Infinite information can be carried using a single quantum particle

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In the beginning was the photon, and the photon was with information. Theoretically speaking, a photon can travel arbitrarily long before it enters into a detector, which then clicks. How much information can a photon carry? We study a bipartite asymmetric "two-way signaling" protocol as the extension of that proposed by Del Santo and Dakić. Suppose that Alice and Bob are distant from each other and each of them has an n-bit string. They are tasked to exchange the information of their local n-bit strings with each other, using only a single photon during the communication. It has been shown that the superposition of different spatial locations in a Mach-Zehnder (MZ) interferometer enables bipartite local encodings. We prove that, after the photon's travelling through a cascade of n-level MZ interferometers in our protocol, the one of Alice or Bob whose detector clicks can access the other's full information of n-bit string, while the other can gain one-bit of information. That is, the wave-particle duality makes two-way signaling possible, and a single photon can carry infinite information as n goes infinite.

Introduction.— Communication is a process of sending and receiving messages from one party to another [1]. More precisely, communication is a physical process with physical information carriers transmitted without violating any physical principle. For instance, electromagnetic waves used in wireless communication are governed by Maxwell's equations in classical physics. As a consequence of special relativity, faster-than-light communication is impossible. Also, the unavoidable energy consumption in the the Maxwell's demon and Landauer's erasure indicates that information is physical [2], and the link between thermodynamics and information has potential to deliver new insights in physics and biology.

The role of information in physics theory has been extensively investigated. For example, it is proposed that quantum theory can be derived and reconstructed from purely informational principles [3, 4]. The effect of uncertainty relation in information processing can be stated in terms of information content principle [5] and No-Disturbance-Without-Uncertainty principle [6]. Therein, a fundamental and interesting concern is the channel capacity in communication. According to the no-signaling principle, there is no information gain without classical or quantum communication; the transmission of the message as the cause that increases the information. It is well known that, in the dense-coding protocol, two bits of information can be carried in one qubit with pre-shared entanglement [7]. For the receiver to obtain n bits of information, at least a total of n qubits have to be exchanged and at least $n/2$ qubits have to be sent from the sender [8–11]. As a generalization of no-signaling principle respected both in classical and quantum physics, information causality states that one cannot gain more information than the number of bits sent via classical communication [12]. Note that the protocols mentioned above are proposed for one-way communication, and quantum entanglement as physical resource is initially distributed between the sender and receiver.

Photons as flying qubits are usually exploited in quantum communication. Generating entangled photon pairs is much more difficult than preparing single-photon sources with near-term photonic technology. Given a photon as an information carrier, its particle-wave duality makes two-way communication possible. Very recently a variant of the "guess your neighbor's input" game [13] was studied by Santo and Dakic [14], which we call the SD game in this article. They proposed a protocol (SD protocol) to win the SD game with certainty, while a classical strategy can win with probability at most 50%. We review the SD game as follows. Two distant agents Alice and Bob are given two input bits $x, y \in \{0, 1\}$, respectively, which are drawn uniformly at random, and they are asked to output two bits $a, b \in \{0, 1\}$, respectively. They win the game if both of them output a bit that is equal to the other's input (i.e., $a = y$ and $b = x$). However, only an information carrier, classical or quantum, can be manipulated. Obviously, one-bit classical communication must be one-way communication, and they cannot win with certainty using a classical information carrier. Using a photon, on the other hand, they can win the SD game with certainty [15]. Notably,one of them can gain one-bit of information even without any signal being detected. According to Renninger's negative result experiment [16, 17] or the bomb-testing problem [18], even if there is no interaction between the quantum object and the measuring device, one still learns definite knowledge of the quantum state.

The concept of SD protocol is explained as follows. To take quantum advantage of a photon, let a referee source emit a photon, which is then injected into the first beam splitter (BS 1) of a Mach-Zehnder (MZ) interferometer. Consequently, this single photon is coherently superposed over different spatial locations. Hence the two local agents can each (i) perform local operations on the incoming parts of the photon as information encoding, and (ii) access a detector to detect the photon at some time window later. According to (i), Alice and Bob encode their bits in the phase of the photon before it reaches the second beam splitter (BS2). With delicate design, the parity of the two input bits completely determines the path of the photon leaving BS2. Consequently, one knows with certainty which detector will detect this photon while the other will detect nothing. For example, Alice's (Bob's) detector clicks if $x = y$ ($x \neq y$) in the ideal case. Once Alice's detector does not receive any photon at some time window (no interaction between the quantum object and the measuring device), she knows that $x \neq y$ and outputs bit $a = x + 1 \mod 2$. As a result, using the spacial superposition of a single photon Alice and Bob can communicate a total of two-bit information and hence win the game with certainty. As opposed to quantum communication, to win the game with certainty using classical communication, Alice and Bob each must send a classical bit to the other.

