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Breather-like collision of gap solitons in Bragg gap regions within nonlocal nonlinear photonic crystals

YuanYao Lin, I-Hong Chen and Ray-Kuang Lee

Institute of Photonics Technologies, National Tsing-Hua University, Hsinchu 300, Taiwan

E-mail: rklee@ee.nthu.edu.tw

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Abstract
We analyze the existence, stability and mobility of gap solitons in a periodic photonic structure with nonlocal nonlinearity. Within the Bragg region of bandgaps, gap solitons exhibit better stability and higher mobility due to the combinations of nonlocality effect and the oscillation nature of Bloch waves. Using linear stability analysis and calculating the Peierls–Nabarro potentials, we demonstrate that gap solitons can revive a nontrivial breather-like collision even in the periodic systems with the help of nonlocal nonlinearity. Such interesting behaviors of gap solitons in nonlocal nonlinear photonic crystals are believed to be useful in optical switching devices.

Keywords: solitons, photonic crystals, nonlocal effect

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Solitary waves are self-guided wavepackets as they propagate in nonlinear media, remaining localized and preserving their own shape. However, they are dramatically altered when they collide with one another. Strictly speaking, only the special case of Kerr nonlinearity is integrable by the inverse scattering transform method. The *soliton* belongs to a special family of solitary waves that are unaffected by collisions. They are particle-like wavepackets supported by nonlinear action and their collision behavior strongly depends upon the relative phase between the interacting solitons [1]. With such an asymptotically linear superposition of nonlinear interactions, a new window of soliton-based photonics can be employed in different optical switching devices and communication systems [2].

During the last decade, photonic crystals, artificial periodic structures with modulation in the refractive index, provide an efficient control of wave transmission and localization, making it possible to tailor dispersion, diffraction and emission of electromagnetic waves [3]. A combination of Kerr nonlinear material and photonic crystals, nonlinear photonic crystals have revealed a wealth of nonlinear optical phenomena and, in particular, self-trapped nonlinear localized modes in the form of so-called gap solitons [4–6]. Gap solitons are unique solutions which can be formed both in focusing or defocusing media, depending on the dispersion/diffraction characteristics caused by the photonic lattices [7]. Current technology in reconfigurable optical lattices, such as photorefractive crystals [8] and nematic liquid crystals [9], also paves a new way to control solitary waves by varying the lattice depth and period.

With both benefits from photonic crystals and solitons, gap solitons are believed to be an important key footstone in soliton-driven photonics. For Kerr-type nonlinear photonic crystals, the bifurcation and stability of gap solitons in the internal reflection (IR) and Bragg gap (BG) regimes are studied with local nonlinear response [10]. However, when the gap soliton exceeds a certain threshold power level, it suffers from a limited mobility in the transverse directions due to the fact that lattice potentials draw great concerns for various switching and routing operations [11–13]. Recently it has been predicted that with *nonlocality* solitons can move across the lattice very easily in the internal reflection region [14]. Such a nonlocal effect comes to play an important role as the characteristic response function of the medium is comparable to the transverse content of the wavepacket [15]. Experimental observations of nonlocal responses have also been demonstrated in various systems, such as photorefractive crystals [16], nematic liquid crystals [17], thermo-optical materials [18] and Bose–Einstein condensates of ^{52}Cr atoms with long range dipole–dipole interactions [19]. The study of nonlocal nonlinearity brings new features in solitons [20], such as modification of modulation instability [21], azimuthal instability [22] and transverse instability [23]. Suppression of collapse in multidimensional solitons [24], change of the soliton interaction [25], formation of soliton bound states [26] and unique families of dark–bright soliton pairs [27] are also recently predicted.

For a nonlocal nonlinear medium, the nonlocality is known to improve the stability of solitons due to the diffusion mechanism of nonlinearity. Nevertheless, to the best of our knowledge, the existence, stabilities, mobilities and collisions of gap solitons in Bragg regions have not been reported. We herein extend the concept of gap solitons in nonlocal nonlinear photonic crystals [14] from internal reflection to Bragg gap regimes. The propagation and stability of gap solitons with an imprinted transverse index modulation under the influence of a nonlocal effect are studied. With the oscillation nature of wavepackets in the Bragg regions, we show that gap solitons are better stabilized and mobilized even with a small degree of nonlocality.

