The Dynamics of Default and Debt Reorganization

Pierre Mella-Barral
London School of Economics

This article documents the fact that when debtors decide to default on their obligations too early, it is in the creditors' collective interest, as residual claimants, to make concessions prior to forcing a costly liquidation. Symmetrically, when debtors prefer to default at an inefficiently late stage, it is in the creditors' interest to propose a departure from the absolute priority rule. This article develops a continuous time pricing model of dynamic debt restructuring that reflects the crucial influence of the two counterparties' relative bargaining power. Simple and intuitive path-dependent pricing formulae are derived for equity and debt. The debt capacity as well as the evolution of the firm's capital structure throughout its existence is provided.

Understanding the potential for debtors to default on their obligations is essential to lenders. Clearly, if debtors do not service promised debt payments, that is, *default*, creditors can use debt collection law to seize the physical assets of the firm, that is, trigger *liquidation*. However, debtors' default rarely coincides with liquidation. Most often the debt is *reorganized*, either through out-of-court arrangements or formal bankruptcy law.[1] Hence the initially agreed priority of debt on the revenues generated (or to be generated) by the firm is not respected. According to Gilson, Kose, and Lang (1990) almost half the companies in financial distress avoid liquidation through out-of-court debt restructuring.

Several explanations have been given to justify the fact that creditors are complaisant: In Franks and Torous (1989) and Bebchuk and Chang (1992), debtors' ability to strategically remain in renegotiations gives them an option to delay the exit from a lengthy, hence costly, formal debt reorganization procedure. In Baird and Jackson (1988), it is management's assumed unique ability to preserve firm value that gives them such a bar-

Comments by Ronald Anderson, Sudipto Bhattacharya, Gerard Gennotte, Hayne Leland, Kjell Nyborg, Jacques Olivier, William Perraudin, Suresh Sundaresan, Pierre Tychon, Paolo Vitale, and Josef Zechner are gratefully acknowledged. Seminar participants at the LSE, Universitat Pompeu Fabra Barcelona, Brunel University, Cambridge University, ESSFM97 Gerzensee, Catholic University of Louvain-la-Neuve, HEC Paris, INSEAD, London Business School, LTCM London, EFA96 Oslo, Oxford University, Paris Dauphine University, Stockholm School of Economics, and Tilburg University also provided helpful comments. Any errors remain my responsibility. Financial support from the Financial Markets Group at the LSE is gratefully acknowledged. Address correspondence to Pierre Mella-Barral, London School of Economics, Houghton St., London WC2A 2AE, UK, or e-mail: pmella@lse.ac.uk.

[1] Liquidation under debt collection law refers to Chapter 7 of the U.S. Bankruptcy Code; in-court debt reorganizations under bankruptcy law refers to Chapter 11 of the U.S. Bankruptcy Code.

The Review of Financial Studies Fall 1999 Vol. 12, No. 3, pp. 535–578

gaining power. Giammarino (1989) gives an alternative explanation based on opportunistic misrepresentation, as creditors suffer from an asymmetric information problem. In Bergman and Callen (1991), debtors use their discretion over the investment decisions of the firm to wrest concessions from creditors by threatening to sap firm value through suboptimal investment policies. Finally, in Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997), debtors' ability to make take-it-or-leave-it offers to their creditors enables them to strategically obtain "debt service holidays."

This article discusses instead the following moral hazard problem: Debt contracts do not induce debtors' ex post optimal timing of default to coincide with the ex ante optimal time to sell off the firm.[2] Our analysis considers situations where cooperatively selling the assets of the firm becomes preferable to continuing operating it. Therefore, unlike previous studies, it recognizes that the ex ante optimal time of liquidation is not just equal to the infinite.[3] Importantly, whereas existing theories are all built upon the idea that creditors are complaisant because debtors can sabotage firm value, here it is rational to restructure the debt, even when debtors have no bargaining power.

With standard debt contracts, this leads to only one level of borrowings for which debtors' ex post optimal time of default actually equals the ex ante cooperatively optimal time of liquidation. For other levels of borrowing, it always becomes in creditors' collective interest, as residual claimants, to eventually reduce their own contractual claims. In doing so creditors avoid bearing the cost of an inefficient liquidation. Debt reorganization essentially enhances the market value of the debt, as it enables creditors to avoid ill-timed liquidation.

On the one hand, an inefficiently "early" default leads self-interested creditors to propose a reduction of their immediate cash flow claims, conditional on debtors restarting to service the reduced debt obligation. We consider the fact that to exit a situation of default it is most often necessary to propose a new debt contract to the investors.[4] Then creditors' ex post optimal behavior consists of gradually writing down their coupon, each time it becomes necessary and in their interest, to extend debtors' willingness to meet their obligations. This reflects empirical evidence showing that (i) reorganizations yield a reduction in leverage and (ii) occur in a repeated

[2] We use ex ante and ex post with respect to the date the debt is issued.

[3] In Nielsen, Saa-Requejo, and Santa-Clara (1993) and Longstaff and Schwartz (1994) liquidation of an unlevered firm would also occur at a finite time. It happens the first time the value of the firms' assets reaches a critical level, which is either a constant or follows a diffusion process. The critical level is, however, not necessarily optimal, as it is just exogenously given.

[4] Technically such "irreversible" concessions are difficult to handle because the problem becomes path dependent. They differ from the simpler state-dependent concessions considered in Mella-Barral and Perraudin (1997).

fashion.[5] To capture the magnitude of creditors' willingness to restructure the debt, we also construct an adapted version of the "debt-relief Laffer curves" encountered in the literature on sovereign debt.[6] A first important result is that debt value can be decreasing in debt service obligation.

On the other hand, the prospect of a "late" default induces creditors to propose a concession on their collateral claim in liquidation, conditional on debtors accepting to immediately declare liquidation. Here creditors are trying to precipitate the event of liquidation. It is therefore rational, even for noncoerced and self-interested creditors to promise debtors a share of the *proceeds* of a liquidation sale. This is a new explanation for what are essentially departures from the absolute priority rule *after* liquidation.[7] Of importance, debt value can also be decreasing in collateral.

Whereas sequential debt forgiveness arises for relatively high levels of leverage, departures from the absolute priority rule in liquidation occur for relatively low levels of leverage. The structure of the model also relates the predominance of one form of debt reorganization or the other to fundamental characteristics of the industry; in particular, the relative contribution to value of inalienable human capital to alienable physical assets.

This article then provides the pricing implications of these forms of debt reorganization. With respect to the pricing literature, and in particular Leland (1994), Leland and Toft (1996), and Mella-Barral and Perraudin (1997), our main contribution consists of dissociating the events of default and liquidation. The analysis is extensive, (i) expanding the strategy space open to debtors and creditors in debt reorganization, allowing for the debt contract to be renegotiated, and (ii) incorporating game theoretic interactions between these two conflicting parties.

The analysis is conducted constructing a flexible structural continuous-time model of the levered firm which allows for *dynamic restructuring of the debt contract*. The *closed form* asset pricing formulae for debt and equity we derive are simple and intuitive *path-dependent* functions of the basic state variable which summarizes economic fundamentals. The dynamic nature of the model yields the expected evolution of the leverage through time, hence a dynamic theory of capital structure. It therefore also contributes to the literature on the dynamics of the capital structure which includes

[5] Gilson, Kose, and Lang (1990) report that three-quarter of out-of-court workouts involve a reduction in debt obligations. Alderson and Betker (1995) find that firms with high bankruptcy costs emerge from debt restructuring procedures with relatively low debt:equity ratios. Furthermore, according to Gilson (1995), between a quarter to a third of all financially distressed firms reenter financial distress within a few years after completing a restructuring.

[6] Krugman (1988, 1989), Sachs (1988a, b), and Froot (1989) have studied the logic behind several market-based reduction schemes.

[7] Franks and Torous (1989, 1994) and Eberhart, Moore, and Roenfeld (1990) find that in three of four corporate liquidations, debtors get some share of the proceeds of the open-market liquidation sale, even though creditors are not completely paid off.

Kane, Marcus, and McDonald (1984) and Fischer, Heinkel, and Zechner (1989).

The influence of the relative bargaining power of debtors and creditors in contract renegotiation is very substantial. Although the option literature ignores this factor, surprisingly, it is not an important one in corporate asset pricing.[8] Relative bargaining power in renegotiation alters the debt risk premium at entry as well as the ex post departures from the absolute priority rule by as much as the threatening implicit cost of liquidation. Similarly the endogenous absolute limit to the amount debtors are able to borrow ex ante, that is, the debt capacity of the firm, diminishes rapidly with debtors' bargaining power.

The article is structured as follows: Section 1 constructs a model of the levered firm where debtors' ex post optimal timing of default differs from the ex ante optimal time of liquidation. Section 2 explains the difference (i) between "deferring" and "inducive" concessions and (ii) between "self-imposed" and "forced" concessions, hence introducing four cases. Section 3 presents the assumptions behind the "benchmark" model we later develop and gives as an example of the structure for which we will derive closed-form pricing formulae. Sections 4 and 5 develop, in the four cases introduced above, the associated asset pricing equations with dynamic debt reorganization. Section 6 characterizes the set of feasible debt contracts that can actually be initially issued and provides a numerical example. Section 7 discusses possible extensions of the analysis. Section 8 concludes.

1. Modeling the Levered Firm

1.1 Operating a firm with the alternative to liquidate it

Consider a set of physical assets that can yield revenues that are positively related to a single state variable, x_t. This uncertain state variable follows a diffusion process:

$$dx_t = \mu(x_t)\,dt + \sigma(x_t)\,dB_t, \tag{1}$$

where B is a standard Brownian motion. This state variable reflects economic fundamentals. It can be the market value of this set of physical assets or the operating income they could generate.[9] It may also be a more

[8] Empirical evidence reported in Jones, Mason, and Rosenfeld (1984) shows that yield spreads substantially exceed those implied by contingent-claims models based on the work of Merton (1974) and Black and Cox (1976). Leland (1994) is able to generate such high levels of yield spreads, but needs to assume excessively high bankruptcy costs. Longstaff and Schwartz (1995) produce significantly boosted risk premia but need to incorporate exogenously specified departures from the absolute priority rule. Here, high debtors' bargaining power in renegotiation can easily lead to dramatic increases in yield spreads.

[9] Most corporate debt valuation models, including Merton (1974), Black and Cox (1976), Brennan and Schwartz (1984), Fischer, Heinkel, and Zechner (1989), Kim, Ramaswamy, and Sundaresan (1993), Leland (1994), Longstaff and Schwartz (1995), and Leland and Toft (1996) take the total value of the firm's assets as their economic fundamental x_t. The cash-flow models of Mello and Parsons (1992), Fries,

industry-specific fundamental, such as the oil price. More generally it can be any summary combination of them.

In combination with the incumbents' human capital, these physical assets can yield a period income flow, which only depends on the level of the state variable x. Operating losses are not ruled out, hence this income flow may be negative in some range. We will denote $\Pi(x)$ as the unlimited liability value of a *perpetual* claim (and obligation) on this income flow. We assume that $\Pi(x)$ is a continuous and twice differentiable function of x. This is a reasonable assumption given that it is the integral over time of a perpetual stream of discounted period income flows.

Although the set of physical assets could be *operated* forever, the incumbents have the alternative to abandon operations and sell these assets. That is, usually a set of physical assets has worthwhile alternative uses in the hands of competitors. This is because the assets of the firm, in the hands of other market participants, could yield an *alternative* period income flow. At any time these alternative uses determine competitors' willingness to pay for the firm's assets, and we will denote $\Pi^*(x)$ as the price the incumbents can expect to sell them for. We will also assume that $\Pi^*(x)$ is a continuous and twice differentiable function of x.

To simplify, we consider that partial asset sales destroy existing economies of scale: If a fraction of the assets of the firm are sold, the remaining income flow from operations is reduced by more than this fraction. Therefore, gradually selling the assets through a sequence of auctions is not profitable, and the assets of the firm are best sold simultaneously. We also consider that selling the assets is an irreversible decision, so that when the incumbents decide to do so, they essentially *liquidate* the firm.

Overall, once it is constituted the firm consists of a set of physical assets which (i) are currently *operated* but (ii) could alternatively be irreversibly *liquidated*.

Distinguishing $\Pi(x)$ from $\Pi^*(x)$ and assuming the irreversibility of the decision enables us to capture in a structural fashion the implicit *cost* of liquidation: The decision would be a perfectly reversible one, if in the case the state variable x_t reversed its course a second after the selling decision is taken, (i) the incumbents would wish to buy back the firm immediately, and (ii) their competitors would actually sell it back for the price they just bought it for. Liquidation would then essentially be a temporary costless decision to redeploy assets.

In practice however, a liquidation sale often involves a partial dismantlement of the technology. Furthermore, interrupting a line of production often diminishes future access to current operations, possibly due to a loss

---

Miller, and Perraudin (1997), and Mella-Barral and Perraudin (1997) take the price of the commodity produced as the driving process. They all assume that x_t follows a geometric Brownian motion.

of human capital, know-how, and competitive edge. In any case, it is difficult to reverse such a decision. Therefore once liquidation is decided, owning back the firm is less attractive for the incumbents than owning it before. In short, reconstituting a working combination of physical assets and human capital is costly.

This interpretation is in the spirit of Hart and Moore (1994), who consider that liquidation avoids future use of the incumbents' inalienable human capital. Our setup is just less drastic: The incumbent has access (and can exclude access) to an opportunity set, which is the support of $\Pi(x)$. Their competitors have a different access protected set of opportunities, which supports $\Pi^*(x)$. The difference between $\Pi(x)$ and $\Pi^*(x)$ reflects the fact that after a liquidation sale the sellers' willingness to buy back the technology is lower than his willingness to hold it before.

Such a framework can capture well the degree to which initial investments are specific to current operations. Clearly, when the specialization of an investment is human capital specific (instead of physical asset specific), which is inalienable, the specialization implies a relative reduction of the outside value of the physical assets. Then $\Pi(x)$ is very different from $\Pi^*(x)$.

Euro-Disney is an example of this case: If Disney was to abandon operations, potential buyers of the site would only be left with an unattractive field distant from Paris. Because competitors do not have access to Mickey Mouse, they would not be willing to pay much. Conversely, if Euro-Tunnel declares bankruptcy, the liquidation value of the tunnel can be expected to be close to the current market value of the existing firm. This because several market participants are able to generate a similar income stream if they held the tunnel, hence $\Pi(x)$ is similar to $\Pi^*(x)$.

1.2 Value of the firm

If it is known that the event of liquidation will occur the first time the state variable x_t reaches a given level y, we can express the present value of the firm as a function of $\Pi(x)$ and $\Pi^*(x)$: The value of the firm prior to the event of liquidation is simply

$$V(x_t \mid y) = \Pi(x_t) + \left[\Pi^*(y) - \Pi(y)\right] \mathcal{P}(x_t \triangleright y). \tag{2}$$

This decomposition of the value of the firm is similar to Black and Cox (1976): The first term on the right-hand side is the value of a perpetual entitlement on the current flow of income, $\Pi(x_t)$. The second term is the product of (i) the change in asset value intervening when liquidation occurs, $[\Pi^*(y) - \Pi(y)]$, and (ii) a probability-weighted discount factor for this event denoted $\mathcal{P}(x_t \triangleright y)$.

Let $T_y \equiv \inf\{T \mid x_T = y\}$ be the first time at which x_t hits the level y, and denote $f_t(T_y)$ as the density of T_y conditional on information at t (that the state is x_t). The random time T_y is well-defined since the sample paths of x_t are continuous almost surely. Assuming risk neutrality, and a constant identical borrowing and lending safe interest rate, ρ,[10] then $\mathcal{P}(x_t \rhd y)$ is the Laplace transform of $f_t(T_y)$,

$$\mathcal{P}(x_t \rhd y) = \int_t^{\infty} e^{-\rho(T_y - t)} f_t(T_y)\, dT_y. \tag{3}$$

This notation should allow an intuitive understanding of the articles' valuation formulas. $\mathcal{P}(x_t \rhd y)$ is a probability-weighted discount factor for payoffs accruing the first time the state variable reaches a level y, given that it is currently equal to x_t.

Now, instead of assuming that liquidation occurs the first time the state variable x_t reaches an exogenously given level y as if it was an uncontrolled event, we would like the liquidation trigger level y to emerge endogenously out of an optimization problem, reflecting the fact that it is the result of a decision-making process.

