-ratios. The overall accuracy of the fit goes from 84% for the strictly absolute value of all variables (regression 1) to 88% when the dependent variable (regression 2) is specified in natural logs, to the same 88% (regression 3) when only the primary independent variable (default rates – ***BLDR***) is specified in natural logs to as much as 91% (unadjusted) and 87% (adjusted) R-squares where both the primary dependent (***BLRR)*** and explanatory variable (***BLDR***) are expressed in natural logs (regression 4). We refer to the model in regression 4 as our "power" model.

In regression 4 of Figure III.8, we see that all of the four explanatory variables have the expected sign (negative for ***BLDR***, ***BDRC***, and ***BOA*** and positive for ***BIR***) and are significant at the 5% or 1% level, except for the Defaulted Bond Index (***BIR***) which has the appropriate sign (+) but a less meaningful t-ratio[31]. ***BLDR*** and ***BDRC*** are extremely significant, showing that the level and change in the default rate are highly important explanatory variables for recovery rates. Indeed the variables ***BDR*** (and ***BLDR***) explain up to 57% (unadjusted) and 53% (adjusted) of the variation in ***BRR*** simply based on a linear or log-linear association. The size of the high yield

[31] ***BIR***s t-ratio is only significant at the 0.25 level. Without the ***BIR*** variable, the R-squared measures are slightly lower at 90% (unadjusted) and 87% (adjusted). On a univariate basis, the ***BIR*** is significant with a t-ratio of 2.34 and explains 25% of the variation in ***BRR***.

market also performs very well and adds about 8% to the explanatory power of the model. When we substitute ***BDA*** for ***BOA***, the explanatory power of multivariate model drops somewhat to 0.89 (unadjusted) and 0.84 (adjusted) R-squared. Still, the sign of BDA is correct (+) and the t-ratio is quite high (1.84 – see regression 6 of Figure III.8). Indeed, on a univariate basis, ***BDA*** is actually far more significant than ***BOA*** (see Figures III.5 and III.6 regression 7-10).

FIGURE III.10 APPROXIMATELY HERE

Figure III.10 shows, graphically, the results for the Figure III.8 regression 4 structure, by comparing the actual Recovery Rate vs. the estimated rates (designated by a "+" sign) for 1987-2000. Note the extremely close accuracy in almost every year between the actual and estimated rates. Figure III.11 shows the same pattern only the model utilizes BDA instead of BOA. As we will discuss at a later point, when we use BDA the expected 2001 recovery rate is somewhat lower.

FIGURE III.11 APPROXIMATELY HERE

The Results for 1982-2000 - Figure III.9 regressions 1-4 show the same regression structures as Figure III.8 regressions 1-4, only the sample period is for the longer 19-year period 1982-2000 and the models do not include the ***BIR*** variable. Regression 1 of Figure III.9 shows that all three explanatory variables are significant at the 1% or 5% level with high t-ratios. All have the correct sign, indicating that recovery rates are negatively correlated with default rates, the change in default rates and the size of the high yield bond market. The R-squared of this straightforward, linear regression is 0.77 (0.73 adjusted). Finally, as with the shorter time period, the highest R-squared explanatory model for the longer time period uses the log specification for both ***BLRR*** and ***BLDR,*** which raises the unadjusted R-squared to 0.87 (Figure III.9, regression 4).

Figure III.12 shows regression 4 of Figure III.9, graphically. And Figure III.13 shows the same model except ***BDA*** is substituted for ***BOA***. Again, the "+" sign indicates the estimate for each year's recovery rate compared to the actual level in that year. These are extremely close in most years for both structures.

FIGURE III.12 APPROXIMATELY HERE

FIGURE III.13 APPROXIMATELY HERE

***Macroeconomic Variables* -** While we are pleased with the accuracy and explanatory power of the regressions described above, we were not very successful in our attempts to introduce several fundamental macroeconomic factors. We assessed these factors both on a univariate as well as a value-added basis for our multivariate structures. We are somewhat surprised by the low contributions of these variables since there are several models that have been constructed that utilize macro-variables, apparently significantly, in explaining annual default rates. For example, Fons (1991), Jonsson and Fridson (1996), Moody's (1999), and Fridson, Garman, and Wu (1997) all find that variables like GDP growth, corporate profits, and other aggregate measures were very helpful in their default rate pattern models. And, Helwege and Kleiman (1997) include GDP growth in their model to better understand the effects of recessions on default rates. They find that their regressions increase the explanatory power by as much as 13% when a GDP growth variable is introduced.

Since variables like GDP growth have been found to influence aggregate default rate patterns and since we now know that default rates are highly associated with recovery rates, we fully expected that GDP growth would be significantly associated with recovery rates. Despite the fact that the growth rate in annual GDP is significantly negatively correlated with the bond default rate, i.e., -0.67 for the period 1987-2000 and -0.50 for 1982-2000, the univariate correlation between recovery rates (***BRR***) and GDP growth is a relatively low 0.06 to 0.18, depending on the sample period. The sign (+) is appropriate, however. These univariate relationships are shown in Figure III.5 regressions 13 and 14 and Figure III.5 regressions 12 and 13. Note that the GDP growth variable has a -0.02 and -0.03 adjusted R-squared with ***BRR*** and ***BLRR*** (Figure III.6 regressions 11 and 12), and a positive but not significant relationship with recovery rates when we utilize the change in GDP growth (***GDPC***, Figure III.6 regressions 13 and 14). When we introduce ***GDP*** and ***GDPC*** to our existing multivariate structures (Figure III.8 and Figure III.9 regressions 7,8, 11, and 12), not only are they not significant, but they have a counterintuitive sign (negative).

The news is not all bad with respect to the multivariate contribution of the GDP growth variable. When we substitute ***GDP*** for ***BDR*** in our most successful regressions (see Figure III.8 and Figure III.9 regressions 9 and 10), we do observe that GDP is significant at .05 level and the sign (+) is correct.

BRR = f (GDP, BDRC, BIR, BOA)

explains 0.76 of ***BRR*** and 0.78 of ***BLRR***. This compares to 0.84 and 0.88 when we use ***BDR*** instead of ***GDP***. No doubt, the high negative correlation (-0.67) between ***GDP*** and ***BDR*** eliminates the possibility of using both in the same multivariate structure and the higher explanatory power of the ***BDR*** structure determined our preferred model (Figure III.8 regression 4).

We are now persuaded that the level of change in GDP growth does contribute to our understanding as to why recovery rates on defaulted bonds vary. As noted earlier, a number of researchers have found that this macroeconomic indicator can assist in explaining default rates, especially where the indicator is specified as growing more or less than some benchmark, arbitrarily chosen amount. For example, Helwege and Kleiman (1997) postulate that, while a change in GDP of say 1% or 2% was not very meaningful in explaining default rates when the base year was in a strong economic growth period, the same change was meaningful when the new level was in a weak economy. When the economy slips below some "critical" level, however, they thought and found that the default rate reacted more. They chose a 1.5% level of GDP growth as the critical level, and so did we. ***GDPI*** takes the value of 1 when GDP grew at less than 1.5% and 0 otherwise.

Figure III.5, regressions 17 and 18, and Figure III.6, regressions 15 and 16 show the univariate ***GDPI*** results and Figure III.8 and Figure III.9 regression 14 add the "dummy" variable ***GDPI*** to the "power" models, discussed earlier. Note that the univariate results show a somewhat significant relationship with the appropriate sign (negative). When the economy grows less than 1.5%, we find that this macroeconomic indicator explains about 0.16 to 0.17 (uadjusted) and 0.11 and 0.12 (adjusted) of the change in recovery rates. The multivariate model with ***GDPI***, however, does not add any value to our already very high explanatory power and the sign (+) now is not appropriate. No doubt, the fact that GDP growth is highly correlated with default rates, our primary explanatory variable, impacts the significance and sign of the GDP indicator (***GDPI***) in our multivariate model.

