Q1:

The famous Michelson-Morley Experiment tests if the speed of light in all inertial frames…

A – …is the same in all directions.

B – …is the same in air and in vacuum.

C – …is the same in accelerating frames.

D – …does depend on the wavelength or color.

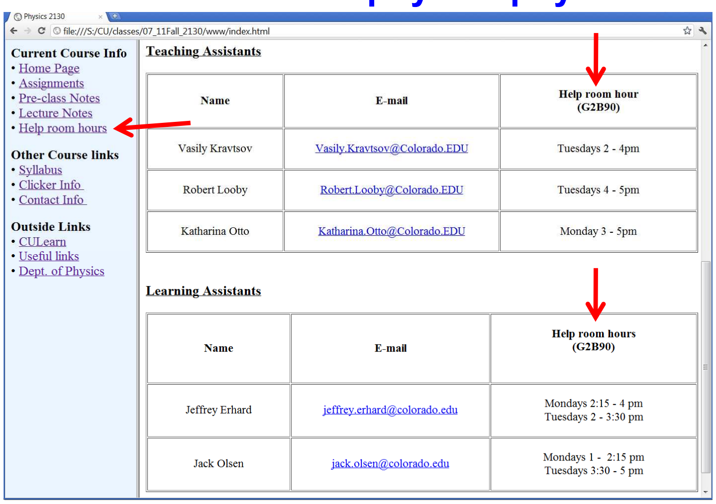
E – …does change when reflected by mirror.

Announcements

- Reading for Wednesday: TZD 1.10 – 1.11
- Homework #1 due on Wednesday noon. (Remember: no late HWs accepted!)
- Register your clicker!
- Help room hours posted on course page:

http://www.colorado.edu/physics/phys2130

www.colorado.edu/physics/phys2130



Physics 2130

file:///S:/CU/classes/07_11Fall_2130/www/index.html

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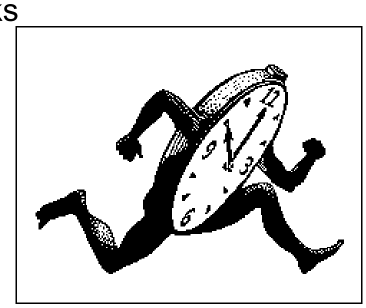
Today

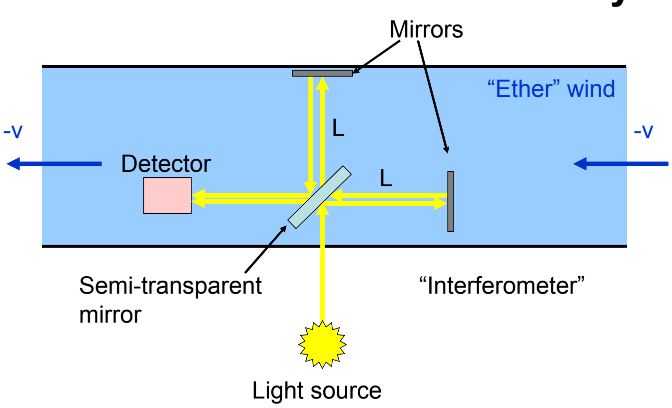
First: Wrap-up Michelson Morley.

Then: Let's talk about time!

- Synchronizing clocks
- Time dilation and other weird stuff!

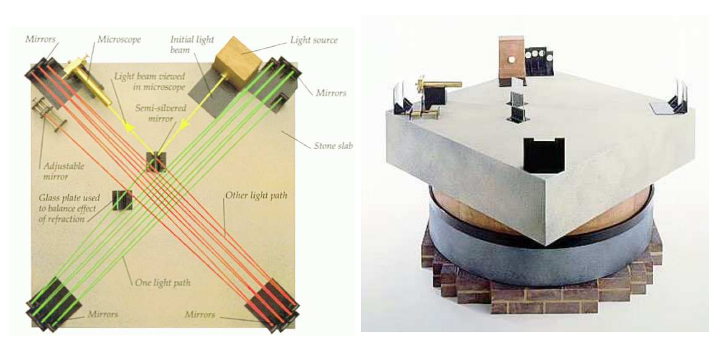


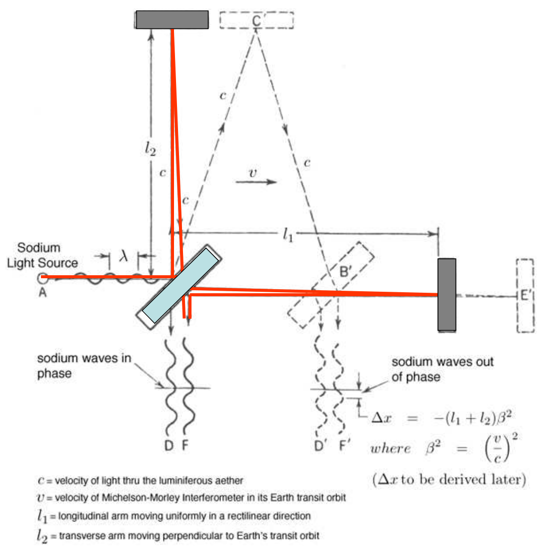
Remember Michelson and Morley?

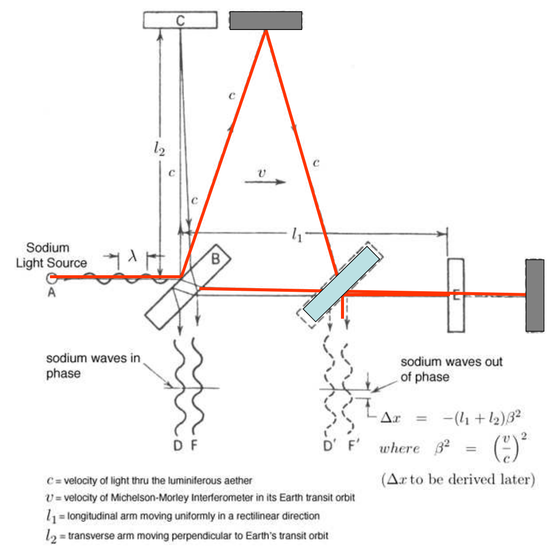


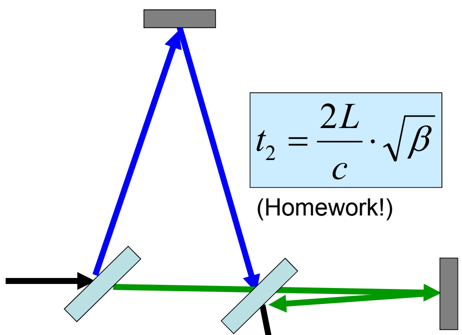
The detector measures differences in the position of the maxima or minima of the light-waves from each of the two interferometer arms.

The 2nd (improved) setup (~1887)









$$t_2 = \frac{2L}{c} \cdot \sqrt{\beta}$$

(Homework!)

$$t_1 = \frac{L}{c-v} + \frac{L}{c+v} = \frac{2L}{c} \cdot \beta \quad , \beta = \frac{1}{1-\frac{v^2}{c^2}}$$

$$\Delta t = t_1 - t_2 \approx \frac{L \cdot v^2}{c^3}, \quad \Delta L \approx L \cdot \frac{v^2}{c^2}, \quad \frac{\Delta L}{\lambda} \approx \frac{L}{\lambda} \cdot \frac{v^2}{c^2}$$

Michelson-Morley experimental results

Over a period of about 50 years, the Michelson-Morley experiment was repeated with increasing levels of sophistication. The overall result is a high level of confidence that **there is absolutely no effect from an 'ether'!!!**

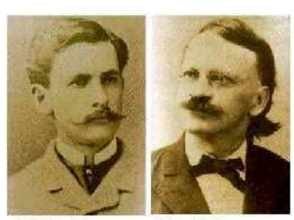
	L (cm)	Calc.	Meas.	Ratio
Michelson, 1881	120	0.04	0.02	2
Michelson & Morley 1887	1100	0.40	0.01	40
Morley & Miller, 1902-04	3220	1.13	0.015	80
Illingworth, 1927	200	0.07	0.0004	175
Joos,1930	2100	0.75	0.002	375
Today (Ligo, optical clocks etc.)				$>10^{12}$

Shankland, et al., Rev. Mod. Phys. 27, 167 (1955)

Michelson and Morley

They thought that the experiment was a complete failure because no effect was found.

Michelson was awarded the Nobel Prize in 1907!!



A.A. Michelson 1852 - 1931 E.W. Morley 1838 - 1923

True result:

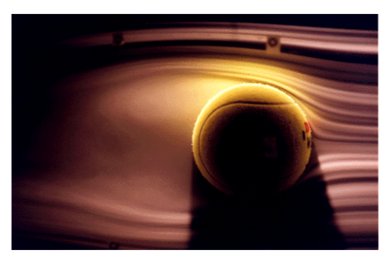
Speed of light is the same in all directions!

Homework (was part of your reading assignment):
Work out the math for this experiment (TDZ, Chapter 1.5)

Yes, but...

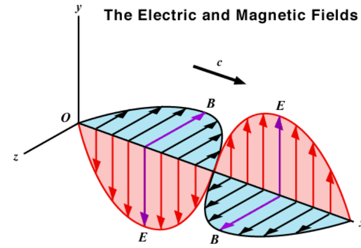
Q: What if the ether is "dragged along" the surface of the earth, like air flowing around a tennis ball?

A: If so, this would require a "viscosity" of the ether, and would require re-writing Maxwell's equations.



Remark: Lots of effort tried to save the idea of the 'ether', but none held up.

There is no ether!



EM waves are wavew without an ocean!

Electromagnetic waves are special! A time-varying electric field induces a magnetic field, and vice-versa. A propagation medium is not necessary/nonexistent.

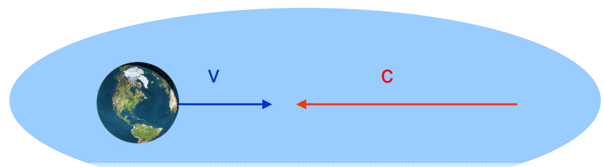
Einstein's Second Postulate of Relativity

The speed of light is the same in all inertial frames of reference.

This was still new in 1905 when Einstein proposed it. Now it has been tested experimentally many times.

Q2:

There is no ether!



Suppose the earth moves through space with speed v. A light wave traveling at speed c with respect to faraway stars is heading in the opposite direction. According to *Einstein's relativity*, what is the **magnitude** of the speed of the light wave as viewed from the earth? (Assume the earth is not accelerating).

a) |c| b) |c|+|v| c) |c|-|v| d) |v|-|c| e) something else!

Q3:

Suppose you're in a spaceship traveling through the solar system at at a constant speed of one-half impulse power, v = 1.5×10^8 m/s. You fire a pulse of laser light out the front of your vessel. (Speed of light = 3.0×10^8 m/s).

Q: How fast do you see the pulse leave your ship?

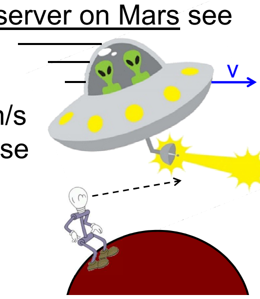
a) 1.5×10^8 m/s b) 3.0×10^8 m/s
c) 4.5×10^8 m/s d) none of these

Q4:

Suppose you're in a spaceship traveling through the solar system at at a constant speed of one-half impulse power, v = 1.5×10^8 m/s. You fire a pulse of laser light out the front of your vessel. (Speed of light = 3.0×10^8 m/s).