An *optimal* liquidation point is, among all candidate trigger levels y, one that maximizes firm value. Therefore, assuming that at the time of entry t_0 incumbents are better off operating the physical assets than liquidating them, an optimal liquidation point $\underline{x}$ solves the first-order optimality condition

$$\frac{\partial V(x \mid \underline{x})}{\partial \underline{x}} = 0. \tag{4}$$

As it is, the model does not guarantee the actual desirability of the liquidation decision; that is, (i) nothing dictates the number of optimal solutions and (ii) nothing restricts their range in value. In other words, (i) there could be zero, one, or several optimal liquidation trigger levels, and (ii) these optimal trigger levels could be less or greater than the entry state, x_0.

We therefore need additional structural assumptions which (i) address the issue of existence and unicity of optimal solutions and (ii) actually reflect the economic situation we are interested in. Assumption 1 addresses these issues as follows:

1. We wish to depict situations where the decision to sell the physical assets becomes eventually profitable. In doing so, our objective is to depart from previous studies which consider that a firm's first best closure time is just infinite. Assumption 1 will ensure that there exists an optimal liquidation point, and to simplify the analysis, will also guarantee its uniqueness.
2. The focus of the article is on financial distress. We would therefore like the liquidation decision to be desirable as economic fundamentals

[10] Harrison and Kreps (1979) show how to extend the results of the article to a non-risk-neutral world, with risk adjusted probabilities, that is, under an equivalent martingale measure.

deteriorate.[11] Given that to start with we have normalized the analysis, assuming that the cash generating ability of the firm's assets is positively related to x_t, liquidation should be desirable as the state variable *falls*.

Assumption 1. *At the entry state x_0, the option value of the decision to trigger liquidation at y,*

$$\left[\Pi^*(y) - \Pi(y)\right]\ \mathcal{P}(x_0 \ \triangleright\ y), \tag{5}$$

is a strictly concave function in y, maximized at a trigger level $\underline{x}$ strictly smaller than x_0.

The firm, $\{x; \Pi(x); \Pi^*(x)\}$, and the economic environment it is evolving in, $\{\mu(x); \sigma(x); \rho\}$, which Assumption 1 portrays, are such that when the firm is setup, it is known that selling off the assets of the firm will eventually become optimal in the future the first time the state variable reaches a threshold level that is lower than the entry state. The unique ex ante optimal liquidation trigger level $\underline{x}$ is obtained solving the first-order optimality condition of Equation (4). In this context the value of the firm under the first-best closure policy is $V(x_t \mid \underline{x})$.

Figure 1 gives a graphical representation of the setup: At entry the incumbent is better off operating the physical assets than liquidating them. If the state of the world deteriorates, the income flow associated under current operations becomes relatively less attractive than the one competing corporations could generate, and selling the assets of the firm eventually becomes preferable to continuation. The unique random optimal time to do so is the first time the state variable falls to the level $\underline{x}$.

1.3 Introducing debt

To finance the initial investment the incumbents have issued infinite maturity debt that is promising an instantaneous flow of coupon payments δ to a cohesive group of creditors. That is, the value of a perpetual stream on creditors contractual income flow is δ/ρ. The remaining part of the income that results from operations accrues to the debtors, and its perpetual value is $\Pi(x) - \delta/\rho$.

In the spirit of recent corporate finance literature, debt can be justified assuming that the incumbents are not wealthy enough to finance this entire initial investment and *need* external financing. If x_t, $\Pi(x_t)$, and $\Pi^*(x_t)$ are observable to both contracting parties but not verifiable to outsiders, as in

[11] A situation where a decision to sell the assets of the firm to competitors is desirable if economic fundamentals improve corresponds more to an opportunity to be taken over. Combining both a financial distress liquidation and a take over opportunity in such a framework may provide interesting insights. This, however, generates a two-sided pricing problem, and obtaining closed-form solutions requires the model to have a very particular geometry.

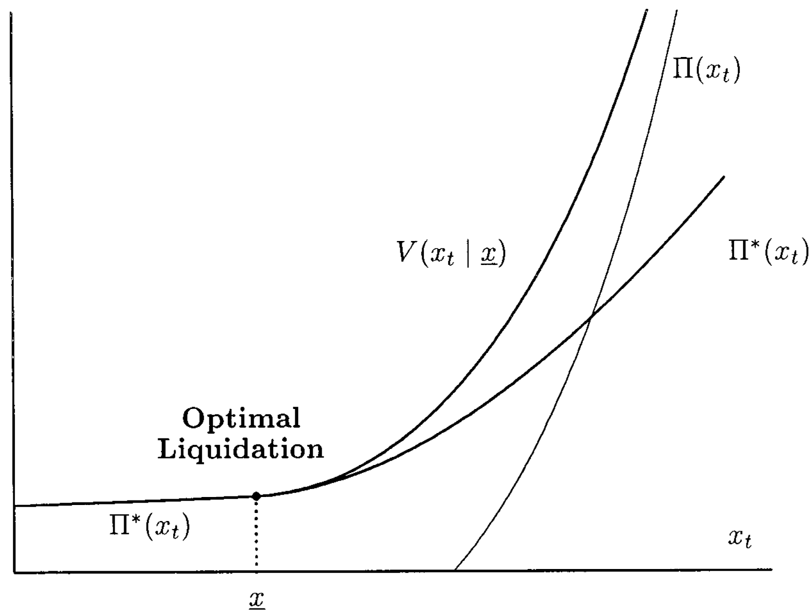

Figure 1
The firm and its economic environment
The firm is economically worth $V(x_t \mid \underline{x})$. $\Pi(x_t)$ is the unlimited liability value of the income flow generated through current operations. The ex ante optimal liquidation decision is taken when the economic fundamental variable, x_t, reaches $\underline{x}$. $\Pi^*(x_t)$ is the expected liquidation value of the firm.

Hart and Moore (1989), they cannot be part of an enforceable contract. The incumbents (agents) have to issue simple debt contracts for creditors (principals) to lend.

Alternatively, and arguably more significantly in practice, debt is justified in the presence of a *tax advantage of debt*. The model could easily be extended to incorporate the effect of a tax relief, τ:[12] The value of a perpetual entitlement on the flow of income to equity and debt would be $(1-\tau)[\Pi(x) - \delta/\rho]$ and δ/ρ, respectively, instead of just $\Pi(x) - \delta/\rho$ and δ/ρ. However, for simplicity and to emphasize the role of the parameters that crucially drive our analysis, the model is developed setting τ to zero.

Debtors can always decide to repudiate the contract and *default* on their contractual obligations. But if the contract is not serviced, creditors can take legal action, going to court. The contract stipulates that if the debt service is not respected, a sharing rule of the proceeds from a *liquidation* sale of the firm can be imposed, invoking debt collection law. This sharing rule defines

[12] As Miller (1977) pointed out, $\tau = 1 - (1 - \tau_c)(1 - \tau_V)/(1 - \tau_L)$, where τ_c is the corporation tax rate. τ_V and τ_L are the personal tax rate on equity and debt income, respectively.

the value of creditors' collateral, which we denote $\underline{C}(x)$. For example, if as is almost exclusively the case, the absolute priority rule (APR) is to be applied, creditors should be paid first out of the proceeds of a liquidation sale, up to a par value P, and therefore $\underline{C}(x) = \Pi^*(x) \wedge P$.

Trading of assets is assumed to occur continuously in perfect and frictionless markets with no asymmetry of information or transaction costs. Management acts in the best interest of the debtors, ignoring the insiders-outsiders principal-agent conflict of interest discussed in Hart (1993). Once debt obligations are serviced, all residual revenues from operations are paid as dividends to the debtors. But for creditors' collateral to be credible, debtors are prohibited to set larger dividends: This prevents them from being tempted to appropriate firm value, selling parts of the assets of the firm before defaulting. Furthermore, the debt contract does not require debtors to make contributions to a sinking fund.

The stylized debt contract we consider is therefore a *perpetual* entitlement (i) to a coupon δ in operations, and (ii) to collect a collateral $\underline{C}(x)$ if the contract is repudiated.

Let us examine the situation if the debt contract is never renegotiated. It is then clear to both groups of claimants that if debtors' default, liquidation will follow immediately. If we know that debtors default the first time the state variable x_t reaches a given level y, the values debtors and creditors claim prior to this event are, respectively,

$$D(x_t \mid y) \;=\; \Pi(x_t) - \frac{\delta}{\rho} + \left[\Pi^*(y) - \Pi(y) - \underline{C}(y) + \frac{\delta}{\rho}\right] \mathcal{P}(x_t \,\rhd\, y), \tag{6}$$

$$C(x_t \mid y) \;=\; \frac{\delta}{\rho} + \left[\underline{C}(y) - \frac{\delta}{\rho}\right] \mathcal{P}(x_t \,\rhd\, y). \tag{7}$$

The structure of these expressions is similar to equation (2): In both cases, the first term on the right-hand side is the value of a perpetual entitlement on the flow of income to equity and debt—$\Pi(x) - \delta/\rho$ and δ/ρ, respectively. The second term is the product of (i) the change in asset value intervening when liquidation occurs, which is the expression in the square brackets, and (ii) the probability-weighted discount factor for this event, $\mathcal{P}(x_t \,\rhd\, y)$.

These valuation equations are derived for a given debtors' default trigger level. We now turn our attention to the determination of the state in which debtors default: The important corporate governance feature which differentiates equity from debt is *ownership*. Debtors have residual control rights:[13] as long as they (i) respect the written contract and, (ii) are willing to meet their contractual obligations, creditors cannot force liquidation. The

[13] Residual control rights are defined by Grossman and Hart (1986) and Hart and Moore (1990) as the right to decide all usages of the physical assets in any way not inconsistent with the contract.

debtors control the event of repudiation, as they can at any time decide not to service the debt. They select noncooperatively the default trigger level in order to maximize the value of their claim.

Consider the simplest scenario, assuming that debtors choose their time of default in an *unconstrained* fashion.[14] A debtors' ex post optimal default trigger level, which we will denote $\underline{x}_{\mathcal{D}}$, is essentially a point where it is optimal for them to irreversibly exchange their current claim for their residual one: not defaulting, debtors are entitled to a perpetual income flow with value $\Pi(x) - \delta/\rho$; defaulting, they expect to receive a residual payment $\Pi^*(x) - \underline{C}(x)$. Debtors' ex post optimal default $\underline{x}_{\mathcal{D}}$ maximizes the value of the equity. As the solution to an unconstrained optimization problem, $\underline{x}_{\mathcal{D}}$ must solve the first-order optimality condition:

$$\frac{\partial D(x_t \mid \underline{x}_{\mathcal{D}})}{\partial \underline{x}_{\mathcal{D}}} = 0. \tag{8}$$

Here again, in the general case, neither existence nor uniqueness of this optimal trigger level, $\underline{x}_{\mathcal{D}}$, is guaranteed. The following assumption addresses these issues in similar fashion:

Assumption 2. *At the entry state x_0, the value of the debtors' option to default at y,*

$$\left[\Pi^*(y) - \Pi(y) - \underline{C}(y) + \delta/\rho\right] \mathcal{P}(x_0 \rhd y), \tag{9}$$

is a strictly concave function in y, maximized at a trigger level, $\underline{x}_{\mathcal{D}}$, strictly smaller than x_0.

The debt contract agreed upon at entry that Assumption 2 portrays involves a creditor residual entitlement, $\underline{C}(x)$, which (i) is credible (such that immediate default is not optimal) and (ii) only induces debtors to use their limited liability option when economic fundamentals deteriorate. The unique debtors' ex post optimal default trigger level is obtained solving the first-order optimality condition of Equation (8).

Let us now compare the closure rule that results from debtors' ex post noncooperative behavior to the first-best policy depicted in the previous section: Debtors' decision to default consists of assessing when it is optimal to abandon a *current* position worth $\Pi(x) - \delta/\rho$ for an *alternative* one worth $\Pi^*(x) - \underline{C}(x)$, knowing that switching is irreversible. In contrast, the ex ante optimal liquidation trigger level $\underline{x}$ results from balancing, in similar fashion, a *current* position worth $\Pi(x)$ against an *alternative* one worth $\Pi^*(x)$. The point is that debtors' ex post optimal default trigger level $\underline{x}_{\mathcal{D}}$ has no reason to be equal to the ex ante optimal one, $\underline{x}$.

14 This is the endogenous closure rule assumed in Leland (1994), Leland and Toft (1996), Fries, Miller, and Perraudin (1997), and Mella-Barral and Perraudin (1997).

Lemma 1. *Debtors' default trigger level,* $\underline{x}_{\mathcal{D}}$*, is increasing in debt service obligations,* δ*. Then, for a given sharing rule in the event of repudiation,* $\underline{C}(x)$*, there exists a unique threshold amount of debt service obligations,* $\tilde{\delta}$*, such that* $\underline{x}_{\mathcal{D}}$ *equals* $\underline{x}$*:*

$$\tilde{\delta} \;=\; \rho\,\underline{C}(\underline{x}) + \rho\,\frac{\left[\Pi^*(\underline{x}) - \Pi(\underline{x})\right]\, d\,\underline{C}(\underline{x})/d\underline{x}}{d\,\Pi(\underline{x})/d\underline{x} - d\,\Pi^*(\underline{x})/d\underline{x}}. \tag{10}$$

In the presence of two groups of claimants, debtors and creditors, the terms of the debt contract induce the debtors to default at $\underline{x}_{\mathcal{D}}$, which is most often different from $\underline{x}$. Consequently the total value of the firm, $D(x_t \mid \underline{x}_{\mathcal{D}}) + C(x_t \mid \underline{x}_{\mathcal{D}}) = V(x_t \mid \underline{x}_{\mathcal{D}})$, can only be less than the value of the firm under the ex ante optimal closure, $V(x_t \mid \underline{x})$. Whereas in the presence of a single group of claimants the first-best policy is obtained, with debt and equity, the debtors control the time of default, but their residual claim on the liquidation value of the firm is only rarely efficient. Lemma 1 splits the set of possible debt contracts in the following three subsets:

1. If the promised interest payment on the outstanding debt δ corresponds exactly to $\tilde{\delta}$, the total firm value, $D(x_t \mid \underline{x}_{\mathcal{D}}) + C(x_t \mid \underline{x}_{\mathcal{D}})$, equals the value of the firm under the ex ante optimal closure, $V(x_t \mid \underline{x})$, *even if* debtors' determine the time of liquidation in a noncooperative fashion. Notice that if the APR is respected, that is, $\underline{C}(x) = \Pi^*(x) \wedge P$, there is only one level of borrowings for which this is the case.
2. If the leverage is higher, δ is greater than $\tilde{\delta}$, and therefore $\underline{x}_{\mathcal{D}}$ is greater than $\underline{x}$. The amount of debt introduced yields an inefficiently "early" liquidation, as the value of the firm is not maximized, that is, $D(x_t \mid \underline{x}_{\mathcal{D}}) + C(x_t \mid \underline{x}_{\mathcal{D}}) < V(x_t \mid \underline{x})$.
3. For lower levels of leverage, δ is smaller than $\tilde{\delta}$, and therefore $\underline{x}_{\mathcal{D}}$ is smaller than $\underline{x}$ (if $\tilde{\delta}$ is nonnegative). Such an amount of debt creates an inefficiently "late" liquidation, because again $D(x_t \mid \underline{x}_{\mathcal{D}}) + C(x_t \mid \underline{x}_{\mathcal{D}}) < V(x_t \mid \underline{x})$.

Clearly debtors' freedom to choose the time of default is often limited either by (i) preestablished covenants, and/or (ii) nonintentional inability to service the coupon. However, any particular default "scenario" results from a set of constraints on debtors' ownership and generates a specific debtors' ex post *constrained* optimal trigger level. Therefore the main point would still remain: $\underline{x}_{\mathcal{D}}$ still has no reason to be equal to $\underline{x}$, and in the absence of renegotiation this generates an inefficiency due to an inappropriate sharing rule of the residual value. This is most generally the case, even under a different prevailing set of constraints on debtors' ownership.

2. Restructuring the Debt

We have just examined an efficiency that emerges when the debt contract is never renegotiated, hence when debtors' default and liquidation are simultaneous. However, default is not necessarily followed by liquidation.

