If the total energy $E$ of the electron is LESS than the work function of the metal, $V_0$, when the electron reaches the end of the wire, it will...

A. stop.
B. be reflected back.
C. exit the wire and keep moving to the right.
D. either be reflected or transmitted with some probability.
E. dance around and sing, “I love quantum mechanics!”
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http://phet.colorado.edu/en/simulation/quantum-tunneling
So, the thinner and/or lower the barrier, the easier it is to tunnel ...

And particle can escape...

Application: Alpha-Decay, Scanning tunneling microscope
Application of Quantum Tunneling:
Radioactive decay
George Gamow: 1928
Radioactive decay
(Quantum tunneling – George Gamow)

Nucleus is unstable → ejects alpha particle (2 neutrons, 2 protons)
Typically found for large atoms with lots of protons and neutrons.

Polonium-210
84 protons,
126 neutrons

- Proton (positive charge)
- Neutron (no charge)

Nucleus has lots of protons and lots of neutrons.

Two forces acting in nucleus:
- Coulomb force .. Protons really close together, so very big repulsion between protons due to coulomb force.
- Nuclear force (attraction between nuclear particles is very strong if very close together) … called the STRONG Force.
Radioactive decay

- Proton (positive charge)
- Neutron (no charge)

In alpha-decay, an alpha-particle is emitted from the nucleus.

Polonium-210
84 protons, 126 neutrons

Lead-206
82 protons, 124 neutrons

This raises the ratio of neutrons to protons … makes for a more stable atom.
(Neutrons are neutral.. no coulomb repulsion, but nuclear force attraction)
How to figure out what's going on?
Starting point: *Always* look at potential energy curve for particle!

Nucleus  \( \rightarrow \) New nucleus  \( + \) Alpha particle  \( \rightarrow \) KE

Nucleus (Z protons, & bunch of neutrons)  \( \rightarrow \) New nucleus (Z-2 protons, bunch of neutrons-2)  \( + \) Alpha particle (2 protons, 2 neutrons)

Now look at this system... as the distance between the alpha particle and the nucleus changes.

As we bring the \( \alpha \) particle closer to the core, what happens to potential energy?
As bring $\alpha$ closer, what happens to potential energy?

V=0 At a great distance

D. Something else
First: Coulomb repulsion

Takes energy to push $\alpha$ towards the nucleus, so potential energy must increase.

Then:
At edge of the nucleus ($\sim 8 \times 10^{-15}$ m), Nuclear (Strong) force starts acting
Strong attraction between nucleons
Potential energy drops dramatically

\[
V(r) = \frac{kq_1q_2}{r} = \frac{k(Z-2)(e)(2e)}{r}
\]
Potential energy curve for the $\alpha$ particle

Nucleus
(Z protons, & bunch of neutrons)

New nucleus
(Z-2 protons, bunch of neutrons)

Alpha particle
(2 protons, 2 neutrons)

Look at this system… as the distance between the alpha particle and the nucleus changes.

As we bring the $\alpha$ particle closer to the core, what happens to potential energy?

Coulomb repulsion:

$$V(r) = \frac{kq_1q_2}{r} = \frac{k(Z-2)(e)(2e)}{r}$$

V=0 for $r \to \infty$
very small $r$ (~1 fm): nuclear force dominates

\[
V(r) \approx 30 \text{ MeV}
\]

‘Large’ $r$: coulomb force dominates

\[
V(r) = \frac{kq_1q_2}{r} = \frac{k(Z-2)(e)(2e)}{r}
\]

Edge of the nucleus (~8x10^{-15} m), Nuclear (‘Strong’) force starts acting strong attraction between nucleons. Potential energy drops dramatically.
What’s the kinetic energy of this particle inside the nucleus?

E: Something else
Edge of the nucleus (~$8 \times 10^{-15}$ m), Nuclear (‘Strong’) force starts acting strong attraction between nucleons. Potential energy drops dramatically.

$$V(r) = \frac{kq_1q_2}{r} = \frac{k(Z-2)(e)(2e)}{r}$$

Energy

Small $r$: Nuclear force dominates

Large $r$: coulomb force dominates

~30 MeV

1 to 10 MeV

Review: Radioactive decay
What’s the kinetic energy of this particle inside the nucleus?

