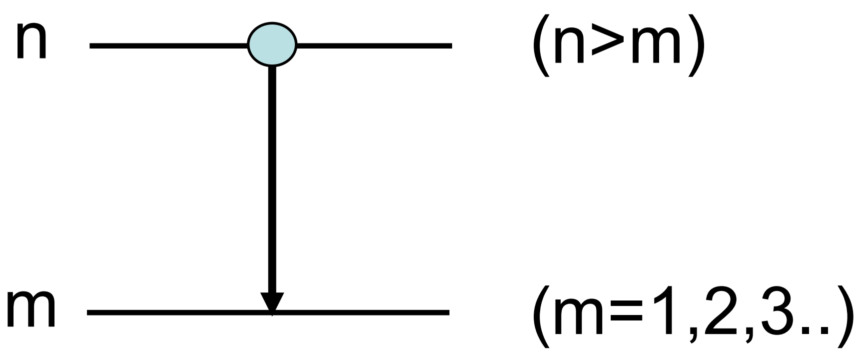
Hydrogen atom – Lyman Series

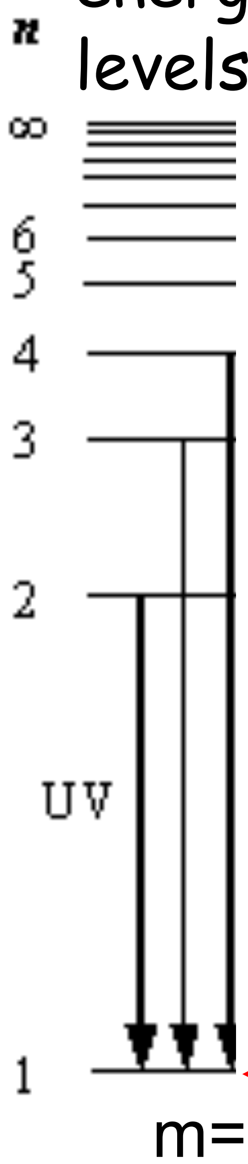
Rydberg's formula

$$\lambda = \frac{91.19\text{nm}}{\dfrac{1}{m^2} - \dfrac{1}{n^2}}$$

Predicts λ of n→m transition:



Hydrogen energy levels



Can Rydberg's formula tell us what ground state energy is?

Balmer-Rydberg formula

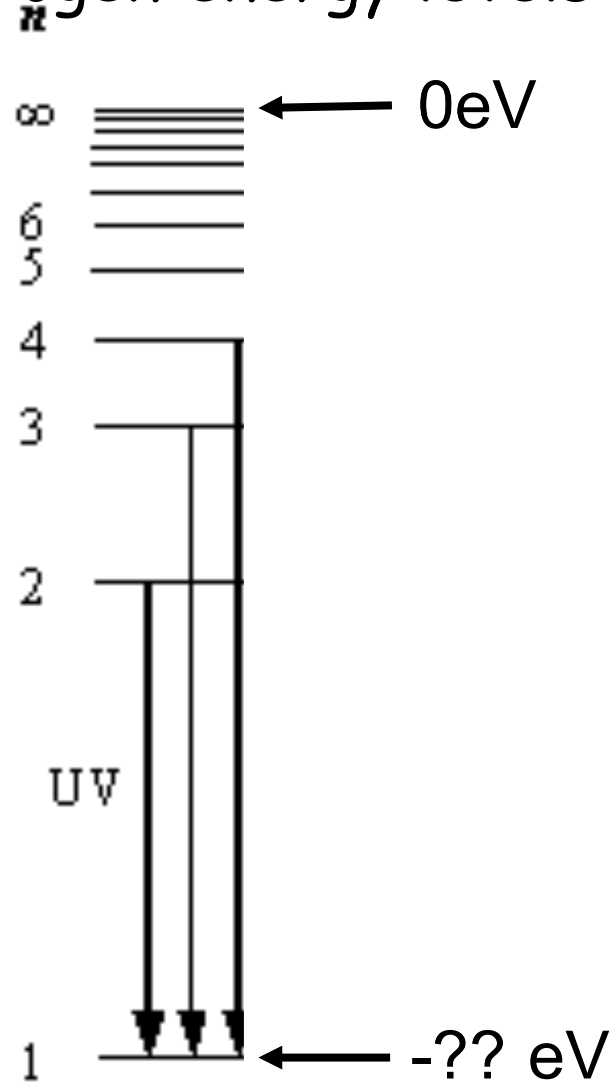
$$\boxed{\lambda = \frac{91.19\text{nm}}{\frac{1}{m^2} - \frac{1}{n^2}}}$$

Look at energy for a transition between n=infinity and m=1

$$\overset{\equiv 0\text{eV}}{\cancel{E_{initial}}} - E_{final} = \frac{hc}{\lambda} = \frac{hc}{91.19nm}\left(\overset{1}{\cancel{\frac{1}{m^2}}} - \overset{0}{\cancel{\frac{1}{n^2}}}\right)$$

$$-E_{final} = \frac{hc}{91.19nm}$$

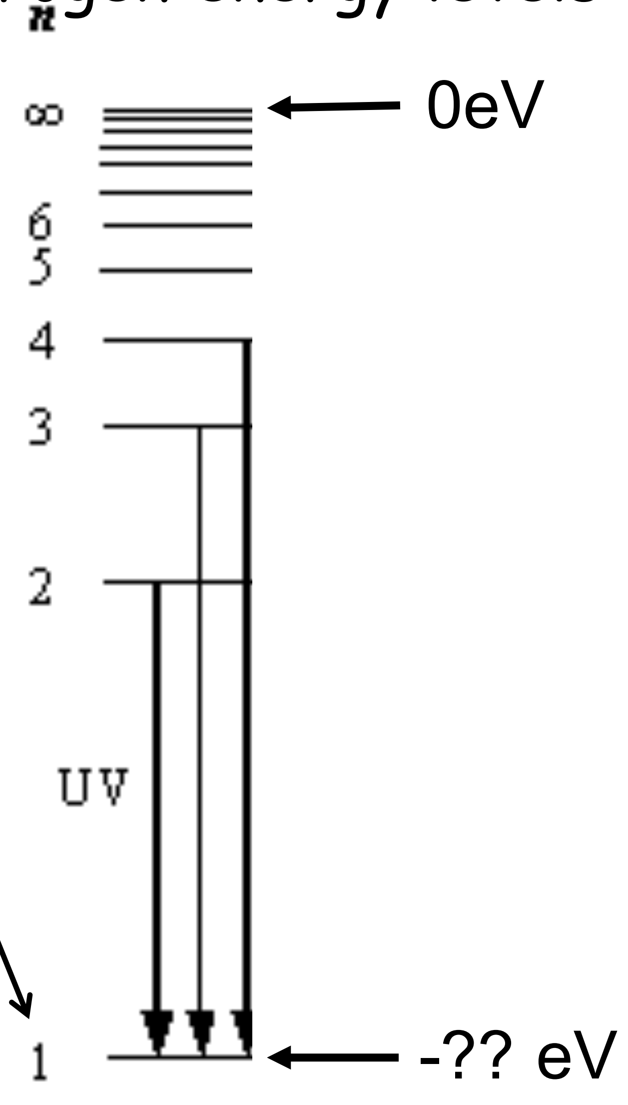
Hydrogen energy levels



$$\boxed{E_1 = -13.6eV}$$

Hydrogen energy levels

<u>A more general case:</u> What is the energy of each level ('m') in hydrogen?