Q: How fast does <u>an inertial observer on Mars</u> see the pulse leave your ship?

a) 1.5×10^8 m/s b) 3.0×10^8 m/s
c) 4.5×10^8 m/s d) none of these



Remember!

The speed of light is the same in all inertial frames of reference. *Einstein, 1905*

Now it's time to talk about time!

- Measuring time in one frame
- Synchronization of clocks
- Measuring time in different frames

Recall 'event' (x, y, z, t)

Last Wednesday we have argued that to describe a physical event, we must specify both: where something is – say at (x, y, z) in some inertial coordinate system – and what time it was there – say at time t according to some clock. **But what clock?**

Q

Q5:

Quiz on the reading

(No discussions during this one)

In a given reference frame, the time of an event (x,y,z,t) is given by…

a) the time the observer at the origin sees it.
b) the time that any observer anywhere in the frame sees it.
c) the time according to the clock nearest the event when it happens.
d) the time according to a properly synchronized clock nearest the event when it happens.
e) the time according to a Swiss watch nearest to the event when it happens.

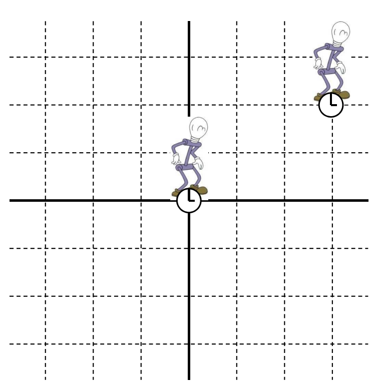
Q6:

Time of an event

Lightning strikes the top of Bear Mountain, generating a clap of thunder. At what time did the lightning strike?

A – At the instant you hear the thunder.
B – At the instant you see the lightning.
C – Very slightly before you see the lightning.
D – Very slightly before you hear the thunder.
E – Some time between when you see the lightning and when you hear the thunder.

'Events' in one reference frame

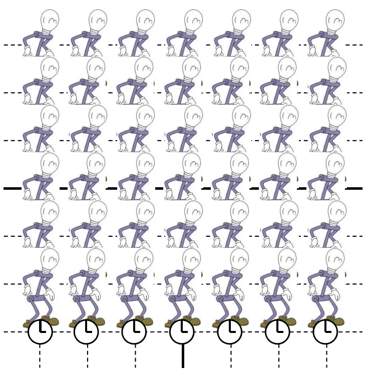


An observer at (0,0) has a clock; events there are covered.

An observer at (3m,2m) had better have a clock too, if we want to know about events there.

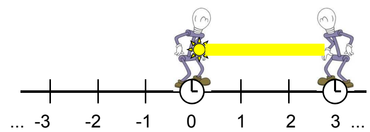
And, the two clocks had better show the same time.

‘Events’ in one reference frame



And there had better be clocks everywhere, so you don’t miss any event.

Synchronizing clocks



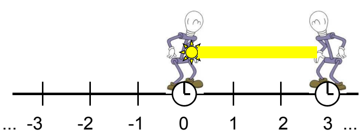
At the origin, at three o’clock, the clock sends out a light signal to tell everybody it’s three o’clock.

Time passes as the signal gets to the clock at x = 3m.

When the signal arrives, the clock at x=3m is set to 3:00.

Does this scheme work?

Well, didn’t quite work…



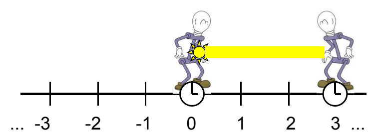
At the origin, at three o’clock, the clock sends out a light signal to tell everybody it’s three o’clock.

Time passes as the signal gets to the clock at x = 3m.

If you do this, then the clock at x = 3m is 10 ns slow, because of the delay!

~~When the signal arrives, the clock at x=3m is set to 3:00.~~

Synchronizing clocks

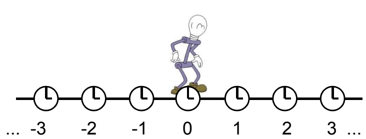


At the origin, at three o’clock, the clock sends out a light signal to tell everybody it’s three o’clock.

Time passes as the signal gets to the clock at x = 3m.

When the signal arrives, the clock at x=3m is set to 3:00 *plus the 10 ns delay*.

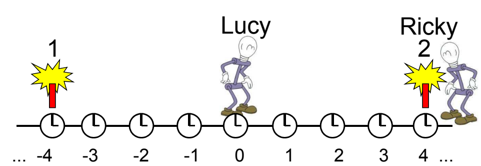
Simultaneity in *one* frame



Using this procedure, it is now possible to say that all the clocks in a given inertial reference frame read the same time. *Even if* we don’t go out there to check it ourselves.

Now I know when events really happen, even if I don’t find out until later (due to finite speed of light).

Q7:



Two firecrackers explode. Lucy, halfway between the firecrackers, *sees* them explode at the same time. Ricky (same reference frame as Lucy) is next to firecracker 2. According to Ricky, which firecracker explodes first in her reference frame?

A. Both explode at the same time
B. Firecracker 1 explodes first
C. Firecracker 2 explodes first

Even though Ricky *sees* the flash from 1 after the one from 2, she knows the local times at which each cracker went off.

Event 1: (x_1, y_1, z_1, t_1) *Event 2:* (x_2, y_2, z_2, t_2), *with* $\boxed{t_1 = t_2}$ “simultaneous”

Q8:

Two firecrackers sitting on the ground explode. This time, Lucy is sitting twice as far from firecracker 1 as from firecracker 2. She *sees* the explosions at the same time. Which firecracker exploded first in Lucy's reference frame?

A. Both explode at the same time
B. Firecracker 1 explodes first
C. Firecracker 2 explodes first

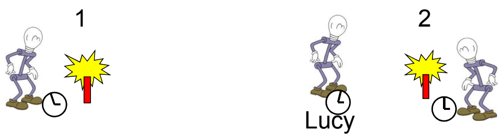
Now we have time under control in *one* frame!

How about if there are *two* frames moving relative to each other?

Answers to clicker questions

Q1: A
Q2: A
Q3: B
Q4: B
Q5: D
Q6: C
Q7: A
Q8: B

Q1:



Two firecrackers sitting on the ground explode. Lucy is standing twice as far from firecracker 1 as from firecracker 2. She *sees* the explosions at the same time. Which firecracker exploded first in Lucy's reference frame?

A. Both explode at the same time
B. Firecracker 1 explodes first
C. Firecracker 2 explodes first

When an event occurred is not a matter of *when you saw* it; it is the time when a *'local observer'* would have seen it.

Announcements

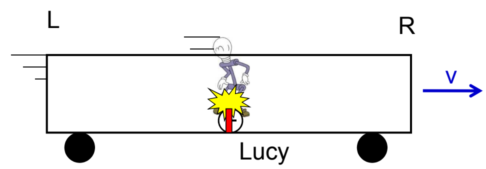
- Reading for Friday: TZD 1.12 – 1.14
- Extra credit class survey due this Friday, midnight.
- If you have not done yet: Please register your clicker! (See link on course web page)

http://www.colorado.edu/physics/phys2130

Today's class

- Measuring time in two different reference frames: Time dilation!!
- Twin Paradox
- Spacetime diagrams

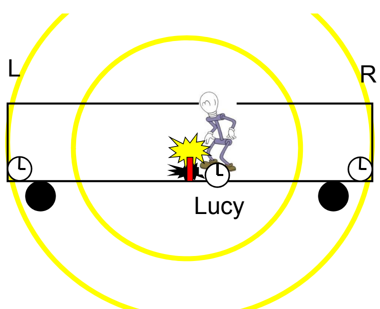
Q2:



Lucy is the middle of a railroad car, and sets off a firecracker. (Boom goes the dynamite!) Light from the explosion travels to both ends of the car. Which end does it reach first according to Lucy?

a) both ends at once
b) the left end, L
c) the right end, R

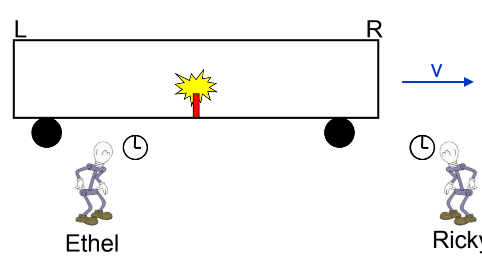
These events are simultaneous in Lucy's frame.



Sure! After the firecracker explodes, a spherical wave front of light is emitted. ('Spherical', because the speed of light is the <u>same in all directions in any inertial frame of reference</u>). A little while later, it reaches both ends of the car.

Sometime *later*, Lucy finds out about it – but that's a different story. The synchronized clocks are all that matter.

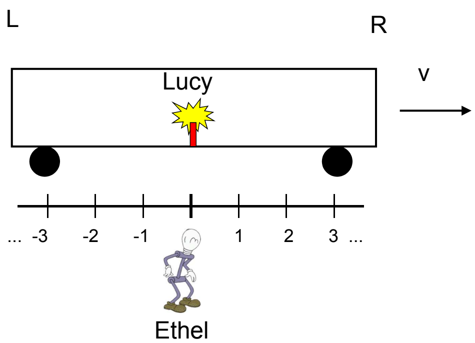
Q3:



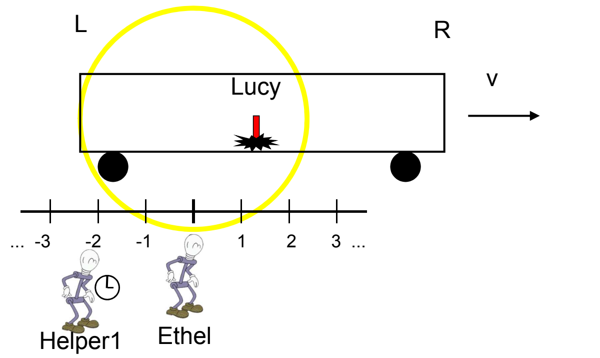
Lucy's friends Ethel and Ricky are standing still next to the tracks, watching the train move to the right. According to Ethel and Ricky, which end of the train car does the light reach first? (As before the firecracker is still in the middle of the car.)

a) both ends at once
b) the left end, L
c) the right end, R

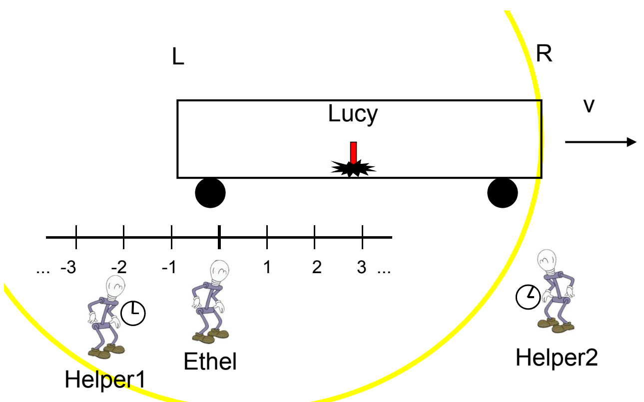
In Ethel & Ricky's frame, these events are *not* simultaneous!