We also postulated that the return of the stock market could impact prices of defaulting bonds in that the stock market represented investor expectations about the future. Figure III.8 and Figure III.9 regression 15 and 16 show the association between the annual S&P 500 Index stock return

(***SR***) (and the change in the return on the Index (***SRC***) are shown in regressions 17 and 18) with recovery rates. Note the extremely low univariate R-squared measures and the insignificant t-ratios in the multivariate model, despite the appropriate signs.

III.2.4 Predicting 2001 Recovery Rates

One of the important reasons for specifying models to explain recovery rates on defaulting bonds is to use them in order to predict these recovery rates given certain expectations about the state of credit markets – particularly expected default rates. Most, if not all, of the existing portfolio credit risk models assume independence of default and recovery rates. It is true that some technical documentation for certain credit portfolio models indicate that researchers are aware of the possible consequences if this assumption is not valid. Indeed, most models allow users to stress test the models under various recovery rate assumptions, but the assumption (e.g., in *CreditMetrics®*) that default rates and recovery rates are independent is the foundation for these basic models. Therefore, it is important to test this assumption and if found not to be valid (as we find), then more precise estimates of recovery rates and loss given default (LGD) will be useful for many participants in credit markets (e.g. investors, regulators, traders, etc.), as well as for researchers.

From Figures III.10-13 we can observe the 2001 expected recovery rate, given certain assumptions about the independent variables and the time frame for the regressions. Specifically, we have assumed default rates for 2001 of 8.5% (and 10%)[32], a change in default rates compared to 2000's 5.1% of 3.44% (and 4.94%), a ***BIR*** of 18.0% (the rate of return as of

[32] These were Altman's (8.5%) and Moody's (10.0%) default rates estimates for 2001 made at the beginning of the year. More recent estimates are higher given the impact from the September 11, 2001 tragedy.

August, 2001), and a ***BOA*** of $630 billion (midyear 2001), or a ***BDA*** of 8.5% ($53 billion) or 10% ($63 billion) of the high yield bond market. This results in an estimated recovery rate of 23% for both time periods (see Figures 10 and 12) when we assume an 8.5% default rate[33]. When we use Moody's 10% expected default rate for 2001, our recovery rate estimate falls to 22% for both time periods. When we substitute ***BDA*** for ***BOA***, the estimates for 2001 recovery rates are, for the shorter time period regression, 20% assuming an 8.5% default rate and 18% assuming a 10% default rate. The estimated 2001 recovery rates from the longer time period regression are 18% assuming an 8.5% default rate and 16% assuming a 10% default rate.

We realize that most of the variables of our model require forecasts in order to estimate recovery rates. These forecasts can either be made based on other models (e.g. models by Altman, Moody's, and Merrill Lynch for default rates) and to use average annual returns on the ***BIR***. As a reasonable alternative to using one forecast, we can estimate recovery rates based on different scenarios of default rates - our key independent/explanatory variable - and then simulate conditional average recovery rates.

III. 3 A Word on Procyclicality

Our models also have implications for the issue of procyclicality with which the BIS, and others, are concerned[34]. Procyclicality involves the regulatory capital impact of changes in reserves for expected losses and capital for unexpected losses based on the rating distribution of bank portfolios. Since average ratings and default rates and amounts are sensitive to business cycle

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[33] As of September 30, 2001, the weighted average recovery rate was approximately 22% when we do not include a very large outlier (FINOVA) and 28% with FINOVA. Most market practitioners agree that FINOVA, while technically a junk bond default, was not held by high yield bond investors.

[34] Indeed, the BIS will be holding a special conference on procyclicality in Basel on March 6, 2002.

effects and several models have found significant correlations between macroeconomic measures and bond rates/defaults, we might expect that low recovery rates when defaults are high would exacerbate bank loan losses (LGD) in those periods. We do not find much of a relationship, however, between GDP growth and recovery rates. When we substitute GDP growth for our primary bond default rate variable, however, the multivariate results are quite meaningful, albeit with lower explanatory power than with the BDR variable.

This clouds the thesis that there is a procyclicality impact of expected and unexpected loss rates, at least with respect to recovery rates. There certainly is a significant negative relationship between ***GDP*** and ***BDR***. We acknowledge that our results are particular to the corporate bond market and not specifically to the bank loan market. We also note that there were only one, possibly two, recessionary periods in our sample periods. Further work on loss rates on bank loans is called for but our results can be added to the body of knowledge on this emerging, possibly important, procyclicality subject.

III.4 Concluding Remarks

Our results clearly show a significant negative correlation between aggregate default rates and recovery rates on corporate bonds. We strongly suspect that the same will be true when we extend our analysis to bank loan defaults. This has important implications for bond portfolios, for securitized instrument analysis, for the credit derivative market and for bank regulatory capital requirements and bank economic capital analysis. It is clear that negative economic cycles and high default periods carry with them higher loss-given-default expectations than if the PD and RR variables were considered stochastic but independent.

Table III.1 – Variables Explaining Annual Recovery Rates on Defaulted Corporate Bonds and their Expected Effects on Recovery Rates

Variable	Description	Expected Effect on Recovery Rates
BDR	Bond Default Rate	–
BLDR	Log of Bond Default Rate	–
BDRC	Bond Default Rate Change	–
BOA	Bond Outstanding Amount	–
BDA	Bond Defaulting Amount	–
BIR	Defaulted Bond Index Return	+
GDP	GPD Growth Rate	+
GDPC	GDP Growth Rate Change	+
GDPI	GDP Recession Indicator	–
SR	Stock Return	+
SRC	Stock Return Change	+

Figure III.1

Historical Default Rates

Straight Bonds Only Excluding Defaulted Issues From Par Value Outstanding, 1971 - September 30, 2001 *(US$ millions)*

Year	Par Value Outstanding[a]	Par Value Defaults	Default Rates *(%)*
3Q 2001	**$649,000**	**$44,930**	**6.923**
2000	$597,200	$30,295	5.073
1999	$567,400	$23,532	4.147
1998	$465,500	$7,464	1.603
1997	$335,400	$4,200	1.252
1996	$271,000	$3,336	1.231
1995	$240,000	$4,551	1.896
1994	$235,000	$3,418	1.454
1993	$206,907	$2,287	1.105
1992	$163,000	$5,545	3.402
1991	$183,600	$18,862	10.273
1990	$181,000	$18,354	10.140
1989	$189,258	$8,110	4.285
1988	$148,187	$3,944	2.662
1987	$129,557	$7,486	5.778
1986	$90,243	$3,156	3.497
1985	$58,088	$992	1.708
1984	$40,939	$344	0.840
1983	$27,492	$301	1.095
1982	$18,109	$577	3.186
1981	$17,115	$27	0.158
1980	$14,935	$224	1.500

Year	Par Value Outstanding[a]	Par Value Defaults	Default Rates *(%)*
1980	$14,935	$224	1.500
1979	$10,356	$20	0.193
1978	$8,946	$119	1.330
1977	$8,157	$381	4.671
1976	$7,735	$30	0.388
1975	$7,471	$204	2.731
1974	$10,894	$123	1.129
1973	$7,824	$49	0.626
1972	$6,928	$193	2.786
1971	$6,602	$82	1.242

		Standard Deviation *(%)*
Arithmetic Average Default Rate		
1971 to 2000	2.713	2.484
1978 to 2000	2.948	2.683
1985 to 2000	3.719	2.829
Weighted Average Default Rate[b]		
1971 to 2000	3.482	2.558
1978 to 2000	3.503	2.563
1985 to 2000	3.582	2.565
Median Annual Default Rate		
1971 to 2000	1.656	

[a] As of mid-year
[b] Weighted by par value of amount outstanding for each year.