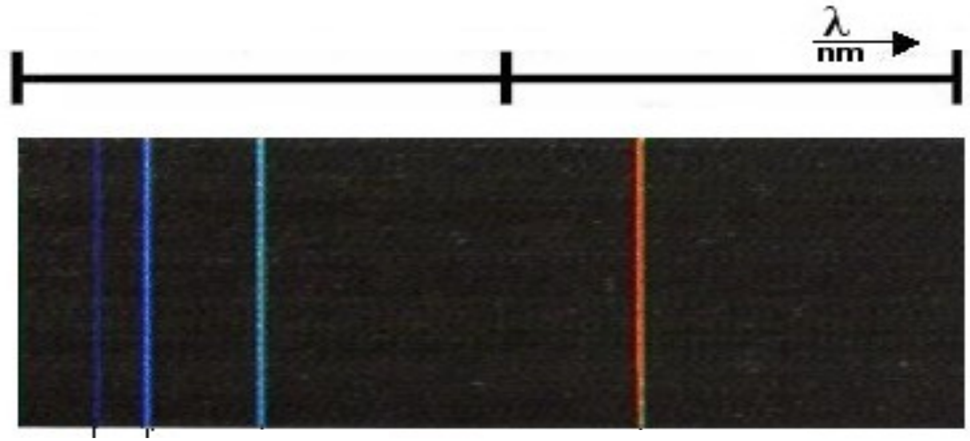
$$\underbrace{\cancel{E_{initial}}}_{\equiv\, 0eV} - E_{final} = \frac{hc}{\lambda} = \frac{hc}{91.19nm}\left(\frac{1}{m^2} - \underbrace{\cancel{\frac{1}{n^2}}}_{0}\right)$$

$$-E_{final} = \frac{hc}{91.19nm}\frac{1}{m^2}$$

$$\boxed{E_m = -13.6eV\,\frac{1}{m^2}}$$

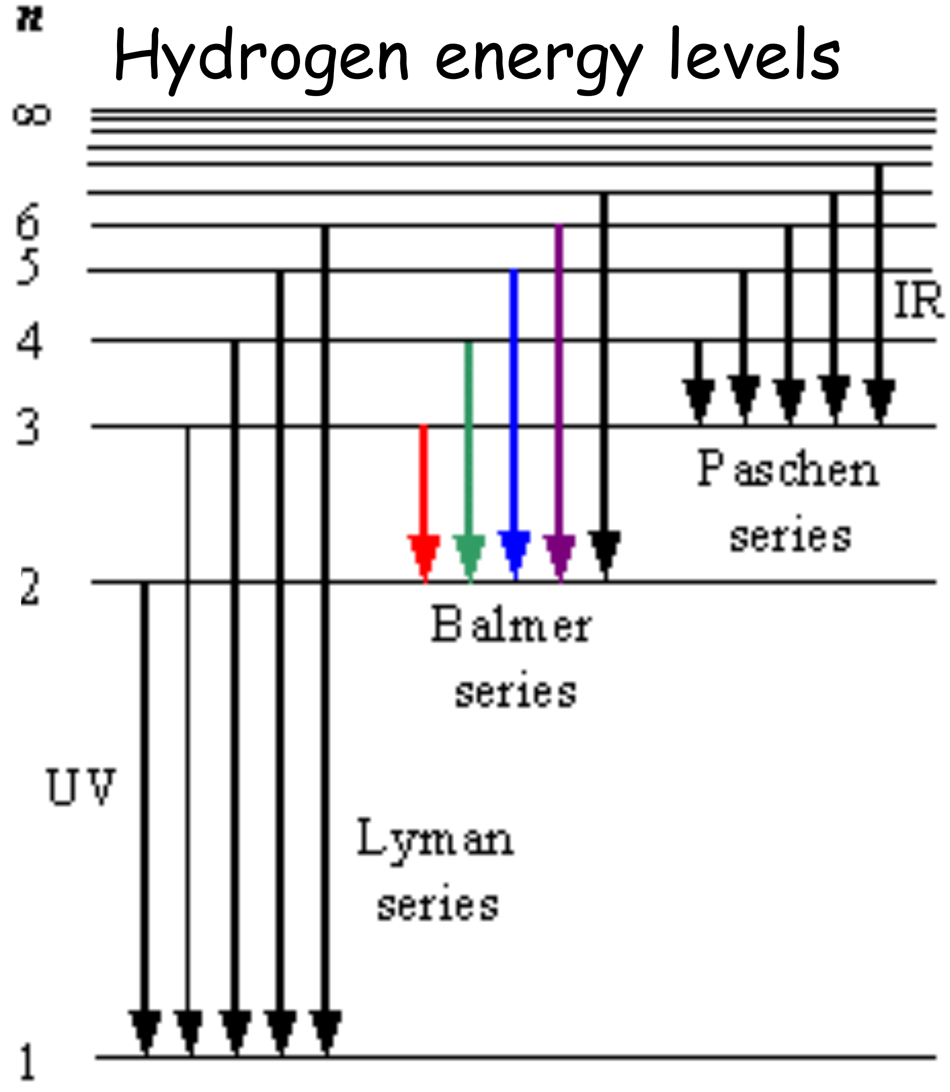
Balmer/Rydberg had a mathematical formula to describe hydrogen spectrum, but no mechanism for why it worked!

Why does it work?



$$\lambda = \frac{91.19\text{nm}}{\frac{1}{m^2} - \frac{1}{n^2}}$$

where m=1,2,3
and where n = m+1, m+2



The Balmer/Rydberg formula correctly describes the hydrogen spectrum!

Is it a good model?

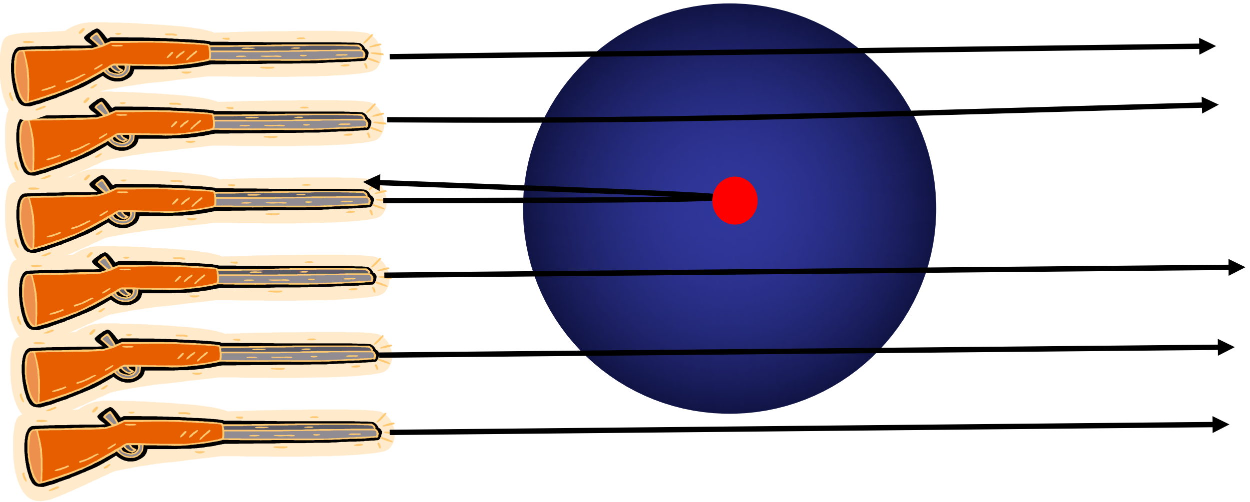
The Balmer/Rydberg formula is a mathematical representation of an empirical observation.

It doesn’t really *explain* anything.

How can we *explain* (not only *calculate*) the energy levels in the hydrogen atom?

Next step: A semi-classical explanation
of the atomic spectra (*Bohr model*)

Rutherford shot alpha particles at atoms and he figured out that a tiny, positive, hard core is surrounded by negative charge very far away from the core.



- One possible model:

  Atom is like a solar system:

  electrons circling the nucleus like planets circling the sun…
- The problem is that accelerating electrons should radiate light and spiral into the nucleus:

***Elapsed time: ~10^{-11} seconds**

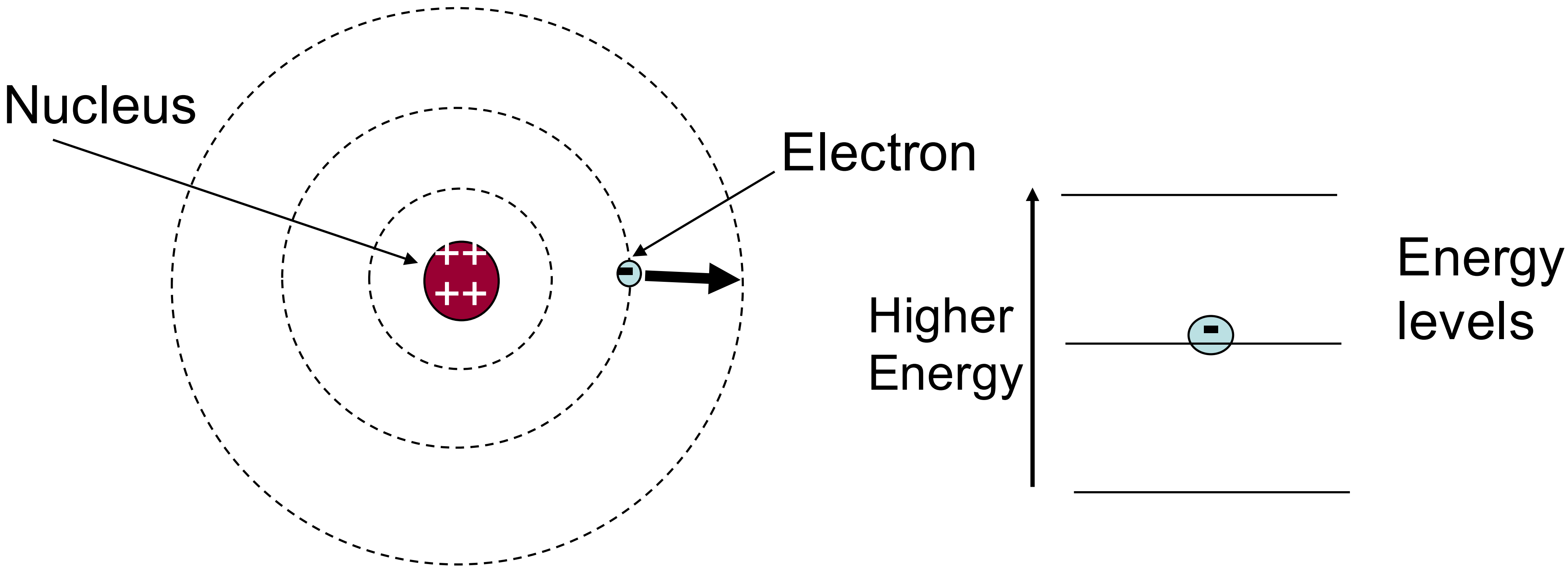
Why don't planets spiral into the sun?