In this paper, we characterize the power of a single photon as an information carrier. Our concerns are twofold: how much information a single photon can carry; and how much information an agent can obtain even if an interaction-free measurement occurs (no photon is detected by the detectors at hand).

Generalized SD game.— We consider a generalized SD (GSD) game as follows. Alice and Bob are assigned two independent input strings $\mathbf{x} = x_1 \cdots x_n$, $\mathbf{y} = y_1 \cdots y_n \in \{0,1\}^n$, respectively, and they are asked to output bit strings $\mathbf{a} = a_1 \cdots a_n$ and $\mathbf{b} = b_1 \cdots b_n$, respectively. The referee is restricted to emit only a single photon flying to a MZ interferometer. Without classical communication, they win the game if $a_i = y_i$ and $b_i = x_i$ for all i. Intuitively if n photons are available, they can repeat the SD protocol n times to win the game with certainty. Equipped with a single photon in the GSD game, it will be shown that there can be non-zero information gain for both Alice and Bob as a result of two-way signaling. In particular, there is non-zero probability to win the game with certainty if one of them can access only a detector.

The experimental setup of our protocol can be unfolded as Fig. 1, which can be schematically depicted as a perfect n-level binary tree. A detector is placed at each leaf node, and a MZ interferometer is placed at each parent node. The evolution of two levels is detailed in Fig. 2. According to the input bits a_i and b_i, Alice and Bob perform phase encoding by inserting a phase modulator (bit value = 0) or not (bit value = 1) into each of the 2^i MZ interferometers at depth i. Hence a single photon injected into the root will travel through one of the 2^n light paths. The light path taken after leaving the MZ interferometer at depth i is completely determined by the parity of a_i and b_i, and then flies to one of the two following MZ interferometers at depth $(i+1)$. After traveling through n MZ interferometers in sequence, the photon finally goes to one of the 2^n detectors, which are locally accessible to either Alice or Bob. (Note that it is not necessary that Alice and Bob have an equal number of detectors.) The detector where the photon flies is completely determined by the bitwise parities of $\mathbf{x}$ and $\mathbf{y}$. The local agent whose detector clicks can learn these n parities and hence knows the other's n input bits exactly.

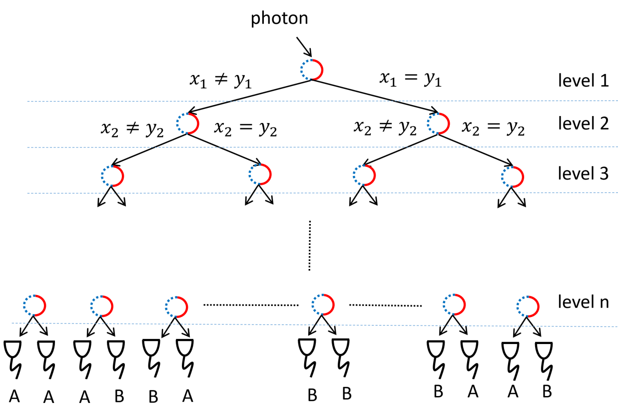

FIG. 1. The unfolding layout of the n-level circuit as a perfect n-level binary tree and the detectors. The photon is initially injected into the level-1 MZ interferometer. After traveling through a cascade of n MZ interferometers, the photon flies into one of the 2^n detectors, each of which is held by Alice (A) or Bob (B). For example, without loss of generality, let the photon goes to the right MZ interferometer at level 2 if $x_1 = y_1$, and the left MZ interferometer, otherwise.

Let us quantify how many detectors Bob should have to optimize his information gain $I(X;B|Y)$, where $I(X;B|Y) = H(B|Y) - H(B|X,Y)$ is the *mutual information* between Alice's input variable X and Bob's output variable B conditioned on Bob's input variable Y; $H(B|Y)$ is the *conditional Shannon entropy*, and $H(X) = -\sum_{\mathbf{x}} p_{\mathbf{x}} \log p_{\mathbf{x}}$. Let m be the number of detectors that belong to Bob. Since X and Y are independent, it is clear that

$$\begin{aligned} I(X;B|Y) =& H(B) = H\left(\left\{\underbrace{1/2^n,\ldots,1/2^n}_{m}, 1-m/2^n\right\}\right) \\ =& n - \left(1-\frac{m}{2^n}\right)\log\left(2^n-m\right). \end{aligned}$$

The total information gain of Alice and Bob is

$$\begin{aligned} & I(Y;A|X) + I(X;B|Y) = H(A) + H(B) \\ & = 2n - \frac{m}{2^n}\log m - \left(1-\frac{m}{2^n}\right)\log\left(2^n-m\right) \leq n+1, \end{aligned}$$