Source: Author's compilation and Salomon Smith Barney

Figure III.2

Default Rates and Losses[a]

1978 - Sept. 30, 2001

Year	Par Value Outstanding[a] *($MM)*	Par Value Of Default *($MMs)*	Default Rate *(%)*	Weighted Price After Default	Weighted Coupon *(%)*	Default Loss *(%)*
3Q 2001	**$649,000**	**$44,930**	**6.92**	**$28.1**	**9.14**	**5.29**
2000	**$597,200**	**$30,248**	**5.06**	**$26.4**	**8.54**	**3.94**
1999	$567,400	$23,532	4.15	$27.9	10.55	3.21
1998	$465,500	$7,464	1.60	$35.9	9.46	1.10
1997	$335,400	$4,200	1.25	$54.2	11.87	0.65
1996	$271,000	$3,336	1.23	$51.9	8.92	0.65
1995	$240,000	$4,551	1.90	$40.6	11.83	1.24
1994	$235,000	$3,418	1.45	$39.4	10.25	0.96
1993	$206,907	$2,287	1.11	$56.6	12.98	0.56
1992	$163,000	$5,545	3.40	$50.1	12.32	1.91
1991	$183,600	$18,862	10.27	$36.0	11.59	7.16
1990	$181,000	$18,354	10.14	$23.4	12.94	8.42
1989	$189,258	$8,110	4.29	$38.3	13.40	2.93
1988	$148,187	$3,944	2.66	$43.6	11.91	1.66
1987[b]	$129,557	$7,486	5.78	$75.9	12.07	1.74
1986	$90,243	$3,156	3.50	$34.5	10.61	2.48
1985	$58,088	$992	1.71	$45.9	13.69	1.04
1984	$40,939	$344	0.84	$48.6	12.23	0.48
1983	$27,492	$301	1.09	$55.7	10.11	0.54
1982	$18,109	$577	3.19	$38.6	9.61	2.11
1981	$17,115	$27	0.16	$12.0	15.75	0.15
1980	$14,935	$224	1.50	$21.1	8.43	1.25
1979	$10,356	$20	0.19	$31.0	10.63	0.14
1978	$8,946	$119	1.33	$60.0	8.38	0.59
Arithmetic Average 1978–2000			2.95	$41.2	11.22	1.95
Weighted Average 1978–2000			3.50			2.45

[a] Excludes defaulted issues.

Source: Authors' compilations and various dealer price quotes.

[b] Includes Texaco, Inc., which was a unique Chapter 11 bankruptcy, with a face value of over $3 billion and an outlier recovery rate of over 80%. Our models do not include this one firm's influence. Without Texaco the default rate is 1.34% and the recovery rate is 62%.

Figure III.3

Weighted Average Recovery Rates

On Defaulted Debt by Seniority Per $100 Face Amount
1978 – Sept. 30, 2001

Default	Senior Secured		Senior Unsecured		Senior Subordinated		Subordinated		Discount and Zero Coupon		All Seniorities	
Year	**No.**	**$**	**No.**	**$**	**No.**	**$**	**No.**	**$**	**No.**	**$**	**No.**	**$**
3Q 2001	**8**	**$40.95**	**110**	**$33.19**	**41**	**$18.12**	**0**	**$0.00**	**28**	**$14.64**	**187**	**$28.02**
2000	**13**	**$39.58**	**47**	**$25.40**	**61**	**$25.96**	**26**	**$26.62**	**17**	**$23.61**	**164**	**$25.83**
1999	14	$26.90	60	$42.54	40	$23.56	2	$13.88	11	$17.30	127	$31.14
1998	6	$70.38	21	$39.57	6	$17.54	0	0	1	$17.00	34	$37.27
1997	4	$74.90	12	$70.94	6	$31.89	1	$60.00	2	$19.00	25	$53.89
1996	4	$59.08	4	$50.11	9	$48.99	4	$44.23	3	$11.99	24	$51.91
1995	5	$44.64	9	$50.50	17	$39.01	1	$20.00	1	$17.50	33	$41.77
1994	5	$48.66	8	$51.14	5	$19.81	3	$37.04	1	$5.00	22	$39.44
1993	2	$55.75	7	$33.38	10	$51.50	9	$28.38	4	$31.75	32	$38.83
1992	15	$59.85	8	$35.61	17	$58.20	22	$49.13	5	$19.82	67	$50.03
1991	4	$44.12	69	$55.84	37	$31.91	38	$24.30	9	$27.89	157	$40.67
1990	12	$32.18	31	$29.02	38	$25.01	24	$18.83	11	$15.63	116	$24.66
1989	9	$82.69	16	$53.70	21	$19.60	30	$23.95			76	$35.97
1988	13	$67.96	19	$41.99	10	$30.70	20	$35.27			62	$43.45
1987	4	$90.68	17	$72.02	6	$56.24	4	$35.25			31	$66.63
1986	8	$48.32	11	$37.72	7	$35.20	30	$33.39			56	$36.60
1985	2	$74.25	3	$34.81	7	$36.18	15	$41.45			27	$41.78
1984	4	$53.42	1	$50.50	2	$65.88	7	$44.68			14	$50.62
1983	1	$71.00	3	$67.72			4	$41.79			8	$55.17
1982			16	$39.31			4	$32.91			20	$38.03
1981	1	$72.00									1	$72.00
1980			2	$26.71			2	$16.63			4	$21.67
1979							1	$31.00			1	$31.00
1978			1	$60.00							1	$60.00
Total/Avg	**134**	**$52.97**	**475**	**$41.71**	**340**	**$29.68**	**247**	**$31.03**	**93**	**$18.97**	**1289**	**$35.85**
Median		**$57.42**		**$42.27**		**$31.9**		**$31.96**		**$17.40**		**$40.05**

Figure III.4 – **Univariate Regressions 1987-2000**

Variables Explaining Annual Recovery Rates on Defaulted Corporate Bonds
Coefficients and T-Ratios (in parentheses)

Regression #	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
R-Squared	0.43	0.46	0.55	0.57	0.50	0.53	0.25	0.26	0.55	0.63	0.31	0.31
Adj. R-Squared	0.38	0.42	0.51	0.53	0.46	0.49	0.19	0.20	0.52	0.60	0.25	0.26
Dependent Variable:												
BRR	X		X		X		X		X		X	
BLRR		X		X		X		X		X		X
Explanatory Variables:												
Constant	0.51	-0.68	0.01	-1.97	0.42	-0.90	0.53	-0.64	0.52	-0.64	0.40	-0.95
	(13.14)	(-7.20)	(0.05)	(-7.22)	(18.11)	(-15.84)	(8.69)	(-4.21)	(15.27)	(-8.24)	(14.21)	(-13.45)
BDR	-2.49	-6.50										
	(-3.00)	(-3.21)										
BLDR			-0.11	-0.29								
			(-3.82)	(-3.96)								
BDRC					-2.95	-7.62						
					(-3.45)	(-3.65)						
BOA							-0.38	-0.97				
							(-2.00)	(-2.04)				
BDA									-9.89	-26.48		
									-3.85	-4.50		
BIR											0.27	0.69
											(2.33)	(2.34)
GDP												
GDPC												
GDPI												
SR												
SRC												
Spread												

Figure III.4 – **Univariate Regressions 1987-2000 (Continued)**
Variables Explaining Annual Recovery Rates on Defaulted Corporate Bonds
Coefficients and T-Ratios (in parentheses)