This work is organized as follows, first we show that in the bandgap region a nonlinear Bloch wave can support bright soliton solutions. Families of even and odd modes of bright gap solitons imprinted onto the Bloch wave in local and nonlocal nonlinearities are found numerically. Then the modulational instability of these nonlocal gap soliton families are analyzed by standard linear stability analysis, and the Peierls–Nabarro (PN) potentials that inhibit the mobility of the gap solitons are also calculated in terms of nonlocality. Finally, we address the transverse mobility and soliton interactions under the influence of the nonlocal effect with the presence of periodic potentials. Based on the dramatic reduction of the PN potential barrier for gap solitons in Bragg regions, we demonstrate a nontrivial breather-like collision between gap solitons, which should be useful for optical switching devices based on soliton collisions.

2. Nonlocal solitons in Bragg gaps

We consider a wavepacket propagating along the z axis in the nonlocal nonlinear photonic crystals with a Kerr-type nonlinearity and an exponential-type nonlocal response, which can be modeled by the modified nonlinear Schrödinger equation:

$$\mathrm{i}\frac{\partial\Psi}{\partial z}+\frac{1}{2}\frac{\partial^2}{\partial x^2}\Psi-V(x)\Psi+n(x,z)\Psi=0, \qquad (1)$$

$$n-d\frac{\partial^2 n}{\partial x^2}=|\Psi|^2, \qquad (2)$$

with corresponding invariants, i.e. average Lagrangian L and soliton power U, respectively,

$$L=\int_{-\infty}^{\infty}\left[\frac{\mathrm{i}}{2}(\Psi_z^*\Psi+\Psi_z\Psi^*)-\frac{1}{2}\left|\frac{\partial\Psi}{\partial x}\right|^2-V(x)|\Psi|^2 + \tfrac{1}{2}|\Psi|^2 n\right]\mathrm{d}x, \qquad (3)$$

$$U=\int_{-\infty}^{\infty}|\Psi|^2\,\mathrm{d}x, \qquad (4)$$

where Ψ is the envelope function of the wavepacket, x is the transverse coordinate and $n(x,z)$ is the refractive index profile induced by the exponential-type kernel function responding to the intensity soliton intensity [28]. $V(x)$ is the periodic potential provided externally in the transverse direction. The coefficient d stands for the degree of nonlocality which governs the diffusion strength of the refractive index.

2.1. Linear band diagram

The periodicity of potential $V(x)$ in equation (1) suggests that the stationary states can be expanded by Bloch waves $\Psi(x,z) = f(x)\mathrm{e}^{\mathrm{i}kx+\mathrm{i}bz}$, where $f(x+T) = f(x)$ is a periodic function with period T. If the medium is linear, $n(x,z) = 0$, we can drop the nonlinear index response and rewrite equation (1) in terms of $f(x)$:

$$\left(\frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}x^2}-\frac{k^2}{2}+\mathrm{i}k\frac{\mathrm{d}}{\mathrm{d}x}-V(x)\right)f=bf, \qquad (5)$$

where k is the transverse wavevector of the wavepacket and b is the longitudinal wavevector. The linear wave spectrum consists of bands of eigenvalues $b_{n,k}$ in which $k(b)$ is a real wavenumber of the amplitude-bounded oscillatory Bloch waves. The bands are separated by gaps where the wavefunctions are not stationary with $\mathrm{Im}(b) \neq 0$. In the absence of nonlinearity, the solution at the band edge is exactly a periodic stationary Bloch wave; in contrast, in the presence of Kerr nonlinearity, bright gap solitons arise from the forbidden gaps that are characterized by a band diagram without any nonlinearities. Figure 1(a) shows the linear bandgap diagram on the plane (b, V_0) that is obtained by solving the linear eigenvalue problem in equation (5). Bloch wave patterns at the band edge of the first, second, third and fourth band are plotted in figure 1(b) for a comparison. In the numerical calculations, we employ $V(x) = -3\cos(4x)$ as an example and the corresponding dispersion relations are depicted in figure 2 where the value of the longitudinal wavevector b for each band edge is addressed. Based on our definitions, a semi-infinite total internal region exists for $b > 0$ while finite Bragg gap regions exist for $b < 0$, as indicated in figure 2.

