

Giant Kerr nonlinearities and slow optical solitons in coupled double quantum-wells

Wen-Xing Yang

Institute of Photonics Technologies,

National Tsing-Hua University, Hsinchu 300, Taiwan and

Department of Physics, Southeast University, Nanjing 210096, China

Ray-Kuang Lee

Institute of Photonics Technologies,

National Tsing-Hua University, Hsinchu 300, Taiwan

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Abstract

We show the formation of slow optical solitons in the asymmetric coupled double quantum-wells (CQW) via a two-photon Raman resonance. With the consideration of real parameters in AlGaAs-based CQW, we indicate the possibility to have cancellation of the linear absorption and giant Kerr nonlinearities. With the controllable balance between dispersion and nonlinear effects in these solid-state based devices, this work may provide a practical platform for nonlinear optical signal processing.

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Third-order Kerr nonlinearities play an important role in nonlinear optics such as cross-phase modulation (XPM) for optical shutters [1] and generation of optical solitons [2], etc. It is desirable to achieve giant Kerr nonlinearities with low light powers [3]. In recent years, both theoretically [4] and experimentally [5], the giant third-order nonlinear susceptibility with reducing linear absorption has been one of the most extensively studied phenomena. In addition, retaining the merits of the giant Kerr nonlinearities, Wu and Deng [6] have theoretically proposed that it is possible to form ultraslow optical bright and dark solitons for weak light by including the self-phase modulation in cold atomic media.

However, it is more advantageous at least from the view point of practical purposes to find, solid media that could permit to realize the giant Kerr nonlinearities with low pump power, low absorptions, and shape-invariant propagation of the optical field instead of the aforementioned cold atom gases. In fact, we note that, in conduction band of semiconductor quantum well (QW) structures, there have been studies on the oscillations and wave propagations such as strong electromagnetically induced transparency (EIT) [7], tunneling-induced transparency [8], ultrafast all-optical switching[9], slow light propagation[10], etc. More recently, the enhancement of Kerr nonlinearities based on Fano-interference with intersubband transitions [11] and a large XPM have been studied in an asymmetric QWs [12].

In this Letter, we show that asymmetric semiconductor coupled double quantum-wells (CQW) also can support the propagation of optical solitons via a two-photon Raman resonance scheme. Besides, under two-photon resonance condition and with appropriate one-photon detuning, we can obtain the cancellation of the linear absorption, enhancement of Kerr nonlinearities, and slow group velocity propagation of the weak probe pulse. Since the conduction subband energy level can be easily tuned by an external bias voltage, the proposed CQW structure also provide another possibility to realize electrically controlled phase modulator at low light levels.

Let us consider an asymmetric semiconductor CQW structure consisting of 10 pairs of a 51-monolayer (145\AA) thick wide well and a 35-monolayer (100\AA) thick narrow well, separated by a $\text{Al}_{0.2}\text{Ga}_{0.8}\text{As}$ buffer layer[13], as shown in Fig. 1. The energy difference 2δ of the bonding state $|3\rangle$ and anti-bonding state $|4\rangle$ is determined by the level splitting in the absence of tunneling and related tunneling matrix element, which can be controlled by an electric field applied perpendicularly to CQW.

We assume the transitions $|2\rangle \leftrightarrow |4\rangle$ and $|2\rangle \leftrightarrow |3\rangle$ are simultaneously coupled by a strong coupling field with the respective one-half Rabi frequencies $\Omega_c = \mu_{42}E_c/2\hbar$ and $(\Omega_c\mu_{32})/\mu_{42}$. At the same time, a weak probe field is applied to the transitions $|1\rangle \leftrightarrow |4\rangle$ with the respective Rabi frequencies Ω_p . And the transitions $|1\rangle \leftrightarrow |3\rangle$ is dipole forbidden transition due to selection rules. E_c and E_p are the amplitude of the strong-coupling and weak probe field, respectively. By adopting the standard approach [14], under the electro-dipole and rotating-wave approximations the system dynamics can be described by equations of motion for the probability amplitudes of the electronic wave functions:

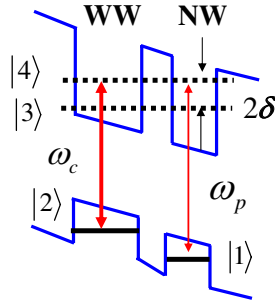


FIG. 1: Conduction subband energy level diagram for an asymmetric coupled quantum wells consisting of a wide well (WW) and a narrow well (NW).