Regression #	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
R-Squared	0.00	0.00	0.13	0.14	0.14	0.15	0.02	0.05	0.03	0.07	0.10	0.11
Adj. R-Squared	-0.08	-0.08	0.06	0.07	0.07	0.08	-0.06	-0.02	-0.05	-0.01	0.03	0.04
Dependent Variable:												
BRR	X		X		X		X		X		X	
BLRR		X		X		X		X		X		X
Explanatory Variables:												
Constant	0.40	-0.95	0.42	-0.92	0.46	-0.81	0.40	-0.98	0.42	-0.90	0.52	-0.65
	(4.60)	(-4.30)	(13.55)	(-11.98)	(11.36)	(-8.09)	(8.86)	(-8.69)	(12.95)	(-11.29)	(5.78)	(-2.93)
BDR												
BLDR												
BDRC												
BOA												
BDA												
BIR												
GDP	0.47	1.13										
	(0.19)	(0.18)										
GDPC			3.00	7.80								
			(1.36)	(1.42)								
GDPI					-0.09	-0.23						
					(-1.42)	(-1.48)						
SR							0.12	0.48				
							(0.53)	(0.84)				
SRC									0.10	0.37		
									(0.61)	(0.94)		
Spread											-1.92	-5.12
											(-1.16)	(-1.24)

Figure III.5 – Univariate Regressions 1982-2000
Variables Explaining Annual Recovery Rates on Defaulted Corporate Bonds
Coefficients and T-Ratios (in parentheses)

Regression #	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
R-Squared	0.45	0.49	0.58	0.60	0.51	0.52	0.20	0.23	0.46	0.54
Adj. R-Squared	0.42	0.46	0.56	0.58	0.48	0.49	0.15	0.18	0.43	0.51
Dependent Variable:										
BRR	X		X		X		X		X	
BLRR		X		X		X		X		X
Explanatory Variables:										
Constant	0.51	-0.67	0.01	-1.94	0.43	-0.87	0.49	-0.72	0.49	-0.71
	(17.40)	(-9.55)	(0.10)	(-9.12)	(24.01)	(-19.39)	(12.83)	(-7.70)	(19.16)	(-12.01)
BDR	-2.62	-6.82								
	(-3.73)	(-4.04)								
BLDR			-0.11	-0.28						
			(-4.86)	(-5.05)						
BDRC					-2.99	-7.51				
					(-4.19)	(-4.25)				
BOA							-0.29	-0.76		
							(-2.06)	(-2.23)		
BDA									-8.53	-23.16
									-3.78	-4.48
GDP										
GDPC										
GDPI										
SR										
SRC										
Spread										

Figure III.5 – **Univariate Regressions 1982-2000 (Continued)**
Variables Explaining Annual Recovery Rates on Defaulted Corporate Bonds
Coefficients and T-Ratios (in parentheses)

Regression #	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
R-Squared	0.03	0.03	0.16	0.14	0.16	0.17	0.02	0.04	0.02	0.05	0.10	0.12
Adj. R-Squared	-0.02	-0.03	0.11	0.09	0.11	0.12	-0.04	-0.01	-0.04	0.00	0.04	0.06
Dependent Variable:												
BRR	X		X		X		X		X		X	
BLRR		X		X		X		X		X		X
Explanatory Variables:												
Constant	0.39	-0.96	0.42	-0.89	0.46	-0.80	0.41	-0.95	0.43	-0.89	0.52	-0.63
	(7.81)	(-7.66)	(18.00)	(-15.02)	(15.48)	(-10.96)	(10.88)	(-10.19)	(16.82)	(-14.27)	(7.12)	(-3.52)
BDR												
BLDR												
BDRC												
BOA												
BDA												
GDP	1.00	2.30										
	(0.77)	(0.70)										
GDPC			1.76	4.11								
			(1.78)	(1.65)								
GDPI					-0.09	-0.22						
					(-1.78)	(-1.84)						
SR							0.11	0.43				
							(0.56)	(0.88)				
SRC									0.08	0.31		
									(0.62)	(0.99)		
Spread											-1.94	-5.27
											(-1.35)	(-1.49)

Figure III.6

Recovery Rate/Default Rate Association

Altman Defaulted Bonds Data Set (1982-2000)

Dollar Weighted Average Recovery Rates to Dollar Weighted Average Default Rates

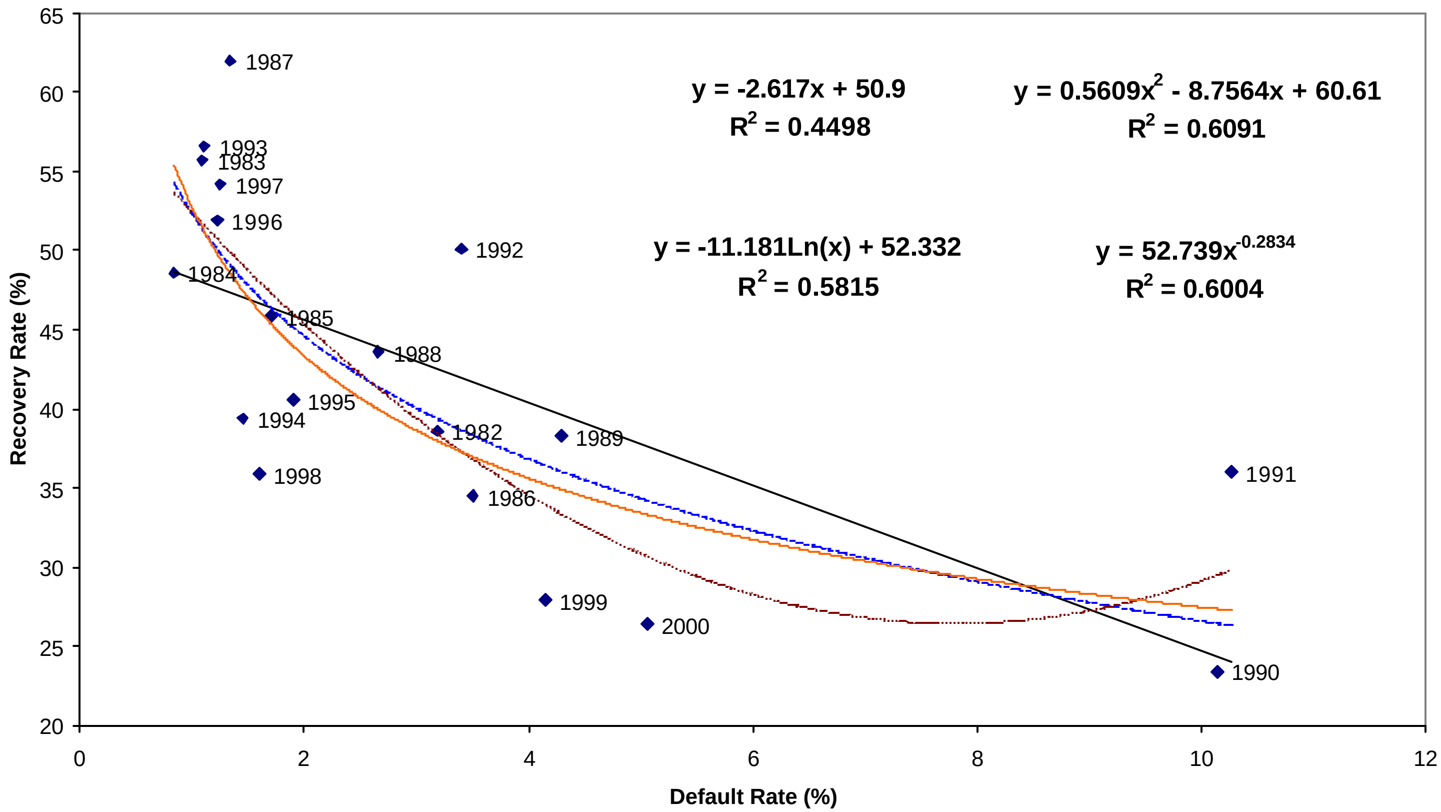


Figure III.7 – **Multivariate Regressions 1987-2000**
Variables Explaining Annual Recovery Rates on Defaulted Corporate Bonds
Coefficients and T-Ratios (in parentheses)