- A
- B
- C
- D
- E: Something else

Energy

V(r)
What would the kinetic energy of that particle be after it tunneled out from the nucleus?

E: Something else
So we found that the particle has less kinetic energy outside than inside the nucleus. Did it lose energy?

A) Yes.
B) No.
C) Impossible to tell. Need to solve Schröd. equ. first.
Wave function picture:

Exponential decay in the barrier

Wave function of the particle inside the potential well: Large KE $\rightarrow$ small Wavelength

Wave function of the free particle: ‘small’ KE $\rightarrow$ Large wavelength

$\sim 100\text{MeV}$ of KE inside the nucleus

$\sim 1-10\text{MeV}$ of KE outside
Observations show Alpha-particles from the same chemical element exit with a range of energies.

Different KE in different isotopes

# neutrons influence nuclear potential
Observe $\alpha$ particles from **different isotopes** (same # protons, different # neutrons), exit with different amounts of energy.

$$\alpha = \sqrt{\frac{2m}{\hbar^2}}(V - E)$$

1. Less distance to tunnel
2. $V-E$ is smaller ($\rightarrow$ smaller $\alpha$) → Wave function doesn’t decay as much before reaches other side … more probable!

Isotopes that emit higher energy alpha particles, have shorter lifetimes!!!
Solving Schrödinger equation for this potential energy is hard!

Square barrier is much easier... and get almost the same answer!
Nuclear Physics Sim

phet.colorado.edu/simulations/sims.php?sim=Alpha_Decay
Application: Scanning Tunneling Microscope
→ 'See' individual atoms!

Use tunneling to measure very(!) small changes in distance. Nobel prize winning idea! (1986)
Limitation: only works on conductive surfaces.
Look at current from sample to tip to measure gap.

Electron tunnels from sample to tip.

How would $V(x)$ look like after an electron tunneled from the sample to the tip if sample and tip were isolated from each other? Note: Sample and tip are electrically isolated from each other.

A. same as before.  
B. $V$ in tip higher, $V$ sample lower.  
C. $V$ in tip lower, $V$ sample higher.  
D. $V$ same on each side as before but barrier higher.

ans. b. electron piled on top (in energy) of many other electrons that contribute to $V(x)$. Add electron, makes higher $V(x)$, remove makes lower. So what does next electron want to do?
Correct picture of STM-- voltage applied between tip and sample → constant potential difference between tip and sample. *Figure out what potential energy looks like in different regions so can calculate current, determine sensitivity to gap distance.*

What does V tip look like?
A. higher than V sample
B. same as V sample
C. lower than V sample
D. tilts downward from left to right
E. tilts upward from left to right
Correct picture of STM-- voltage applied between tip and sample.

*Potential energy in different regions so can calculate current, determine sensitivity to gap distance.*

What is potential in air gap approximately?

Linear connection!
Note: changing V will change barrier, and hence tunneling current.
Attached to +0V supply

Attached to +5V supply

(a)

(b)

(c)

(d)

(e) None of the above is correct
cq. if tip is moved closer to sample which picture is correct?

a. 

b. 

c. 

d. 

tunneling current will go: (a) up, (b) stay same, (c) go down
How sensitive to distance?  
Need to look at numbers.

Tunneling rate: \( T \sim (e^{-\alpha d})^2 = e^{-2\alpha d} \)

How big is \( \alpha \)?

\[
\alpha = \frac{\sqrt{2m(V_0 - E)}}{\hbar}
\]

If \( V_0 - E = 4 \text{ eV} \), \( \alpha = 1/(10^{-10} \text{ m}) \)

So if \( d \) is \( 3 \times 10^{-10} \text{ m} \), \( T = e^{-6} = .0025 \)

add 1 extra atom (\( d \sim 10^{-10} \text{ m} \)), how much does \( T \) change?

\( T = e^{-4} = 0.018 \)

→ Decrease distance by diameter of one atom:
→ Increase current by factor 7!
In typical operation, STM moves tip across surface, adjusts distance to keep tunneling current constant. Keeps track of how much tip moves up and down to keep current constant. 

→ Scan in x+y directions. 