Suppose Lucy's firecracker explodes at the origin of Ethel's reference frame.

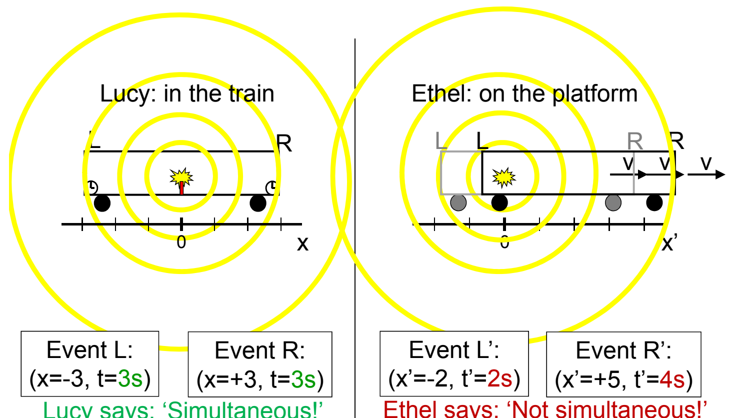


The light spreads out in Ethel's frame from the point she saw it explode. Because the train car is moving, the light in Ethel's frame arrives at the left end first.



Sometime later, in Ethel's frame, the light catches up to the right end of the train.

Summary: Timing of events depend on the choice of the inertial frame!!



An important conclusion

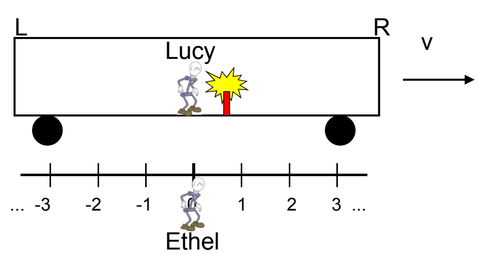
Given two spacetime events:

1) Light hits the right end of the train car
2) Light hits the left end of the train car

Lucy finds that the events ***are*** simultaneous.

Ethel (in a different reference frame) finds that they ***are not*** simultaneous.

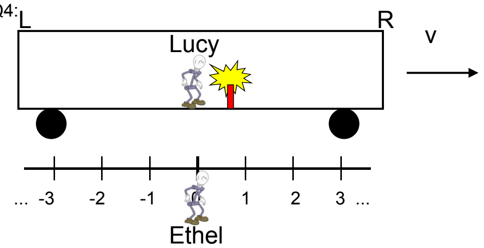
And they're both right!



Now suppose Lucy's firecracker is *just slightly* toward the right side of the train, so slightly that Ethel still measures the light hitting the left end first.

According to Lucy, which end gets hit first?

Q4:



Now suppose Lucy's firecracker is *just slightly* toward the right side of the train, so slightly that Ethel still measures the light hitting the left end first. According to Lucy, which end gets hit first?

a) both at the same time
b) the left end, L
c) the right end, R

An important conclusion

- In Lucy's frame:
  - Firecracker explodes (event 1)
  - Light gets to the right end of the train (event R)
  - A little later, light gets to the left end (event L)
- In Ethel's frame:
  - Firecracker explodes (event 1')
  - Light gets to the left end of the train (event L')
  - A little later, light gets to the right end (event R')

And they're both right!

Again: This is not only a matter of which event they *see* first. The events really do occur in a different sequence in their respective reference frames!!

An important conclusion

Not only can observers in two different inertial frames disagree on whether two events are simultaneous, they may not even agree which event came first.

And that's the relativity of simultaneity.

C. L. Freeman, etal., "Structural Control of Crystal Nuclei by an Eggshell Protein," Angewandte Chemie, International Edition, **49,** 5135–5137 (2010).

Like races? Check this out!

The winner of a horse race may not be the same for different reference frames (if they are **REALLY** close in time or **REALLY** far apart spatially, or both)

Q

Q5:

Quiz on the reading

(No talking during this one please)

What does **Proper Time** refer to?

A – *Time* when an event occurs, measured by an accurate clock that is at rest in an inertial frame.

B – *Time* when an event occurs, measured by a correctly synchronized clock closest to the event and at rest in the inertial frame in which the event occurs.

C – *Time interval* between two events that occur at the same place in one inertial frame measured by a clock in that inertial frame and close to these events.

D – *Time interval* between any two events that take place in the same inertial frame, measured by two correctly synchronized clocks that are closest to each of these two events.

Proper time (see TZD p. 18)

If two events have the same spatial coordinates in a specific inertial frame, then the time between them measured by a clock at rest in the same inertial frame is the *proper time*.

Example: Any given clock never moves with respect to itself. It keeps proper time in its own frame.

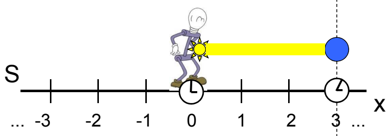
Same location

Mathematically: Event 1: (x_1,y_1,z_1,t_1)
Event 2: (x_1,y_1,z_1,t_2)

→ Proper time: $\Delta t = t_2 - t_1$

Up next: Any observer moving with respect to this clock sees it run slow (i.e., time intervals are longer). This is time dilation.

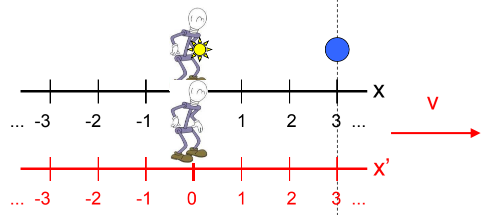
Measuring speed of light



An observer and a ball are at rest in reference frame S.
At t = 0, the observer in S emits a light pulse to be received at x = 3 m.
At Δt = 10 ns, the light is received. Observer S measures a distance Δx = 3 m, so the speed of light in frame S is

$$c = \frac{\Delta x}{\Delta t} = \frac{3m}{10ns} = 0.3m/ns$$

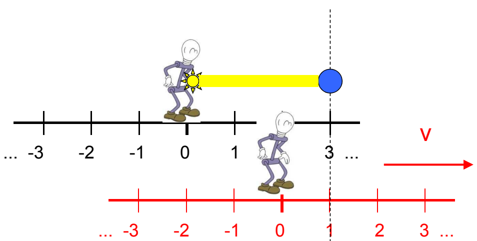
Measuring speed of light



S' is moving with respect to S at v = 0.2 m/ns.
At t = 0, observer in S flashes a light pulse to be received at x = 3 m.

Q6:

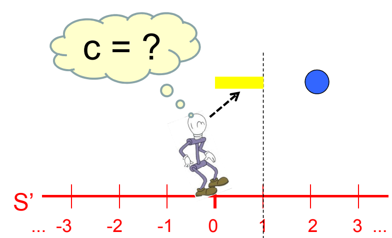
Ten nanoseconds later…



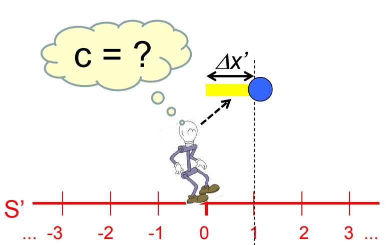
S' is moving with respect to S at v = 0.2 m/ns.
At Δt = 10 ns, the light is received. According to an observer in S': how far did the light travel in the S' frame?

a) 3 m b) 2 m c) 1 m d) 0 m

As seen by observer in S'



Ten nanoseconds later



The observer in S' would therefore measure the speed of light as:

$$c' = \frac{\Delta x'}{\Delta t'} = \frac{1m}{10ns} = 0.1m/ns$$

Uh-oh!

Uh-oh!

If we are to believe Einstein's second postulate ($c' = c$), then:

In frame S $c = \frac{\Delta x}{\Delta t}$

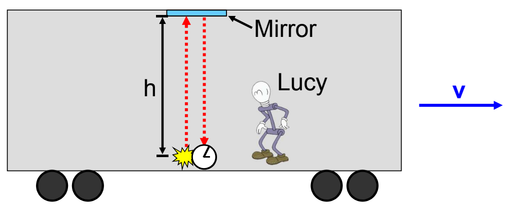
In frame S' $c = \frac{\Delta x'}{\Delta t'}$

$$c = \frac{\Delta x}{\Delta t} = \frac{\Delta x'}{\Delta t'}$$

Since we found that $\Delta x \neq \Delta x'$, and we accepted Einstein's second postulate of relativity (*'c' is the same in all inertial frames*), we conclude that $\Delta t \neq \Delta t'$.

In other words: **Time passes at different rates** in the two different frames of reference!! This is **time dilation**.

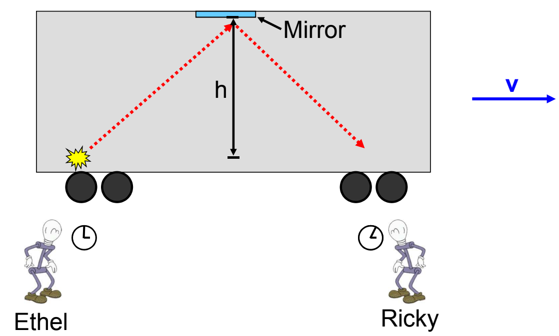
More ‘evidence’ for time Dilation



Lucy measures the time interval: $\Delta t = {}^{2h}/_{c}$

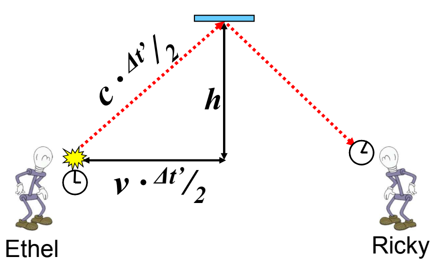
(Not a big surprise!)

Let’s look at it from the tracks:



Note: This experiment requires two observers.

More evidence for ‘Time Dilation’



Ethel and Ricky measure the time interval:

$$\Delta t' = \frac{2h}{c}\gamma, \quad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

But Lucy measured $\Delta t = 2h/c$!!

Time dilation in moving frames

Lucy measures: Δt

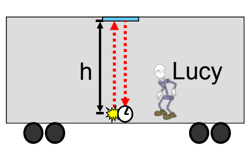
Ethel and Ricky: $\Delta t' = \gamma \Delta t$, with

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \geq 1$$ “Lorentz Factor”

Ethel concludes that time runs slower for Lucy!

(Lucy is moving relative to Ethel and Ricky)

In which of the two frames do we measure the proper time between the two events?

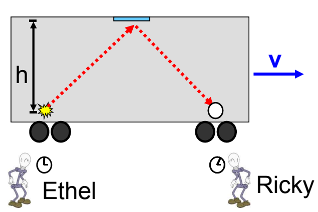


Lucy measures:

$\Delta t' = {}^{2h}/_{c}$

Proper time!

Both events occur at the same location!



Ethel and Ricky:

$\Delta t = \gamma\, {}^{2h}/_{c}$, with $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

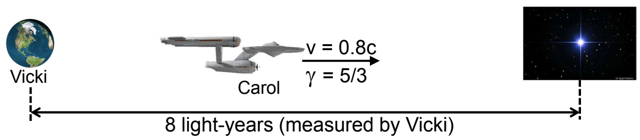
In any other frame: time interval between events is longer than the proper time

Let’s have some fun!