2.2. Bright gap soliton in nonlocal medium

By applying the Bloch solution near the band edge of the linear eigenvalue problem as an initial trial solution, we find different families of bright gap solitons $\Psi(x,z) = u(x)\mathrm{e}^{\mathrm{i}bz}$ numerically with the conventional relaxation technique and the boundary conditions $u(\pm\infty) \approx 0$. Families of nonlinear gap soliton solutions are found, as shown in figure 3(a). Here the bifurcation curves for gap solitons of odd mode and even mode in the internal reflection and the first Bragg gap regions through the relations of b and soliton power U as defined in equation (4) are shown by dashed and solid lines, respectively. The two distinct types of solitons, on-site (odd mode) and off-site (even mode), are defined by their relative position of the center of the wave functions with respect to the external periodic potential [14, 29]. In other words, those that are centered on the minimum and maximum potential of the lattice are classified as

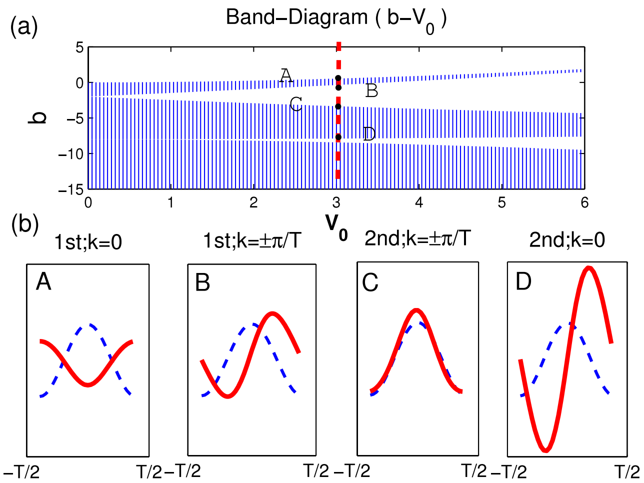

Figure 1. (a) The bandgap spectrum of a Bloch wave at band edge in the linear region for different wavevector b and potential depth V_0. The shaded area shows allowed bands for longitudinal wave vector b. (b) Solid lines correspond to wavefunctions of the Bloch state at the band edges, marked by A, B, C and D; while dashed lines indicate periodic potentials, $V(x) = -V_0 \cos(2\pi x/T)$.

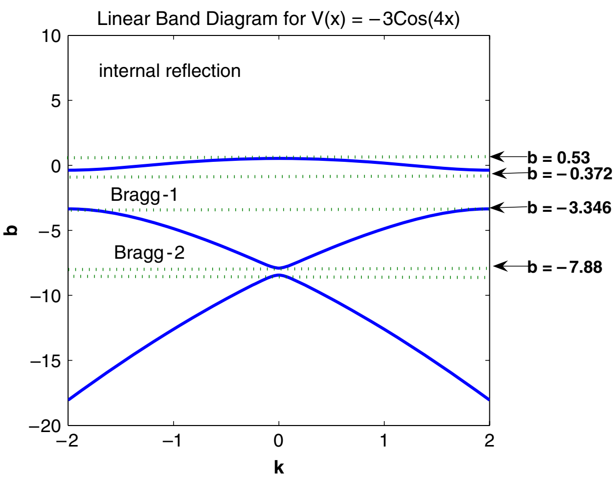

Figure 2. The linear bandgap diagram for the longitudinal and transverse wavevectors, b–k, with the periodic function $V(x) = -3\cos(4x)$ used. Different gap regions are marked as internal reflection and Bragg gap regions.