When debtors exercise their option to default, creditors have in turn an *option* to use debt collection law and force liquidation. This is not an obligation to do so. Although the firm is set up as a single investment decision, at any time, nothing prohibits one group of claimants from starting out-of-court debt reorganization discussions with the other. This, after or even before default.

If asset-holders are able to renegotiate perfectly and costlessly their contract, they will internalize all surplus their is eventually to be gained from renegotiation. Here, with an implicitly costly liquidation, but perfect costless renegotiation, a capital structure irrelevance result should hold. The renegotiation surplus to be generated equals the difference between the value of the firm under the ex ante optimal closure, $V(x_t \mid \underline{x})$, and its value with debtors' noncooperative liquidation decision, $D(x_t \mid \underline{x}_{\mathcal{D}}) + C(x_t \mid \underline{x}_{\mathcal{D}})$.

2.1 Deferring versus inducive creditors' concessions

Consider first an amount of leverage for which associating an immediate bankruptcy to debtors' default would lead to an inefficiently "early" liquidation. This corresponds to high levels of outstanding debt, generating an $\underline{x}_{\mathcal{D}}$ greater than $\underline{x}$. Such a situation is represented in Figure 2. Clearly the debt would be worth more if liquidation occurred at $\underline{x}$ instead of $\underline{x}_{\mathcal{D}}$, hence creditors stand to gain from *postponing* the event of liquidation.

If debtors default at $\underline{x}_{\mathcal{D}}$, it is always profitable for creditors, in the absence of renegotiation costs, to make concession offers which increase the payoffs to the debtors under current operations, relative to liquidation. Such concessions decrease debtors' noncooperatively optimal default trigger level. It provides an incentive for debtors to prefer current operations, hence generating an interest to restart servicing the debt. Define a net transfer of wealth from the creditors to their debtors, which only applies if default does not occur, as a "deferring" concession. In providing this incentive, creditors' self-interested objective is to internalize to their own profit the surplus to be generated through renegotiation. This is the rationale for the existence of *debt forgiveness* we will develop in this article.

On the other hand, a low level of leverage possibly creates an inefficiently "late" liquidation. Creditors would then benefit from *precipitating* liquidation. Such a mirror situation is represented in Figure 3, where $\underline{x}_{\mathcal{D}}$ is smaller than $\underline{x}$. Define an "inducive" concession as a net transfer of wealth from the creditors to their debtors, to be perceived once and only once liquidation is triggered. In the absence of renegotiation costs, it is optimal for creditors to self-impose such concessions in order to appropriate the renegotiation

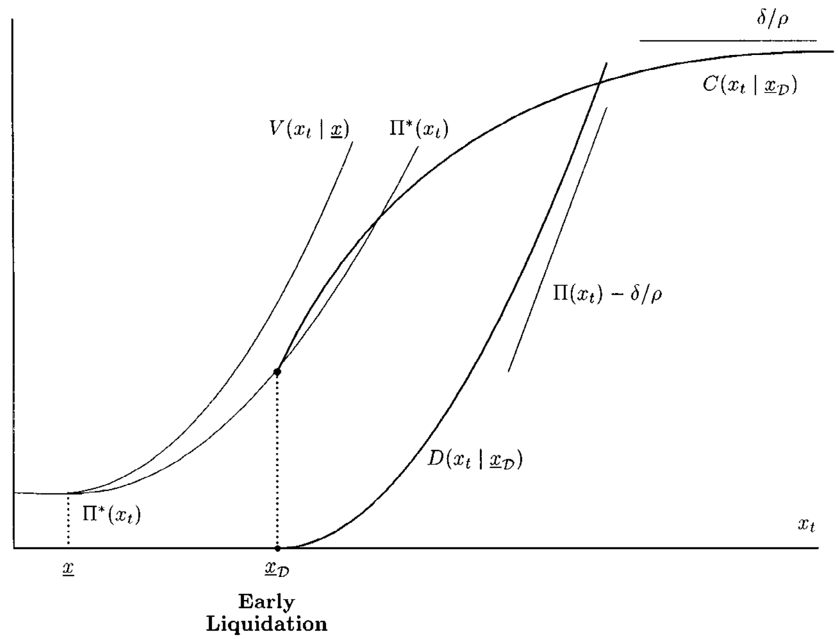

Figure 2
Inefficiently early liquidation

surplus. This is the rationale for the existence of creditors' self-imposed departures from the absolute priority rule we develop in the article.[15]

2.2 Self-imposed versus forced concessions

Debtors are always in favor of concessions. However, instead of passively accepting or rejecting such offers as they are, debtors can actually obtain more. If they can make credibly threatening offers, opportunistic debtors are in a position to extract part of the renegotiation surplus.

Therefore another important dimension to incorporate is the relative bargaining power of each group of claimants when it comes to renegotiating the contracts. The renegotiation surplus will be appropriated not exclusively by the creditors, but also by the debtors. In the absence of regulation, it is their relative bargaining power which determines the allocation of this surplus.

Differential games, such as the one played here by debtors and creditors, are difficult to solve for. In order to introduce in a tractable fashion such game-theoretic elements into this continuous-time pricing model, we restrict

[15] Notice that inducive concessions have the opposite effect to deferring concessions, hence creditors will not combine them.

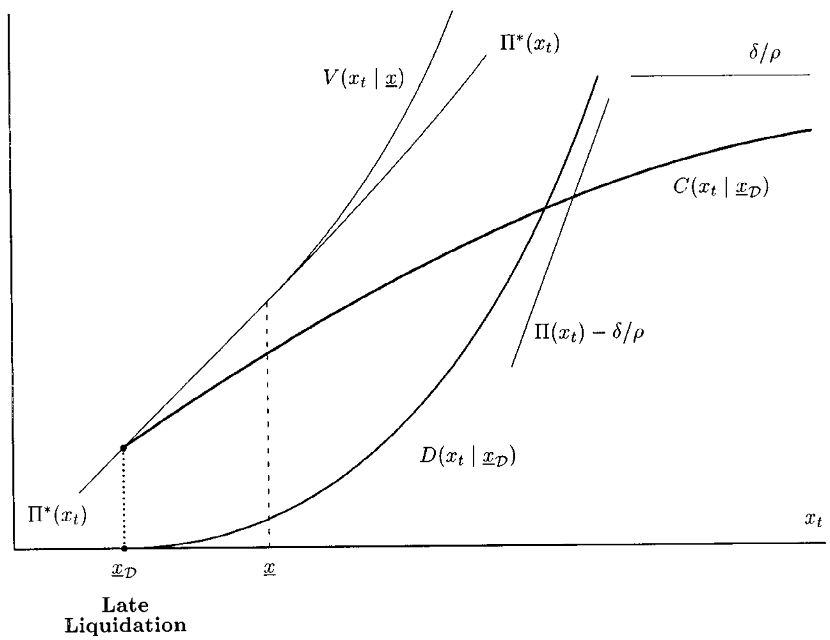

Figure 3
Inefficiently late liquidation

our attentions to the two limiting cases. We will only consider the equilibria that result if (i) the creditors are in a position to make take-it-or-leave-it offers to the debtors, or (ii) vice versa. Notice that noncoerced agreements can only be reached if resulting gains to one side do not imply losses to the other. This is to say that in these simple games, the follower player is nevertheless entitled to reject the take-it-or-leave-it offer made by the leader.

The first extreme case corresponds to the intuition developed so far: Creditors make "self-imposed" concessions, and debtors are passive in that they are not pushing creditors to make even larger concessions through strategic default. The alternative case is the mirror situation: Opportunistic debtors select their time of default to "force" the maximum concessions that creditors are ready to make in order to avoid the ill-timed liquidation.

Such situations result in hierarchical Stackelberg equilibria [see Basar and Olsder (1994) for an extensive discussion]. In the case of self-induced concessions, the leader creditors commit to a particular strategy and the follower debtors then react optimally, taking the leader's strategy as given. The strategies are Markov, open loop (state dependent), and perfect state (perfect information). Our assumption of risk neutrality, and the fact that asset-holders wish to extract all of the surplus, enables us to solve explicitly for the values and strategies.

3. Developing a "Benchmark" Pricing Model

3.1 The "Benchmark" model's assumptions

We set out the assumptions under which we will develop, in Sections 4–6, a pricing model with debt restructuring. Our choice, although disputable, intends to correspond best to a reference case. Section 7 provides a further discussion of these assumptions, suggesting future extensions of this "benchmark" case. The model of this article could then be used as a reference to assess the relative impact of alternative assumptions.

1. We consider that debtors select their time of default in an *unconstrained* fashion. This default scenario yields the simplest expressions because it corresponds to the absence of particular constraints on debtors' ownership, and seems the most appropriate one for a benchmark model. Implicitly it involves assuming that

   - Liquidity problems do not influence debtors' decision to default. This is arguably a reasonable assumption: When debtors face a liquidity problem, solving it would increase the value of their claim, which is nonnegative in the first instance. But then it is feasible and in their interest to issue additional shares in order to overcome the cash shortage: The new shares always have a positive price, because they consist of sharing a positive equity value.
   - There are no covenants triggering liquidation before debtors wish to default, because they cannot be enforced. This is justified in an environment where x_t, $\Pi(x_t)$ and $\Pi^*(x_t)$ are observable to both parties, but not verifiable to outsiders.

2. We assume that contracts are *perfectly* renegotiated. This implicitly assumes that debtors and creditors constitute two distinct but cohesive negotiating units, and silences the influence of the conflict of interest there exists among creditors. Roe (1987) and Gertner and Scharfstein (1991) discuss the fact that publicly traded debt exchange offers suffer from a "hold-out" problem as nonexchanging bondholders will be enriched at the expense of those who tender. When tendering, bondholders accept new bonds with reduced terms; the probability of ill-timed liquidation is also reduced for nontendering bondholders.

   This moral hazard problem is typically due to the noncohesiveness of the public debtholders' negotiating unit, and can freeze the out-of-court debt restructuring process. Therefore, in the presence of multiple creditors, it is often difficult to renegotiate contracts. In the extreme case, if debtors and creditors cannot restructure the debt at all, they face *forced liquidation*. When default is directly followed by liquidation, the following sections do not apply, and the valuation of assets is instead provided by Equations (6) and (7). With the endogenous closure rule, debtors' optimal default trigger level, $\underline{x}_{\mathcal{D}}$, simply solves Equation (8).

3. Debt reorganization is assumed *costless*. Clearly, if restructuring is costly, creditors face a trade-off between (i) bearing the immediate costs of liquidation in a nonrenegotiation strategy and (ii) incurring the aggregate amount of renegotiation costs associated with a dynamic restructuring strategy. Whereas here the firm will always ultimately be liquidated at the ex ante optimal time.
4. The concessions control variables are the coupon δ and the collateral $\underline{C}(x)$, that is, the terms of the debt contract. We only consider *infinite maturity debt*, and restructuring does not involve changing the maturity of the contract.
5. The model is developed with *no tax advantage of debt*, largely to isolate (and not exaggerate) the impact of debt reorganization on pricing.

3.2 Asset valuation technique

Throughout the article, asset values are derived using the same two-step procedure. We actually already employed this procedure to express the value of the firm under the first-best closure policy $U(x_t)$ and the value of equity and debt in the absence of renegotiation, $D(x_t \mid \underline{x}_{\mathcal{D}})$ and $C(x_t \mid \underline{x}_{\mathcal{D}})$.

First we express the value of the asset prior to an "event" as the *sum* of the value of the asset if the event was never to occur, that is, the asymptotic value of the asset, *plus* the product of (i) the change in asset value, relative to its asymptotic value, that occurs when the "event" occurs and (ii) a probability-weighted discount factor for this "event." This is close to the Black and Cox (1976) decomposition of corporate asset values, and is precisely the method employed to write Equations (2), (6), and (7). The pricing exercise is formulated as a stochastic stopping time problem for a *given* regime switch trigger level.[16]

Second, considering *all* possible trigger levels as a candidate solution, we *optimize* over these potential triggers to reflect the choice of the decision maker. This is in the spirit of optimization methods employed in the real-options literature.[17] Obtaining the *optimal* trigger level involves solving the relevant first-order optimality condition, optimizing the decision-maker's claim value over the trigger level he controls. This is precisely how Equations (4) and (8) characterize the ex ante optimal closure level, $\underline{x}$, and the debtors' ex post optimal default trigger level, $\underline{x}_{\mathcal{D}}$.

[16] There is extensive literature dealing with the first time a diffusion process reaches a level. Froot and Obstfeld (1991) and Smith (1991) give closed-form solutions for some applications in economics.

[17] This refers to the literature on optimal decisions under uncertainty started by Dumas (1988), Pindyck (1988), Bertola (1989), and Dixit (1989). Dixit (1991) and Dumas (1991) discuss the "high order" or "smooth pasting" boundary conditions resulting from the optimal regulation of a Brownian motion.

3.3 Structures that yield closed-form solutions

Clearly a desirable feature of any pricing model is to yield closed-form solutions. This requires additional structure. The parametrization of the firm $\{x; \Pi(x); \Pi^*(x)\}$ and the uncertainty in the environment $\{\mu(x); \sigma(x); \rho\}$ must be selected to perform the following:

First, the description of the environment, $\{\mu(x); \sigma(x); \rho\}$, should enable us to express the Laplace transform, $\mathcal{P}(x_t \rhd y)$, introduced in Equation (3). Depending on the specific type of diffusion process we assume to be driving the uncertainty, the probability density function, $f_t(T_y)$, of the random variable T_y yields more or less complex expressions. However, the solution for most commonly assumed diffusion processes such as arithmetic or geometric Brownian motion is simple [see Karlin and Taylor (1975)].

Second, the formulation of the asymptotic value of the firm's alternative cash flows, $\Pi(x)$ and $\Pi^*(x)$, as well as the collateral agreed in the debt contact, $\underline{C}(x)$, should enable us to solve explicitly for the different trigger levels using the relevant first-order optimality conditions.

These are the only basic calculations we need to be able to carry out in order to obtain closed solutions throughout the model. There are several model specifications for that this remains a fairly simple task. The following is an example structure that is easy to implement. Actually the setup of most previously cited corporate debt valuation models can be nested by this structure:

Structure 1. *The uncertain state variable, x_t, describing the current status of the firm follows a geometric Brownian motion,*

$$dx_t = \mu\, x_t\, dt + \sigma\, x_t\, dB_t, \tag{11}$$

where $\mu < \rho$ and σ are constants, and B_t is a standard Brownian motion. Both the unlimited liability value of a perpetual claim on the income flow from operations, $\Pi(x)$, and the price competitors are willing to pay for the assets of the firm, $\Pi^*(x)$, are linear in x. That is, there exists four constants Θ_0, Θ_1, Θ_0^*, and Θ_1^*, where $\Theta_0 > \Theta_0^*$ and $\Theta_1 < \Theta_1^*$, such that

$$\Pi(x) = \Theta_0 + \Theta_1\, x \qquad \textit{and} \qquad \Pi^*(x) = \Theta_0^* + \Theta_1^*\, x. \tag{12}$$

Debt contracts are agreed upon under the absolute priority rule (APR): In the event of repudiation, creditors are to be paid first out of the proceeds of a liquidation sale, up to a par value, P, and therefore $\underline{C}(x) = \Pi^*(x) \wedge P$.