The Twin ‘Paradox’

Q7:

A little journey

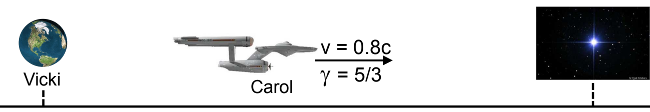


Carol and Vicki are identical twins. While Vicki stays on Earth, Carol departs for the star Sirius, 8 light-years away, traveling at a speed v = 0.8 c (Note $\gamma = 5/3$). According to Vicki, how long does the trip take? (Assume Earth and Sirius are not moving relative to each other.)

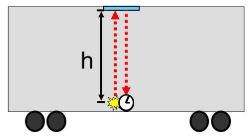
a) 6 years b) 8 years c) 10 years d) 16.67 years

Q8:

A little journey

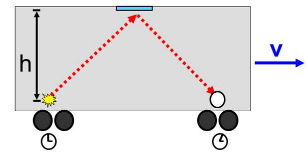


Hint: Follow the proper time!



Proper time:

$$\Delta t = {}^{2h}/_{c}$$

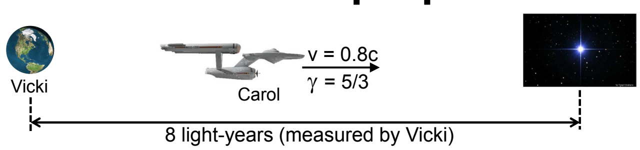


Any other frame:

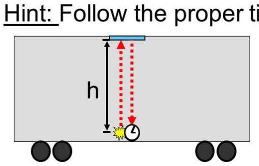
$$\Delta t' = \gamma \cdot \Delta t, \text{ with } \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

a) 6 years b) 8 years c) 10 years d) 16.67 years

Hint: Follow the proper time!

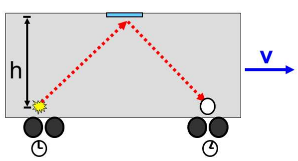


Hint: Follow the proper time!



Proper time:

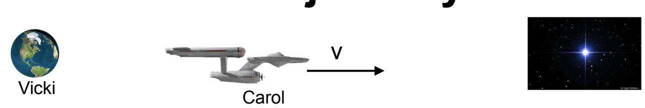
$$\Delta t = {}^{2h}/_{c}$$



Any other frame:

$$\Delta t' = \gamma \cdot \Delta t, \text{ with } \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

A little journey



Why? Because Carol's clock is present at both events:

Event 1: Carol leaves Earth.
Event 2: Carol arrives at Sirius.

← Time between these two events is the proper time

So if Δt is Carol's proper time between these events, and $\Delta t'$ is the time in the Earth-Sirius system, we have:

$$\Delta t = \frac{\Delta t'}{\gamma} = \frac{10y}{5/3} = 6y$$

← Proper time

Remember: Proper time ≤ Δt' in any other frame!

A little journey

Upon arriving at Sirius, Carol immediately turns around and heads home at v = 0.8c. When she returns, she has aged 12 years, while Vicki has aged 20 years!

Vicki

Carol

But wait....

From Carol's point of view, it was Vicki who was moving. So we should expect:

In Vicki's frame (the Earth), Carol ages 12 years while Vicki ages 20.

~~In Carol's frame (the spacecraft), Vicki ages 12 years while Carol ages 20.~~

And they can't both be right!

Resolution: Carol has to turn around (not an inertial frame!)

Answers to clicker questions

Q1: B
Q2: A
Q3: B
Q4: C
Q5: C
Q6: C
Q7: C
Q8: A

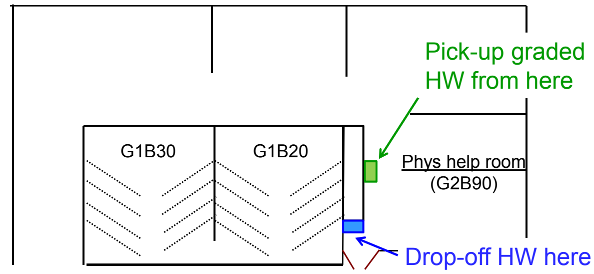
Did you read the pre-lecture notes for today's class? ("Pre-lectures" are *not* the reading assignments from the book.)

A) Yes, I read them regularly.
B) I download them only occasionally.
C) I know where to get them, but I never download or read them.
D) What pre-lecture notes?

Pre-lectures (and more) are available **for free** from here:
http://www.colorado.edu/physics/phys2130

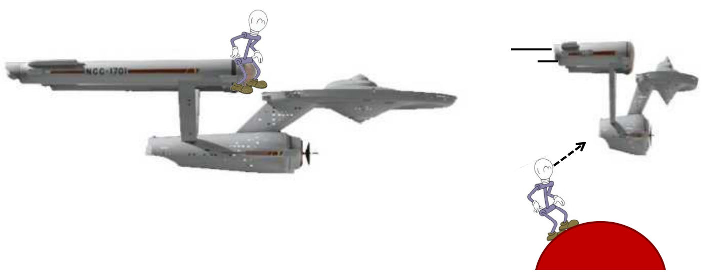
Announcements

- No class on Monday!! (Labor Day)
- Reading for Wednesday: Review Chapter 1.
- HW2 due Wednesday noon.



Today's class

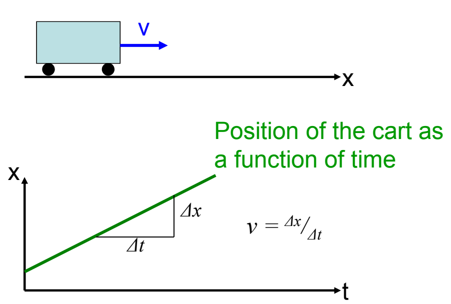
- Spacetime diagrams
- Length contraction
- Lorentz transformation



Spacetime diagrams
(also called "Minkowski diagrams")
(very useful in SR!)

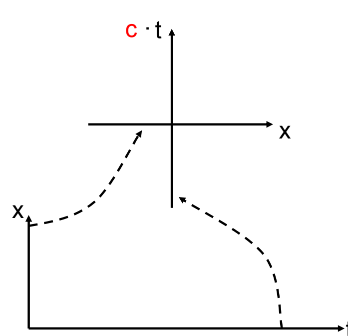
Spacetime Diagrams (1D in space)

In PHYS 1110:



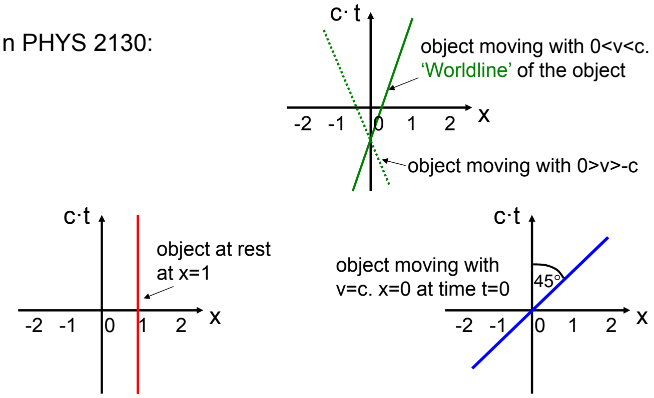
Spacetime Diagrams (1D in space)

In PHYS 2130:

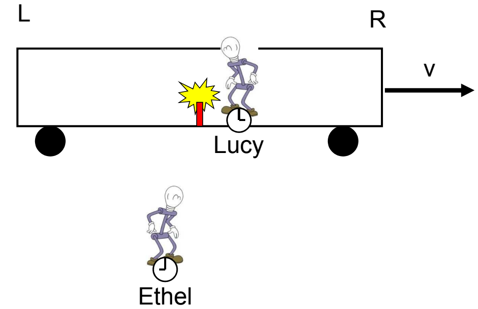


Spacetime Diagrams (1D in space)

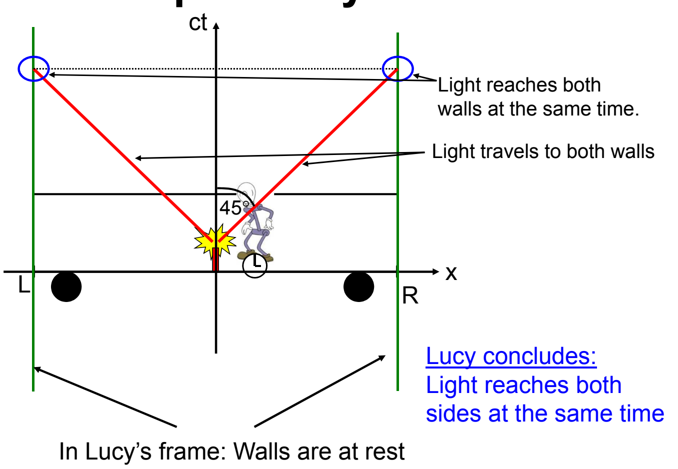
In PHYS 2130:



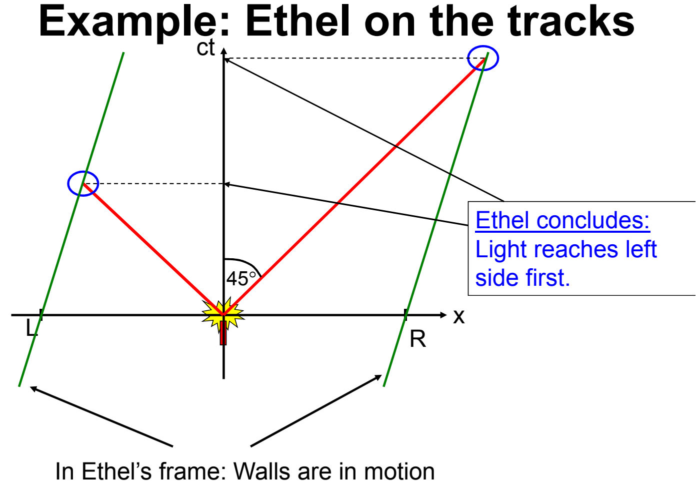
Recall: **Lucy plays with a fire cracker in the train.**
Ethel watches the scene from the track.