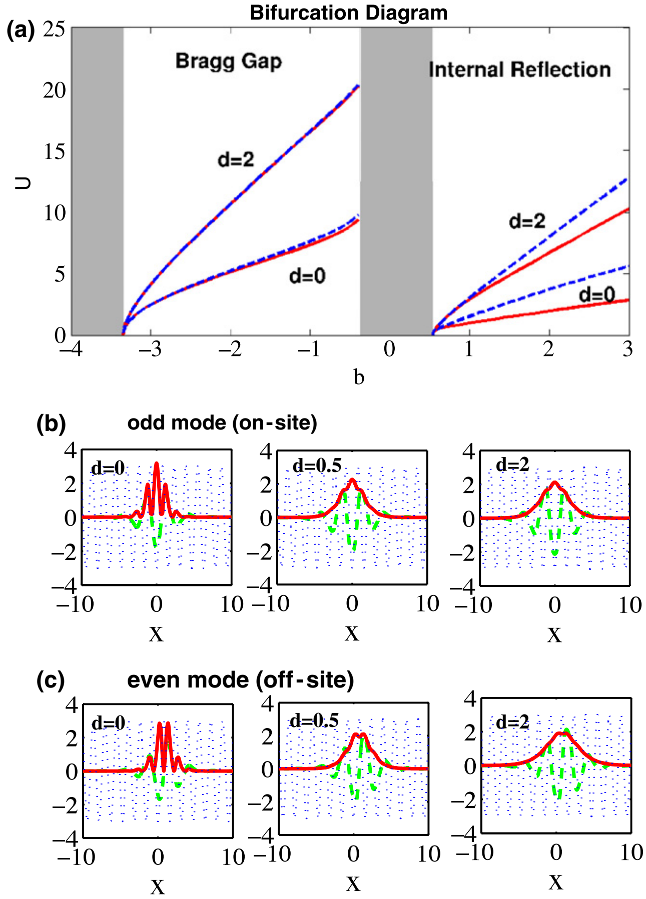

Figure 3. (a) Families of even modes (solid) and odd modes (dashed) of gap solitons for different nonlocality in the internal reflection and the first Bragg gap regions. Field profiles of odd modes (on-site) (b) and even modes (off-site) (c) in the first Bragg gap region for different strength of nonlocalities, $d = 0, 0.5$ and 2 are shown with a fixed wavenumber vector $b = -2$. The dashed, solid and dotted lines are the field $u(x)$, nonlinear index modulation $n(x)$ and periodical potential $V(x)$, respectively.

on-site and off-site accordingly. Solutions of these gap solitons for on-site and off-site modes in the first Bragg gap region for different strengths of nonlocality $d = 0, 0.5$ and 2 are plotted in figures 3(b) and (c), corresponding to the dashed and solid curves in figure 3(a). However, we adopt a negative potential in the calculations herein, then the solutions of on-site and off-site modes differ from those given in [14, 29].

In comparison to the local nonlinear medium, $d = 0$ in figure 3(a), the nonlocality effect increases the formation power for gap solitons as expected. As a result, the gap soliton solutions, both even and odd modes, have a broader width of profile and a smaller level of amplitude. Compared to the solitons in the first Bragg gap region, gap soliton solutions in the internal reflection region have a smooth envelope function and no oscillation tails [14]. These Bragg gap soliton solutions are not only localized wavepackets, but also have similar oscillatory tails as linear Bloch modes in the bands. In particular, the oscillatory tails are significant near the edge of the linear band. Additionally, the nonlinear refractive index distribution $n(x)$ in the internal reflection region has a smooth symmetric bell-like shape without pronounced local maxima on top of it.

When physical realization is concerned, the nonlocal parameter d varies over a wide range of values. For example, in a lead glass which is a nonlinear thermal–optical material [18], the nonlocal refractive response is proportional to the temperature distribution induced by absorption of optical power. Therefore, the nonlinear refractive index distribution can be described by a heat transfer equation that equivalently provides an ‘infinite range of nonlocality’ [18]. In separate experiments of nonlocal dark soliton interactions [30, 31], the nonlocal response is modeled by a modified Bessel function of the second kind, which has a characteristic width of 22 μm while the corresponding soliton width is 105 μm. As suggested by [30], for a given wavelength of solitary wave λ, normalizing