Of interest, notice that by simply requiring $\Theta_0 > \Theta_0^*$ and $\Theta_1 < \Theta_1^*$, Assumption 1 is satisfied. Furthermore, when debt contracts are written applying the APR, Assumption 2 is satisfied. This should illustrate the fact that Assumptions 1 and 2 are reasonable and not very restrictive. With the state variable following a geometric Brownian motion, the expression of

$\mathcal{P}(x \rhd y)$ becomes

$$\mathcal{P}(x \rhd y) = \left(\frac{x}{y}\right)^{\lambda},$$

where $\lambda \equiv \sigma^{-2}[-(\mu-\sigma^2/2)-((\mu-\sigma^2/2)^2+2\rho\sigma^2)^{1/2}]$. This structure is particularly attractive because not only all asset values but also *all* decision trigger levels and threshold levels have very simple closed-form solutions: The value of the firm under the first-best closure policy is

$$V(x_t \mid \underline{x}) \;=\; \Theta_0 + \Theta_1 x_t + \left[\Theta_0^* - \Theta_0 + (\Theta_1^* - \Theta_1)\underline{x}\right]\left(\frac{x_t}{\underline{x}}\right)^{\lambda}. \quad (13)$$

Similarly, if contracts are not renegotiable, the values debtors and creditors claim are, respectively,

$$D(x_t \mid \underline{x}_{\mathcal{D}}) \;=\; \Theta_0 + \Theta_1 x_t - \frac{\delta}{\rho} + \left[-\Theta_0 - \Theta_1 \underline{x}_{\mathcal{D}} + \frac{\delta}{\rho}\right]\left(\frac{x_t}{\underline{x}_{\mathcal{D}}}\right)^{\lambda}, \quad (14)$$

$$C(x_t \mid \underline{x}_{\mathcal{D}}) \;=\; \frac{\delta}{\rho} + \left[\Theta_0^* + \Theta_1^* \underline{x}_{\mathcal{D}} - \frac{\delta}{\rho}\right]\left(\frac{x_t}{\underline{x}_{\mathcal{D}}}\right)^{\lambda}. \quad (15)$$

The ex ante optimal liquidation trigger level, $\underline{x}$, and the debtors' ex post optimal default trigger level in the absence of renegotiation, $\underline{x}_{\mathcal{D}}$, are

$$\underline{x} = \frac{-\lambda}{1-\lambda}\left(\frac{\Theta_0^* - \Theta_0}{\Theta_1 - \Theta_1^*}\right) \text{ and } \underline{x}_{\mathcal{D}} = \frac{-\lambda}{1-\lambda}\left(\frac{\delta/\rho - \Theta_0}{\Theta_1}\right). \quad (16)$$

Furthermore, the threshold level of debt obligations introduced in Lemma 1, $\tilde{\delta}$, which corresponds to the unique level of leverage such that $\underline{x}_{\mathcal{D}}$ equals $\underline{x}$ is

$$\tilde{\delta} \;=\; \rho\,\frac{\Theta_0^*\,\Theta_1 - \Theta_0\,\Theta_1^*}{\Theta_1 - \Theta_1^*}. \quad (17)$$

4. Creditors' Self-Imposed Concessions

In Section 2 we introduced two dimensions to the problem of debt restructuring: First, the amount borrowed determines whether concessions are intended to "defer" or "induce" liquidation. Second, the relative bargaining power between debtors and creditors determines whether the concessions will be "self-imposed" or "forced." This leads to four cases.

In this section we first consider cases where creditors make "self-imposed" concessions where debtors are not in a position to be very active in renegotiations. In Section 5 we will examine the opposite situation, where debtors opportunistically extract the largest possible concessions through strategic default, so creditors' concessions are "forced."

We analyze the moral hazard problem working backwards in time: Here we examine the ex post behavior once the debt is issued, hence for a given contract coupon, δ, and collateral, $\underline{C}(x)$. In Section 6 we look at the ex ante implications of debtors ex post residual control rights and derive the ex ante endogenous limits to creditors willingness to lend. This will provide a characterization of the set of contracts acceptable at the date of issue, t_0.

4.1 Deferring concessions: Debt forgiveness

We start considering large amounts of outstanding debt where, in the absence of renegotiation, debtors default inefficiently "early." More precisely, $\delta > \tilde{\delta}$ which leads to $\underline{x}_{\mathcal{D}} > \underline{x}$, as in Figure 2. Here creditors have a collective interest in unilaterally reducing their own cash flow claims prior to liquidation. Reorganization offers consist of "deferring" net transfers from creditors to debtors. The contractual concession is aimed at providing an incentive for the debtors to prefer servicing the revised debt obligation instead of staying in default.

We consider that to exit a situation of default it is necessary to propose a new debt contract to investors. Formal contracts are needed for the implementation of agreements to be credible. The alterations are therefore not temporary. Now, after a new contract is established, the problem may arise again. Asset-holders may have similar reasons to reorganize this new contract later on. Accordingly, an essential structural feature of a dynamic model is to allow for an unlimited sequence of reorganizations.

If creditors perfectly control the renegotiation process, that is, only make "self-imposed" concessions, they will only propose these concessions as late as possible. If renegotiation is perfect and costless, they will offer a *marginal* deferring concession just as debtors default on the existing contract. The timing of the whole sequence of marginal concessions is solely designed to provide debtors with the minimum incentive to stay in operations. The optimal reorganization process therefore involves agreements to *marginally* write down the contractual coupon by $d\delta$. Debt is therefore an entitlement to an income flow, given the current level of the coupon, *plus* a continuum of options to *either* (i) lower the coupon level by incremental amounts as the output price changes, or (ii) collect the residual value of the firm, when it becomes optimal to stop deferring liquidation.

Once the state variable falls below the first reorganization trigger level, $x_{\mathcal{C}}^{f}$, further debt forgiveness options will be exercised by creditors as x_t reaches new historical lows. At each reorganization time, the creditors' optimal reduction leaves their debtors marginally preferring continuation to default. In this way the entire renegotiation surplus is appropriated by the creditors, who avoid a costly liquidation. Accordingly, the concessions control variable is only reduced when x_t hits a new minimum level, $\check{x}_t$, where

$$\check{x}_t \equiv \inf_{0 \le \kappa \le t} \{x_\kappa\}. \tag{18}$$

However, creditors are only interested in making such concessions as long as they gain from avoiding "early" liquidations. Therefore this process of successive concessions stops when the coupon is reduced to $\tilde{\delta}$, which is the coupon corresponding to the unique amount of outstanding debt for which the contract is optimal, even in the absence of renegotiation.

When a new creditors' concession occurs, it leaves the equity worth just slightly more than $\Pi^*(x) - \underline{C}(x)$ (which is equal to zero when the initial contract involves the APR). Consequently, at the times of restructuring, the creditors' claim is worth the complement, which is $V(x \mid \underline{x}) - \Pi^*(x) + \underline{C}(x)$ (which is equal to $V(x \mid \underline{x})$ when the contract respects the APR). Given that we know the value of both assets at the times of reorganization, we directly derive the valuation formulas prior to the next reorganization, using the pricing technique already employed.

We provide a formal exposition of the argument above in the Appendix. We write the two conflicting parties optimization problems: The follower debtors' optimization problem, which takes the leader creditors' strategy as given, is nested in the leader creditors' optimization problem. We derive the resulting sequence of events with the associated evolution of the coupon. Assumptions 1 and 2, stationarity of the process x_t, and time homogeneity of the setup are sufficient ingredients for the proof. The results are expressed in the following proposition:

Proposition 1. *Consider, under Assumptions 1 and 2, the case where (i) the level of borrowings is large enough for δ to be greater than $\tilde{\delta}$, and (ii) creditors make purely "self-imposed" concessions. Outside of reorganizations debtors and creditors hold claims respectively worth*

$$\check{D}_{\mathcal{C}}(x_t, \check{x}_t) = \Pi(x_t) - \frac{\delta_{\mathcal{C}}(\check{x}_t)}{\rho} + \left[\Pi^*(x_{\mathcal{C}}) - \underline{C}(x_{\mathcal{C}}) - \Pi(x_{\mathcal{C}}) + \frac{\delta_{\mathcal{C}}(\check{x}_t)}{\rho}\right] \mathcal{P}(x_t \rhd x_{\mathcal{C}}) \tag{19}$$

$$\check{C}_{\mathcal{C}}(x_t, \check{x}_t) = \frac{\delta_{\mathcal{C}}(\check{x}_t)}{\rho} + \left[V(x_{\mathcal{C}} \mid \underline{x}) - \Pi^*(x_{\mathcal{C}}) + \underline{C}(x_{\mathcal{C}}) - \frac{\delta_{\mathcal{C}}(\check{x}_t)}{\rho}\right] \mathcal{P}(x_t \rhd x_{\mathcal{C}}). \tag{20}$$

The next default occurs the first time x_t hits $x_{\mathcal{C}} \equiv x_{\mathcal{C}}^f \wedge [\check{x}_t \vee \underline{x}]$, where $T_{x_{\mathcal{C}}} \equiv \inf\{T \mid x_T = x_{\mathcal{C}}\}$. When $x_t = x_{\mathcal{C}}$, creditors make defering concessions, and debtors and creditors hold claims respectively worth $\Pi^*(x_t) - \underline{C}(x_t)$ and $V(x_t \mid \underline{x}) - \Pi^*(x_t) + \underline{C}(x_t)$.

At any time, the prevailing coupon is $\delta_{\mathcal{C}}(\check{x}_t) = \delta \wedge [\overline{\delta}_{\mathcal{C}}(\check{x}_t) \vee \tilde{\delta}]$, where $\overline{\delta}_{\mathcal{C}}(x)$ denotes the maximum coupon acceptable to debtors, that is, the highest instantaneous debt obligation that does not drive them to pursue in default. This function solves the optimality condition

$$\frac{\partial \check{D}_{\mathcal{C}}(x_{\mathcal{C}}, x_{\mathcal{C}})}{\partial x_{\mathcal{C}}} = \frac{\partial \Pi^*(x_{\mathcal{C}})}{\partial x_{\mathcal{C}}} - \frac{\partial \underline{C}(x_{\mathcal{C}})}{\partial x_{\mathcal{C}}}. \tag{21}$$

The first reorganization trigger level $x_{\mathcal{C}}^f$ solves $\overline{\delta}_{\mathcal{C}}(x_{\mathcal{C}}^f) = \delta$. The liquidation trigger level is the first-best closure point $\underline{x}$.

Pricing using first hitting time methods proves to be particularly advantageous here. This procedure preserves the structure of the pricing problem, even after some debt is forgiven. As a consequence, allowing for an unlimited sequence of reorganizations (i.e., an infinite series of embedded options) does not complicate much the valuation exercise. The expressions for $\check{D}_{\mathcal{C}}(.)$ and $\check{C}_{\mathcal{C}}(.)$ are derived directly, in a similar fashion to Equation (2).

In both expressions $\check{D}_{\mathcal{C}}(.)$ and $\check{C}_{\mathcal{C}}(.)$, the first term on the right-hand side ($\Pi(x) - \delta_{\mathcal{C}}(\check{x}_t)/\rho$ and $\delta_{\mathcal{C}}(\check{x}_t)/\rho$, for equity and debt, respectively) is the value of a perpetual entitlement on the asset's flow of income, under the prevailing agreement, and if there was no further reorganization of the debt. The second term is the product of (i) the change in asset value intervening when the next creditors' concession occurs, which is the expression in the square brackets, and (ii) the probability-weighted discount factor for this event, $\mathcal{P}(x_t \rhd x_{\mathcal{C}})$. In the context of the example in Structure 1, the following closed-form solutions are obtained:

Corollary 1 (Proposition 1 with Structure 1). *Consider (i) higher levels of borrowings, such that $\delta > \tilde{\delta}$, where the initial contract involves the APR, and (ii) creditors make purely "self-imposed" concessions. Outside of reorganizations, debtors and creditors hold claims respectively worth*

$$\check{D}_{\mathcal{C}}(x_t, \check{x}_t) = \Theta_0 + \Theta_1 x_t - \frac{\delta_{\mathcal{C}}(\check{x}_t)}{\rho} + \left[-\Theta_0 - \Theta_1 x_{\mathcal{C}} + \frac{\delta_{\mathcal{C}}(\check{x}_t)}{\rho}\right]\left(\frac{x_t}{x_{\mathcal{C}}}\right)^{\lambda}, \tag{22}$$

$$\check{C}_{\mathcal{C}}(x_t, \check{x}_t) = \frac{\delta_{\mathcal{C}}(\check{x}_t)}{\rho} + \left[\Theta_0 + \Theta_1 x_{\mathcal{C}} + \left[\Theta_0^* - \Theta_0 + (\Theta_1^* - \Theta_1)\underline{x}\right]\left(\frac{x_{\mathcal{C}}}{\underline{x}}\right)^{\lambda} - \frac{\delta_{\mathcal{C}}(\check{x}_t)}{\rho}\right]\left(\frac{x_t}{x_{\mathcal{C}}}\right)^{\lambda}. \tag{23}$$

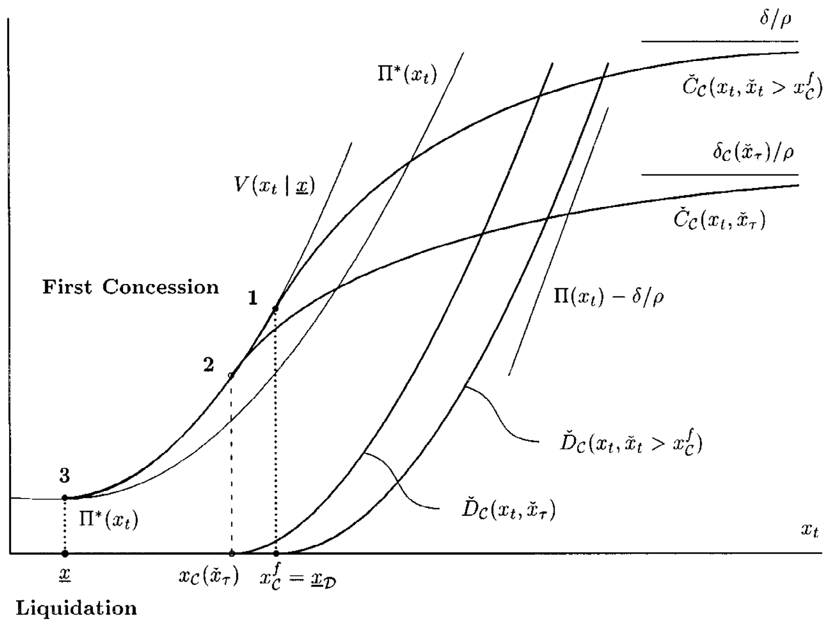

Figure 4
Creditors' self-imposed debt forgiveness
The initial coupon is δ, and $\check{D}_{\mathcal{C}}(x_t, \check{x}_t > x^f_{\mathcal{C}})$ and $\check{C}_{\mathcal{C}}(x_t, \check{x}_t > x^f_{\mathcal{C}})$ represent the values of equity and debt prior to any concession. Debtors' default and a first concession occur when the minimum level of x_t recorded, $\check{x}_t \equiv \inf_{0 \le \kappa \le t}\{x_\kappa\}$, reaches $x^f_{\mathcal{C}}$ (point **1**). After the first concession, debtors default again each time the state variable reaches a new minimum, and the coupon is successively marginally reduced to $\delta_{\mathcal{C}}(\check{x}_t)$. With each new contract asset values irreversibly shift to a new pair of curves $(\check{D}_{\mathcal{C}}; \check{C}_{\mathcal{C}})$. $\check{D}_{\mathcal{C}}(x_t, \check{x}_\tau)$ and $\check{C}_{\mathcal{C}}(x_t, \check{x}_\tau)$ illustrate an intermediate case where $\inf_{0 \le \kappa \le t}\{x_\kappa\} = \check{x}_\tau$ and $x^f_{\mathcal{C}} > \check{x}_\tau > \underline{x}$ (point **2**). This gradual and irreversible process ends when creditors prefer liquidation to further debt coupon forgiveness. This occurs when $\inf_{0 \le \kappa \le t}\{x_\kappa\} = \underline{x}$, the ex ante optimal liquidation trigger level (point **3**).