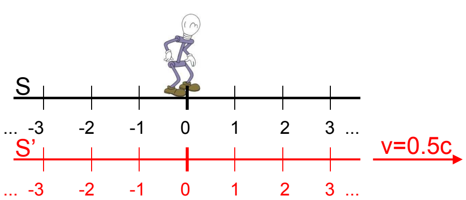
Example: Lucy in the train



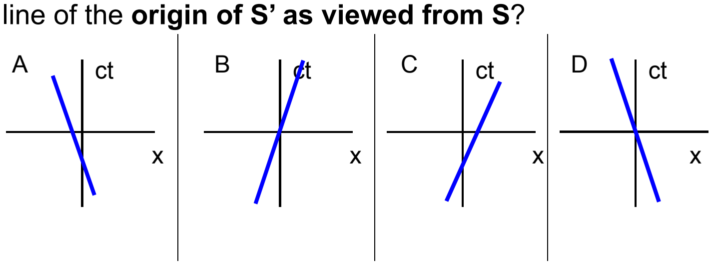
Example: Ethel on the tracks



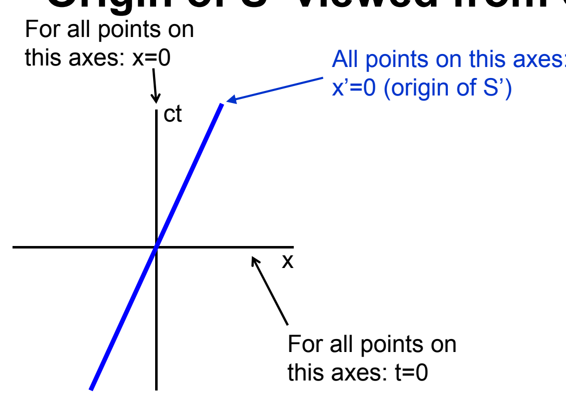
Q1:



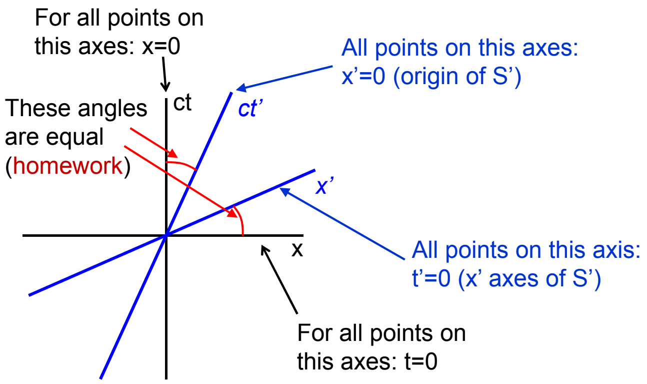
Frame S' is moving to the right at v = 0.5c. The origins of S and S' coincide at t=t'=0. Which shows the world line of the **origin of S' as viewed from S**?



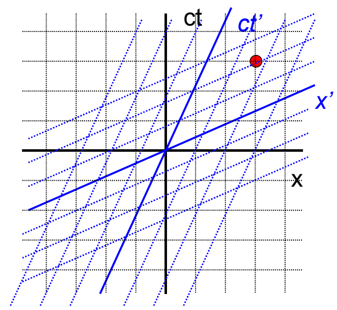
Origin of S' viewed from S



Frame S' as viewed from S



Both frames are adequate for describing events, but will generally give different spacetime coordinates. In S: (x,t), or in S': (x',t')

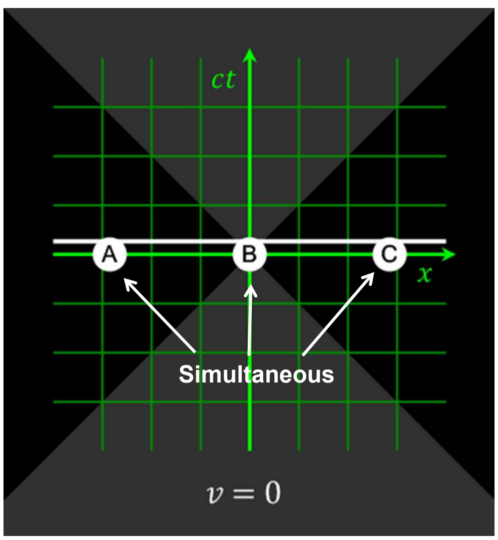


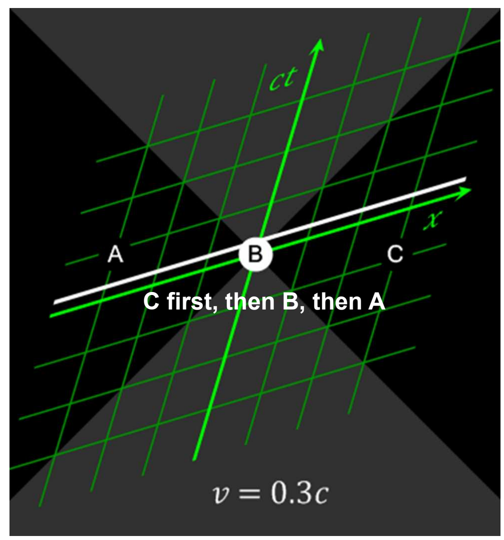
In S: (3,3)
In S': (1.8,2)

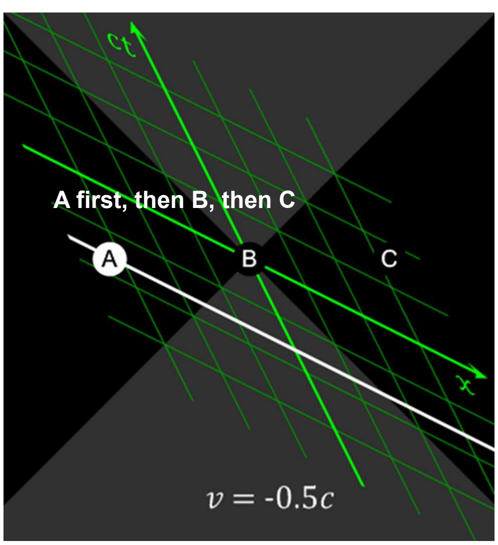
In classical physics we had something similar: The Galileo transformations.

x' = x - v·t
t' = t

(We know that they are no good for light. We'll fix them soon!)

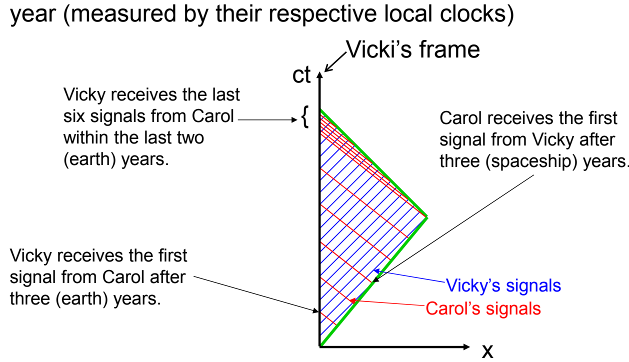






Example: Spacetime diagram for the twin paradox

Carol and Vicky send out radio signals at the end of every year (measured by their respective local clocks)

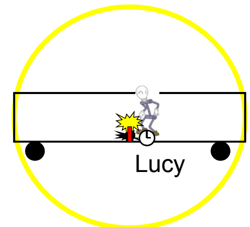


(End of Space time diagrams)

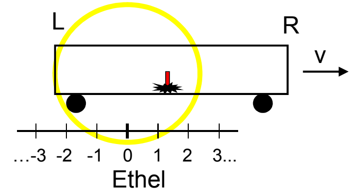
Now back to Time dilation!

What we found so far:

Simultaneity of two events depends on the choice of the reference frame



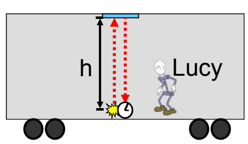
<u>Lucy concludes:</u>
Light hits both ends at the same time.



<u>Ethel concludes:</u>
Light hits left side first.

Time Dilation

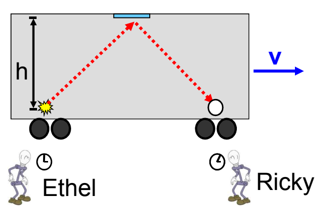
Time Dilation: Two observers (moving relative to each other) can measure different durations between two events.



Lucy measures:

$$\Delta t' = 2h/c$$

Here: $\Delta t'$ is the **proper time**



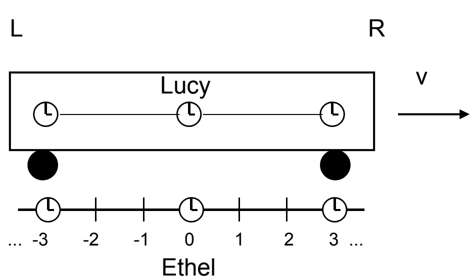
Ethel and Ricky:

$$\Delta t = \gamma\, 2h/c, \text{ with } \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

More about time dilation:

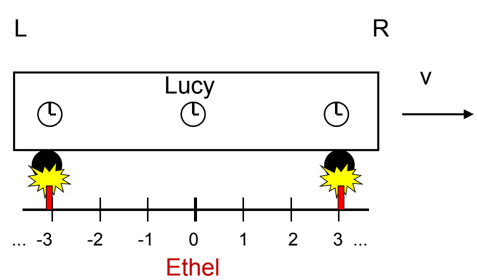
Are your clocks *really* synchronized?

They sure don't seem to be!…



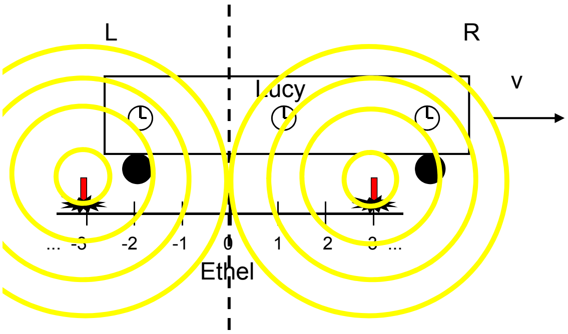
Now Lucy and Ethel each have a set of clocks. Lucy's are synchronized in her frame (the train), while Ethel's are synchronized in her frame (the tracks).

How do the clocks of one frame read in the other frame?



At 3 o'clock in Ethel's frame, two firecrackers go off to announce the time. It so happens that these firecrackers are at the left and right ends of the train, in Ethel's frame.

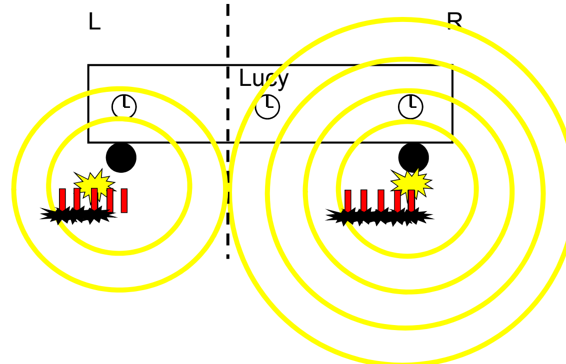
Event 1: firecracker 1 explodes at 3:00
Event 2: firecracker 2 explodes at 3:00



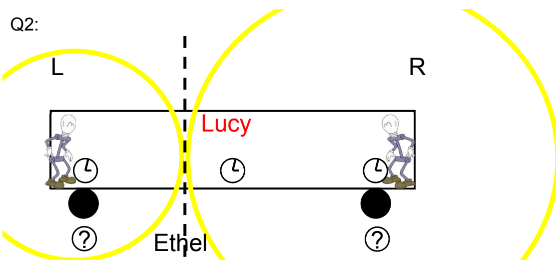
Sometime later, the wave fronts meet. The meeting point is halfway between the firecrackers in Ethel's frame, but is somewhere in the left of the train car, in Lucy's frame.

Event 3: two light pulses meet, shortly after 3:00.