A. Well, they do, but very, very slowly.
B. Because planets obey quantum mechanics, not classical mechanics.
C. Because planets obey classical mechanics, not quantum mechanics.
D. Because planets are much bigger than electrons they don't emit anything.

Answer is A. "Gravitational radiation" (i.e. gravitational waves) is much, much weaker than electromagnetic radiation.

('Tidal forces' likely dominate the energy loss of the planets).

Electrostatic potential energy

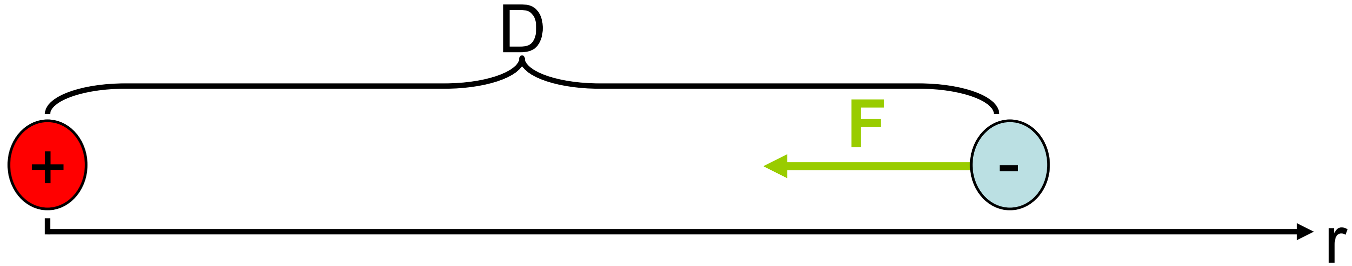


When an electron moves to location <u>further away</u> from the nucleus its <u>energy increases</u> because energy is required to separate positive and negative charges, and there is an <u>increase in the electrostatic potential energy</u> of the electron.

→ ***Force* on electron is less, but *Potential Energy* is higher!**

→ Electrons at higher energy levels are further from the nucleus!

Potential energy of the electron in hydrogen



We define electron's PE as 0 when far, far away from the proton!

→ Electron's PE = -work done by electric field from $r_1=\infty \ldots r_2=D$

Coulomb's constant

$$\int_\infty^D \mathbf{F}\bullet d\mathbf{r} = \int_\infty^D \frac{kq_{elect}q_{prot}}{r^2}\,dr$$

$$PE = kq_{elect}q_{prot}\int_\infty^D \frac{dr}{r^2} = kq_{elect}q_{prot}\left.\frac{1}{r}\right|_\infty^D = -\frac{ke^2}{D}$$

-e e

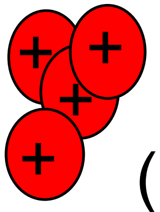
(for hydrogen)

($k = {}^{1}/_{(4\pi\varepsilon 0)}$: Coulomb force const.)

Potential energy of a single electron in an atom

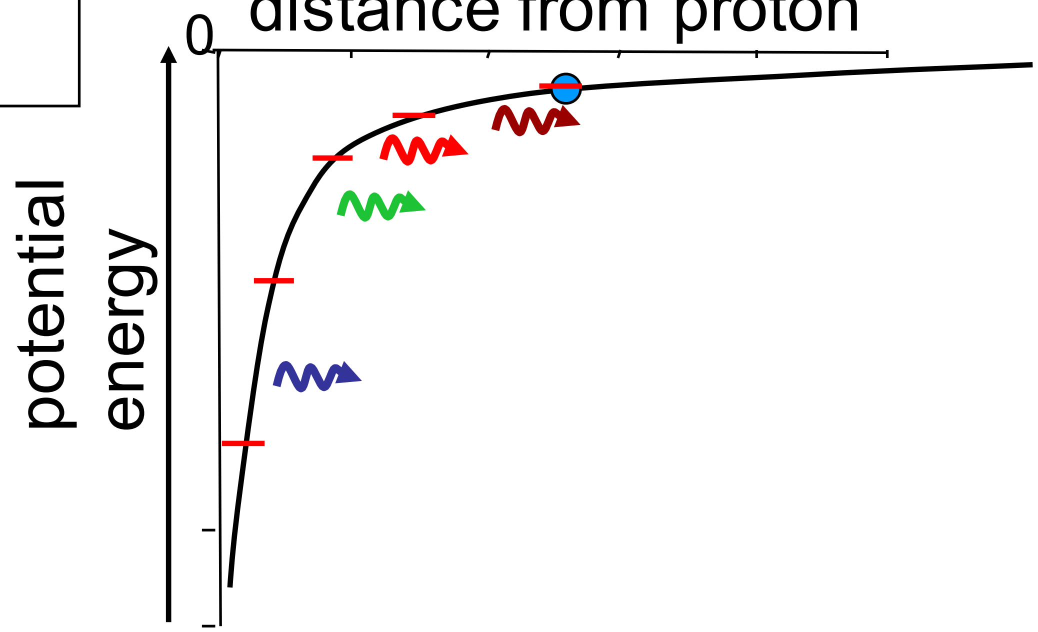
PE of an electron at distance D from the proton is

$$PE = -\frac{ke^2}{D}$$, $ke^2 = 1.440\,\text{eV}\cdot\text{nm}$



$$PE = \frac{-ke(Ze)}{D}$$

(For Z protons)



How can we *calculate* the energy levels in hydrogen?

Balmer-Rydberg
(from last class)

$$\lambda = \frac{91.19\text{nm}}{\dfrac{1}{m^2} - \dfrac{1}{n^2}}$$

Step 1: Make precise, quantitative observations!
Step 2: Be creative & come up with a model.

How to avoid the Ka-Boom?

***Elapsed time: ~10^{-11} seconds**

Bohr Model

- When Bohr saw Balmer’s formula, he came up with a new model that would predict it and 'solve' the problem of electrons spiraling into the nucleus.
- The Bohr model has some problems, but it's still useful.
- Why doesn’t the electron fall into the nucleus?
  - According to classical physics, *It should!*
  - According to Bohr, *It just doesn’t.*
  - Modern QM will give a more satisfying answer, but you’ll have to wait till next week.

Original paper: Niels Bohr: *On the Constitution of Atoms and Molecules,* Philosophical Magazine, Series 6, Volume 26, p. 1-25, July 1913.)

Bohr's approach:

#1: Treat the mechanics classical (electron spinning around a proton):

- Newton's laws assumed to be valid
- Coulomb forces provide centripetal acceleration.

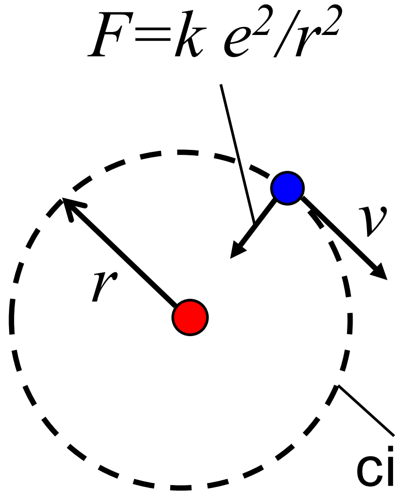
#2: Bohr's hypothesis (Bohr had no proof for this; he just assumed it – leads to correct results!):

- The angular momentum of the electrons is quantized in multiples of $\hbar$.
- The lowest angular momentum is $\hbar$.

$$\boxed{\hbar = h/_{2\pi}}$$

Bohr Model. # 1: Classical mechanics

The centripetal acceleration
$a = v^2 / r$ is provided by the coulomb
force $F = k \cdot e^2/r^2$.
($k = 1/(4\pi\varepsilon_0)$: Coulomb force const.)