The next default occurs the first time x_t hits

$$x_{\mathcal{C}} \;\equiv\; \frac{-\lambda}{1-\lambda}\left(\frac{\delta/\rho - \Theta_0}{\Theta_1}\right) \wedge \left[\check{x}_t \vee \frac{-\lambda}{1-\lambda}\left(\frac{\Theta_0^* - \Theta_0}{\Theta_1 - \Theta_1^*}\right)\right]. \qquad (24)$$

When $x_t = x_{\mathcal{C}}$, creditors make defering concessions, and debtors and creditors hold claims respectively worth 0 *and $V(x_t \mid \underline{x})$. At any time the prevailing coupon is*

$$\delta_{\mathcal{C}}(\check{x}_t) \;=\; \delta \wedge \left[\rho\left(\Theta_0 + \Theta_1 \frac{1-\lambda}{-\lambda}\check{x}_t\right) \vee \rho\,\frac{\Theta_0^*\Theta_1 - \Theta_0\Theta_1^*}{\Theta_1 - \Theta_1^*}\right]. \quad (25)$$

Figure 4 illustrates the *path-dependent* nature of the pricing formulae. In the context of the example in Structure 1, the implied evolution of the

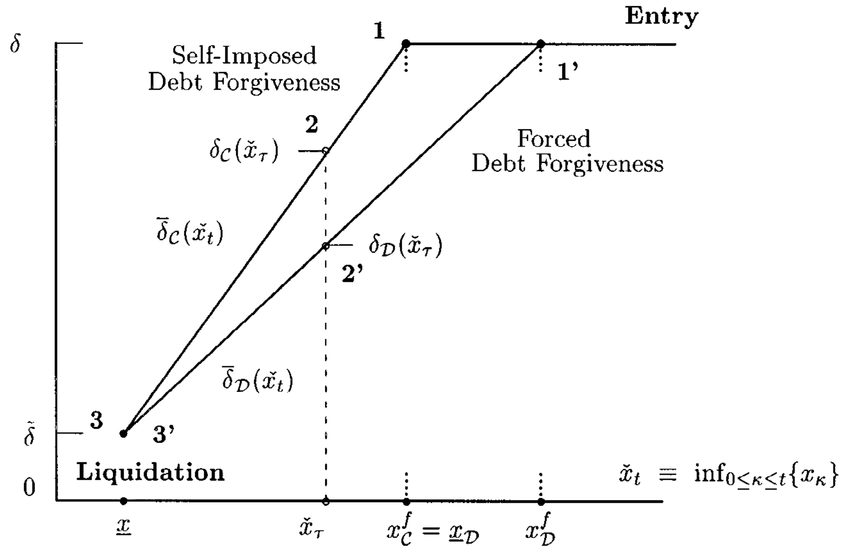

Figure 5
Debt forgiveness: evolution of contract coupons

coupon, $\delta_{\mathcal{C}}(\check{x}_t)$, is shown in Figure 5. Notice that under Structure 1, $\overline{\delta}_{\mathcal{C}}(\check{x}_t)$ is linear in $\check{x}_t$. In these figures, $\{x; \Pi(x); \Pi^*(x)\}$, $\{\mu(x); \sigma(x); \rho\}$, and $\{\delta; \underline{C}(x)\}$ correspond to the initial inefficient situation depicted in Figure 2. The figures gain in being compared.

The first reorganization actually occurs when the state variable reaches the trigger level of debtors' default, in the absence of concessions ($x^f_{\mathcal{C}} = \underline{x}_{\mathcal{D}}$). After the first renegotiation, the coupon is marginally reduced each time the state variable reaches a new minimum. $\delta_{\mathcal{C}}(\check{x}_t)$ equals $\overline{\delta}_{\mathcal{C}}(\check{x}_t)$, the maximum acceptable coupon the creditors can set without pushing debtors to persist in default, in other words the highest one debtors will cope with. When x_t increases, coupon reductions are unnecessary. Contracts remain unchanged until x_t reaches a new minimum level. This gradual and irreversible coupon reduction process ends when creditors prefer liquidation to further debt coupon forgiveness. This takes place when x_t first hits $\underline{x}$, the ex ante optimal liquidation trigger level, corresponding to $\overline{\delta}_{\mathcal{C}}(\underline{x})$ equal to $\tilde{\delta}$.

The magnitude of creditors' willingness to restructure the contract can be visualized in Figure 6, which is just a three-dimensional version of Figure 4. Plane P_{x_τ} shows (i) that it is in creditors interest to reorganize the debt before liquidation, and (ii) that the prevailing coupon $\delta_{\mathcal{C}}(\check{x}_\tau)$ is their optimal choice. The curve $L_{\mathcal{C}}(\delta \mid \check{x}_\tau)$ plots, for a given state of the world, the debt value as a function of possible coupons (contractual debt obligations). To use a similar terminology from Krugman (1989), $L_{\mathcal{C}}(\delta \mid \check{x}_\tau)$ is a corporate finance version of the "debt-relief Laffer curve".[18]

A smaller coupon reduces the likelihood of default because debtors' financial burden is smaller. However, it also reduces the asymptotic value of the debt, which is the value of a riskless debt with a similar coupon. When $\delta > \tilde{\delta}$, lowering the coupon prior to triggering liquidation is always advantageous. Reducing the likelihood of liquidation increases the market value of the debt more than the prospect of a lower future income stream decreases it. Whereas for high levels of x_t, creditors prefer a higher coupon, a smaller contractual coupon eventually yields a higher market value. The value of the creditors' option on the stream of future payments with a lower coupon outside liquidation becomes greater than the value of their option on the residual value of the firm in liquidation.

Lemma 2. *When there exists an incentive to defer liquidation, that is,* $\delta > \tilde{\delta}$, *the corporate debt-relief Laffer curves,* $L_{\mathcal{C}}(\delta \mid x)$, *are negatively sloped in some range.*

4.2 Inducing concessions: Departures from the absolute priority rule

Let us now consider the opposite situation where, in the absence of renegotiation, debtors' default would lead to inefficiently "late" liquidations. Such situations arise for low levels of leverage, when $\delta < \tilde{\delta}$, which leads to $\underline{x}_{\mathcal{D}} < \underline{x}$ as in Figure 3. Reorganization offers consist of "inducing" net transfers from creditors to debtors. The contractual concession is now aimed at providing an incentive for the debtors to stop withholding liquidation to keep servicing the debt. The rationale is therefore the same as the one leading to debt forgiveness.

The problem of timing is different, however, and the mechanism to solve for it is simpler. Whereas the immediate possibility of an inefficiently "early" default led creditors to *gradually* write down immediate cash-flow claims such as their coupon, here the future prospect of a "late" default induces creditors to propose a *single* concession which is only applicable if liquidation is triggered. This sort of concession is in essence a proposal to depart from the APR.

If an agreement to declare liquidation at the ex ante optimal trigger level $\underline{x}$ can be reached, the surplus to be internalized equals $\Pi^*(\underline{x}) - [D(\underline{x} \mid \underline{x}_{\mathcal{D}}) + C(\underline{x} \mid \underline{x}_{\mathcal{D}})]$. As creditors have most of the bargaining power in renegotiations, they will offer a departure from the APR which makes debtors

[18] "Debt-relief Laffer curves" have been used to argue that creditors should forgive part of the external debt of heavily indebted countries and that, in doing so, they will serve both their interests and those of the borrowers. Leland (1994) examined the comparative statics of corporate debt value with respect to different coupon levels under different assumptions about what triggers bankruptcy. His model does not allow for debt renegotiation, but when bankruptcy is imminent, he obtains greater values of both debt and equity with smaller coupons.

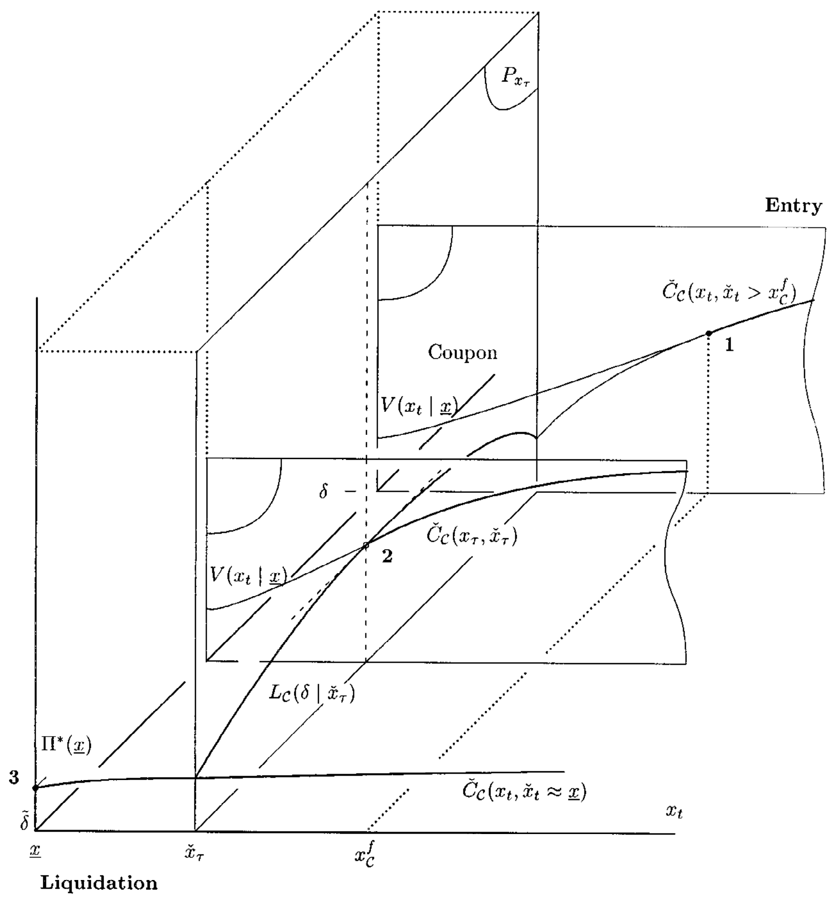

Figure 6
Corporate debt-relief Laffer curves
The center plane shows the debt value $\check{C}_{\mathcal{C}}(x_\tau, \check{x}_\tau)$, the asymptotic value of the debt $\delta_{\mathcal{C}}(\check{x}_\tau)/r$, and the firm's economic worth, $V(x_t \mid \underline{x})$. In the bottom plane, the extension of the curve $\check{C}_{\mathcal{C}}(x_t, \check{x}_t > x^f_{\mathcal{C}})$ shows the value of the debt if the coupon had not been reduced to $\delta_{\mathcal{C}}(\check{x}_\tau)$, between date $t = 0$ and $t = \tau$. For high output prices, creditors prefer a higher coupon. But for prices below x_τ, a smaller contractual coupon yields a higher market value. In plane P_{x_τ}, the corporate debt-relief Laffer curve $L_{\mathcal{C}}(\delta \mid \check{x}_\tau)$ shows the value of the debt value for different possible coupons. The maximum level corresponds to the prevailing coupon $\delta_{\mathcal{C}}(\check{x}_\tau)$.

slightly better off defaulting instead of continuing to service the debt. Given that in the latter case debtors' claim is then worth $D(\underline{x} \mid \underline{x}_{\mathcal{D}})$, creditors' optimal strategy is as proved in the Appendix.

Proposition 2. *Consider, under Assumptions 1 and 2, the case where (i) the level of borrowings is small enough for δ to be smaller than $\tilde{\delta}$, and (ii) creditors make purely "self-imposed" concessions. Before x_t reaches*

$\underline{x}$, anticipating the future concession, debtors and creditors hold claims respectively worth

$$\hat{D}_{\mathcal{C}}(x_t) = \Pi(x_t) - \frac{\delta}{\rho} + \left[\Delta_{\mathcal{C}} - \Pi(\underline{x}) + \frac{\delta}{\rho}\right]\mathcal{P}(x_t \; \triangleright \underline{x}), \tag{26}$$

$$\hat{C}_{\mathcal{C}}(x_t) = \frac{\delta}{\rho} + \left[\Pi^*(\underline{x}) - \Delta_{\mathcal{C}} - \frac{\delta}{\rho}\right]\mathcal{P}(x_t \; \triangleright \underline{x}). \tag{27}$$

When x_t reaches $\underline{x}$, creditors propose an inducing concession to transfer a sum $\Delta_{\mathcal{C}}$ equal to $D(\underline{x} \mid \underline{x}_{\mathcal{D}})$ to the debtors, if they accept to liquidate the firm. For the promise to be credible, both parties agree upon a departure from the APR, and liquidation occurs immediately.

If there is no departure from the APR stipulated in the initial debt contract, the departure from the APR that creditors propose when x_t reaches $\underline{x}$ amounts to a fraction $\Delta_{\mathcal{C}}/\Pi^*(\underline{x})$ of the proceeds from liquidation. In the context of the example in Structure 1, the following closed-form solutions are obtained:

Corollary 2 (Proposition 2 with Structure 1). *Consider (i) lower levels of borrowings, such that $\delta < \tilde{\delta}$, where the initial contract involves the APR, and (ii) creditors make purely "self-imposed" concessions. Before x_t reaches $\underline{x}$, debtors and creditors hold claims respectively worth*

$$\hat{D}_{\mathcal{C}}(x_t) = \Theta_0 + \Theta_1 x_t - \frac{\delta}{\rho} + \left[\Delta_{\mathcal{C}} - \Theta_0 - \Theta_1 \underline{x} + \frac{\delta}{\rho}\right]\left(\frac{x_t}{\underline{x}}\right)^{\lambda}, \tag{28}$$

$$\hat{C}_{\mathcal{C}}(x_t) = \frac{\delta}{\rho} + \left[\Theta_0^* + \Theta_1^* \underline{x} - \Delta_{\mathcal{C}} - \frac{\delta}{\rho}\right]\left(\frac{x_t}{\underline{x}}\right)^{\lambda}. \tag{29}$$

When x_t reaches $\underline{x}$, the creditors propose to transfer to their debtors a sum

$$\Delta_{\mathcal{C}} = \Theta_0 + \Theta_1 \underline{x} - \frac{\delta}{\rho} + \left[-\Theta_0 - \Theta_1 \underline{x}_{\mathcal{D}} + \frac{\delta}{\rho}\right]\left(\frac{\underline{x}}{\underline{x}_{\mathcal{D}}}\right)^{\lambda} \tag{30}$$

out of the proceeds from a liquidation sale. Liquidation is immediately triggered.

5. Debtors' Opportunistic Offers: Forced Concessions

In the previous section debtors were simply in a position to passively accept the offers because they had no bargaining power in reorganization. That is, creditors were not credibly threatened to bear the consequences of an inefficient liquidation. Let us now examine the mirror game in which debtors have most of the bargaining power. In the resulting Stackelberg equilibrium, the follower creditors react optimally, taking the leader debtors' strategy as given. Debtors are in a position to blackmail their creditors.

5.1 Deferring concessions: Debt forgiveness

On the one hand, if creditors stand to loose from an "early" liquidation, debtors can default strategically very early on. Debtors make what is de facto a "take-it-or-leave-it" demand to have the contracted coupon reduced. Creditors can either (i) accept the reduction, in which case a new contract is written or (ii) reject the offer and trigger liquidation. When debtors perfectly control this renegotiation process, they gradually request marginal deferring concessions, as soon as creditors are ready to make it. These marginal options are therefore exercised as soon as they become acceptable to creditors, that is, when making such concessions do not leave creditors worse off than if they triggered an inefficient liquidation.

Proposition 3. *Consider, under Assumptions 1 and 2, (i) higher levels of borrowings, such that $\delta > \tilde{\delta}$, where the initial contract involves the APR, and (ii) creditors' concessions are "forced." Outside of reorganizations, debtors and creditors hold claims respectively worth*

$$\check{D}_{\mathcal{D}}(x_t, \check{x}_t) = \Pi(x_t) - \frac{\delta_{\mathcal{D}}(\check{x}_t)}{\rho} + \Bigg[V(x_{\mathcal{D}} \mid \underline{x}) - \underline{C}(x_{\mathcal{D}}) - \Pi(x_{\mathcal{D}}) + \frac{\delta_{\mathcal{D}}(\check{x}_t)}{\rho} \Bigg] \mathcal{P}(x_t \rhd x_{\mathcal{D}}) \quad (31)$$

$$\check{C}_{\mathcal{D}}(x_t, \check{x}_t) = \frac{\delta_{\mathcal{D}}}{\rho} + \left[\underline{C}(x_{\mathcal{D}}) - \frac{\delta_{\mathcal{D}}(\check{x}_t)}{\rho} \right] \mathcal{P}(x_t \rhd x_{\mathcal{D}}). \quad (32)$$

Repeatedly debtors opportunistically default in order to obtain gradual writedowns of their debt obligations. The next default always occurs the first time x_t hits $x_{\mathcal{D}} \equiv x^f_{\mathcal{D}} \wedge [\check{x}_t \vee \underline{x}]$, where $T_{x_{\mathcal{D}}} \equiv \inf\{T \mid x_T = x_{\mathcal{D}}\}$. When $x_t = x_{\mathcal{D}}$, the debt is being reorganized, and debtors and creditors hold claims respectively worth $V(x_t \mid \underline{x}) - \underline{C}(x_t)$ and $\underline{C}(x_t)$.