The situation as seen by Lucy



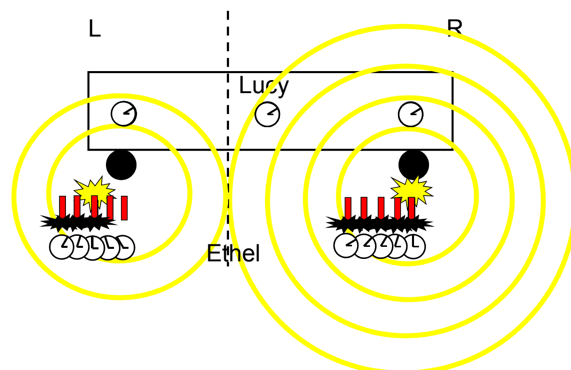
Q2:



In *Lucy's frame*, light left first from the right firecracker. But in Ethel's frame, both went off exactly at 3 o'clock!
According to Lucy's reference frame, which of the following is true:

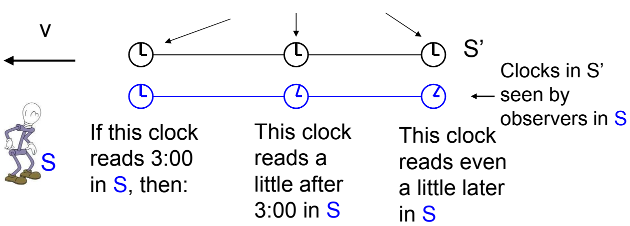
A) Ethel's clock on the left reads a later time than Ethel's clock on the right.
B) Ethel's clock on the right reads a later time than Ethel's clock on the left.
C) Both of Ethel's clocks read the same time.

In Lucy's frame:



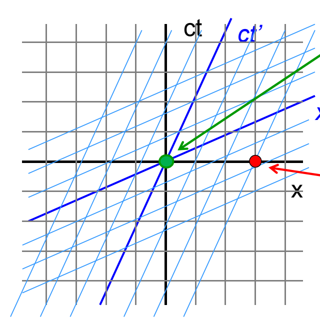
Important conclusion

Clocks in S' (synchronized in S') (Ethel) moving to the left with respect to S (Lucy)



Even though the clocks in S' are synchronized in S', observers in S find that they each show a different time!!
…and in addition, they seem to run slow due to time dilation!

The same events in two different frames:
Spacetime ("Minkowski") diagram:



Clocks at the origin agree at t=0:

In S: (x =0, t = 0)
In S': (x'=0, t' = 0)

Red event:

In S: (x =3, t = 0)
In S': (x'=3, t' = -1.8)

Clock at the same location but other frame shows an earlier time!

Answers to clicker questions

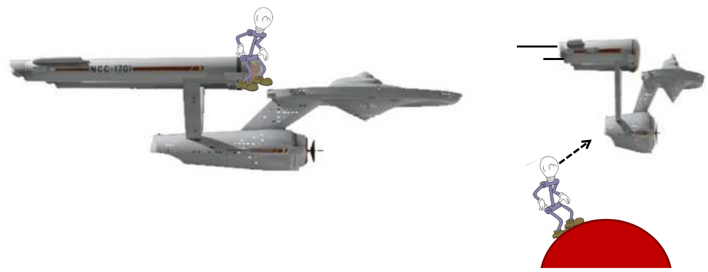
(Correct answer for slide 1: “A” ☺)
Q1: B
Q2: B

Announcements

- Reading for Friday: 2.1 - 2.5
- HW 3 is posted. Due next Wed. noon.
- Exam 1 in less than 2 weeks. It will cover Chapters 1 & 2.

Today's class

- Length contraction
- Lorentz transformation



Length contraction

(Consequence of time dilation and vice versa)

Q1:

Quiz on the reading

Proper length of a rigid object is the length of the object measured…

(see TDZ p. 23)

A – …in any inertial frame.

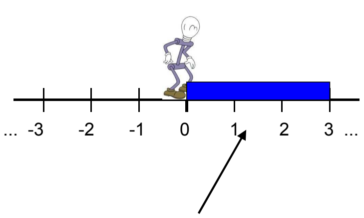
B – …by the speed of light.

C – …in the one inertial frame in which both ends of the object have the same event coordinates.

D – …in the frame in which the object is not rotating.

E – …in the inertial frame in which the object is at rest.

Length of an object



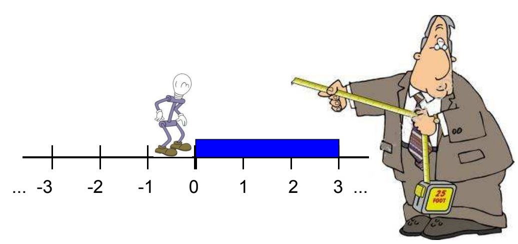
This stick is 3m long. I measure both ends at *the same time* in my frame of reference.

"Same time" or not doesn't matter, if we are in the object's rest frame.

This length, measured in the stick's rest frame, is its proper length.

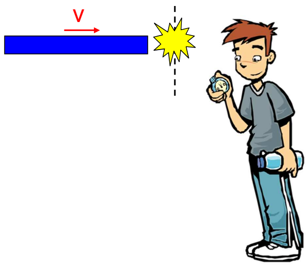
'Proper length'

Proper length: Length of object measured at rest / object measured in the frame where it is at rest (use a ruler)

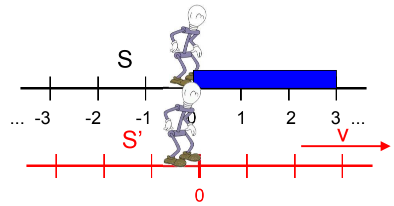


Remember 'proper time'

Proper time: Time interval $\Delta t = t_2 - t_1$ between two events measured in the frame, in which the two events occur at the same spatial coordinate, i.e. time interval that can be **measured with one clock**.



Length of an object

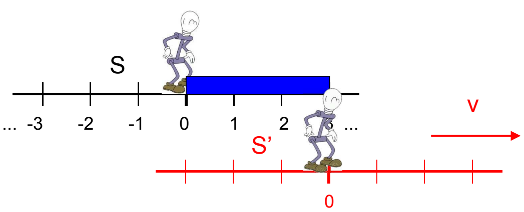


Observer in S measures the proper length L of the blue object.

Another observer comes whizzing by at speed v. This observer measures the length of the stick, *and keeps track of time*.

Event 1 – Origin of S' passes left end of stick.

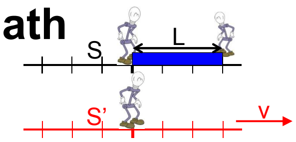
Length of an object



Event 1 – Origin of S' passes left end of stick.
Event 2 – Origin of S' passes right end of stick.

Q2:

A little math



In frame S: (rest frame of the stick)
- length of stick = L (this is the proper length)
- time between events = Δt
- speed of frame S' is $v = L/\Delta t$

In frame S':
- length of stick = L' (this is what we're looking for)
- time between events = $\Delta t'$
- speed of frame S is $v = L'/\Delta t'$

Q: a) $\Delta t = \Delta t'$ b) $\Delta t = \gamma\Delta t'$ c) $\Delta t' = \gamma\Delta t$

A little math

Speeds are the same (both refer to the relative speed). And so

$$|v| = \frac{L'}{\Delta t'} = \frac{L}{\Delta t} = \frac{L}{\gamma \Delta t'}$$

$$L' = \frac{L}{\gamma}$$

Length in moving frame

Length in stick's rest frame (proper length)

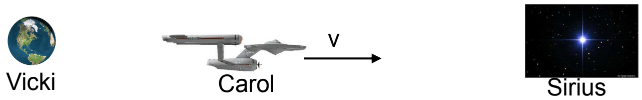
Length contraction is a consequence of time dilation (and vice-versa).

Q

The Twin Paradox revisited

Q3:

Quiz on proper time/length

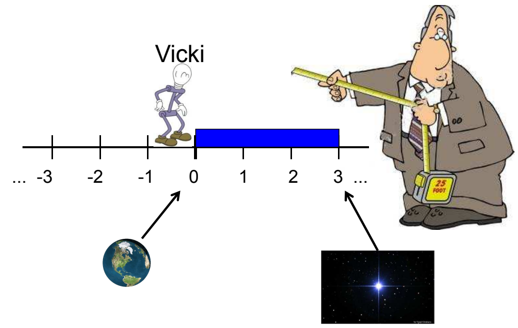


Carol travels from the Earth to Sirius. Which of the following statements is correct? (Assume that Earth and Sirius are not moving relative to each other)

A – Vicky measures proper time and proper length of the journey.
B – Carol measures proper time and proper length of the journey.
C – Vicky measures proper time and Carol measures proper length of the journey.
D – Carol measures proper time and Vicky measures proper length of the journey.
E – none of the above

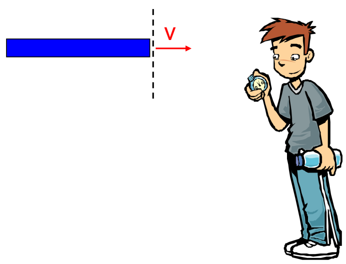
Review: Proper length

Proper length: Length of object measured in the frame, where it is at rest (use a ruler)

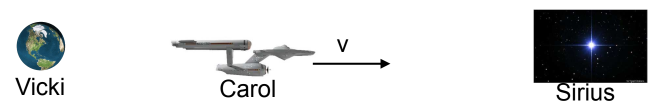


Review: Proper length

Proper time: Time interval $\Delta t = t_2 - t_1$ between two events measured in the frame, in which the two events occur at the same spatial coordinate, i.e. time interval that can be **measured with one clock**.



Proper time & proper length



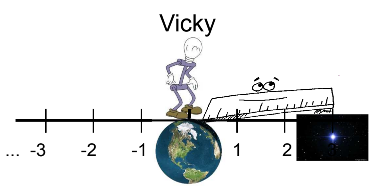
Now we know the following about this journey:

- Vicki measures the proper length: 8 light-years.
- Carol measures the proper time: 6 years.
- Both agree that Carol travels at a speed of v=0.8c relative to the earth.

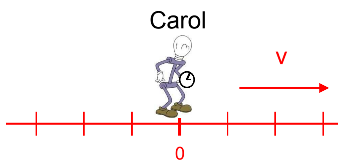
From Carol's perspective:
Carol finds that she traveled only 6y*0.8c = 4.8 ly. But why does she find herself at Sirius after 6 years??

→ Length contraction!!

Length contraction vs. time dilation



Vicky measures:
Proper length: L_{Vicky} = 8 ly
Time: $\Delta t_{Vicky} = \gamma\, \Delta t_{Carol}$ = 10y

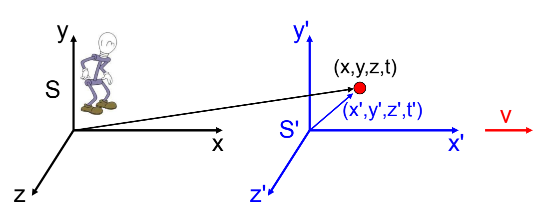


Carol measures:
Length: $L_{Carol} = L_{Vicky}/\gamma$ = 4.8 ly
Proper time: Δt_{Carol} = 6 y

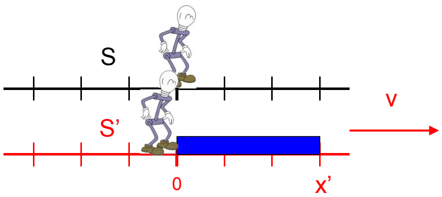
Q

Lorentz transformation

(Relativistic version of Galileo transformation)



The Lorentz transformation

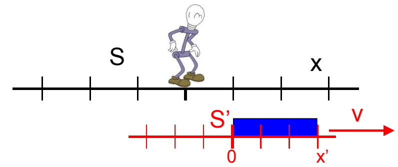


A stick is at rest in S'. Its endpoints are the events (x',t') = (0,0) and (x',0) in S'. S' is moving to the right with respect to frame S.
Event 1 – left end of stick passes origin of S. Its coordinates are (0,0) in S and (0,0) in S'.