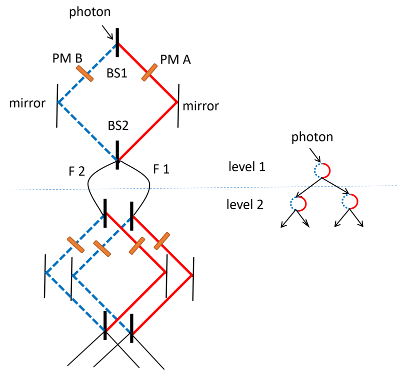

FIG. 2. Left: The two-level evolution circuit. The photon is initially injected into the level-1 MZ interferometer. After traveling through the first 50/50 beam splitter (BS1), the incident photon is half-reflected and half-transmitted in a coherent way. Alice and Bob can locally access the halves depicted in the red solid line and blue dash line, respectively. For the local encoding at level 1, Alice inserts the π-phase modulator (PM A) if $x_1 = 0$ and does nothing if $x_1 = 1$; similarly, Bob inserts the π-phase modulators, (PM B) if $y_1 = 0$ and does nothing if $y_1 = 1$. After the interference of these two coherent halves meeting at the second 50/50 beam splitter (BS2), the photon leaves the BS2, and is then injected into one of the two fibers (F1 and F2), and enters one of these two MZ interferometers at level 2. Similarly, Alice (Bob) inserts PMs into these two MZ interferometers at level 2 if $x_2 = 1$ ($y_2 = 1$) and does nothing if $x_2 = 0$ ($y_2 = 0$). Here the bit values x_1, y_1, x_2, and y_2 are all set to 1. Right: The topological unfolding of the 2-level circuit as a full 2-level binary tree. The nodes therein denote the MZ interferometers, where the photon is spatially superposed, while the directed edges between nodes indicates the possible travelling paths of the photon.

where the equality holds when $m = 2^{n-1}$. The main result can be stated as follows:

The optimal total information gain is $n + 1$.

Without loss of generality, we may assume that one of Alice's (Bob's) detectors always clicks (never click). In this case, Alice knows y and hence gains n-bit of information. On the other hand, even though there is no interaction between Bob's measuring devices and the photon, he still can obtain one-bit of information. In other words, a single photon can carry infinite information as n goes to infinity. To reach optimal total information gain, Alice and Bob each can access half of these 2^n detectors, and Bob can learn one-bit information of $\mathbf{x}$. In addition, what local information can be learned is closely related to the local accessibility of these 2^n detectors between Alice and Bob. As a concrete example, suppose that Alice and Bob can access left and right halves of the detectors shown in Fig. 1, respectively. Since none of Bob's detectors clicks, he knows the relation $x_1 = y_1$ must be false and hence outputs the bit $b_1 = y_1 + 1 \mod 2$. With dedicated initial assignment of these 2^n detectors between Alice and Bob, they can exchange the input bit pair (x_k, y_k) with certainty for some specific k.

It is noteworthy to mention the following detector assignment. Let Alice occupy only one detector and Bob occupy the other $2^n - 1$ ones. With the probability 2^{-n}, Alice's detector receives a photon. In this case, the no-click on Bob's side makes him exclude the possibility of $2^n - 1$ parity relation sets and hence learn the input string $\mathbf{x}$. Alice and Bob can win the GSD game with certainty with probability 2^{-n}. As for the $n = 1$ case, Alice and Bob can always exchange one-bit information [14].

Moreover, the implementation of Fig. 1 can be refined in the case that the detectors at the left and right leaves belong to Alice and Bob, respectively. Specifically, we can use only two detectors (one for Alice and the other for Bob) and add a time domain coordinate to save the massive number of 2^n detectors required. This is done as shown in Fig. 3, where the left path at level i has a time delay D and the right light path at level i has an additional delay of $2^{n-i-1}D$ and both the left and right paths the level n have delay D. Consequently, the 2^n light paths from left to right in Fig. 1 will have delays $nD, nD, (n+1)D, (n+1)D, \ldots, (2^{n-1}-1)D, (2^{n-1}-1)D$ in Fig. 3, respectively. An important observation is that at the same level each of Alice and Bob has the same input bit and applies the same PMs to the corresponding light paths. Also a beam splitter has two input ports, which allows us to connect both branches to the same beam splitter. Therefore, from the time of clicking, one can deduct the corresponding light path in the circuit of Fig. 1 and learn the n-bit string.

As a comparison, the previous scheme by Santo and Dakic [14] uses one single-photon source and two detectors to exchange two bits of information. Our scheme is able to transmit more information at the cost of additional fibers, MZ interferometers, and beam splitters.