At any time, the prevailing coupon is $\delta_{\mathcal{D}}(\check{x}_t) = \delta \wedge [\overline{\delta}_{\mathcal{D}}(\check{x}_t) \vee \tilde{\delta}]$, where $\overline{\delta}_C(x)$ denotes denotes the minimum coupon acceptable to the creditors, that is, the lowest one they prefer to liquidate. This function solves the optimality condition

$$\frac{\partial \check{C}_{\mathcal{D}}(x_{\mathcal{D}}, x_{\mathcal{D}})}{\partial x_{\mathcal{D}}} = \frac{\partial \underline{C}(x_{\mathcal{D}})}{\partial x_{\mathcal{D}}}. \quad (33)$$

The first reorganization trigger level x^f_C solves $\overline{\delta}_{\mathcal{D}}(x^f_{\mathcal{D}}) = \delta$. The liquidation trigger level is the first-best closure point $\underline{x}$.

The proof of this proposition is similar to that of Proposition 1. The formal expression of the two conflicting parties optimization problem is just the mirror one. We now provide the closed-form solutions that are obtained in the context of the example in Structure 1.

Corollary 3 (Proposition 3 with Structure 1). *Consider (i) higher levels of borrowings, such that $\delta > \tilde{\delta}$, where the initial contract involves the APR, and (ii) creditors' concessions are "forced." Outside of renegotiations, debtors and creditors hold claims respectively worth*

$$\check{D}_{\mathcal{D}}(x_t, \check{x}_t) \;=\; \Theta_0 + \Theta_1\, x_t - \frac{\delta_{\mathcal{D}}(\check{x}_t)}{\rho} + \left[-\Theta_0^* - \Theta_1^*\, x_{\mathcal{D}} + \left[\Theta_0^* - \Theta_0 + (\Theta_1^* - \Theta_1)\, \underline{x}\right] \left(\frac{x_{\mathcal{D}}}{\underline{x}}\right)^{\lambda} - \frac{\delta_{\mathcal{D}}(\check{x}_t)}{\rho} \right] \left(\frac{x_t}{x_{\mathcal{D}}}\right)^{\lambda}, \tag{34}$$

$$\check{C}_{\mathcal{D}}(x_t, \check{x}_t) = \frac{\delta_{\mathcal{D}}(\check{x}_t)}{\rho} + \left[\Theta_0^* + \Theta_1^*\, x_{\mathcal{D}} - \frac{\delta_{\mathcal{D}}(\check{x}_t)}{\rho}\right] \left(\frac{x_t}{x_{\mathcal{D}}}\right)^{\lambda} \tag{35}$$

The next default occurs the first time x_t hits

$$x_{\mathcal{D}} \;\equiv\; \frac{-\lambda}{1-\lambda} \left(\frac{\delta/\rho - \Theta_0^*}{\Theta_1^*}\right) \wedge \left[\check{x}_t \vee \frac{-\lambda}{1-\lambda} \left(\frac{\Theta_0^* - \Theta_0}{\Theta_1 - \Theta_1^*}\right) \right]. \tag{36}$$

When $x_t = x_{\mathcal{D}}$, the debt is being reorganized, and debtors and creditors hold claims respectively worth $V(x_t \mid \underline{x}) - \Pi^*(x_t)$ and $\Pi^*(x_t)$. At any time, the prevailing coupon is

$$\delta_{\mathcal{D}}(\check{x}_t) \;=\; \delta \wedge \left[\rho \left(\Theta_0^* + \Theta_1^* \frac{1-\lambda}{-\lambda}\, \check{x}_t \right) \vee \rho\, \frac{\Theta_0^*\, \Theta_1 - \Theta_0\, \Theta_1^*}{\Theta_1 - \Theta_1^*} \right]. \tag{37}$$

Figure 7 illustrates this alternative Stackelberg equilibrium. In Figure 5, the associated evolution of the coupon, $\delta_{\mathcal{D}}(\check{x}_t)$, can be compared against that in the case of self-imposed debt forgiveness. Whereas $\overline{\delta}_{\mathcal{C}}(x)$ is the highest debt burden debtors can accept, $\overline{\delta}_{\mathcal{D}}(x)$ is the lowest interest payment creditors will accept. Notice that $x_{\mathcal{D}}^f > x_{\mathcal{C}}^f$, and that the last prevailing coupon, $\overline{\delta}_{\mathcal{D}}(\underline{x})$, equals $\tilde{\delta}$.

5.2 Inducing concessions: Departures from the absolute priority rule

On the other hand, if creditors stand to loose from a "late" liquidation, the debtors can threaten to keep on servicing the debt. Here debtors demand a much larger departure from the APR to trigger liquidation than creditors would to self-impose. They make a "take-it-or-leave-it" offer that leaves

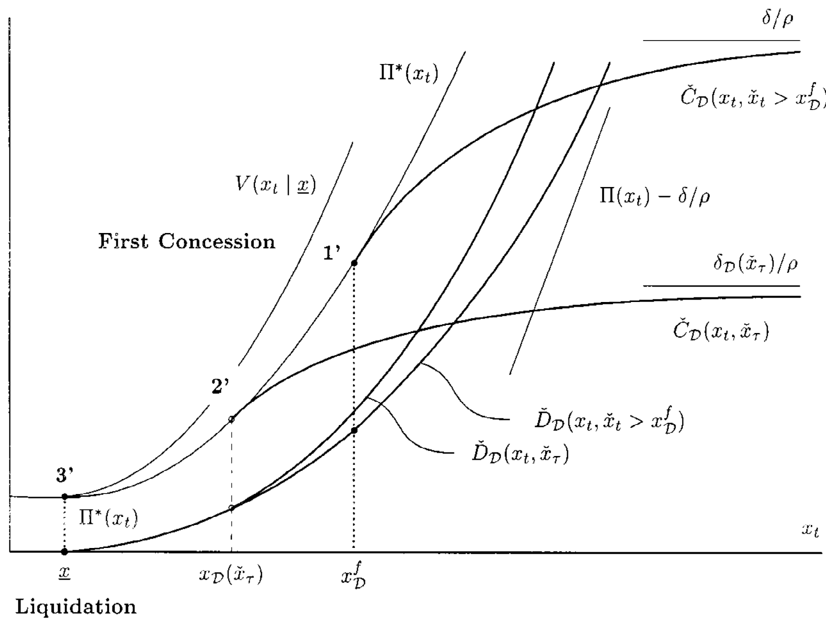

Figure 7
Forced debt forgiveness
The initial coupon is δ, and $\check{D}_{\mathcal{D}}(x_t, \check{x}_t > x^f_{\mathcal{D}})$ represents the values of equity and debt prior to any concession. Debtors default and force a first concession as the minimum level of x_t recorded, $\check{x}_t \equiv \inf_{0 \le \kappa \le t}\{x_\kappa\}$, reaches $x^f_{\mathcal{D}}$ (point **1**). They successively obtain smaller coupons $\delta_{\mathcal{D}}(\check{x}_t)$ each time the state variable reaches a new minimum, and asset values gradually shift to new pairs of curves $(\check{D}_{\mathcal{D}}; \check{C}_{\mathcal{D}})$. $\check{D}_{\mathcal{D}}(x_t, \check{x}_\tau)$ and $\check{C}_{\mathcal{D}}(x_t, \check{x}_\tau)$ illustrate an intermediate case, where $\inf_{0 \le \kappa \le t}\{x_\kappa\} = \check{x}_\tau$ and $x^f_{\mathcal{D}} > \check{x}_\tau > \underline{x}$ (point **2**). Creditors' willingness to postpone liquidation ceases when $\inf_{0 \le \kappa \le t}\{x_\kappa\} = \underline{x}$, the ex ante optimal liquidation trigger level (point **3**).

creditors marginally better off accepting than reject it, knowing that in the latter case creditors have $C(\underline{x} \mid \underline{x}_{\mathcal{D}})$. In doing so debtors internalize to their own profit the renegotiation surplus $\Pi^*(\underline{x}) - [D(\underline{x} \mid \underline{x}_{\mathcal{D}}) + C(\underline{x} \mid \underline{x}_{\mathcal{D}})]$.

Proposition 4. *Consider, under Assumptions 1 and 2, the case where (i) the level of borrowings is small enough for δ to be smaller than $\tilde{\delta}$, and (ii) creditor concessions are "forced." Before x_t reaches $\underline{x}$, anticipating the future concession, debtors and creditors hold claims respectively worth*

$$\hat{D}_{\mathcal{D}}(x_t) = \Pi(x_t) - \frac{\delta}{\rho} + \left[\Delta_{\mathcal{D}} - \Pi(\underline{x}) + \frac{\delta}{\rho}\right] \mathcal{P}(x_t \vartriangleright \underline{x}), \qquad (38)$$

$$\hat{C}_{\mathcal{D}}(x_t) = \frac{\delta}{\rho} + \left[\Pi^*(\underline{x}) - \Delta_{\mathcal{D}} - \frac{\delta}{\rho}\right] \mathcal{P}(x_t \vartriangleright \underline{x}). \qquad (39)$$

When x_t reaches $\underline{x}$, debtors opportunistically request a payment for accepting to trigger liquidation. Creditors are forced to grant an inducing *concession to transfer a sum $\Delta_{\mathcal{D}}$ equal to $\Pi^*(\underline{x}) - C(\underline{x} \mid \underline{x}_{\mathcal{D}})$ to the debtors for them to liquidate the firm. For the promise to be credible, both parties agree upon a departure from the APR, and liquidation occurs immediately.*

If there is no initial departure from the APR, the final forced departure from the APR will now equal a fraction $\Delta_{\mathcal{D}}/\Pi^*(\underline{x})$ of the residual value of the firm. Reassuringly, $\Delta_{\mathcal{D}} - \Delta_{\mathcal{C}} = \Pi^*(\underline{x}) - [D(\underline{x} \mid \underline{x}_{\mathcal{D}}) + C(\underline{x} \mid \underline{x}_{\mathcal{D}})]$, which is strictly positive, otherwise $\delta \not< \tilde{\delta}$. As expected, if debtors have the bargaining power, the concession they obtain is larger than the one creditors would self-impose if the latter had the bargaining power instead. In the context of the example in Structure 1 we obtain the following expressions.

Corollary 4 (Proposition 4 with Structure 1). *Consider (i) lower levels of borrowings, such that $\delta < \tilde{\delta}$, where the initial contract involves the APR, and (ii) creditors concessions are "forced." Before x_t reaches $\underline{x}$, debtors and creditors hold claims respectively worth*

$$\hat{D}_{\mathcal{D}}(x_t) = \Theta_0 + \Theta_1 x_t - \frac{\delta}{\rho} + \left[\Delta_{\mathcal{D}} - \Theta_0 - \Theta_1 \underline{x} + \frac{\delta}{\rho}\right]\left(\frac{x_t}{\underline{x}}\right)^{\lambda}, \tag{40}$$

$$\hat{C}_{\mathcal{D}}(x_t) = \frac{\delta}{\rho} + \left[\Theta_0^* + \Theta_1^* \underline{x} - \Delta_{\mathcal{D}} - \frac{\delta}{\rho}\right]\left(\frac{x_t}{\underline{x}}\right)^{\lambda}. \tag{41}$$

When x_t reaches $\underline{x}$, the creditors agree to give to their debtors a sum

$$\Delta_{\mathcal{D}} = \Theta_0^* + \Theta_1^* \underline{x} - \frac{\delta}{\rho} - \left[\Theta_0^* + \Theta_1^* \underline{x}_{\mathcal{D}} - \frac{\delta}{\rho}\right]\left(\frac{\underline{x}}{\underline{x}_{\mathcal{D}}}\right)^{\lambda} \tag{42}$$

out of the proceeds from a liquidation sale. Liquidation is immediately triggered.

6. Issuing Debt and Numerical Results

6.1 Debt capacity

In the two previous sections we considered the evolution of the firm's capital structure for a *given* initial debt contract. That is, we examined the *ex post* optimal behavior of debtors and creditors, assuming that the terms of the debt contract initially issued at the date of entry t_0 (coupon δ_0, residual claim $\underline{C}(x)$) are within an "acceptable" range. Proceeding backwards in time, let us now consider the *ex ante* implications of the moral hazard problem we have analyzed, characterizing the set of debt contracts that can be initially issued.

Clearly, creditors will not lend unlimited amounts, and the set of possible initial terms of the contract is bounded by creditors' ex ante willingness to lend. This endogenous absolute limit to the amount debtors are able to borrow, referred to as the *debt capacity* of the firm, determines the limits of this set. For any given firm, economic environment, and counterparties' relative bargaining power, the debt capacity can be seen as the envelope of possible debt value functions.

On the one hand, if creditors only make "self-imposed" concessions, hence control perfectly the renegotiation process, renegotiation occurs when the firm is at *full* debt capacity: Close examination of Proposition 1 reveals that for any state x_t, the debt value is maximal when $x_t = \check{x}_t$. This is easily visualized in Figures 4 and 7. For a given residual claim, $\underline{C}(x)$, the highest level of the debt value corresponds to the point where the trade-off between increasing the stream of coupon payment and postponing a costly liquidation is optimized. The associated coupon corresponds to the highest level of the debt Laffer curve, $L_{\mathcal{C}}(\delta \mid x_0)$. The debt capacity when creditors make "self-imposed" concessions therefore equals

$$\check{C}_{\mathcal{C}}(x, x) = V(x \mid \underline{x}). \tag{43}$$

For any given state of the world, the highest achievable debt value equals the ex ante optimal value of the firm, $V(x \mid \underline{x})$. When they perfectly control renegotiation, creditors have no reluctance to lend at all, and the firm could be 100% levered.

On the other hand, when debtors are in a strong position and will default opportunistically, creditors are not inclined to lend as much money at entry. Concessions occur earlier as they are "forced" by opportunistic debtors, rather than "self-imposed" by the creditors. Now, ex ante, creditors anticipate the implications of debtors' ex post opportunistic behavior. Therefore the maximum amount debtors can borrow is reduced. Creditors are aware of their weakness, and know that the value of the debt will not remain above the residual value of their claim, $\underline{C}(x)$, without debtors blackmailing them. Then the debt capacity when creditor concessions are "forced" equals

$$\check{C}_{\mathcal{D}}(x, x) = \underline{C}(x). \tag{44}$$

This can be seen in Proposition 3, and it is graphically exhibited in Figure 6.

To summarize, in this benchmark model there is no tax advantage of debt, and renegotiation is assumed perfect and costless, therefore a capital irrelevance result holds. However, this result only applies in a more or less limited range of levels of leverage. When creditors know they will be able to make self-imposed concessions, the debt capacity is fully equal to the first-best value of the firm. But this debt capacity is substantially reduced if the debtors are known to have a strong bargaining power in debt reorganizations. When it is clear that debtors are in a position to behave opportunistically, the initial amount they can borrow is limited to the residual value of the firm, if liquidation was triggered just after the firm is setup.

As far as P, the par value of the debt that is written in the contract, is concerned, it must be higher than the value of the debt at entry for the promise to service the debt to be *credible*. Otherwise debtors would immediately default, pay P to the creditors, and get hold of the difference. Creditors initial contribution would amount to a net transfer to the debtors. Actually, if the promise to pay a coupon δ is also to be credible, not only at entry, but also if the firm becomes more profitable, the contracted par value of the debt, P, must be greater than any possible future value of the debt. Therefore P must be greater than the riskless value of the coupon payments, δ/ρ.

Clearly this holds in the absence of growth opportunities, because debtors have no reason to prefer a "good" reputation which would enable them to borrow again from the same creditors.

6.2 Measuring the impact on value of relative bargaining power

Of interest, we can *quantitatively* assess whether debtors and creditors' relative bargaining power is an important factor in pricing. That is, for the first time we are equipped to measure the importance of renegotiation bargaining power in corporate debt pricing:

For higher levels of leverage, the bond's default risk premium with self-imposed concessions is $p_{\mathcal{D}}(x_t, \check{x}_t) \equiv \delta(\check{x}_t) - \rho\, \check{C}_{\mathcal{D}}(x_t, \check{x}_t)$. Similarly, with forced concessions it is $p_{\mathcal{C}}(x_t, \check{x}_t) \equiv \delta(\check{x}_t) - \rho\, \check{C}_{\mathcal{C}}(x_t, \check{x}_t)$. The relative difference between the risk premia in one extreme situation versus the other one, $[p_{\mathcal{D}}(x_t, \check{x}_t) - p_{\mathcal{C}}(x_t, \check{x}_t)]/p_{\mathcal{C}}(x_t, \check{x}_t)$, gives us, at any time, an interesting measure of the potential impact of the balance of bargaining power.