Q4:

Lorentz transformation

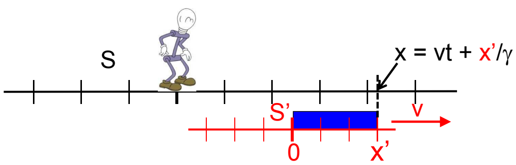
An observer at rest in frame S sees a stick flying past her with velocity v:



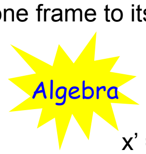
As viewed from S, the stick's length is x'/γ. Time t passes. According to S, where is the *right* end of the stick? (Assume the *left* end of the stick was at the origin of S at time t=0.)

a) $x = \gamma vt$ b) $x = vt + x'/\gamma$ c) $x = -vt + x'/\gamma$
d) $x = vt - x'/\gamma$ e) something else

The Lorentz transformation

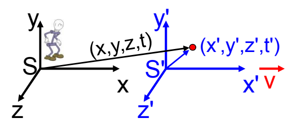


$x = vt + x'/\gamma$. This relates the spatial coordinates of an event in one frame to its coordinates in the other.



$x' = \gamma(x-vt)$

Transformations



If S' is moving with speed v in the positive x direction relative to S, then the coordinates of the same event in the two frames are related by:

Galilean transformation
(classical)

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

Lorentz transformation
(relativistic)

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma(t - \frac{v}{c^2}x)$$

See homework #3

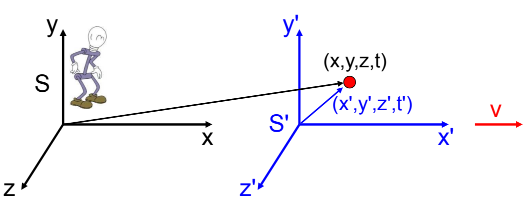
Note: This assumes (0,0,0,0) is the same event in both frames.

A note of caution:

The way the Lorentz and Galileo transformations are presented here (and in the textbook) assumes the following:

An observer in S would like to express an event (x,y,z,t) (in his frame S) with the coordinates of the frame S', i.e. he wants to find the corresponding event (x',y',z',t') in S'. The frame S' is moving along the x-axes of the frame S with the *velocity* v (measured relative to S) and we assume that the origins of both frames overlap at the time t=0.

Note:
Velocity
has a sign!



Answers to clicker questions

Q1: E
Q2: B
Q3: D
Q4: B

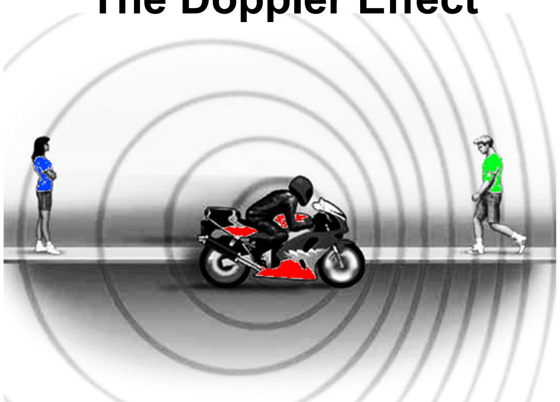
Announcements

- Reading for Wednesday: 3.1 – 3.6
- HW 3 Due in 2 days @ noon.
- Exam 1 in 8 days. It will cover Chapters 1 & 2. Start preparations now!
- Practice exam available on CUlearn. (NOTE: our exam will be all multiple choice)

Today's class

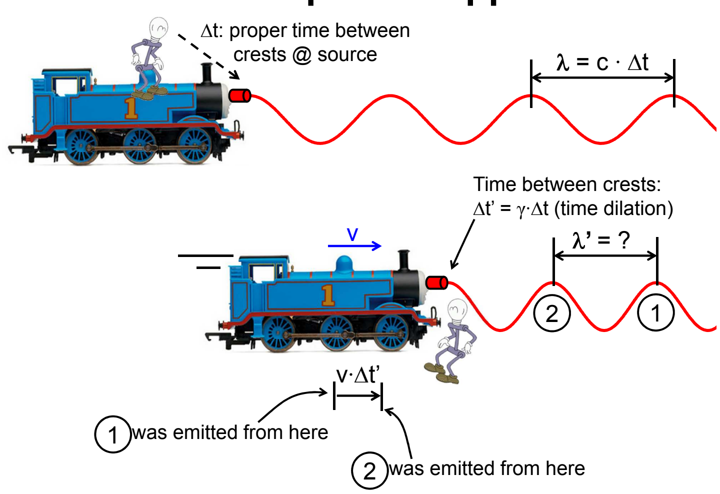
- The optical Doppler effect
- Excursion to the universe!

The Doppler Effect

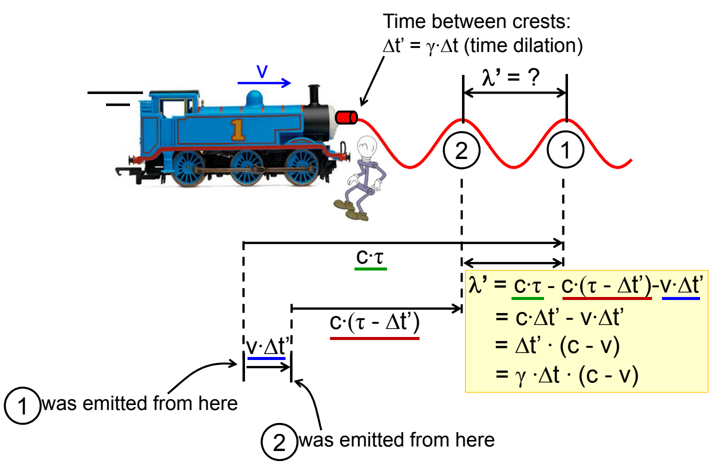


We are all familiar with the acoustic Doppler effect. It relies on the speed of sound in air. For light: No ether! We expect some differences. (Also must include time dilation!)

How about the optical Doppler effect?



How about the optical Doppler effect?



How about the optical Doppler effect?

$$\lambda' = c\cdot\tau - c\cdot(\tau - \Delta t') - v\cdot\Delta t'$$
$$= c\cdot\Delta t' - v\cdot\Delta t'$$
$$= \Delta t'\cdot(c - v)$$
$$= \gamma\cdot\Delta t\cdot(c - v)$$

The observed frequency (from the track) is:

$$f_{obs} = \frac{c}{\lambda'} = \frac{c}{\gamma(c-v)\Delta t} = \frac{1}{\gamma(1-\beta)\Delta t} \quad \text{with} \quad \beta = \frac{v}{c}$$

Note that the frequency of the source in its rest-frame is:

$$f_{source} = \frac{c}{\lambda} = \frac{c}{c\Delta t} = \frac{1}{\Delta t}$$

Therefore: $f_{obs} = \frac{f_{source}}{\gamma(1-\beta)}$

The optical Doppler effect

with $\beta = \frac{v}{c}$

$$f_{obs} = \underbrace{\sqrt{\frac{1+\beta}{1-\beta}}} f_{source}$$

If source is approaching

$\geq 1 \rightarrow f_{obs} \geq f_{source}$, if source and observer are approaching each other

$$f_{obs} = \underbrace{\sqrt{\frac{1-\beta}{1+\beta}}} f_{source}$$

If source is receding

$\leq 1 \rightarrow f_{obs} \leq f_{source}$, if source and observer are receding from each other

Questions?

Applications in astronomy

- How far are galaxies away?
- Is the universe inflating or collapsing?
- Are there planets outside our solar system? (aka. "exoplanets")
- Is there dark energy & dark matter?

How many stars in this picture?

← This spiral galaxy probably contains about 200 billion stars (= 200,000,000,000 stars!)

There are galaxies that contain ~1000 times more stars than this spiral galaxy!

There are several hundred galaxies in this picture!

So there are at least 100 trillion stars in this picture!!

~100,000,000,000,000 stars!

(The visible universe contains approximately 10^{24} stars: **1,000,000,000,000,000,000,000,000**)

Edwin Hubble: Use the Doppler effect!

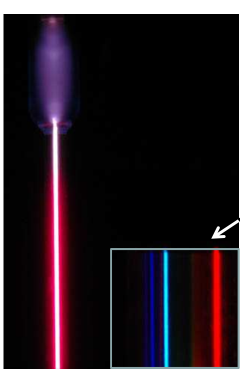
1889 - 1953

If a star is receding, it's light should look reddish (longer wavelength)	v →
If a star is approaching, it's light should look bluish (shorter wavelength)	← v

But how can we measure this?

Quantum mechanics comes to the rescue!!

Spectra of elements are discrete



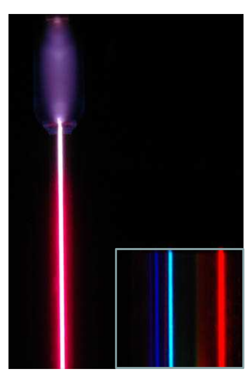
Hydrogen has very well known lines.

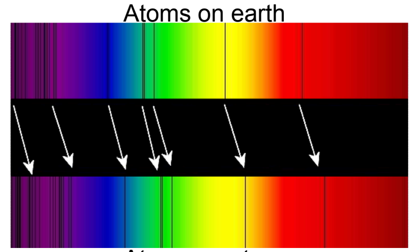
Seen through diffraction grating we can distinguish individual, narrow lines. (More about this in the second half of this course!)

Now we just have to measure the position of these lines.

Each of these hydrogen atoms is like having an accurate clock on a distant star!

Spectra of elements are discrete

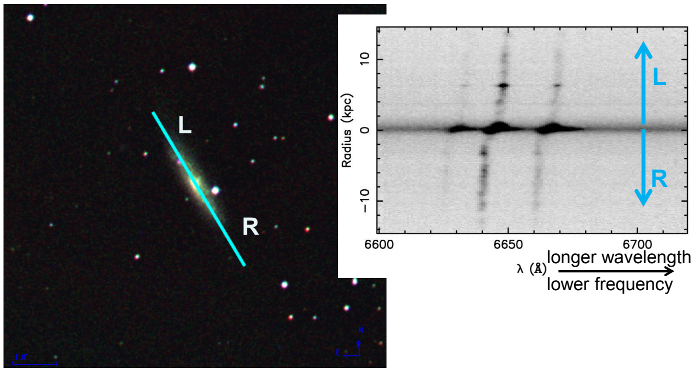




Measure by how much the lines have shifted due to Doppler effect.