Newton's second law → $mv^2/r = k \cdot e^2/r^2$

or: $\boxed{mv^2 = k \cdot e^2/r}$

The electron's kinetic energy is $KE = ½\, m\, v^2$
The electron's potential energy is $PE = -\, ke^2/r$
$\Big\}$ + → E

→ $\boxed{E = KE + PE = -½\, ke^2/r = ½\, PE}$

Therefore: If we know r, we know E and v, etc…

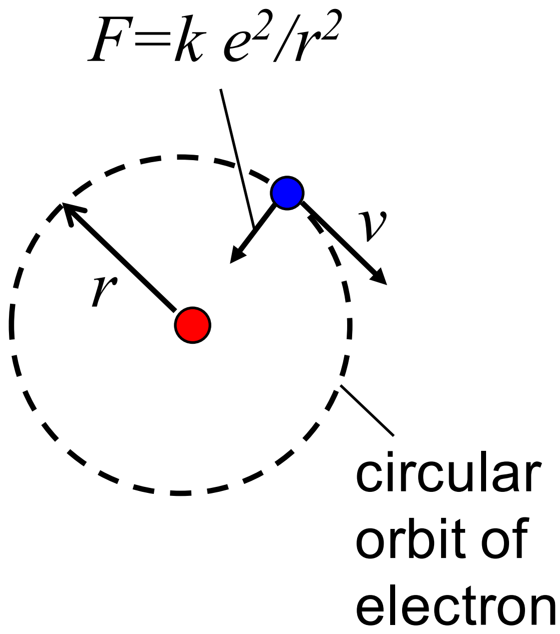
Bohr Model. #2: Quantized angular momentum

Bohr *assumed* that the angular momentum of the electron could only have the quantized values of:

$$L = n\hbar$$

And therefore: $mvr = n\hbar,\ (n=1,2,3\ldots)$

or: $v = n\hbar/(mr)$



Substituting this into $mv^2 = k\cdot e^2/r$ leads to:

$$r_n = r_B n^2, with\ r_B = \frac{\hbar^2}{ke^2 m} = 52.9\,\mathrm{pm}$$

, r_B: Bohr radius

$$E_n = E_R / n^2, with\ E_R = \frac{m(ke^2)^2}{2\hbar^2} = 13.6\,\mathrm{eV}$$

, E_R: Rydberg Energy

Bohr Model. Results

$$r = r_B n^2, with\ r_B = \frac{\hbar^2}{ke^2 m} = 52.9\,\mathrm{pm}$$

, r_B: Bohr radius

$$E_n = E_R / n^2, with\ E_R = \frac{m(ke^2)^2}{2\hbar^2} = 13.6\,\mathrm{eV}$$

, E_R: Rydberg Energy

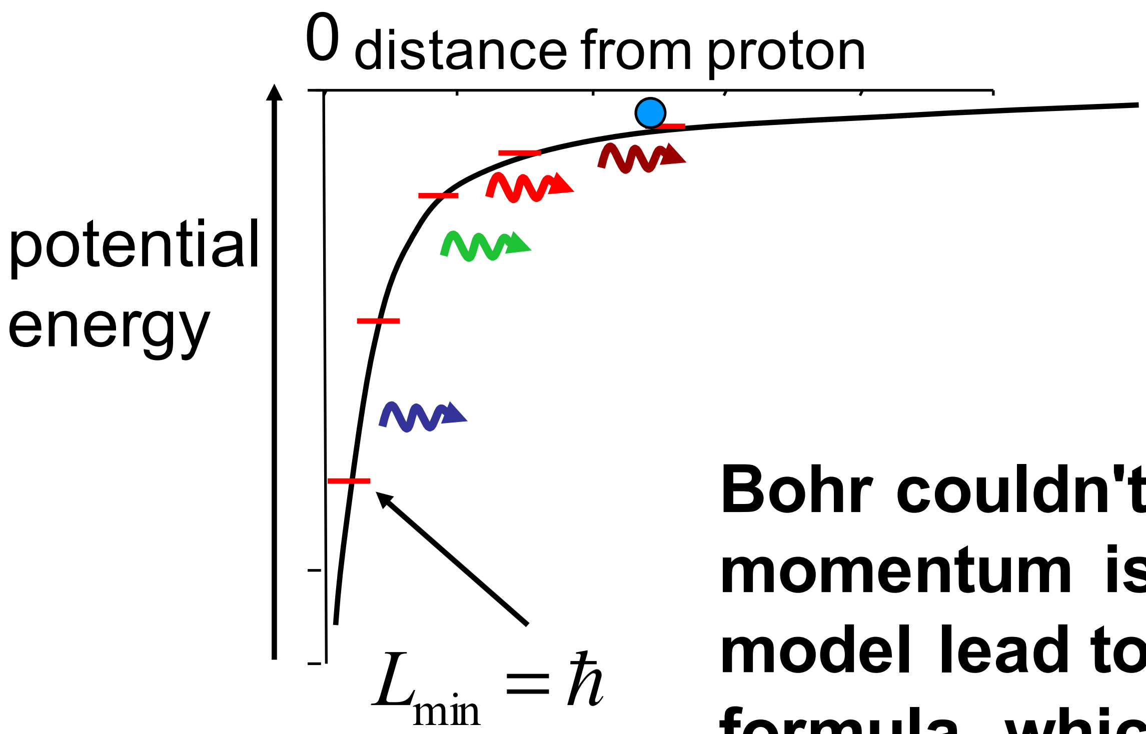
The Bohr model not only predicts a reasonable atomic radius r_B, but it also predicts the energy levels in hydrogen to 4 digits accuracy!

Possible photon energies:

$$E_\gamma = \underset{(n>m)}{E_n - E_m} = E_R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

→ The Bohr model 'explains' the Rydberg formula!!

Only discrete energy levels possible.
Electrons hop down towards lowest level, giving off photons during the jumps. Atoms are stable in lowest level.



Bohr couldn't explain *why* the angular momentum is quantized but his model lead to the Rydberg-Balmer formula, which matched to the experimental observations very well!

He also predicted atomic radii reasonably well and was able to calculate the Rydberg constant.

Successes of Bohr Model

- 'Explains' source of Balmer formula and predicts empirical constant R (Rydberg constant) from fundamental constants: $R={}^{1}/_{91.2\ nm}=mk^2e^4/(4\pi c\hbar^3)$
- Explains why R is different for different single electron atoms *(*called *hydrogen-like ions).*
- Predicts approximate size of hydrogen atom
- Explains (sort of) why atoms emit discrete spectral lines
- Explains (sort of) why electron doesn't spiral into nucleus

Shortcomings of the Bohr model:

- Why is angular momentum quantized yet Newton’s laws still work?
- Why don’t electrons radiate when they are in fixed orbitals yet Coulomb’s law still works?
- No way to know *a priori* which rules to keep and which to throw out…
- Can't explain shapes of molecular orbits and how bonds work
- Can’t explain doublet spectral lines