In the context of the example in Structure 1, we can actually derive a closed-form expression for this measure, at any time *before* the first reorganization, that is, for $\check{x}_t > x^f_{\mathcal{D}}$: The relative difference between the bonds' risk premium with self-imposed concessions versus forced concessions has then the simple expression

$$\frac{p_{\mathcal{D}}(x_t, \check{x}_t) - p_{\mathcal{C}}(x_t, \check{x}_t)}{p_{\mathcal{C}}(x_t, \check{x}_t)} = \left(\frac{\Theta_1}{\Theta_1^*}\right)^{-\lambda} \frac{(\delta - \Theta_0^*)^{1-\lambda}}{(\delta - \Theta_0)^{1-\lambda} + (\Theta_0 - \Theta_0^*)(\tilde{\delta} - \Theta_0)^{-\lambda}} - 1. \tag{45}$$

This expression is valid at entry, until the firm enters the financial distress area. It therefore does not exaggerate the importance of the bargaining power issue. Incidentally, it does not depend on the level of x_t or $\check{x}_t$.

For lower levels of leverage, we can also measure whether the two counterparties' relative bargaining power influences the size of the departure from the APR. The relative difference between self-imposed versus forced departures from the APR, $[\Delta_{\mathcal{D}} - \Delta_{\mathcal{C}}]/\Delta_{\mathcal{C}}$, gives us the potential impact of the balance of bargaining power. Let us now examine a simple numerical application:

Numerical Example (with Structure 1). *The firm's gross income under current operations, x_t, fluctuates with $\mu = 0$ and $\sigma = 15\%$; after liquidation, a quarter of the current gross income could not be generated by competitors (normalizing the current productivity to $\Theta_1 = 1$, then $\Theta_1^* = 0.75$), but the overall cost of production would be halved (normalizing costs to $\Theta_0 = -1$, then $\Theta_0^* = -0.5$). The interest rate is $\rho = 5\%$.*

With these input parameters, if the amount of borrowings gives a debt service obligation δ, which is double the threshold coupon $\tilde{\delta}$, the balance in bargaining power can modify the debt risk premium at entry by $[p_{\mathcal{D}}(x_t, \check{x}_t) - p_{\mathcal{C}}(x_t, \check{x}_t)]/p_{\mathcal{C}}(x_t, \check{x}_t) = 17\%$.

If the debt obligation δ is instead half the threshold coupon $\tilde{\delta}$, it will become rational for creditors to self-impose a $\Delta_{\mathcal{C}}/\Pi^*(\underline{x}) = 24\%$ departure from the APR. If creditors behave opportunistically, they can extract a $\Delta_{\mathcal{D}}/\Pi^*(\underline{x}) = 29\%$ departure from the APR instead. The balance in bargaining power can therefore modify the ex post departure from the APR by $[\Delta_{\mathcal{D}} - \Delta_{\mathcal{C}}]/\Delta_{\mathcal{C}} = 21\%$.

The figures considered in this numerical example are rather conservative. They are more or less equivalent to an aggregate cost of liquidation of 20%. One could easily generate more substantial differences. The overall message is that, as suspected, the two counterparties' relative bargaining power in contract renegotiation is truly an important factor in debt pricing.

7. Extensions

We have developed, under a set of "benchmark" assumptions introduced in Section 3, an asset pricing model that yields simple closed-form formulae for debt and equity, and reflects the crucial importance of debtors and creditors bargaining power in renegotiations. Before concluding, we discuss further these assumptions, and suggest possible extensions of this reference model.

First of all, we have considered that debtors choose when to default in an unconstrained fashion. A fairly convincing argument we gave in favor of this assumption is that (i) debtors can overcome problems of liquidity issuing additional equity, and (ii) it is hard to implement protective covenants. This unconstrained default scenario warrants the following observations:

1. When debtors default too "early," that is, for higher levels of leverage, setting up ex ante debt protective covenants that restrict debtors' choice would actually be inefficient (even if the state of the world was easily verifiable). Here, restricting the range of choices open to debtors, hence expanding the set of constraints they face in their optimization, would only be counterproductive.[19] Creditors can only benefit if their debtors

[19] For any nonempty set of constraints, the resulting constrained optimal debtors' default trigger level can only be greater than the unconstrained one. Any expansion of the set of constraints can only increase

do not face liquidity constraints. Of interest, the additional equity issued by cash-constrained debtors is very likely to be purchased by the creditors themselves.[20]

2. Conversely, for lower levels of leverage creditors would wish, if possible, (i) to setup ex ante protective covenants that force an earlier liquidation, even in the absence of default, and (ii) to prevent debtors from issuing additional equity (and hence will certainly not be among the new equity holders).
3. The unconstrained default scenario is therefore particularly relevant for higher levels of leverage. In a world where it is (i) difficult to issue new assets while in financial distress and (ii) possible to verify the state at some cost, we should expect to see relatively more debt forgiveness and fewer departures from the APR in liquidation than suggested here.
4. Given that debtors are unable to appropriate the creditors' collateral through final boosts in dividends, creditors do not benefit from requiring ex ante that debtors contribute to a sinking fund. Such a fund does not help solve debtors' liquidity problems, and does not help guarantee the value of creditors' collateral. It may be interesting to allow for sinking funds, and they would certainly increase the value of debt and decrease that of equity. We did not consider sinking funds purely for reasons of tractability,[21] but they actually serve no purpose under the unconstrained default scenario.
5. Debtors are not pushed to gradually sell the assets of the firm through time in order to overcome cash shortages. This has substantially simplified the analysis because no particular series of structural assumptions concerning the implications of suboptimal sequences of partial asset sales is needed. We have, however, assumed that partial asset sales destroy existing economies of scale, so that actually they are not optimal ex ante.

Second, the pricing model assumes that contracts are (i) either perfectly and costlessly renegotiated, or that (ii) debt restructuring is impossible altogether (in which case default is immediately followed by liquidation). The model does not consider intermediate cases.

Clearly the fact that public debt-holders constitute a noncohesive negotiating unit disturbs, but does not paralyze altogether, the debt reorganization

---

debtors' default trigger level, $\underline{x}_{\mathcal{D}}$, which is already greater than the ex ante optimal closure rule, $\underline{x}$.

[20] Notice that albeit voting rights, a strategy which consists of (i) writing off debt and (ii) taking on newly issued equity, greatly resembles a debt-for-equity exchange.

[21] With sinking funds, the pricing problem involves an additional state variable, $w_t \equiv \int_0^t s(x_\tau)[\pi(x_\tau)-\delta]\,d\tau$, where $s(x)$ is the share of the earnings contributing to the sinking fund. Instead of being path dependent in one variable, the problem becomes path dependent in this second state variable.

process. An important but difficult topic for future research consists of either (i) allowing for strategies of repeated exchange offers in the presence of multiple creditors, and/or (ii) reflecting the fact that debt restructuring is neither instantaneous nor impossible, but occurs through *lengthy* procedures. Integrating such features in a structural fashion would *mitigate* problems stemming from the conflict of interests among creditors. This would be more convincing than simply introducing a set of exogenous cost parameters.[22]

Notice that Chapter 11 of the U.S. Bankruptcy Code grants debtors with the quasi-exclusive right to design reorganization plans.[23] Conversely, UK Administration gives most of the bargaining power to creditors. Therefore, albeit the shortfalls mentioned above, this article provides the pricing of debt and equity when *both* a structured bargaining procedure (U.S. Chapter 11 or UK Administration) and a cash auction procedure (U.S. Chapter 7 or UK Liquidation) are in place. The parameter capturing the difference between the two systems is the relative bargaining power in reorganization: The U.S. system corresponds to one extreme case where creditors' concessions are "forced," whereas the UK system corresponds to the alternative one, where creditors "self-impose" the concessions they make.

Finally, this model has only considered the case of infinite maturity debt. Pricing maturity based concessions in financial distress is certainly a difficult but exciting topic for future research.[24] On the one hand, there would be an increased scope for opportunistic debtors to extract large concessions just prior to the maturity date, when the principal is supposed to be repaid. On the other hand, if financing this principal repayment involves rolling over the debt, debtors need to preserve some "good" reputation to find new creditors. Then the influence of allowing for dynamic reorganization of the debt on the ex ante default risk premium is not obvious.

8. Conclusion

Most debt contracts do not induce debtors' ex post optimal timing of default to coincide with the ex ante optimal time to sell the assets of the firm. In the

[22] A fairly simple way to account for imperfections and costs of reorganization consists of assuming that (i) each time the coupon is marginally lowered by $d\delta$, a proportional cost $\phi\, d\delta$ is incurred, and (ii) reaching an agreement on the size of a departure from the APR involves a lump-sum cost Φ.

[23] The debtor in possession provision gives them an exclusive entitlement to propose a reorganization plan. After the first four months, this exclusivity period is in most cases largely extended by the judges, and control of the firm remains with current management in the meantime.

[24] Pricing finite maturity debt with default risk, but ruling out renegotiation, is already complicated. One way forward involves assuming that the debt is constantly rolled over, so that the pricing problem remains time homogeneous. Under this assumption, Leland and Toft (1996) have obtained analytical solutions, which are much more intricate than the earlier related ones of Leland (1994) for infinite maturity. Here we are adding sequential debt reorganization, and it appears difficult to convert the problem in a time-homogeneous one.

absence of renegotiation, this suboptimal ex post default policy results in a loss of value. There exists a threshold level of borrowings for which this moral hazard becomes insignificant, because there is actually no divergence of timing. For other levels of borrowings, debt value can be decreasing in debt service obligation, as well as in the collateral.

Allowing for contract renegotiation, we have then characterized the dynamics of debt reorganization as the firm's status deteriorates. Different dynamics emerge depending on the leverage of the firm at entry, that is, the initial participation that is requested from lenders. To conclude and summarize the article, we now highlight the most interesting qualitative and quantitative testable implications it yields.

1. High-leverage firms experience a series of debt contract reorganizations prior to liquidation. Through these financial restructurings the aggregate debt service obligation of the debtors is successively reduced. This sequence of reorganization takes place as the firm's profitability gradually reaches new record low levels. Ultimately, when liquidation occurs the creditors share the proceeds of the liquidation sale.
2. In contrast, low-leverage firms are liquidated without prior debt reorganization. However, liquidation involves departures from the absolute priority rule, that is, debtors receive a substantial share of the proceeds of the liquidation sale, even though creditors do not receive full payment of their collateral.

This structural pricing model allows us to *quantify* the impact on value of ex post reorganizations. Of importance, the closed-form valuation expressions for debt and equity we derived (i) take into account game theoretic interactions between the two conflicting parties, and (ii) are closely linked to fundamental characteristics of the industry the firm is evolving in.

1. Our numerical results suggest that relative bargaining power between debtors and creditors is a key factor in corporate asset pricing. It can alter a bond's risk premium as well as departures from the absolute priority rule by as much as the potential loss in firm value in liquidation (which in most empirical studies is more than 20%). This has direct consequences on the ex ante borrowing ability of the firm, which by backward induction is altered in similar proportions.
2. Our structural model relates the threatening cost of liquidation we have just mentioned to fundamental characteristics of the industry. Consequently if (i) the firm's initial investment is largely specific to current operations, and/or (ii) the relative contribution to value of inalienable human capital to alienable physical assets is large, the potential impact of debtors' bargaining power on risk premia, observed departures from the APR, and debt capacity is large. An example of this class of firms is Euro-Disney, whereas an example of the opposite class is Euro-Tunnel.

Appendix

Proof of Lemma 1. Assumption 2 implies that the derivative of the value of debtors' option to default at y, with respect to this trigger level,

$$H(y) \;\equiv\; \frac{\partial}{\partial y}\left[\,\left(\Pi^*(y) - \Pi(y) - \underline{C}(y) + \delta/\rho\right)\,\mathcal{P}(x_0 \;\triangleright\; y)\,\right] \tag{46}$$

is strictly negative. The single root of $H(y) = 0$ yields the debtors' ex post optimal default level $\underline{x}_{\mathcal{D}}$. Take the derivative of $H(y)$ with respect to δ,

$$\begin{aligned}&\frac{\partial}{\partial\delta}\left[\frac{\partial}{\partial y}\left[\left(\Pi^*(y) - \Pi(y) - \underline{C}(y) + \delta/\rho\right)\mathcal{P}(x_0 \;\triangleright\; y)\right]\right]\\ &\quad = \frac{\partial}{\partial y}\left[\mathcal{P}(x_0 \;\triangleright\; y)\right] > 0.\end{aligned} \tag{47}$$

We see that the root of $H(y) = 0$ increases with δ. In other words, $\partial\underline{x}_{\mathcal{D}}/\partial\delta > 0$.

Express the respective first-order optimality conditions which determine $\underline{x}$ and $\underline{x}_{\mathcal{D}}$:

$$\left[\frac{d\Pi^*(y)}{\partial y} - \frac{d\Pi(y)}{\partial y}\right]\mathcal{P}(x_t \;\triangleright\; y) + \left[\Pi^*(y) - \Pi(y)\right]\frac{\partial\mathcal{P}(x_t \;\triangleright\; y)}{\partial y} = 0, \tag{48}$$

and

$$\begin{aligned}&\left[\frac{d\Pi^*(y)}{\partial y} - \frac{d\Pi(y)}{\partial y} - \frac{d\underline{C}(y)}{\partial y}\right]\mathcal{P}(x_t \;\triangleright\; y)\\ &\quad + \left[\Pi^*(y) - \Pi(y) - \underline{C}(y) + \frac{\delta}{\rho}\right]\frac{\partial\mathcal{P}(x_t \;\triangleright\; y)}{\partial y} = 0.\end{aligned} \tag{49}$$

If $\underline{x}$ equals $\underline{x}_{\mathcal{D}}$, both conditions have the same solution. Replacing the first one in the second yields

$$\begin{aligned}&\left[\Pi^*(y) - \Pi(y)\right]\left[\frac{d\Pi^*(y)}{\partial y} - \frac{d\Pi(y)}{\partial y} - \frac{d\underline{C}(y)}{\partial y}\right]\mathcal{P}(x_t \;\triangleright\; y)\\ &\quad + \left[\Pi^*(y) - \Pi(y) - \underline{C}(y) + \frac{\delta}{\rho}\right]\\ &\quad \times\left[\frac{d\Pi(y)}{\partial y} - \frac{d\Pi^*(y)}{\partial y}\right]\mathcal{P}(x_t \;\triangleright\; y) = 0.\end{aligned} \tag{50}$$

After simplification we see that for $\underline{x}$ to equal $\underline{x}_{\mathcal{D}}$, the coupon obligation $\tilde{\delta}$ must be such that

$$\tilde{\delta} \;=\; \rho\frac{d\underline{C}(\underline{x})}{d\underline{x}}\left[\Pi^*(\underline{x}) - \Pi(\underline{x})\right]\left[\frac{d\Pi(\underline{x})}{d\underline{x}} - \frac{d\Pi^*(\underline{x})}{d\underline{x}}\right]^{-1} + \rho\underline{C}(\underline{x}). \tag{51}$$

Proof of Propositions 1 and 2. Our proofs consider the evolution of the optimization problem through time. To do so, let us expand the notation introduced in the text to keep track of time. If at time t the prevailing coupon is $\delta(t)$, and creditors' residual claim in repudiation is $\underline{C}(x,t)$, this means (i) that debtors are contractually facing an obligation to service each unit of time $\delta(u) = \delta(t)$ for $u \in [t; +\infty)$, and (ii) if debtors repudiate at any time $u > t$ (when the state is x_u), creditors could claim $\underline{C}(x_u, t)$.

The follower debtors react optimally, taking the leader's strategy as given. That is, at any time t, the debtors consider the perpetual obligation to pay the prevailing coupon $\delta(t)$ and creditors' residual claim in repudiation $\underline{C}(x,t)$ as given, and optimize over their default decision trigger level y:

$$D(x_t, y) \; = \; \max_y \Bigg[\Pi(x_t) - \frac{\delta(t)}{\rho} + \left(\Pi^*(y) - \Pi(y) - \underline{C}(y,t) + \frac{\delta(t)}{\rho} \right) \mathcal{P}(x \,\triangleright\, y) \Bigg]. \tag{52}$$

Under Assumption 2, the *optimal* default decision trigger level is unique. This optimal trigger level depends on the prevailing coupon obligation, that is, $\tilde{y} \equiv \tilde{y}[\delta(t), \underline{C}(x,t)]$, and solves the first-order optimality condition

$$\frac{\partial D(x_t, \tilde{y})}{\partial \tilde{y}} = 0. \tag{53}$$

Furthermore, $\partial \tilde{y} / \partial \delta(t) > 0$ (the proof is identical to that of Lemma 1).