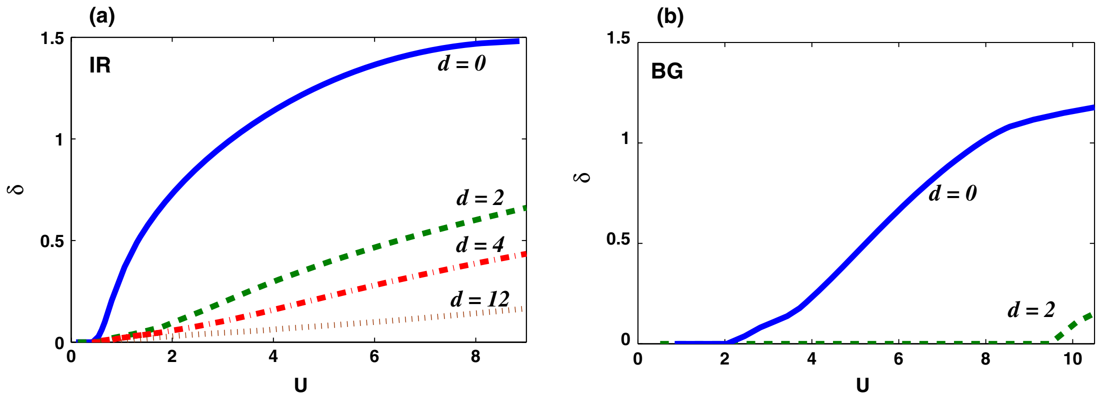

Figure 4. Modulation instability of gap soliton with different nonlocalities, d. The growth rates of the small perturbations Im(δ) in the internal reflection (IR) and the first Bragg gap (BG) regions are shown in (a) and (b), respectively.

the length with respect to the wavevector $k = \frac{2\pi}{\lambda}$ of the optical field yields the degree of nonlocality 197.4 and soliton width 942.5. Although the nonlocal coefficient is very large, it is comparable to the soliton width, which we discussed intensively in this paper. Compared to simulations herein, the width of gap solitons is roughly 3 (for on-site modes) and 4 (for off-site modes) when the characteristic widths of the response function range from 0.5 to 2. The ratio of soliton width and characteristic width of the response function in the experiment by Fischer *et al* [31] matches our simulation parameters in a satisfactory manner. Therefore the experimental conditions are in good agreement with the current model and parameters used under a fair comparison. Moreover, in a Bose–Einstein condensate, such as ^{52}Cr atoms [19, 32], the manipulation of the strength of the long range interaction can be achieved within a very wide range from zero to infinity by an external magnetic field through Feshbach resonances [33]. The experimental evidence and realization provide great support for the current model and the parameters discussed herein.

In the following, we would show that the oscillation nature of a Bloch wave makes solitons in the Bragg gap regions exhibit better stability and higher mobility.

2.3. Stability of gap solitons

The stability of gap soliton solutions is calculated through linear stability analysis, with the perturbed gap soliton solutions

$$u = u_0(x)e^{ibz} + \epsilon[p(x)e^{i\delta z} + q(x)e^{-i\delta^* z}]e^{ibz}, \tag{6}$$

$$n = n_0 + \Delta n, \tag{7}$$

where $\epsilon \ll 1$, $u_0(x)$ is the unperturbed solution and Im$\{\delta\}$ indicates the growth rate of the perturbations. We linearize equations (6) and (7) around the stationary solution and obtain, to the first order in ϵ, the linear eigenvalue problem for the perturbation modes:

$$\begin{pmatrix} \hat{\mathbf{L}}_0 + \hat{\mathbf{N}}_0 & \hat{\mathbf{N}}_0 \\ -\hat{\mathbf{N}}_0 & -\hat{\mathbf{L}}_0 - \hat{\mathbf{N}}_0 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \delta \begin{pmatrix} p \\ q \end{pmatrix}, \tag{8}$$

where

$$\hat{\mathbf{L}}_0 \equiv \mathbf{n_0} - \mathbf{V(x)} - \mathbf{b} + \frac{\mathbf{1}}{\mathbf{2}}\frac{\partial^2}{\partial \mathbf{x}^2},$$

and

$$\hat{\mathbf{N}}_0 \equiv |\mathbf{u_0}|^2 \left(\mathbf{1} - \mathbf{d}\frac{\partial^2}{\partial \mathbf{x}^2}\right)^{-1}.$$

We solve the eigenvalue problem in equation (8) for unstable eigenfunctions with imaginary or complex propagation constant δ by a conventional linear matrix solver. It turns out that the off-site (even mode) soliton family is modulationally stable while the on-site (odd mode) soliton family is not [10]. Figures 4(a) and (b) illustrate that the nonlocal effect significantly reduces the growth rate of the unstable spectral mode for on-site solitons in the internal reflection [14] as well as the first Bragg gap regions. Furthermore, on-site gap solitons in the first Bragg gap region experience stronger suppression of instability than the case in the internal reflection region.