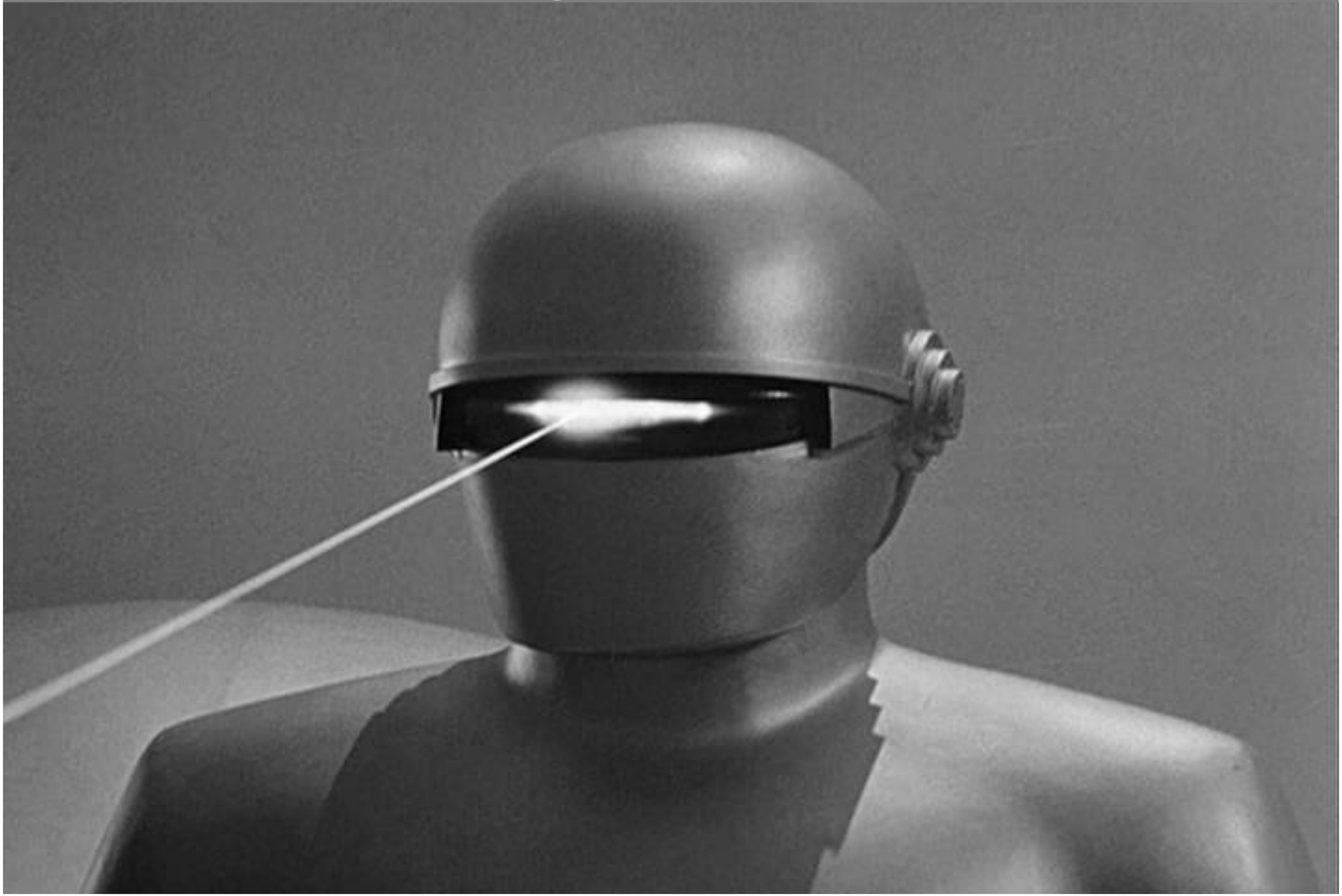
Questions?

Which of the following principles of classical physics is violated in the Bohr model?

A. Coulomb’s law
B. Newton’s F = ma
C. $\boxed{\text{Accelerating charges radiate energy.}}$
D. Particles always have a well-defined position and momentum (Heisenberg’s uncertainty principle)
E. All of the above.

Note that both A & B are used in derivation of Bohr model.

Today: LASERs!




SINGLE-PHOTON DETECTION | LASER CRIMES | DISTANCE LEARNING
May 2011 | Vol. 22 No. 5
An OSA Member Benefit
OPN
Optics & Photonics News
The Life and Legacy of Arthur L. Schawlow
Ultrahigh-Speed Processing Using Space-Time Duality
Acousto-Optic Devices
Applied Optics and the Laser
OSA

What's so special about LASER light?

A) It doesn't diffract when it goes through two slits.

B) All the photons in the laser beam oscillate in-phase with each other.

C) The photons in the laser beam travel a little bit faster because they all go the same direction.

D) Laser light is pure quantum light, and therefore, cannot be described with classical EM theory.

E) Laser light is purely classical light, and therefore, it is incompatible with the photon picture.

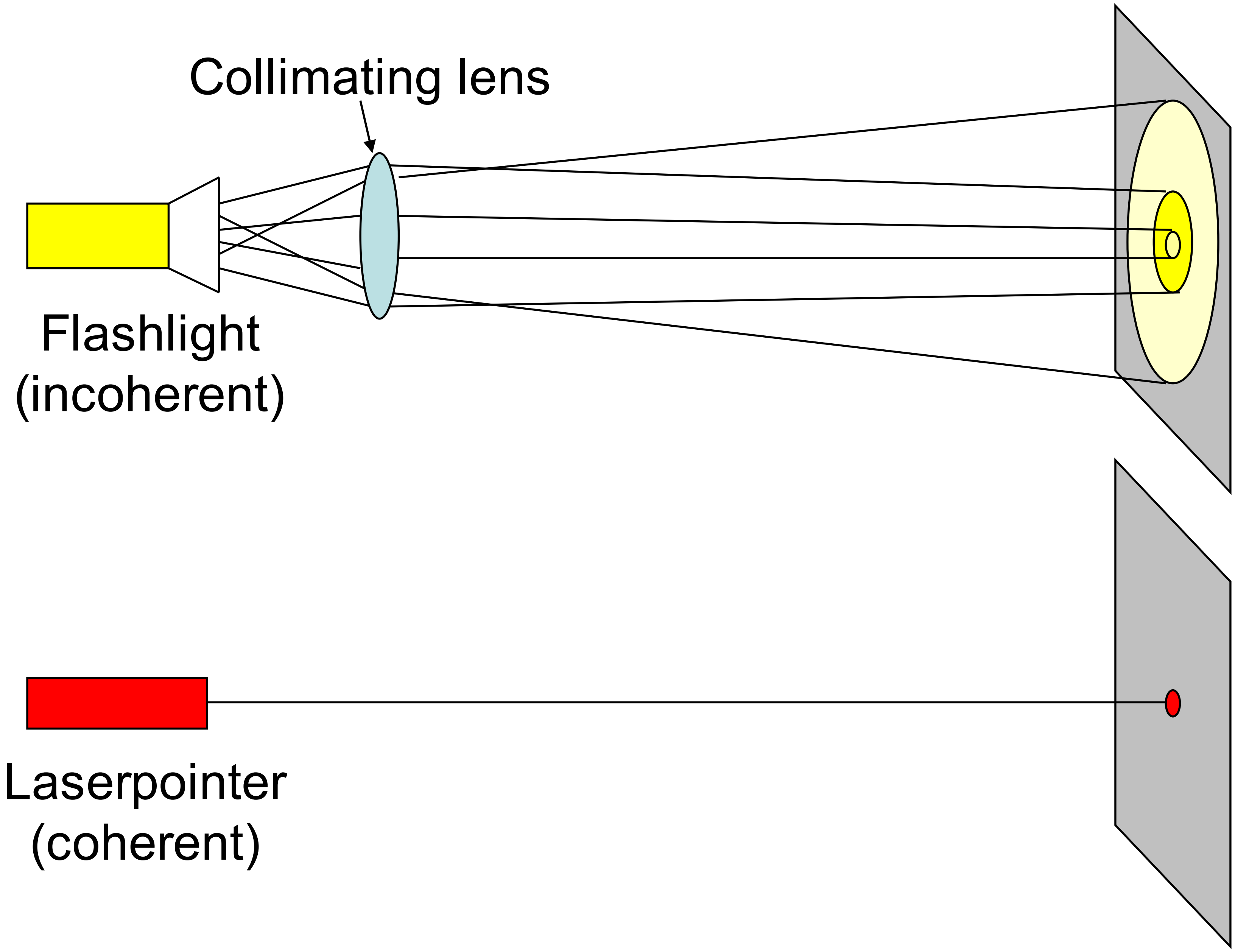
What’s so special about LASER light?

Remember the word 'coherence'?
(We used it occasionally in context with interferometers, diffraction and with photons.)

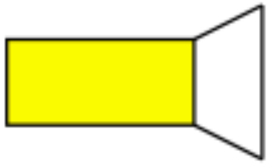
<u>There are two types of coherence:</u>
- Spatial coherence
- Temporal coherence

LASERs produce light with excellent spatial and temporal coherence.