Given that the debtors cannot be forced to accept a change in debt contract, the creditors' strategy cannot involve increases in coupon obligations, nor increases in creditor residual claim. Therefore $D(x_t, \tilde{y})$ is essentially at any given time t the *debtors' reservation value*.

A creditors' concession strategy, $\{\delta(t), \underline{C}(x,t),$ for $t \in [0; T_z]\}$, consists of controlling (i) a nonincreasing coupon function $\delta(t)$ and (ii) a nonincreasing function $\underline{C}(x,t)$ through time, taking into account what the followers' reaction will be. By decreasing $\delta(t)$ creditors prolong their debtors' willingness to operate the firm, whereas by decreasing $\underline{C}(x,t)$ creditors accelerate debtors' willingness to abandon. With these tools creditors select the time length of this concessions policy, controlling the time, T_z, when such concessions should stop (which triggers liquidation). Controlling T_z amounts to controlling its associated trigger level z, where $T_z \equiv \inf\{T \mid x_T = z\}$:

$$C(x_t, t) \; = \; \max_{\delta(t), \underline{C}(x,t), z} \Big[V(x_t \mid z) - D(x_t, \tilde{y}) \Big], \qquad \text{subject to} \tag{54}$$

$$\begin{aligned}
&(a) \qquad D(x, \tilde{y}) \geq \Pi^*(x) - \underline{C}(x), \\
&(b) \qquad \delta(t) < \delta(s), \quad \text{for } t > s, \quad \text{and} \\
&(c) \qquad \underline{C}(x,t) < \underline{C}(x,s) \quad \text{for all } x, \quad \text{for } t > s.
\end{aligned}$$

From the creditors' perspective, for any given path $\{x(t), t \in [0; T]\}$, a concessions' strategy $\{\delta(t)\underline{C}(x,t)$ for $t \in [0; T]\}$ is *preferable* to another concessions' strategy

$\{\delta^*(t), \underline{C}^*(x,t)$ for $t \in [0; T]\}$ if

1. For all $t \in [0; T]$, $\delta(t) \geq \delta^*(t)$, and there exists at least one time $t \in [0; T]$ such that $\delta(t) > \delta^*(t)$,
2. Or, for all $t \in [0; T]$, $\underline{C}(x,t) \geq \underline{C}^*(x,t)$ for all x, and there exists at least one time $t \in [0; T]$ such that $\underline{C}(x,t)$ is strictly greater than $\underline{C}^*(x,t)$ for all x.

In other words, at any time t, minimizing the equity value $D(x, \tilde{y})$ involves setting the highest possible coupon and the highest possible creditor residual claim.

Overall the most creditors could possibly achieve consists of (i) maximizing the total value of the firm $V(x \mid z)$ to $V(x \mid \underline{x})$, while (ii) minimizing the equity value to its reservation value $D(x, \tilde{y})$ throughout the time interval $[0; T_{\underline{x}}]$. A concession strategy $\{\delta(t), \underline{C}(x,t)$, for $t \in [0; T_{\underline{x}}]\}$, which (i) maintains operations until $T_{\underline{x}}$, satisfying conditions (a), (b), and (c), and (ii) is preferable to any other concessions' strategy $\{\delta^*(t), \underline{C}^*(x,t)$, for $t \in [0; T_{\underline{x}}]\}$, which also satisfies conditions (a), (b), and (c), is necessarily a *most preferable* strategy, hence an optimal creditors' strategy.

At any time $t \in (0; T_{\underline{x}}]$, condition (a) is either binding or not. Under Assumption 2, debtors do not default immediately at entry, hence at $t = 0$ condition (a) is not binding. We now consider the remaining possible cases until the random time $T_{\underline{x}}$, establishing in each case the most preferable strategy:

Case A. At $t \in (0; T_{\underline{x}})$, condition (a) is not binding. Debtors are therefore servicing the prevailing coupon $\delta(t)$, and creditors would like operations to continue. The highest possible coupon that (i) maintains operations running for the next instant and (ii) is preferable to any other one is the current one, hence $d\,\delta(t) = 0$.

Case B. At $t \in (0; T_{\underline{x}})$, condition (a) is binding. Debtors default but creditors would like operations to continue. For any prevailing $\delta(t)$, this occurs the first time x_t equals $\tilde{y}$, so

$$D(\tilde{y}, \tilde{y}) \;=\; \Pi^*(\tilde{y}) - \underline{C}(\tilde{y}). \tag{55}$$

For operations not to be interrupted, a coupon reduction such that condition (a) is not binding is necessary. The smallest reduction in coupon $d\,\delta(t)$ for this to be the case is therefore such that condition (a) is binding at a marginally lower debtors' optimal default trigger level, that is, $\tilde{y}[\delta(t) + d\,\delta(t), \underline{C}(x,t)] = \tilde{y}[\delta(t), \underline{C}(x,t)] - dx$. In other words, a reduction in coupon $d\,\delta(t)$ such that

$$\frac{\partial}{\partial\,\delta(t)}\Big[D(\tilde{y}, \tilde{y}) - [\Pi^*(\tilde{y}) - \underline{C}(\tilde{y}, t)]\Big] = 0. \tag{56}$$

Therefore as $\partial\tilde{y}/\partial\delta(t) > 0$, the minimum reduction in coupon to exit Case B (hence most preferable) is

$$\frac{\partial D(\tilde{y}, \tilde{y})}{\partial\,\tilde{y}} - \left[\frac{\partial\,\Pi^*(\tilde{y})}{\partial\,\tilde{y}} - \frac{\partial \underline{C}(\tilde{y}, t)}{\partial\,\tilde{y}}\right] = 0. \tag{57}$$

Case C. At $t = T_{\underline{x}}$, condition (a) is binding. Debtors default and creditors do not want operations to continue. Both debtors and creditors are happy to trigger liquidation.

Case D. At $t = T_{\underline{x}}$, condition (a) is not binding. Debtors are therefore servicing the prevailing coupon $\overline{\delta}(t)$, but now creditors would like them to abandon operations. For operations to be interrupted, a reduction in the creditor's residual claim in repudiation such that condition (a) is binding is necessary. The smallest reduction *Dep* in $\underline{C}(x, t)$ for this to be the case is therefore such that

$$D(\underline{x}, \tilde{y}) \;=\; \Pi^*(\underline{x}) - [\underline{C}(\underline{x}) - Dep]. \tag{58}$$

We are then instantaneously in Case C and liquidation is triggered.

These four cases determine the sequence and the timing of events during the time interval $(0; T_{\underline{x}})$. They also yield the corresponding levels of the coupon for each time:

Proposition 1. Given that $\delta(0) = \delta$ is greater than $\tilde{\delta}$, Lemma 1 implies that (i) at the entry state we are in Case A and (ii) Case B *always* arises before $T_{\underline{x}}$. This first occurrence of Case B is denoted $\inf\{T \mid x_T = x_C^f\}$ in the proposition.

Given the stationarity of the process and the time homogeneity of the optimization problem, further occurrences of Case B arise when and only when the state x equals $\tilde{y}[\delta(t)]$. As in each occurrence of Case B the coupon is marginally lowered, this threshold level decreases. Therefore Case B only reoccurs when a new historical minimum is reached. These further occurrences are similarly denoted $\inf\{T \mid x_T = \check{x}_t\}$ in the proposition. As long as $\delta(t)$ is greater than $\tilde{\delta}$, Lemma 1 implies that Case B will occur again.

In between occurrences of Case B, the coupon remains unchanged (Case A). When $\delta(t)$ finally equals $\tilde{\delta}$, $\tilde{y}$ equals for the first time $\underline{x}$, and we are in Case C. Consequently, given that at time $T_{\underline{x}}$ a new minimum is reached, we are then in Case C.

In the proposition the sequence of events and prevailing coupons are expressed, replacing all the random first-hitting times mentioned here by their associated threshold levels in the state space. The valuation formulas for debt and equity are derived directly as in previous sections. This is because the valuation exercise remains *at any time* a one-sided (below) stochastic process switching problem.

Proposition 2. Given that $\delta(0) = \delta$ is smaller than $\tilde{\delta}$, Lemma 1 implies that (i) at the entry state we are in Case A and (ii) Case B *never* arises for $t \in (0; T_{\underline{x}})$. Furthermore, at time $T_{\underline{x}}$ we are in Case D. In the proposition, Δ_C is the sum transfered to debtors in liquidation, hence it denotes $\Pi^*(\underline{x}) - \underline{C}(\underline{x}) + Dep$.

References

Alderson, M. J., and B. L. Betker, 1995, "Liquidation Costs and Capital Structure," *Journal of Financial Economics*, 39, 45–69.

Anderson, R. W., and S. Sundaresan, 1996, "Design and Valuation of Debt Contracts," *Review of Financial Studies*, 9, 37–68.

Baird, D. G., and T. H. Jackson, 1988, "Bargaining after the Fall and the Contours of the Absolute Priority Rule," *University of Chicago Law Review*, 55, 738–789.

Basar, T., and G. J. Olsder, 1994, *Dynamic Noncooperative Game Theory* (2nd ed.), Academic Press, London.

Bebchuk, L. A., and H. F. Chang, 1992, "Bargaining and the Division of Value in Corporate Reorganization," *Journal of Law, Economics & Organization*, 8, 253–279.

Bergman, Y .Z., and J. L. Callen, 1991, "Opportunistic Underinvestment in Debt Renegotiation and Capital Structure," *Journal of Financial Economics*, 29, 137–171.

Bertola, G., 1989, "Irreversible Investment," working paper, Princeton University.

Black, F., and J. C. Cox, 1976, "Valuing Corporate Securities: Some Effects of Bond Indenture Provisions," *Journal of Finance*, 31, 351–367.

Brennan, M. J., and E. S. Schwartz, 1984, "Optimal Financial Policy and Firm Valuation," *Journal of Finance*, 39, 593–607.

Dixit, A., 1989, "Entry and Exit Decisions under Uncertainty," *Journal of Political Economy*, 97, 620–638.

Dixit, A., 1991, "A Simplified Exposition of the Theory of Optimal Control of Brownian Motion," *Journal of Economic Dynamics and Control*, 15, 657–673.

Dumas, B., 1988, "Hysteresis in Capital Formation," working paper, University of Pennsylvania.

Dumas, B., 1991, "Super Contact and Related Optimality Conditions," *Journal of Economic Dynamics and Control*, 15, 675–685.

Eberhart, A. C., W. T. Moore, and R. L. Roenfeld, 1990, "Security Pricing and Deviations from the Absolute Priority Rule in Bankruptcy Proceedings," *Journal of Finance*, 45, 1457–1469.

Fisher, E. O., R. Heinkel, and J. Zechner, 1989, "Dynamic Capital Structure Choice: Theory and Tests," *Journal of Finance*, 44, 19–40.

Franks, J. R., and W. N. Torous, 1989, "An Empirical Investigation of U.S. Firms in Renegotiation," *Journal of Finance*, 44, 747–769.

Franks, J. R., and W. N. Torous, 1994, "A Comparison of Financial Recontracting in Distressed Exchanges and Chapter 11 Reorganization," *Journal of Financial Economics*, 35, 349–370.

Fries, S. M., M. Miller, and W. R. M. Perraudin, 1997, "Debt Pricing in Industry Equilibrium," *Review of Financial Studies*, 10, 39–68.

Froot, K. A., 1989, "Buybacks, Exit Bonds, and the Optimality of Debt and Liquidity Relief," *International Economic Review*, 30, 49–70.

Froot, K. A., and M. Obstfeld, 1991, "Stochastic Process Switching: Some Simple Solutions," *Econometrica*, 59, 237–239.

Gertner, R., and D. Scharfstein, 1991, "A Theory of Workouts and the Effects of Reorganization Law," *Journal of Finance*, 46, 1189–1222.

Giammarino, R. M., 1989, "The Resolution of Financial Distress," *Review of Financial Studies*, 2, 25–47.

Gilson, S. C., 1995, "Transactions Costs and Capital Structure Choice: Evidence from Financially Distresses Firms," working paper, Harvard Business School.

Gilson, S. C., J. Kose, and L. H. Lang, 1990, "Troubled Debt Restructuring: An Empirical Study of Private Reorganization of Firms in Default," *Journal of Financial Economics*, 27, 315–353.

Grossman, S., and O. Hart, 1986, "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration," *Journal of Political Economy*, 94, 691–719.

Harrison, M. J., and D. M. Kreps, 1979, "Martingales and Arbitrage in Multiperiod Securities Markets," *Journal of Economic Theory*, 20, 381–408.

Hart, O., 1993, "Theories of Optimal Capital Structure: The Managerial Discretion Perspective," in Margaret Blair (ed.), *The Deal Decade: What Takeovers and Leveraged Buyouts Mean for Corporate Governance*, The Brookings Institution: Washington, D.C., pp. 19–53.

Hart, O., and J. Moore, 1989, "Default and Reorganization: A Dynamic Model of Debt," Working Paper 520, MIT.

Hart, O., and J. Moore, 1990, "Property Rights and the Nature of the Firm," *Journal of Political Economy*, 98, 1119–1158.

Hart, O., and J. Moore, 1994, "A Theory of Debt Based on the Inalienability of Human Capital," *Quarterly Journal of Economics*, 109, 841–879.

Jones, P. E., S. P. Mason, and E. Rosenfeld, 1984, "Contingent Claims Analysis of Corporate Capital Structures: An Empirical Ananlysis," *Journal of Finance*, 39, 611–625.

Kane, A., A. Marcus, and R. McDonald, 1984, "How Big is the Tax Advantage to Debt?," *Journal of Finance*, 39, 841–852.

Karlin, S., and H. M. Taylor, 1975, *A First Course in Stochastic Processes*, (2nd ed.), Academic Press: New York.

Kim, I. J., K. Ramaswamy, and S. Sundaresan, 1993, "Does Default Risk in Coupons Affect the Valuation of Corporate Bonds?: A Contingent Claims Model," *Financial Management*, Autumn, 117–131.

Krugman, P., 1988, "Market-based Debt-Reduction Schemes," in Frenkel, J. A., M. P. Dooley, and Wickham, P. (eds.), *Analytical Issues in Debt*, International Monetary Fund, Washington, D.C., pp. 258–278.

Krugman, P., 1989, "Financing vs. Forgiving a Debt: Some Analytical Notes," *Journal of Development Economics*, 29, 253–268.

Leland, H. E., 1994, "Risky Debt, Bond Covenants and Optimal Capital Structure," *Journal of Finance*, 49, 1213–1252.

Leland, H. E., and K. B. Toft, 1996, "Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads," *Journal of Finance*, 51, 987–1019.

Longstaff, F. A., and E. S. Schwartz, 1995, "A Simple Approach to Valuing Risky and Floating Rate Debt," *Journal of Finance*, 50, 789–819.

Mella-Barral, P., and W. R. M. Perraudin, 1997, "Strategic Debt Service," *Journal of Finance*, 52, 531–556.

Mello, A. S., and J. E. Parsons, 1992, "The Agency Costs of Debt," *Journal of Finance*, 47, 1887–1904.

Merton, R. C., 1974, "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance*, 29, 449–470.

Miller, M. H., 1977, "Debt and Taxes," *Journal of Finance*, 32, 261–275.

Nielsen, L. T., J. Saa-Requejo, and P. Santa-Clara, 1993, "Default Risk and Interest Rate Risk: The Term Structure of Default Spreads," mimeo, INSEAD.

Pindyck, R. S., 1988, "Irreversible Investment, Capacity Choice, and the Value of the Firm," *American Economic Review*, 79, 969–985.

Roe, M., 1987, “The Voting Prohibition in Bond Workouts,” *Yale Law Journal*, 97, 232–279.

Sachs, J., 1988a, “Conditionality, Debt Relief, and the Developing Country Crisis,” mimeo, Harvard University.

Sachs, J., 1988b, “The Debt Overhang of Developing Countries,” in De Macedo, and Findlay, (eds.), *Diaz Memorial Volume*, Wider Institute, Helsinki.

Smith, G. W., 1991, “Solution to a Problem of Stochastic Process Switching,” *Econometrica*, 59, 237–239.