$$\frac{\partial A_1}{\partial t} = i\Omega_p^* A_4, \quad (1)$$

$$\frac{\partial A_2}{\partial t} = -[\gamma_2 - i(\Delta_p - \Delta_c)]A_2 + i\Omega_c^* A_4 + iq\Omega_c^* A_3, \quad (2)$$

$$\frac{\partial A_3}{\partial t} = -[\gamma_3 - i(\Delta_p - \delta)]A_3 + iq\Omega_c A_2 + \kappa A_4, \quad (3)$$

$$\frac{\partial A_4}{\partial t} = -(\gamma_4 - i\Delta_p)A_4 + i\Omega_p A_1 + i\Omega_c A_2 + \kappa A_3, \quad (4)$$

where A_j ($j = 1, 2, 3$) being the amplitudes of subbands $|j\rangle$. Here $\Omega_p = \mu_{41}E_p/(2\hbar)$ denotes one half Rabi frequencies for the transition $|1\rangle \leftrightarrow |4\rangle$, the coefficient $q = \mu_{42}/\mu_{32}$ describes the ratio of a pair of dipole moments with μ_{ij} being the dipole moment for the corresponding transitions $|i\rangle \leftrightarrow |j\rangle$. $2\delta = E_4 - E_3$ is the energy splitting between the upper levels. $\Delta_c = \omega_c - \omega_{42}$ and $\Delta_p = \omega_p - \omega_{41}$ are the probe detunings of the coupling and probe fields with transitions $|2\rangle \leftrightarrow |4\rangle$ and $|1\rangle \leftrightarrow |4\rangle$. The total decay rates γ_i are given by $\gamma_i = \gamma_{il} + \gamma_i^{dph}$, where γ_i^{dph} , determined by intrasubband phonon scattering, electron-electron

scattering, and inhomogeneous broadening due to scattering on interface roughness, is the dephasing decay rates. The population decay rates γ_{il} , determined by longitudinal optical (LO) phonon emission events at low temperature, can be calculated by solving the effective mass Schrödinger equation. For the temperatures up to 10 K, the carrier density smaller than 10^{12} cm^{-2} , the dephasing decay rates γ_i^{dph} can be estimated according to Ref.[13]. $\kappa = \sqrt{\gamma_{3l}\gamma_{4l}}$ represents the cross-coupling of states $|3\rangle$ and $|4\rangle$ via the LO phonon decay. Note that a more complete theoretical treatment taking into account these processes for the dephasing is though interesting [15] but beyond the scope of this work.

Under weak probe approximation ($(|\Omega_p| \ll |\Omega_c|)^2$), almost all the electrons are populated in the ground state $|1\rangle$. The excited state $|4\rangle$ can be adiabatically eliminated when the variation of the probe field's envelope is slow compared to the excited state lifetime, so there is no population transfer of the ground state $|1\rangle$. With these assumptions, it is can be shown that $|A_1|^2 \approx 1$, $A_{2,3,4}^{(0)} = 0$. With two-photon resonance condition ($\Delta_p = \Delta_c = \Delta$), we obtain the solutions of A_j to the first order of Ω_p from Eqs. (1-4)

$$A_2^{(1)} = -\frac{(b - q\kappa)\Omega_c^*\Omega_p}{abc - a\kappa^2 + (b + cq^2 - 2q\kappa)|\Omega_c|^2}, \quad (5)$$

$$A_3^{(1)} = -\frac{i(a\kappa + q|\Omega_c|^2)\Omega_p}{-abc + a\kappa^2 - (b + cq^2 - 2q\kappa)|\Omega_c|^2}, \quad (6)$$

$$A_4^{(1)} = -\frac{i(ab + q^2|\Omega_c|^2)\Omega_p}{abc - a\kappa^2 + (b + cq^2 - 2q\kappa)|\Omega_c|^2}, \quad (7)$$

with $a = -\gamma_2$, $b = -[\gamma_3 - i(\Delta + \delta)]$, and $c = -(\gamma_4 - i\Delta)$. The first-order $\chi^{(1)}$ and third-order $\chi^{(3)}$ susceptibility of the probe pulse are given by [6, 16]