As seen in figure 4 clearly, with a small strength of nonlocality the modulation instability of gap solitons in the Bragg gap region is impressively suppressed. A simple reason is that the nonlocal effect can smooth over the oscillation tails of refractive index n due to the diffusion mechanism. In other words, the smoothness of refractive index n indicates that the effective potential $V_{\text{eff}} = V(x) - n(x)$ becomes more broadened. Therefore, as the strength of the long range interaction increases, soliton solutions become more stable due to the broadening effect of the effective potential, especially in the Bragg gap region. Because gap soliton solutions in the Bragg gap regions possess oscillation tails and their profiles extend more apparently in space, in such a way the nonlocal effect becomes more apparent to smooth out the effective potential and to stabilize solitons.

2.4. Mobility and collision of gap solitons

In this section we study the mobility of these gap soliton solutions by calculating their Peierls–Nabarro (PN) potential barrier which is introduced as the height of an effective periodic potential generated by the lattice discreteness. The

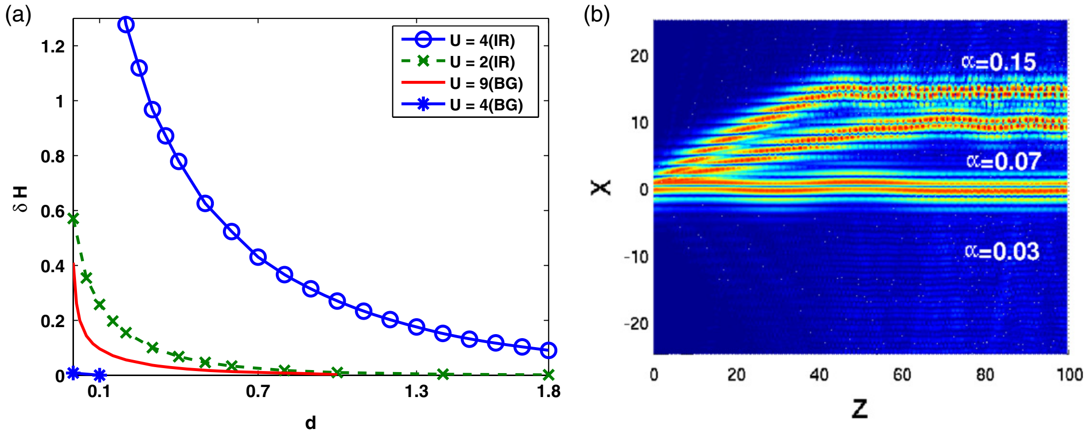

Figure 5. (a) PN potential height δH versus different nonlocality d with a comparison of gap solitons in the internal reflection (IR) and the first Bragg gap (BG) regions. (b) Gap solitons' propagation trajectories in the Bragg gap (BG) region with fixed $d = 0.5$ and $U = 9$ but different initial kinetic energies ($\alpha = 0.03$, 0.07 and 0.15).

PN potential barrier defines the minimum energy required to move the center of mass of a localized wavepacket by one lattice site [34]. Extending this definition to the continuous system, we can define the PN potential barrier as the difference of the system Hamiltonian between odd modes (on-site) and even modes (off-site), i.e.

$$\delta H = H_{\mathrm{odd}} - H_{\mathrm{even}}, \tag{9}$$

where

$$H = \int_{-\infty}^{\infty} \left[\left| \frac{\partial u}{\partial x} \right|^2 - \frac{1}{2} |u|^2 n \right] \mathrm{d}x. \tag{10}$$

Consequently, the PN potential states the smallest amount of energy that a gap soliton needs to gain in order to start moving along the lattice. In terms of PN potential, the threshold power for forming localized gap solitons [13] in nonlocal nonlinear media is considered as the maximum power with vanishing PN barrier. For example, figure 5(a) illustrates that changing the nonlocality from $d = 0.1$ and 1.0 leads to the increase of threshold power to form a localized gap soliton from $U = 4$ up to $U = 9$. It is clearly seen that in the first Bragg gap region the PN potential barrier is drastically reduced in comparison to the local nonlinearity $d = 0$, as well as the internal reflection region, as shown in figure 5(a). With the oscillation tails similar to the linear Bloch waves, nonlocal solitons in the Bragg gap regions are more stable and more movable than those in the internal reflection band [14]. The higher the barrier δH, the larger the incident kinetic energy required to overcome the barrier. The reduction of the PN barrier is confirmed by numerical simulations of equation (1) for gap solitons with fixed power but different initial kinetic energies. The initial conditions for figure 5(b) are set with $u(x, z = 0) = u_0 \mathrm{e}^{\mathrm{i}\alpha x}$, where u_0 is the stationary solution in the Bragg gap region and α stands for the transverse momentum at incidence. When the soliton crosses the lattice it radiates and loses energy. Eventually these gap solitons are captured by one of the lattice channels. However, while the nonlocality effect comes into play, the total effective potential is reduced by the long range interaction. These nonlocal gap solitons are more free to move in the transverse direction due to a lower PN potential height.