Regression #	**(1)**	(2)	(3)	**(4)**	(5)	**(6)**	(7)	(8)	(9)	**(10)**
R-Squared	**0.84**	0.88	0.88	**0.91**	0.81	**0.89**	0.84	0.91	0.76	**0.78**
Adj. R-Squared	**0.76**	0.83	0.83	**0.87**	0.73	**0.84**	0.74	0.86	0.65	**0.68**
Dependent Variable:										
BRR	**X**		X		X		X		X	
BLRR		X		**X**		**X**		X		**X**
Explanatory Variables:										
Constant	**0.56**	-0.55	0.16	**-1.55**	0.49	**-1.29**	0.58	-1.54	0.37	**-1.02**
	(11.65)	(-5.33)	(2.12)	**(-9.27)**	(16.43)	**(-4.51)**	(4.89)	(-8.41)	(5.52)	**(-6.36)**
BDR	**-2.02**	-5.28			-1.04		-2.20			
	(-3.40)	(-4.15)			(-1.29)		(-2.01)			
BLDR			-0.09	**-0.22**		**-0.13**		-0.21		
			(-4.41)	**(-5.18)**		**(-1.96)**		(-3.51)		
BDRC	**-1.17**	-3.06	-1.31	**-3.51**	-1.26	**-3.45**	-1.13	-3.51	-1.82	**-4.85**
	(-1.64)	(-2.01)	(-2.26)	**(-2.82)**	(-1.65)	**(-2.45)**	(-1.48)	(-2.66)	(-2.29)	**(-2.55)**
BOA	**-0.26**	-0.67	-0.23	**-0.59**			-0.25	-0.60	-0.33	**-0.85**
	(-2.08)	(-2.52)	(-2.16)	**(-2.60)**			(-1.66)	(-2.23)	(-2.04)	**(-2.18)**
BDA					-4.37	**-10.60**				
					-1.56	**-1.84**				
BIR	**0.11**	0.26	0.08	**0.20**	0.15	**0.33**	0.10	0.21	0.18	**0.43**
	(1.29)	(1.44)	(1.18)	**(1.28)**	(1.81)	**(2.14)**	(0.94)	(1.10)	(1.61)	**(1.63)**
GDP							-0.56	0.41	3.95	**9.81**
							(-0.20)	(0.11)	(2.17)	**(2.26)**
GDPC										
GDPI										
SR										
SRC										
Spread										

Figure III.7 **– Multivariate Regressions 1987-2000 (Continued)**
Variables Explaining Annual Recovery Rates on Defaulted Corporate Bonds
Coefficients and T-Ratios (in parentheses)

Regression #	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
R-Squared	0.85	0.92	0.84	0.91	0.84	0.93	0.84	0.92	0.85	0.92
Adj. R-Squared	0.75	0.86	0.74	0.86	0.74	0.88	0.74	0.87	0.75	0.87
Dependent Variable:										
BRR	X		X		X		X		X	
BLRR		X		X		X		X		X
Explanatory Variables:										
Constant	0.56	-1.56	0.56	-1.56	0.55	-1.54	0.56	-1.53	0.49	-2.15
	(11.29)	(-8.91)	(10.60)	(-7.17)	(10.23)	(-9.54)	(11.03)	(-8.93)	(4.50)	(-3.28)
BDR	-2.16		-2.06		-1.98		-2.05		-3.71	
	(-3.39)		(-2.88)		(-3.07)		(-3.22)		(-1.39)	
BLDR		-0.22		-0.22		-0.21		-0.21		-0.32
		(-5.00)		(-4.44)		(-4.98)		(-4.95)		(-2.73)
BDRC	-1.64	-4.23	-1.17	-3.52	-1.20	-3.73	-1.12	-3.74	-0.79	-3.27
	(-1.70)	(-2.36)	(-1.55)	(-2.65)	(-1.58)	(-3.08)	(-1.44)	(-2.87)	(-0.85)	(-2.56)
BOA	-0.25	-0.58	-0.26	-0.58	-0.26	-0.61	-0.26	-0.58	-0.29	-0.58
	(-1.98)	(-2.44)	(-1.90)	(-2.37)	(-1.98)	(-2.78)	(-1.98)	(-2.48)	(-2.11)	(-2.57)
BDA										
BIR	0.08	0.16	0.11	0.20	0.10	0.17	0.12	0.15	0.02	0.02
	(0.87)	(0.91)	(1.21)	(1.20)	(1.16)	(1.14)	(1.23)	(0.89)	(0.16)	(0.10)
GDP										
GDPC	-1.33	-1.87								
	(-0.75)	(-0.58)								
GDPI			0.01	0.01						
			(0.13)	(0.09)						
SR					0.03	0.26				
					(0.29)	(1.31)				
SRC							-0.02	0.13		
							(-0.27)	(0.80)		
Spread									2.70	4.55
									(0.65)	(0.95)

Figure III.8 – **Multivariate Regressions 1982-2000**
Variables Explaining Annual Recovery Rates on Defaulted Corporate Bonds
Coefficients and T-Ratios (in parentheses)

Regression #	**(1)**	(2)	(3)	**(4)**	(5)	**(6)**	(7)	(8)	(9)	**(10)**
R-Squared	**0.77**	0.82	0.83	**0.87**	0.74	**0.84**	0.78	0.88	0.66	**0.68**
Adj. R-Squared	**0.73**	0.79	0.80	**0.84**	0.69	**0.81**	0.71	0.85	0.59	**0.62**
Dependent Variable:										
BRR	**X**		X		X		X		X	
BLRR		X		**X**		**X**		X		**X**
Explanatory Variables:										
Constant	**0.53**	-0.61	0.20	**-1.46**	0.49	**-1.20**	0.54	-1.55	0.46	**-0.79**
	(20.03)	(-10.46)	(2.75)	**(-9.17)**	(21.99)	**(-5.01)**	(12.89)	(-9.45)	(13.06)	**(-9.29)**
BDR	**-1.62**	-4.36			-0.69		-1.75			
	(-3.02)	(-3.69)			(-0.94)		(-2.71)			
BLDR			-0.07	**-0.19**		**-0.11**		-0.23		
			(-4.16)	**(-4.74)**		**(-1.96)**		(-4.92)		
BDRC	**-2.02**	-4.92	-1.88	**-4.67**	-2.12	**-4.81**	-2.03	-4.64	-2.63	**-6.60**
	(-3.49)	(-3.87)	(-3.75)	**(-4.20)**	(-3.46)	**(-3.96)**	(-3.40)	(-4.34)	(-3.98)	**(-4.17)**
BOA	**-0.22**	-0.58	-0.19	**-0.51**			-0.20	-0.40	-0.26	**-0.69**
	(-2.72)	(-3.33)	(-2.67)	**(-3.27)**			(-2.31)	(-2.42)	(-2.56)	**(-2.83)**
BDA					-4.94	**-11.73**				
					(-2.20)	**(-2.55)**				
GDP							-0.32	-2.42	0.87	**2.06**
							(-0.37)	(-1.50)	(0.98)	**(0.97)**
GDPC										
GDPI										
SR										
SRC										
Spread										

Figure III.8 – **Multivariate Regressions 1982-2000 (Continued)**
Variables Explaining Annual Recovery Rates on Defaulted Corporate Bonds
Coefficients and T-Ratios (in parentheses)