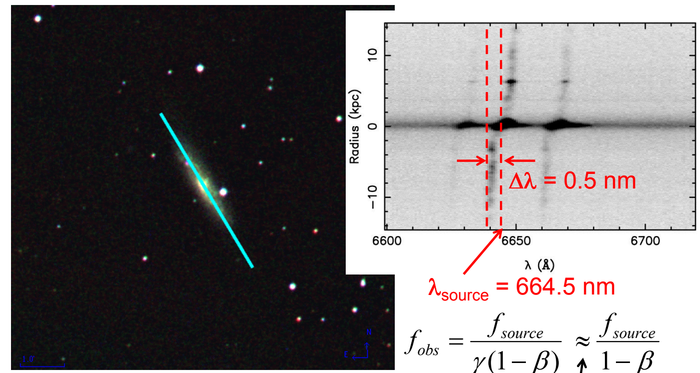
Q1:

Velocity of stars in a galaxy



Which side is moving towards us?
a) Left b) Right

How fast are the stars at the edges?



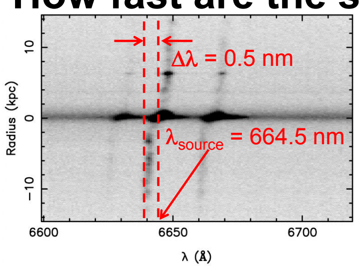
λ_{source} = 664.5 nm

$$f_{obs} = \frac{f_{source}}{\gamma(1-\beta)} \approx \frac{f_{source}}{1-\beta}$$

for v << c

Q2:

How fast are the stars at the edges?



$$f_{obs} \approx \frac{f_{source}}{1-\beta} \quad \Bigg| \quad f = \frac{c}{\lambda}$$

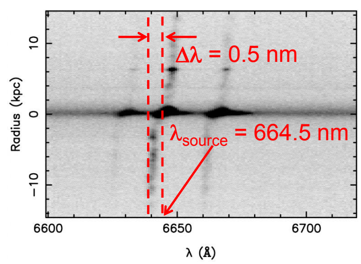
Now we need to convert this to an expression in λ! Which of the following is the correct relation?

A) $\lambda_{obs} \approx \frac{\lambda_{source}}{1-\beta}$ **B)** $\lambda_{obs} \approx \lambda_{source}(1-\beta)$

C) $\lambda_{obs} \approx c \cdot \lambda_{source}(1-\beta)$ **D)** $\lambda_{obs} \approx \frac{\lambda_{source}}{c}(1-\beta)$

E) None of the above

How fast are the stars at the edges?



$$\lambda_{obs} \approx \lambda_{source}(1-\beta)$$
$$\approx \lambda_{source} - \Delta\lambda$$
$$\text{with } \Delta\lambda \approx \lambda_{source}\beta$$
$$\approx \lambda_{source}\frac{v}{c}$$

$$\Rightarrow v \approx \frac{\Delta\lambda}{\lambda_{source}} c$$

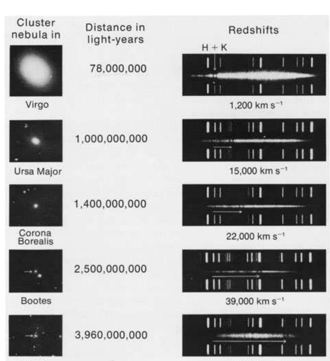
for v << c

Numeric value:

$$v \approx \frac{0.5nm}{664.5nm} c = \underline{\underline{226 km/s}}$$

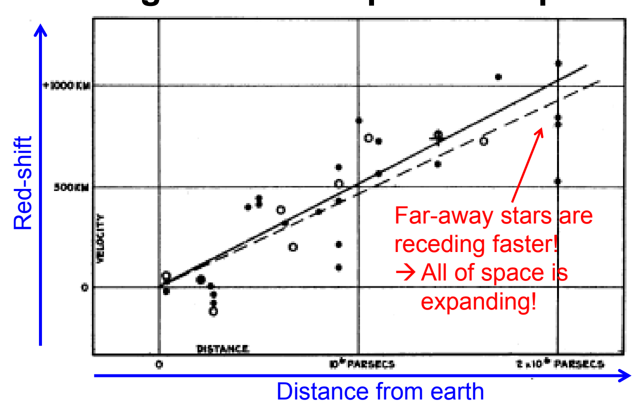
Dark matter required for this much centripetal acceleration! $\frac{GM}{r^2} = \frac{v^2}{r}$

Hubble and the big bang



Edwin Hubble, PNAS March 15, 1929 vol. 15 no. 3 168-173

The big bang: Far-away objects are receding faster: All space is expanding!



Next step: Is there dark energy, i.e. is the expansion accelerating?

Exoplanets (extra-solar planets)

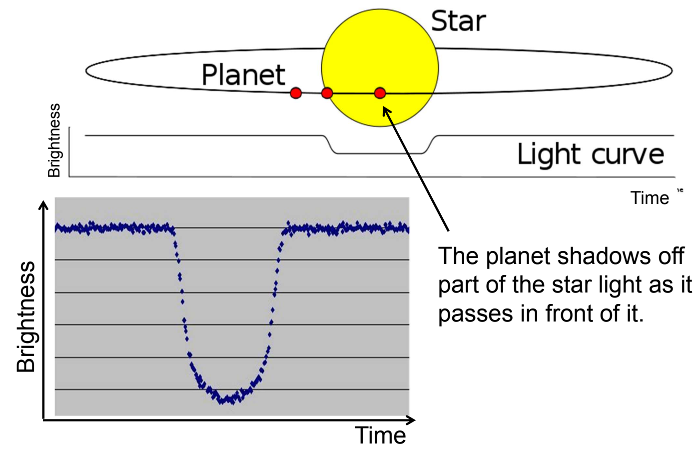
Are there any solar systems (i.e. stars with planets around them) besides our own? Of course there are!
(It would be a bit foolish not to assume this, considering that there are ~10^{24} suns out there!)

But how to prove it?!

We typically cannot see the planet directly. (Imagine you try to see something really tiny and dark next to a huge and bright object.)

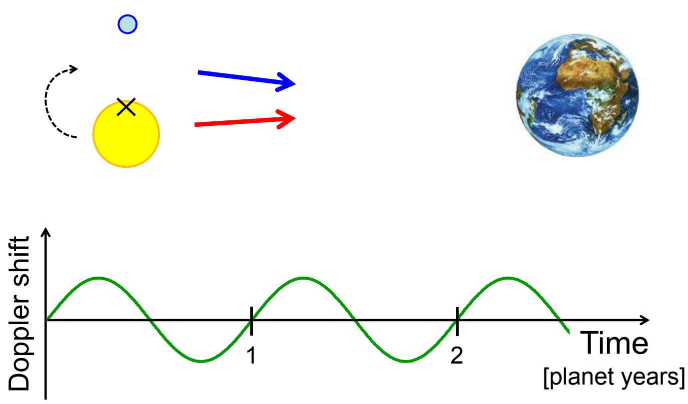
→ Most methods focus on the star, instead of the planet.

Shadowing due to the planet



The planet shadows off part of the star light as it passes in front of it.

Doppler shift due to re-coil



That's the end of our little excursion to the vast universe.

(And all I have on relativistic spacetime)

Questions?

Chapter 2

Relativistic mechanics

Previously:
- Ideas of space and time
- Simple algebra

Next:
- Ideas of momentum and energy
- Slightly more involved algebra

Momentum

The classical definition of the momentum **p** of a particle with mass m is: **p**=m**u**.

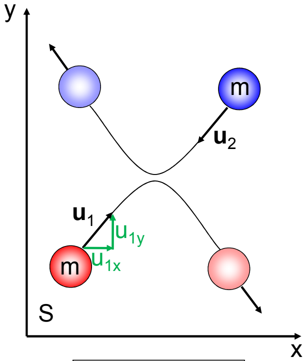
In absence of external forces the total momentum is conserved (Law of conservation of momentum):

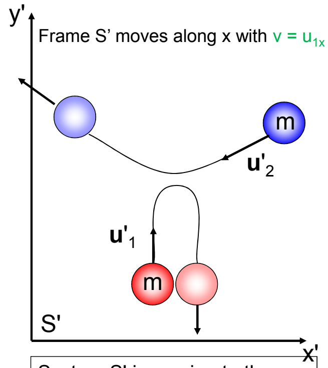
$$\sum_{i=1}^{n} \mathbf{p}_i = const.$$

Due to the velocity addition formula, the definition **p**=m**u** is not suitable to obtain conservation of momentum in special relativity!!

→ **Need new definition for relativistic momentum!**

Conservation of Momentum

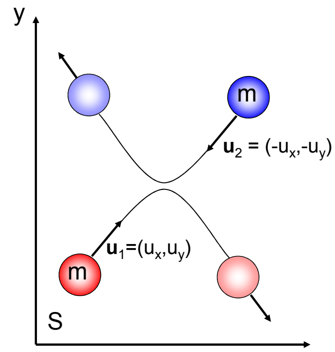




If $\mathbf{u}_1 = -\mathbf{u}_2$ we find:
$p_{tot,before} = 0$
$p_{tot,after} = 0$

System S' is moving to the right with the velocity $v = u_{1x}$. We will use relativistic velocity transformations here.

Classical momentum



$p_{1,\,before} = m(u_x, u_y)$
$p_{2,\,before} = m(-u_x, -u_y)$

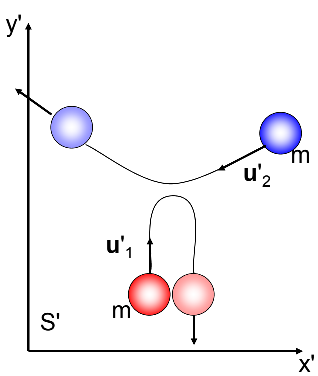
$p_{tot,\,before} = m(0, 0)$

$p_{1,\,after} = m(u_x, -u_y)$
$p_{2,\,after} = m(-u_x, u_y)$

$p_{tot,\,after} = m(0, 0)$

→ $p_{tot,\,before} = p_{tot,\,after}$

Galileo (classical):



$p_{1,\,before} = m(0, u_y)$
$p_{2,\,before} = m(-2u_x, -u_y)$

$p_{tot,\,before} = m(-2u_x, 0)$

$p_{1,\,after} = m(0, -u_y)$
$p_{2,\,after} = m(-2u_x, u_y)$

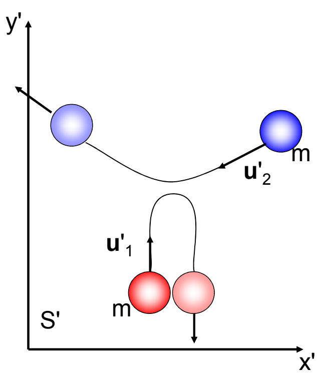
$p_{tot,\,after} = m(-2u_x, 0)$

→ $p_{tot,\,before} = p_{tot,\,after}$

Velocity transformation (3D)

Classical:	Relativistic:
$u'_x = u_x - v$	$u'_x = \frac{u_x - v}{1 - u_x v/c^2}$
$u'_y = u_y$	$u'_y = \frac{u_y}{\gamma(1 - u_x v/c^2)}$
$u'_z = u_z$	$u'_z = \frac{u_z}{\gamma(1 - u_x v/c^2)}$

Lorentz transformation



Use: $u'_x = \frac{u_x - v}{1 - u_x v/c^2}$

$u'_y = \frac{u_y}{\gamma(1 - u_x v/c^2)}$

Algebra
Part of HW#4

→ $p_{tot,\,before} \neq p_{tot,\,after}$

Answers to clicker questions

Q1: B
Q2: B