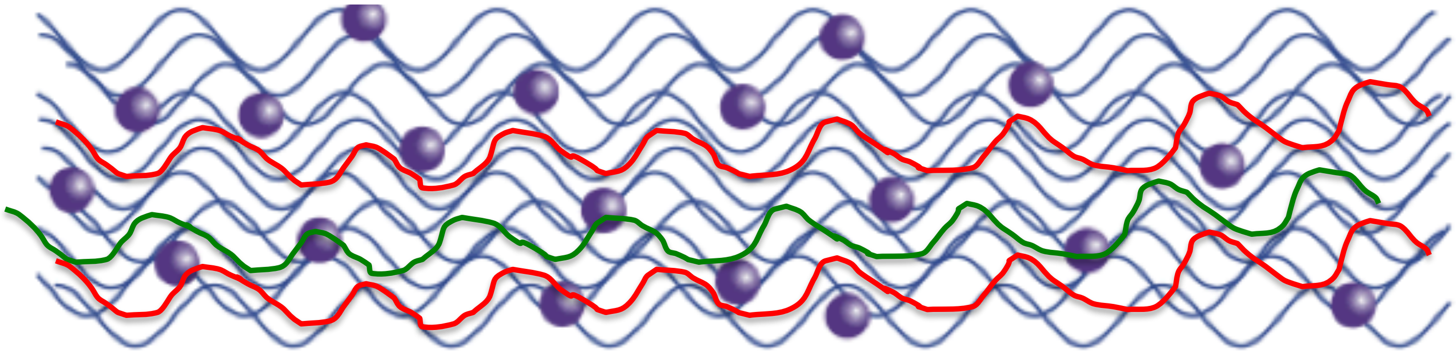
Spatial coherence



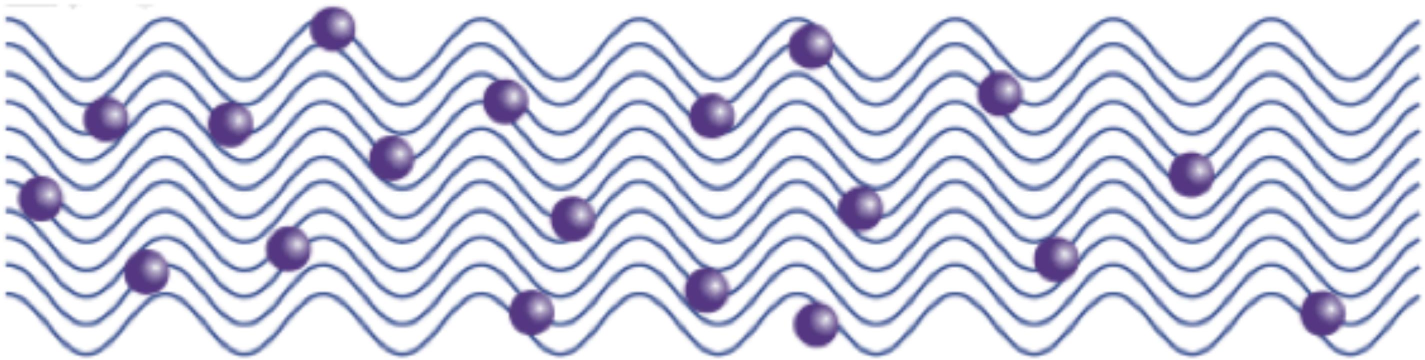
Spatial Coherence 空間同調性



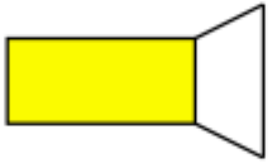
Flashlight
(incoherent)



Laser
(coherent)



不同方向
不同顏色

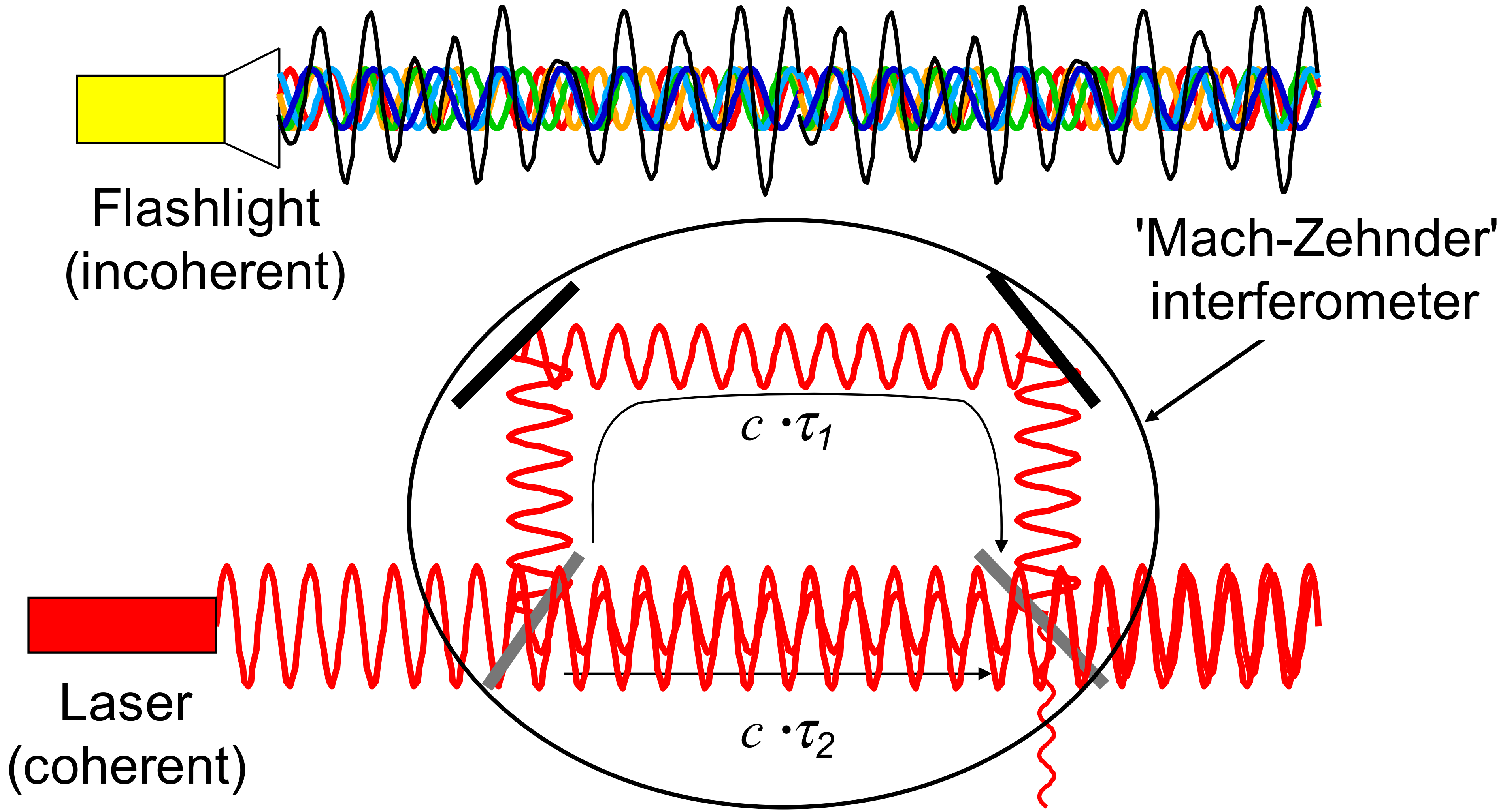


Flashlight
(incoherent)

同方向
同顏色

Laser
(coherent)

Temporal coherence



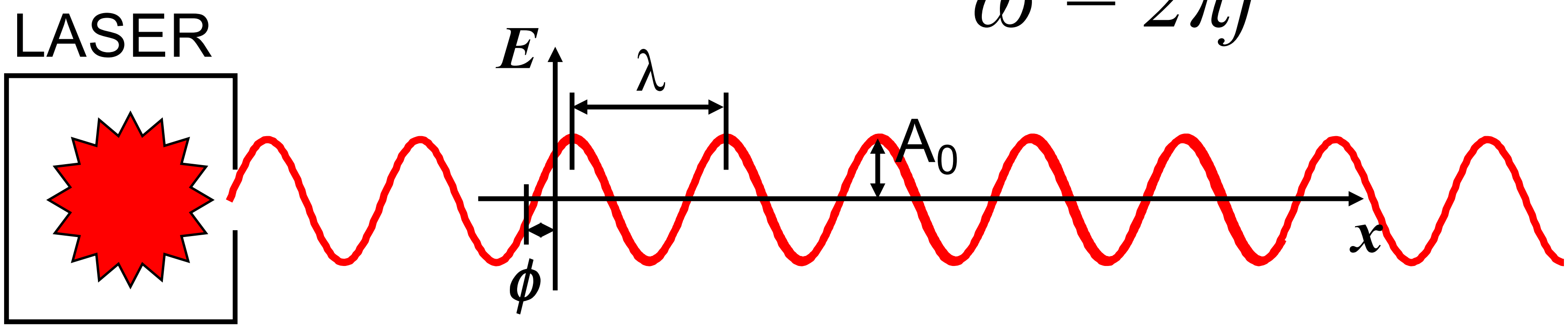
For very stable lasers: $(\tau_1 - \tau_2)_{max}$ ~ 1 second.
(This corresponds to 300'000 km path difference!)