Regression #	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
R-Squared	0.78	0.88	0.77	0.87	0.78	0.89	0.78	0.89	0.71	0.79
Adj. R-Squared	0.71	0.85	0.71	0.83	0.72	0.86	0.71	0.86	0.66	0.75
Dependent Variable:										
BRR	X		X		X		X		X	
BLRR		X		X		X		X		X
Explanatory Variables:										
Constant	0.53	-1.51	0.53	-1.51	0.52	-1.47	0.53	-1.47	0.39	-2.11
	(19.38)	(-9.54)	(19.25)	(-7.79)	(15.95)	(-9.88)	(18.81)	(-9.93)	(6.29)	(-5.32)
BDR	-1.66		-1.68	-0.20	-1.56		-1.60		-3.80	
	(-2.95)		(-2.66)	(-4.30)	(-2.79)		(-2.88)		(-2.69)	
BLDR		-0.20				-0.18		-0.18		-0.28
		(-5.10)				(-4.76)		(-5.03)		(-3.78)
BDRC	-2.15	-5.53	-2.04	-4.74	-2.06	-4.83	-2.07	-4.89	-1.32	-4.03
	(-3.10)	(-4.48)	(-3.38)	(-4.11)	(-3.47)	(-4.63)	(-3.44)	(-4.70)	(-1.60)	(-2.69)
BOA	-0.21	-0.48	-0.22	-0.50	-0.22	-0.51	-0.20	-0.43		
	(-2.58)	(-3.14)	(-2.63)	(-3.15)	(-2.66)	(-3.52)	(-2.39)	(-2.85)		
BDA										
GDP										
GDPC	-0.24	-1.84								
	(-0.35)	(-1.43)								
GDPI			0.01	0.03						
			(0.19)	(0.46)						
SR					0.06	0.33				
					(0.61)	(1.79)				
SRC							0.04	0.23		
							(0.50)	(1.85)		
Spread									3.30	3.77
									(1.63)	(1.33)

Figure III.9

**Actual vs. Estimated Recovery Rates on
Defaulted Corporate Bonds
Based on Multivariate Regression Model
(see Figure 5a-b)
1987–2000**

$$ln(\boldsymbol{BRR}) = b_0 + b_1 \times ln(\boldsymbol{BDR}) + b_2 \times \boldsymbol{BDRC} + b_3 \times \boldsymbol{BIR} + b_4 \times \boldsymbol{B}$$

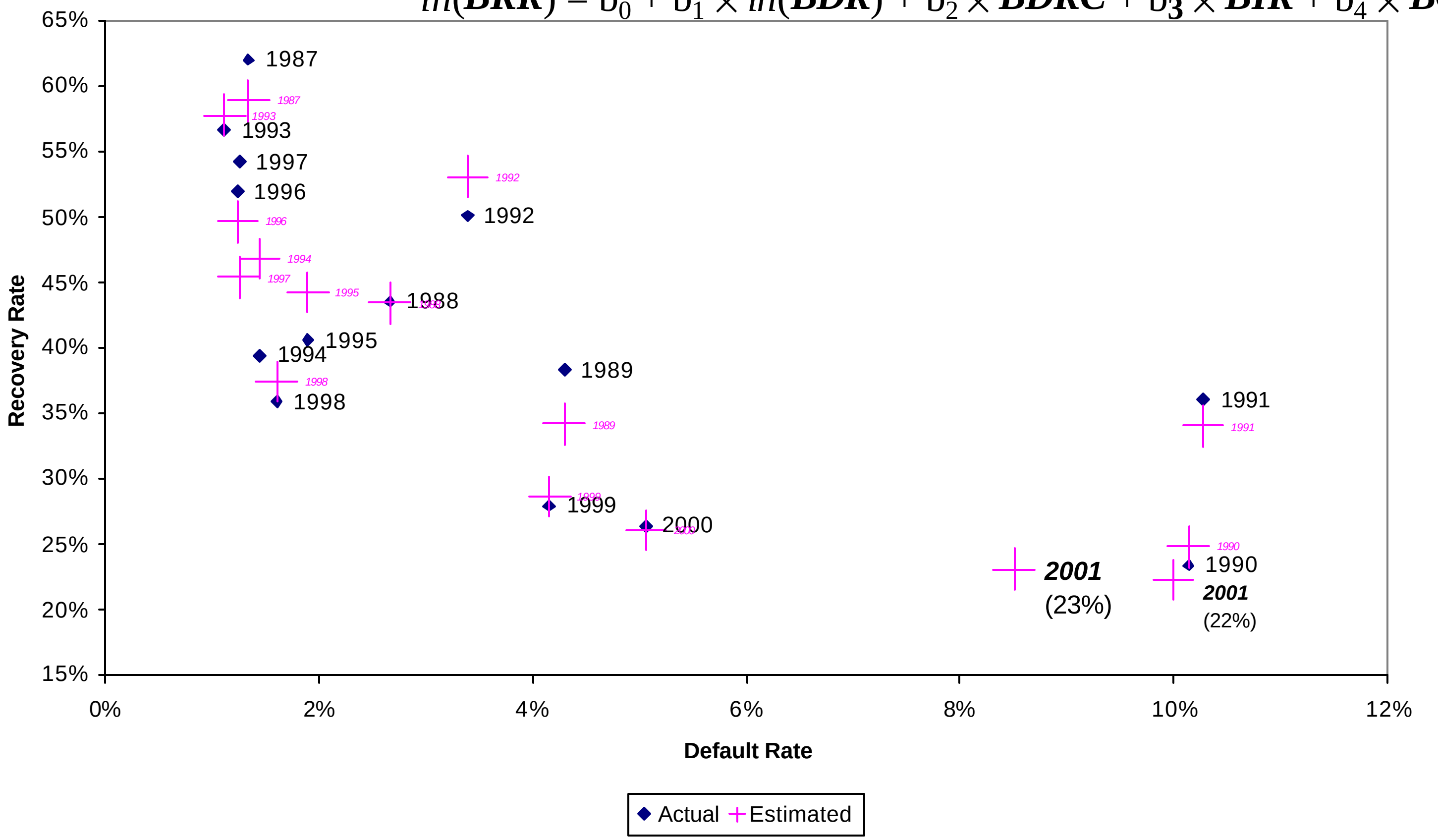


Figure III.10

Actual vs. Estimated Recovery Rates on Defaulted Corporate Bonds Based on Multivariate Regression Model (see Figure 5b-b) 1987–2000

$$ln(\boldsymbol{BRR}) = b_0 + b_1 \times ln(\boldsymbol{BDR}) + b_2 \times \boldsymbol{BDRC} + b_3 \times \boldsymbol{BIR} + b_4 \times \boldsymbol{BDA}$$