Draw a 2D map of surface
STM (picture with reversed voltage, works exactly the same)

end of tip always atomically sharp
Scanning Tunneling Microscope

Requires very precise control of the tip position and height. How to do it?

With a piezoelectric actuator!

Typical piezo: 1V $\rightarrow$ 100nm displacement.
Applying 1mV moves tip by one atom diameter (~100pm)
“See” $|\Psi|^2$ of electrons!

The probability to find an electron that is trapped inside this ring of atoms is highest at the place, where the square of the amplitude of the electron wave function is largest.
Crystal of Ni atoms

(Remember the Davisson-Germer electron diffraction experiment?)

Fe atoms on Cu surface
• Quantum harmonic oscillator
• Brief review of “Eigenstates”
Flash memory

Ref: http://volga.eng.yale.edu/index.php/FlashDrives/MethodsAndMaterials
Flash memory

![Diagram of Flash memory]

Control gate
Gate oxide
Floating gate

S
D

$n^+$
$p$-$Si$
$n^+$

Cause different conductivity

Ref: http://volga.eng.yale.edu/index.php/FlashDrives/MethodsAndMaterials
Programming a bit
(“Fowler-Nordheim tunneling”)

VCG
Erasing a bit
(“Fowler-Nordheim tunneling”)
Another application: Quantum Tunnel Transistor (‘Controlled tunneling’)
In past classes we did:
• Free particle: \( V=0 \) everywhere
• Particle in rigid box: \( V=0 \) inside, \( V=\infty \) outside
• Particle in non-rigid box: \( V=0 \) ins., \( 0<V<\infty \) outside

Next:
• Harmonic oscillator: \( V(x) \propto x^2 \)
One last 1D example: The harmonic oscillator

Classical harmonic oscillator:
Mass ‘m’ experiences restoring force \( F = k \cdot x \) (x: displacement from equilibrium point).

In QM: don’t want to deal with forces. What can we do?

Can derive corresponding PE function:
\[
F = k \cdot x \quad \rightarrow \quad V(x) = \int_{0}^{x} F(x')dx' = \frac{1}{2} kx^2
\]
Why is this important?
Vibration in molecules!! (Used for molecular detection)

Potential close to the ground-state looks very similar to $x^2$

$V(x) \approx \frac{1}{2} kx^2$
Real time molecular detection
Real time molecular detection
Simple harmonic oscillator

All we have to do is to solve the Schrödinger equation with a parabolic potential:

\[
-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + \frac{1}{2} kx^2 \psi(x) = E \psi(x)
\]

(... or a lengthy calculation...)

\[
\psi_n(x) = \sqrt{\frac{1}{2^n n! \cdot \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}}} \cdot e^{-\frac{m\omega x^2}{2\hbar}} \cdot H_n \left(\sqrt{\frac{m\omega}{\hbar}} x\right), \quad n = 0, 1, 2, \ldots
\]

\[
H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} \left(e^{-x^2}\right)
\]

("Hermite polynomials")

\[
E_n = \hbar \omega \left(n + \frac{1}{2}\right)
\]

Energy levels
Simple harmonic oscillator (cont.)

For \( n = 0 \):
\[
E_0 = \frac{1}{2} \hbar \omega, \quad \text{with } \omega = \sqrt{\frac{k}{m}}
\]

‘Ground state energy’

\[
\psi_0(x) = A_0 e^{-\frac{x^2}{2L^2}}, \quad \text{with } L = \sqrt{\frac{\hbar}{m\omega}}
\]

Heisenberg: \( \Delta x \cdot \Delta p \geq \frac{\hbar}{2} \)

For \( \psi_0 \) we find: \( \Delta x = \frac{1}{2} L = \frac{1}{2} \sqrt{\frac{\hbar}{m\omega}} \)

And: \( \Delta p \approx \sqrt{\hbar m \omega} \rightarrow \Delta x \cdot \Delta p \approx \frac{\hbar}{2} \)

Fulfills Heisenberg uncertainty principle. (Of course!)
Simple harmonic oscillator (cont.)

\[ E_n = \hbar \omega \left( n + \frac{1}{2} \right) \]
Probability density
(“Where is the particle most likely to be found”)

Classical probability $1/v_x$
Quantum probability $\Psi^2$

$n=0$

$n=2$

$n=1$

$n=3$

$n=10$