$$\chi^{(1)} = -\frac{N|\mu_{14}|^2}{\hbar\varepsilon_0} \frac{A_4^{(1)}A_1^{(1)*}}{\Omega_p}, \quad (8)$$

$$\chi^{(3)} = -\frac{N|\mu_{14}|^4}{3\hbar^3\varepsilon_0} \frac{A_4^{(1)}(|A_4^{(1)}|^2 + |A_3^{(1)}|^2 + |A_2^{(1)}|^2)}{|\Omega_p|^2\Omega_p}, \quad (9)$$

where N is the electron volume density. For the CQW structure considered here, we take $\gamma_2 = 0$ for the lifetime of level $|2\rangle$ by consulting Ref. [17]. Based on Eq. (8), one can find that the first-order susceptibility is mainly caused by cross coupling of the driving field Ω_c . When probe detuning $\Delta_p = 0$, it shows that the absorption $\text{Im}[\chi^{(1)}]$ and the dispersion $\text{Re}[\chi^{(1)}]$ of the probe field do not depend on Ω_c . We show in Fig. 2(a) their dependences

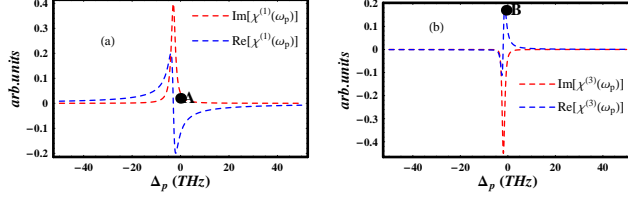


FIG. 2: (a) Dependence of $\text{Im}[\chi^{(1)}]$ and $\text{Re}[\chi^{(1)}]$ and (b) dependence of $\text{Im}[\chi^{(3)}]$ and $\text{Re}[\chi^{(3)}]$ versus the probe detuning Δ_p . We have set $N|\mu_{14}|^2/\hbar\varepsilon_0$ and $N|\mu_{14}|^4/3\hbar^3\varepsilon_0$ as units in plotting, respectively. The other parameters used are $\gamma_{3l} = 0.7$ THz, $\gamma_{4l} = 0.9$ THz, $\gamma_3^{dph} = \gamma_4^{dph} = 0.1$ THz, $\delta = 5$ THz, $\Omega_c = 1$ THz, and $q = 1.25$.

versus the probe detuning Δ_p with the same parameter values used in Ref. [13]. One can clearly see that far away from the point of absorption peak, the linear absorption will be closed to zero.

With two-photon Raman resonance condition, the coupling of the driving field with transition $|2\rangle \leftrightarrow |3\rangle$ destroy the coherence between state $|1\rangle$ and $|2\rangle$, which causes the linear absorption of the probe field and also leads to the nonlinear effect. As a result, $q \neq 0$ indicates constructive interference in XPM nonlinearities. We perform a numerical calculation of the third-order nonlinear susceptibility in Eq. (9). As shown in Fig. 2(b), for a certain probe detuning, for example, at the marker **B** in Fig. 2(b) (corresponds to the marker **A** in Fig. 2(a) at the same detuning frequency), linear absorption is vanished while the strength of XPM is large, which suggests that large XPM can be achieved with vanishing linear absorption. This interesting result is produced by the cross coupling in the nonlinear susceptibility associated with XPM. Unlike the cold atomic systems with specific four-level atoms, the conduction subband energy varies with the bias voltage. When we adjust the energy level of the bonding state $|3\rangle$ and the antibonding state $|4\rangle$ at different bias voltages, different nonlinear phase shifts can be obtained by such giant Kerr nonlinearity. Thus our proposed CQW structures could be provided as a flexible device to realize voltage control, solid-based phase modulators at low light powers.