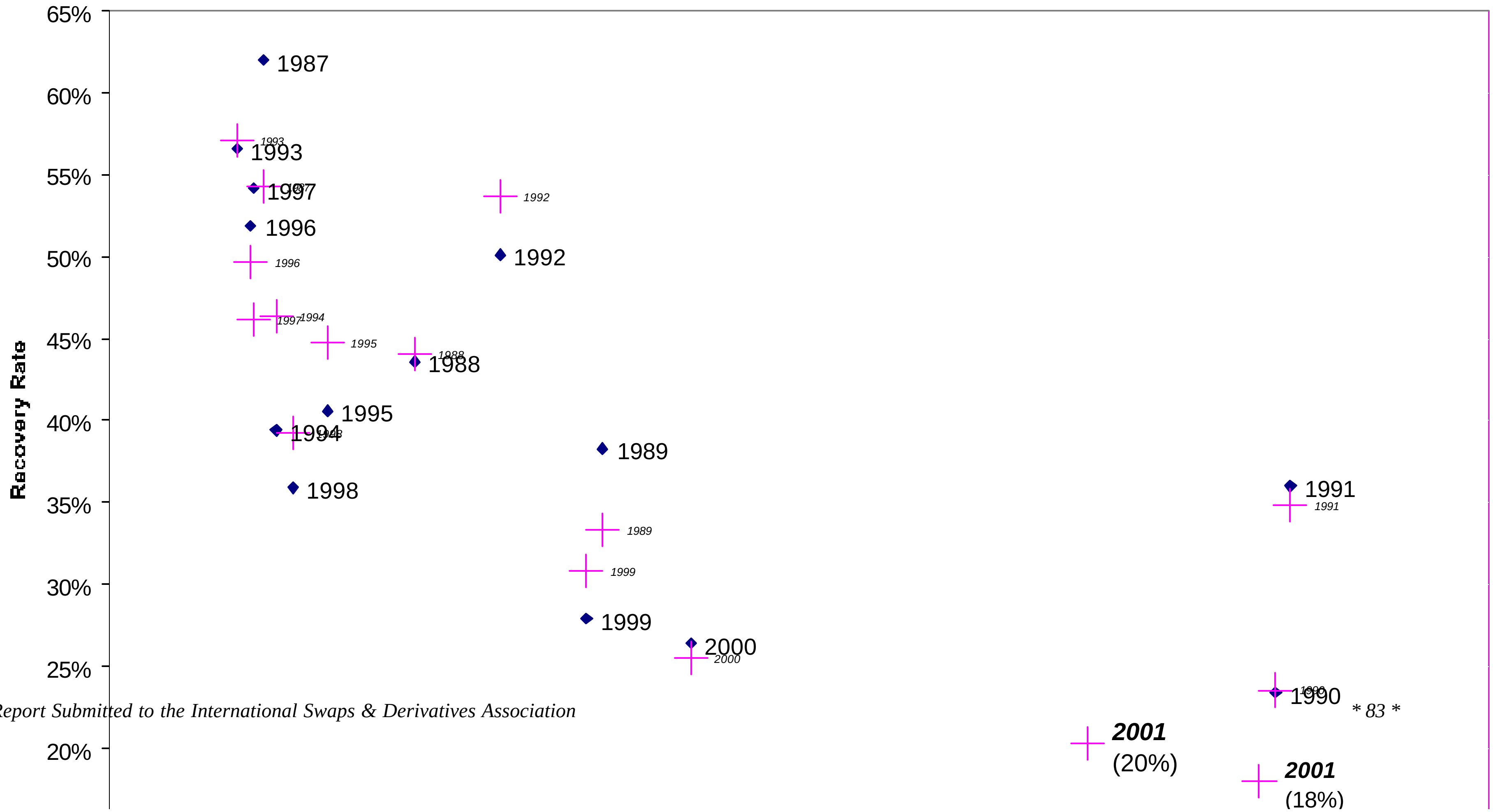


Figure III.11

Actual vs. Estimated Recovery Rates on Defaulted Corporate Bonds Based on Multivariate Regression Model (see Figure 7b-b) 1982–2000

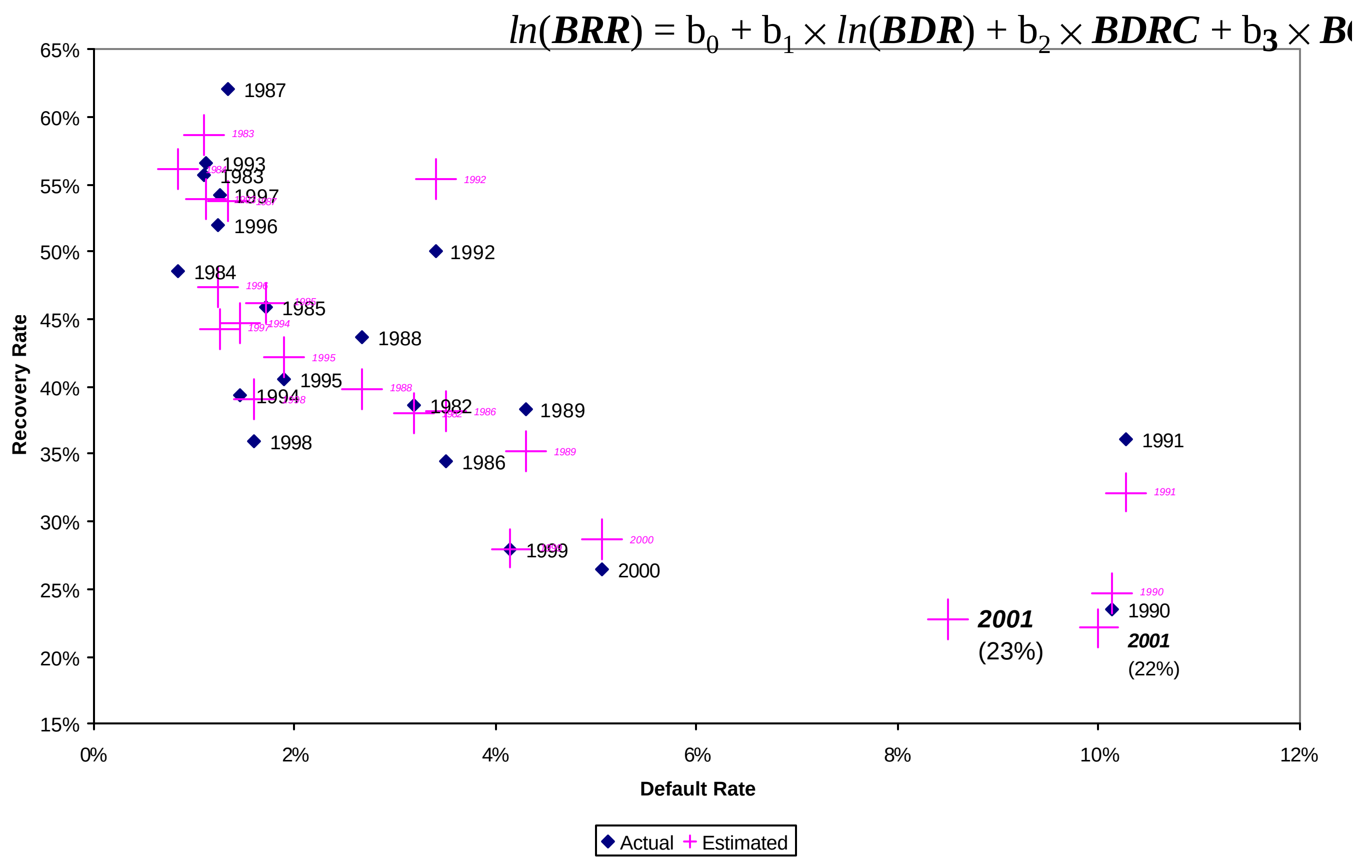
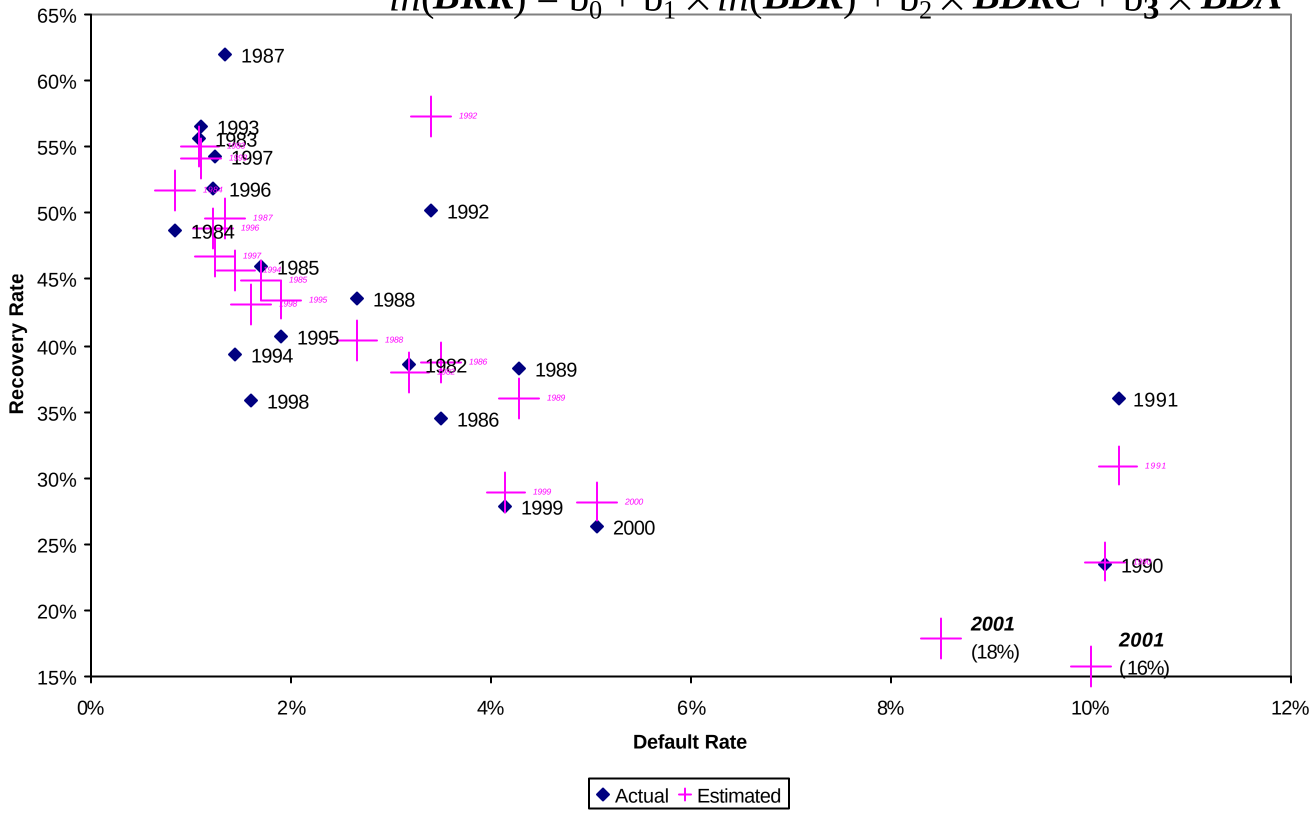