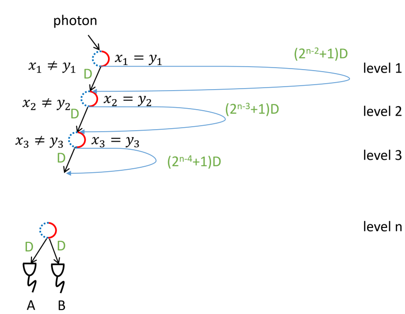

FIG. 3. An effective circuit with two detectors. All the left paths have a time delay D, while the right paths have additional delays denoted by longer optical fibers.

Discussion.— A lesson learned from the dense coding is that sending one qubit is equivalent to sending two classical bits; another lesson from information causality is that, if there is no quantum communication, the information gain is equal to the amount of classical communication. Notably, the dense coding and random access code each

(i) are one-way communication, and (ii) exploit quantum entanglement as physical resource. To the best of our knowledge, the protocols of SD and GSD games are the first ones for two-way signaling quantum communication. Therein, the spatial coherent superposition and wave-particle duality can be regarded as physical resources. From the two-way signaling aspect, these two quantum properties of a photon are more beneficial than quantum entanglement. In the proposed two-way signaling protocol, sending a photon with an n-level circuit is equivalent to sending $n+1$ bits, where n can be arbitrarily large. As a result, given a photon emitted at the beginning of the universe, it is feasible that it carries infinite information. Which agent can obtain the others information depends on the local bit strings $\mathbf{x}$ and $\mathbf{y}$, and the pre-assignment of these 2^n detectors to Alice or Bob. In any way, there is always a detector that clicks, which indicates either $I(Y;A|X) = n$ or $I(X;B|Y) = n$ must hold, and hence we can conclude that $n \leq I(Y;A|X)+I(X;B|Y) \leq n+1$.

From the causal perspective, the optimal information gain in the GSD game can be explained in a two-fold way. Firstly, an information carrier is consumed therein. Notably, regarding the classical communication, information causality states that the information gain cannot exceed the amount of classical communication. Thus sending-and-receiving a photon can result in one-bit information gain. Secondly, the two distant local operations at the same level fully determines into which way the photon enters in the next level, and this contributes the one bit information. In other words, only when the coherent superposed parts meet at BS2 as shown in Fig. 1, the which-way uncertainty between these two beam splitters in the MZ interferometer vanishes, and consequently produces one-bit information. That is, a level contributes one-bit information gain. At the end, at most $(n+1)$ bits of information can be generated during a photon entering an n-level circuit.

For example, in the Elitzur-Vaidman bomb tester, a single photon is emitted, but one of its coherent parts is blocked and there is no interference at the second beam splitter of a MZ interferometer [18]. In this case, only a bit of information (whether the bomb explodes) is accessible. On the other hand, in the simple one-way SD game, assume that the bit $y_1 = 1$ is public, and the bit x_1 is unknown to Bob [14]. To inform Bob, Alice performs local operations on the accessible coherent superposed part. It is the interference at the second beam splitter brings Bob the bit value of x_1.

Implemetation.— Here we estimate the performance of the protocol when it is implemented under realistic experimental conditions. We consider the following error sources. A realistic pulsed single-photon source has a photon number probability $P(n)$ to generate n photons per pulse. A quantum dot single-photon source can achieve $P(1) = 0.72$ [19]. The beam splitters in experiments may not have perfectly even split ratio between transmission and reflection, but this uneven split ratio can be compensated with experimental techniques, such as using wave plates together with polarized beam splitters. Therefore, we assume the split ratio is perfectly even. We also assume the phase errors given by phase shifter/modulators are negligible compared to other error sources. This is justifiable when using piezoelectric phase shifters, which can achieve a phase accuracy better than $2\pi/500$. We consider the optical loss to be a dominating error source, which can result from the non-100% reflectivity of mirrors and the non-perfect anti-reflection coatings of all transmissive optical components. We estimate the optical loss ϵ per stage to be 1.5%. For example, using two AR coated surface for a wave plate and one AR coated surface for a beam splitter, each of 0.5% loss. We assume the detection efficiency η_D of the detectors to be 85%, which is achievable using superconducting nanowire single photon detectors (SNSPD). The contribution from the dark counts of the detectors can be negligible by using low-dark-count detectors such as SNSPD or by applying gating techniques. Using these numbers, we obtain the success rate of our protocol for n stages to be $P(1)\,(1-\epsilon)^n\,\eta_D$, as shown in Fig. 4.

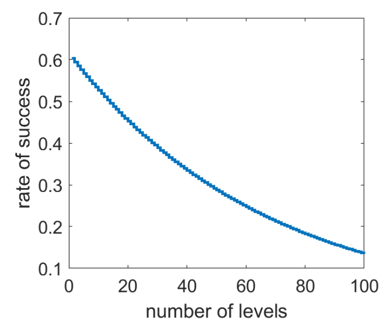

FIG. 4. The rate of success of our GSD protocol versus the number of circuit levels.

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