If the losses of the probe pulse are small enough to be neglected, the balance between the nonlinear self-phase modulation and group velocity dispersion (GVD) may keep a pulse with shape-invariant propagation. From above discussion, as long as probe detuning Δ_p is far away from the point corresponding to the absorption peak, the linear absorption of the

probe pulse is negligible and the nonlinear self-phase modulation is enhanced. In the slowly varying amplitude approximation, the wave equation of the slowly varying envelope $E_p(z, t)$ of the probe pulse along z -axis is given by [16]

$$\left(\frac{\partial}{\partial z} + \frac{1}{v_g} \frac{\partial}{\partial t}\right)E_p + \frac{i\beta}{2} \frac{\partial^2}{\partial t^2}E_p = \frac{2i\omega_p}{c}n|E_p|^2 E_p, \quad (10)$$

together with $v_g = c/\text{Re}[n_0 + \omega_p dn_0/d\omega]$, $n_0 = \sqrt{1 + 4\pi\chi^{(1)}}$, $\beta = d^2k/d\omega^2$, $n = 3\pi\chi^{(3)}/n_0$, and $k = \omega_p n_0/c$ being the group velocity, linear index of refraction, GVD, Kerr-nonlinear refractive index, and wave vector, respectively, where c is the light velocity in vacuum. v_g and β are mainly determined by $\text{Re}[d\chi^{(1)}/d\omega_p]$ and $\text{Re}[d^2\chi^{(1)}/d\omega_p^2]$, respectively. We get the transformation of Eq. (10) by defining $\xi = z$ and $\tau = t - z/v_g$,

$$\frac{\partial\Omega_p}{\partial\xi} + \frac{i\beta}{2} \frac{\partial^2\Omega_p}{\partial\tau^2} = iW|\Omega_p|^2\Omega_p, \quad (11)$$

with $W = -2\pi\omega_p\chi^{(3)}/cn_0$, from Eq.(9). We can choose reasonable and realistic set of parameters to satisfy $\beta = \beta_r + i\beta_i \simeq \beta_r$ and $W = W_r + iW_i \simeq W_r$, so that Eq. (11) reduced to a standard nonlinear Schrödinger equation which admits dark ($\beta_r W_r > 0$) and bright ($\beta_r W_r < 0$) solitons. The fundamental dark soliton takes the form

$$\Omega_p = \Omega_{p0} \tanh(\tau/\tau_0) \exp(-i\beta_r \xi/2\tau_0^2). \quad (12)$$

with $|\Omega_{p0}\tau_0|^2 = -\beta_r/W_r$. As an example, by taking $\Delta_p = 10$ THz, we obtain $v_g = 0.9 \times 10^{-4}c$, $|\Omega_{p0}\tau_0| = \sqrt{|\beta_r/W_r|} \simeq 42.8$, and the linear absorption coefficient $\alpha \simeq 0.0066\text{cm}^{-1}$. There are four adjustable parameters in our proposed system, i.e. the intensity of the driving field, probe detuning Δ_p , the energy splitting 2δ between the two upper levels, and the relative coupling ratio q . From our numerical calculations, we find that decreasing the Rabi frequency of the control field will decrease the group velocity of the probe pulse and increase the values of β_r , W_r and $|\Omega_{p0}\tau_0|$. The control field only need to be strong enough to couple two transitions $|2\rangle \leftrightarrow |4\rangle$ and $|2\rangle \leftrightarrow |3\rangle$, on the other hand relatively lower intensity of driving field can lead to better effects in formation of slow optical solitons. In addition, we have used assumption of $|\Omega_{p0}|^2 \ll |\Omega_c|^2$ in our calculations, so the pulse width of the probe field τ_0 should be chosen to satisfy $|\Omega_{p0}\tau_0|^2 = -\beta_r/W_r \ll |\Omega_c\tau_0|^2$. Fig. 2 shows that the parameter Δ_p has a very large range of validity. For the parameters δ and q , smaller separation δ will be better, which is adjusted and there is no strict requirement for q .

It is worth noting that the cross coupling of control field may be viewed as the perturbation to the two-photon resonance condition, which comes from the closely separated two upper levels instead of coming from the perturbation by introducing another laser field or taking two-photon detuning [6], thus our scheme is a very stable system to form slow optical solitons.

In conclusion, based on the two-photon Raman resonance scheme in the asymmetric coupled double quantum-wells, we have shown that the quantum interference caused by cross coupling of a strong CW laser field not only suppresses linear absorption loss, but also enhances Kerr nonlinearities of the weak probe pulse. With the unique feature of controllable balance between linear dispersion and nonlinear effects in these solid-state devices, we also demonstrate the possibility to form ultraslow optical solitons.

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