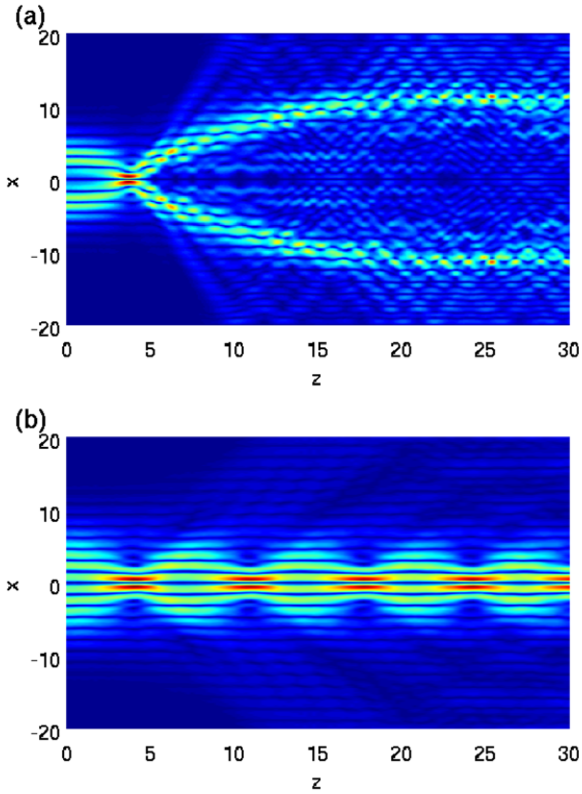

Figure 6. Collision of two solitons in photonic crystals with local (a) and nonlocal (b) nonlinearities. The degree of nonlocality is set as $d = 0.5$.

Based on the results of the PN potential barrier reduction for solitons in the Bragg gap regions with nonlocality, we demonstrate a potential-free collisions between two gap solitons within the photonic crystals. It is well known that, without periodic potentials, solitons experience periodic collisions as a breather when they are relatively in-phase. The breather collision between two localized gap solitons incident at 0° with respect to its propagation direction is destroyed in the periodic systems due to the confinement of the PN potential barrier, as shown in figure 6(a) for the local case with $d = 0$. In this case the soliton power $U = 4$ exceeds the limit that results in the acceleration of the soliton during the collision process [35]. But with a nonlocal nonlinear response, in figure 6(b) we show that gap solitons ($U = 4$) can revive a breather-like collision even in the periodic systems and expand over 10 lattice cycles due to increased long range interaction. In principle, the collisions between gap solitons are very complicated and unpredictable due to the interplays among nonlinearity, periodic potential height and dispersion/diffraction effects. Nonetheless, in this simulation, with only a small strength of nonlocality, $d = 0.5$, one clearly sees that the gap soliton interaction in nonlocal nonlinearity tends to exhibit a lattice-free breather-like wavepacket as a result of decreased PN barrier and increased long range interaction.

3. Conclusion

In conclusion, we demonstrate the existence of gap soliton solutions in nonlocal nonlinear photonic crystals in the internal reflection and Bragg gap regions. The stability and mobility of such novel gap solitons are obtained by calculating the linear stability spectrum and Peierls–Nabarro potential barrier. Compared to the internal reflection region, nonlocal gap solitons in the Bragg gap regions become not only more stable but also more movable due to the oscillation tails of Bloch wavepackets. Moreover we reveal that it is possible to have breather-like collisions between gap solitons with the help of a small value nonlocal effect. Supported by the result of this study and current technology on controllable nonlocal nonlinear media, such as photorefractive crystals, nematic liquid crystals and thermo-optical materials, we believe that the results in this work should provide a new way for soliton-based photonic devices.

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