Figure III.12

Actual vs. Estimated Recovery Rates on Defaulted Corporate Bonds Based on Multivariate Regression Model (see Figure 7b-d) 1982–2000

$$ln(\boldsymbol{BRR}) = b_0 + b_1 \times ln(\boldsymbol{BDR}) + b_2 \times \boldsymbol{BDRC} + b_3 \times \boldsymbol{BDA}$$



Appendix III.A - Bond and Bank Loan Recovery Rates on Defaulted Securities (Results from Various Studies)

1. Altman & Arman (2002) – Period Covered 1978-2001, Prices at Default

Bond Seniority	Number of Issues	Median %	Mean %	Standard Deviation
Senior Secured	134	57.42	52.97	23.05
Senior Unsecured	475	42.27	41.71	26.62
Senior Subordinated	340	31.90	29.68	24.97
Subordinated	247	31.96	31.03	22.53
Discount	93	17.40	18.97	17.64
Total Sample	1289	40.05	35.85	24.87

2. Altman (2002) – Investment Grade vs. Non-Investment Grade (Original Rating) – Period Covered 1971-2001, Prices at Default

Seniority	Number of Observations	Median Price (%)	Average Price (%)	Weighted Price (%)	Standard Deviation (%)
Senior Secured					
Investment Grade	35	57.00	62.00	66.00	19.70
Non-Investment Grade	113	30.00	38.65	32.89	29.46
Senior Unsecured					
Investment Grade	159	50.00	53.14	55.88	26.14
Non-Investment Grade	275	31.00	33.16	30.17	25.28
Senior Subordinated					
Investment Grade	25	27.54	39.54	42.04	24.23
Non-Investment Grade	283	28.00	33.31	29.62	24.84
Subordinated					
Investment Grade	10	35.69	35.64	23.55	32.05
Non-Investment Grade	206	28.00	31.72	28.87	22.06

3. Altman (2002) – Bank Loan (1996-2001) – Prices at or Just After Default

Year	**Number of Facilities**	**Median**	**Weighted Average**	**Average**	**Standard Deviation**
1996	9	86.00	80.42	73.34	24.41
1997	4	87.88	94.86	87.92	12.61
1998	5	72.00	84.70	75.77	18.34
1999	40	51.25	54.44	56.31	22.34
2000	41	65.00	59.36	66.06	16.69
2001	73	60.00	61.06	59.20	21.74
Total	172	64.13	60.39	62.44	21.71

4. FITCH (2001) – Period Covered 1997-2000, Prices One Month After Chapter 11 and Confirmation Dates (Plus one month) of Emerged Companies

Debit Type/ Seniority	**Number of Observations**	**Average Recovery at Chapter 11**	**Recovery at Confirmation**	**Bank Debt Greater Than 45% of Debt**	**Bank Debt Less Than 45% of Debt**
Senior Secured Loans	35	--	73.0%	63.0%	81.0%
Senior Unsecured Bonds	17	--	35.0%	--	–
Subordinated Bonds	35	--	17.0%	--	–
All Bonds	52	--	22.0%	16.0%	28.0%

5. FITCH (1997) – Period Covered June 1991–June 1997, Prices One Month After Confirmation

Bank Loans	60	--	82.0%	--	--
Senior Subordinated Bond	45	--	42.0%	--	--
Subordinated Bonds	45	--	39.0%	--	--

6. Moody's (2000) – Period Covered 1970-2000, Prices One Month After Default

	Median %	Average %	Standard Deviation %
Senior Secured Bank Loans	72.0%	64.0%	24.4%
Senior Unsecured Bank Loans	45.0	49.0	28.4
Senior Secured Bonds	53.8	52.6	24.6
Senior Unsecured Bonds	44.0	46.9	28.0
Senior Subordinated Bonds	29.0	34.7	24.6
Subordinated Bonds	28.5	31.6	21.2
Jr. Subordinated Bonds	15.1	22.5	18.7

7. Standard & Poor's (2000) – Period Covered 1981-1999, Prices Shortly After Default and at Emergence

	Number of Observations	Weighted Average	Simple Average	Standard Deviation	Weighted Average at Emergence
Senior Secured Bonds	91	49.32%	54.28%	24.25%	86.71% (52)
Senior Unsecured Bonds	237	47.09	46.57	25.24	76.66 (157)
Subordinated Bonds	177	32.46	35.20	24.67	47.88 (94)
Jr. Subordinated Bonds	144	35.51	34.98	27.32	32.48 (96)
Total (All Bonds)	649	40.23	41.88	25.23	56.91 (399)

8. Standard & Poor's (2001) – Period Covered (1981-2001), Discounted and Nominal Prices at Confirmation of Chapter 11

Nominal Discounted

	Number of Observations	Simple Average	Simple Average	Standard Deviation
Senior Bank Loans	455	88.32%	77.05%	28.54%
Senior Secured Bonds	196	79.03	69.30	31.04
Senior Unsecured Bonds	208	64.29	53.21	35.57
Senior Subordinated Bonds	251	43.41	36.60	33.20
Subordinated Bonds	352	36.22	31.11	35.13

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