Remember this one (from class 3):

E-field (for a single color):

$$E(x,t) = A_0 \sin(\omega t + {}^{(x \cdot \omega)}/_{c} + \phi)$$

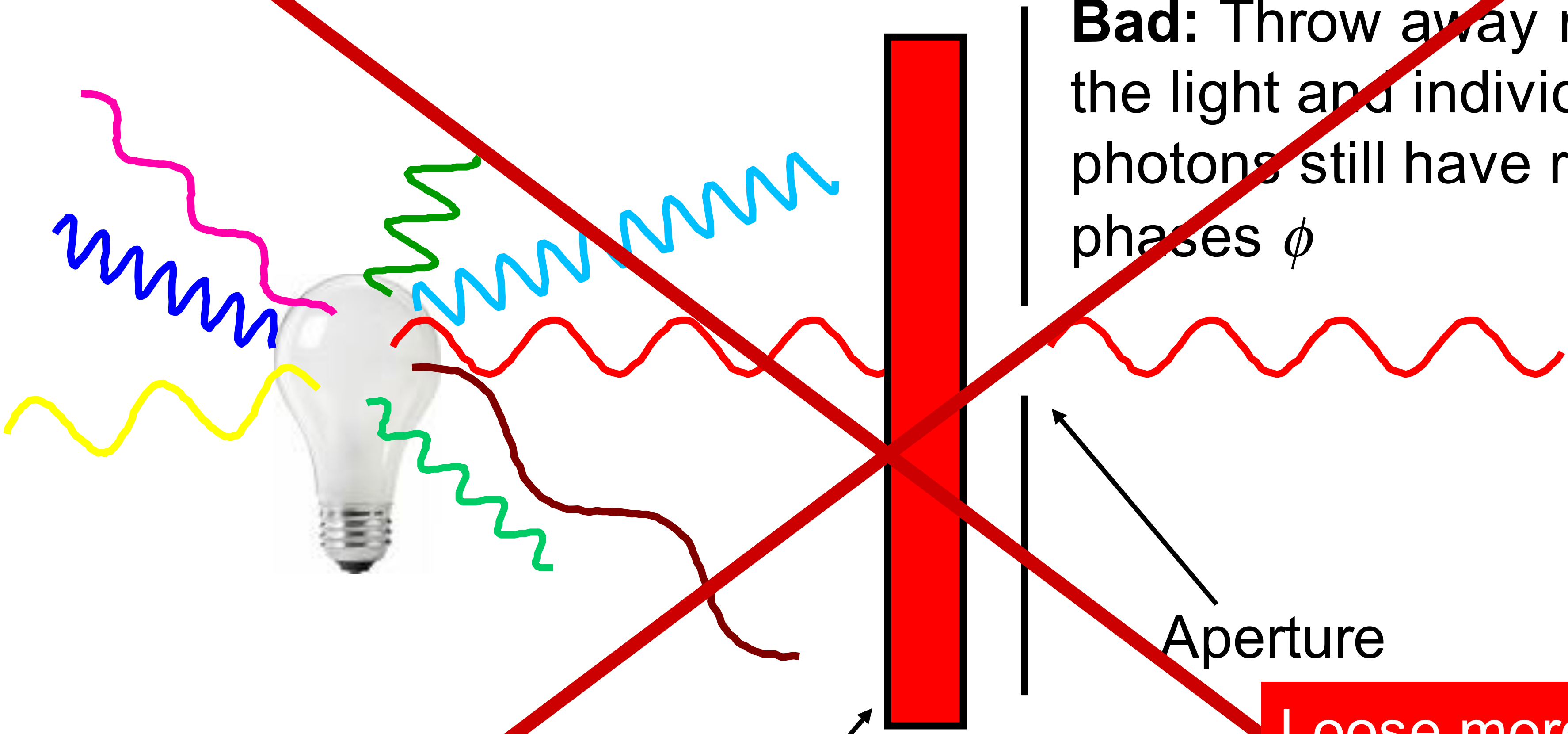
$$\omega = 2\pi f$$



<u>Temporal coherence:</u> ω and ϕ are very well defined constants (i.e. time independent).

Let's make a coherent light pulp!

Bad: Throw away most of the light and individual photons still have random phases ϕ



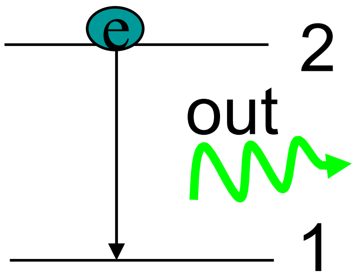
Loose more than a trillion to one in power!(=bad idea)

Remember: Coherent light requires that ω and ϕ are constants.

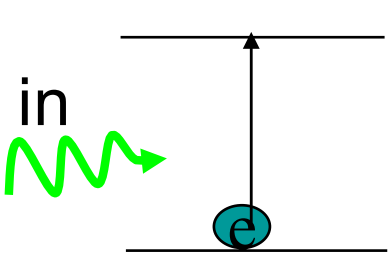
How can we make identical photons?
（製造相同的光場）

Clone them! （複製它！！）

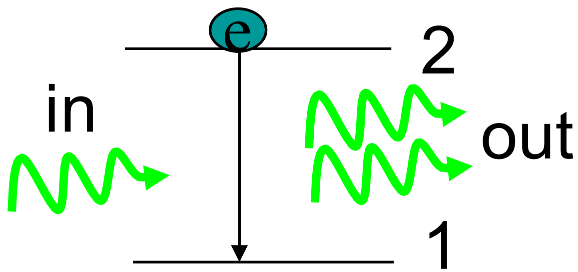
How light interacts with atoms



spontaneous emission of light: Excited atom emits one photon.



absorption of light: Atom absorbs one photon



stimulated emission: clone the photon

-- A. Einstein

Surprising fact: Chance of stimulated emission of excited atom **EXACTLY** the same as chance of absorption by lower state atom. Critical when making a laser.

Laser: Stimulated emission to clone photon many times (~10^{20}/s)

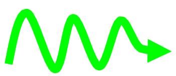
Light **A**mplification by **S**timulated **E**mission of **R**adiation

Spontaneous emission

Random phase
Random direction
Similar energy (as absorbed photon)

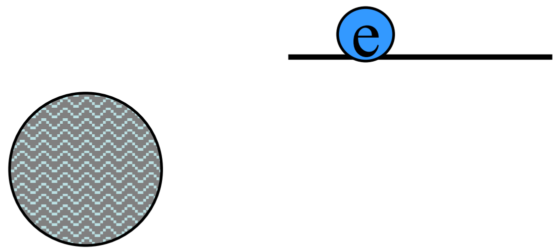
Legend:

Photon



Atom in ground state

Atom in excited state

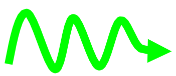


Stimulated emission

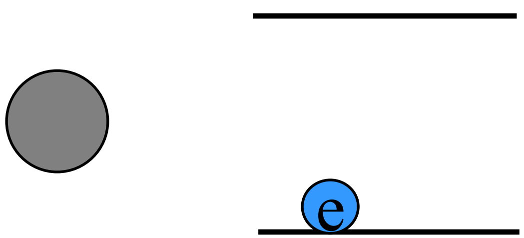
Identical phase
Identical energy
Identical momentum

Legend:

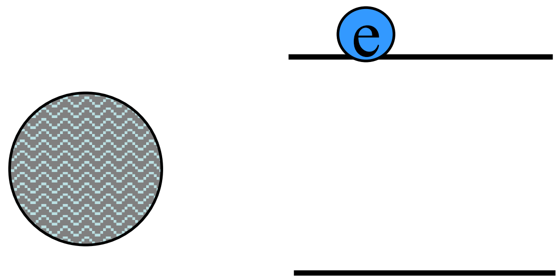
Photon

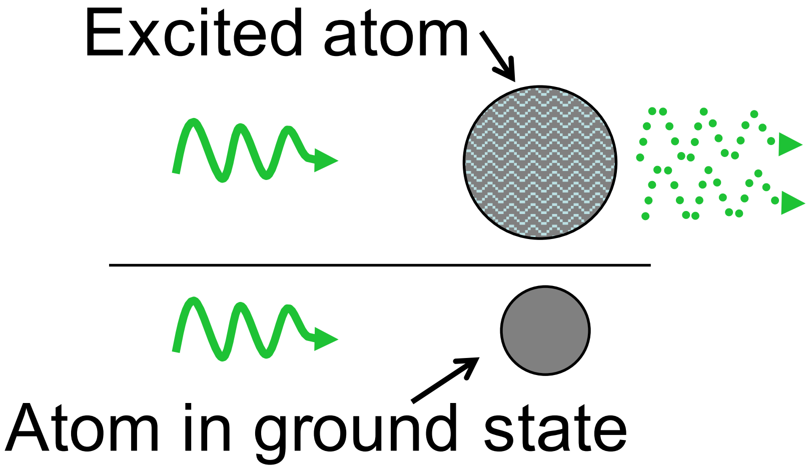


Atom in ground state



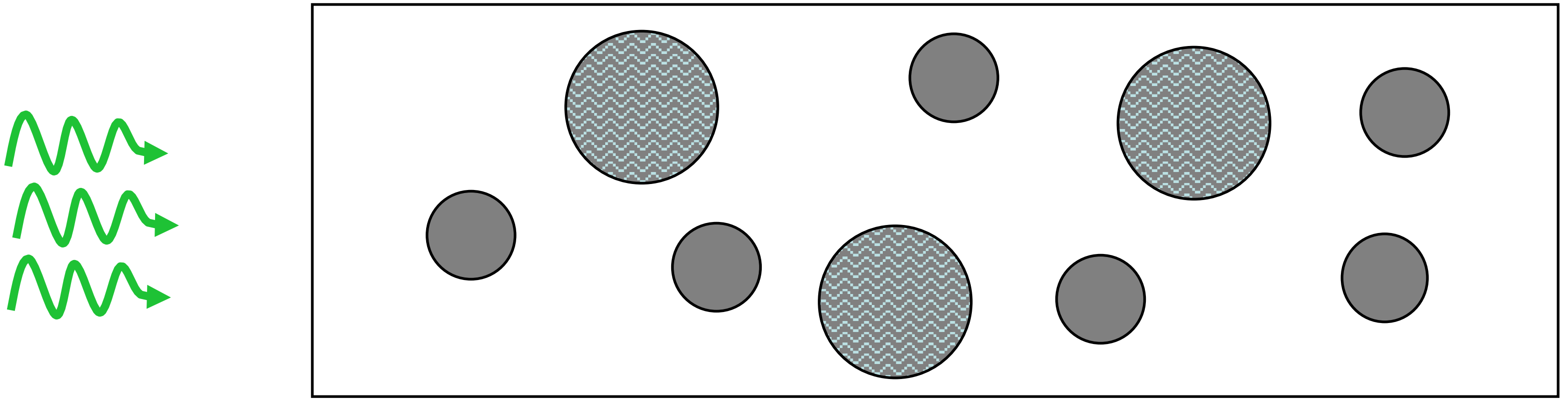
Atom in excited state





Chance of stimulated emission of excited atom is **EXACTLY** the same as chance of absorption by ground state atom.

Glass tube below contains 9 atoms. Some are excited some not excited (as shown). Light enters the tube on the left:



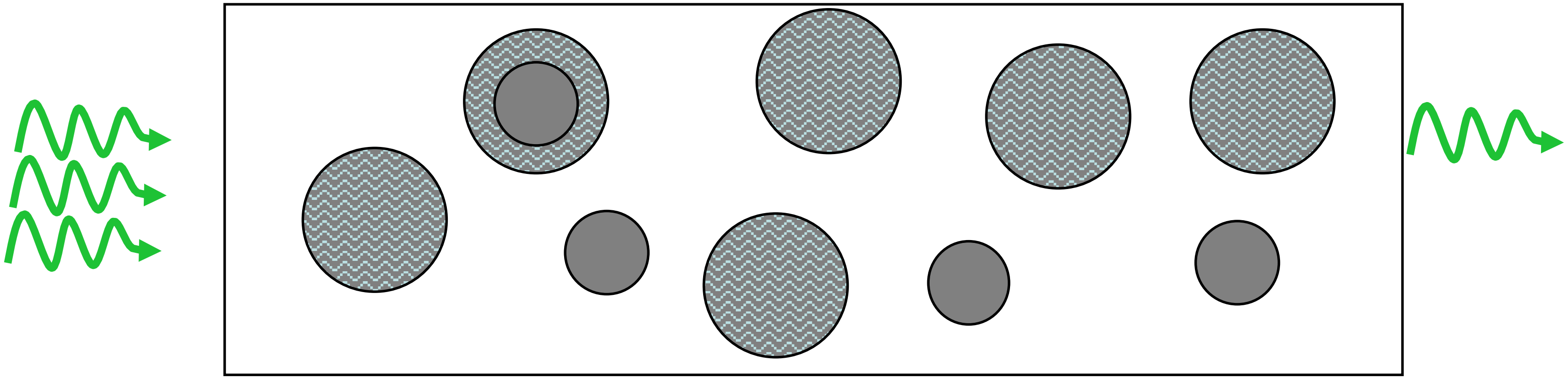
For the condition above: what do you expect?

a. More photons will come out (on the right) than go in.

b. Fewer photons will come out (on the right) than go in.

c. Same number as go in,

d. None will come out.



b. less come out right

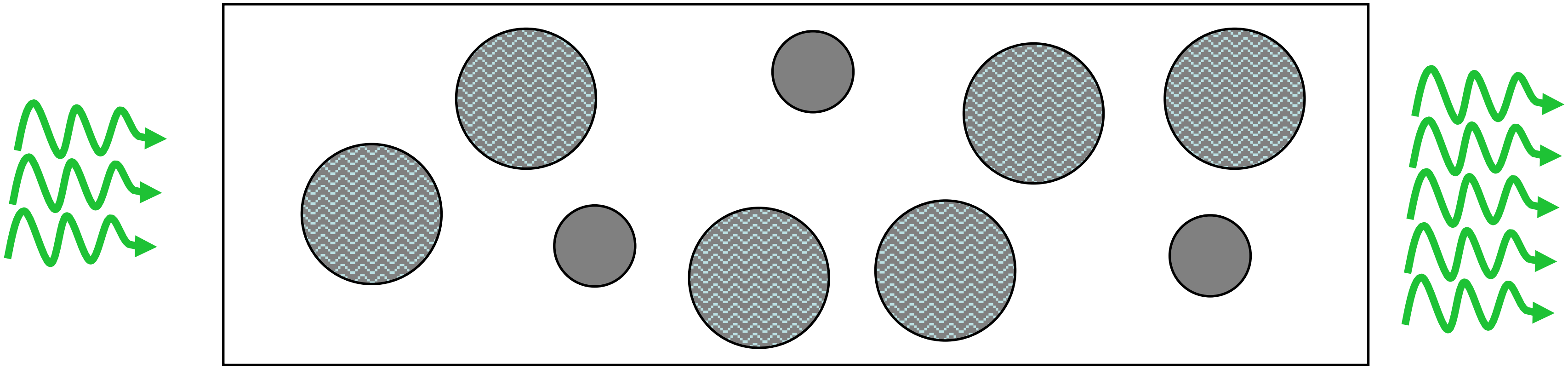
3 excited atoms can emit photons,
6 ground state atoms can absorb. **Absorption wins.**

Think about statistics / probabilities

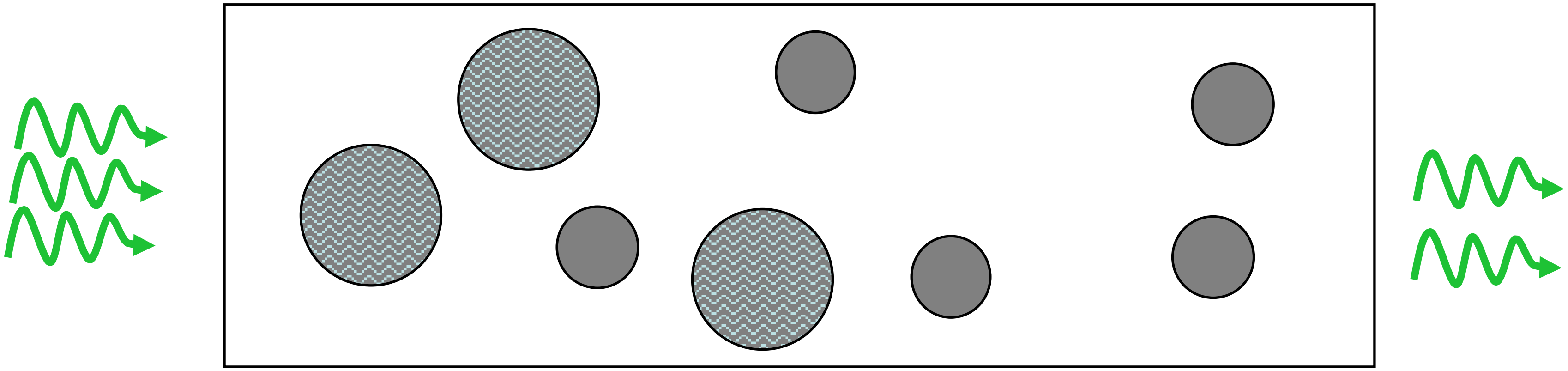
To increase the number of photons when going through the atoms, more atoms need to be in the upper energy level than in the lower.

→ Need a "Population inversion"

(This is the hard part of making laser, b/c atoms jump down so quickly.)



$N_{upper} > N_{lower}$, more cloned than eaten.



$N_{upper} < N_{lower}$, more eaten than cloned.