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Slow optical solitons via intersubband transitions in a semiconductor quantum well

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Abstract – We show the formation of bright and dark slow optical solitons based on intersubband transitions in a semiconductor quantum well (SQW). Using the coupled Schrödinger-Maxwell approach, we provide both analytical and numerical results. Such a nonlinear optical process may be used for the control technology of optical delay lines and optical buffers in the SQW solid-state system. With appropriate parameters, we also show the generation of a large cross-phase modulation (XPM). Since the intersubband energy level can be easily tuned by an external bias voltage, the present investigation may open possibilities for electrically controlled phase modulators in solid systems.

Solitons describe a class of fascinating shaping-preserving wave propagation phenomena in nonlinear media. Over the past few years, the subject of extensive theoretical and experimental investigations on solitons in optical fibers [1,2], cold-atom media [3–7], Bose-Einstein condensates (BEC) [8,9], and other nonlinear media [10], have received a great deal of attention mainly due to the fact that these special types of wave packets are formed as a result of the interplay between nonlinearity and dispersion properties of a medium under excitations, and can lead to undistorted propagation over extended distance. In the optical domain, most optical solitons are produced with intense electromagnetic fields, and far–off-resonance excitation schemes are generally employed in order to avoid unmanageable optical-field attenuation and distortion [1]. As a result, optical solitons produced in this way generally travel with a propagation speed very close to the speed of light in vacuum. As well known, the wave propagation velocity in a highly resonant medium can be significantly reduced via the electromagnetically induced transparency (EIT) technique [11] or Raman-assisted interference effects. Recently, ultraslow optical solitons including two-color solitons with very low group velocities based on the EIT technique or on Raman-assisted interference effects, have been studied in an atomic medium [3–7].

There is a great interest in extending these studies to semiconductors, not only for the understanding of the nature of quantum coherence in semiconductors, but also for the possible implementation of optical devices such as XPM phase shifters [12], switches [13], etc. It is well known, in the conduction band of a semiconductor quantum structure, that the confined electron gas exhibits atomiclike properties. For example, it has been shown that they can lead to gain without inversion [14–16], coherently controlled photocurrent generation [17], electron intersubband transmissions [18], and EIT [19,20], slow light [21], interferences [22], optical bistability [23], etc. Devices based on intersubband transitions in SQW structures have many inherent advantages such as large electric dipole moments due to the small effective electron mass, high nonlinear optical coefficients, and a great flexibility in device design by choosing the materials and structure dimensions. Furthermore, the transition energies, dipoles

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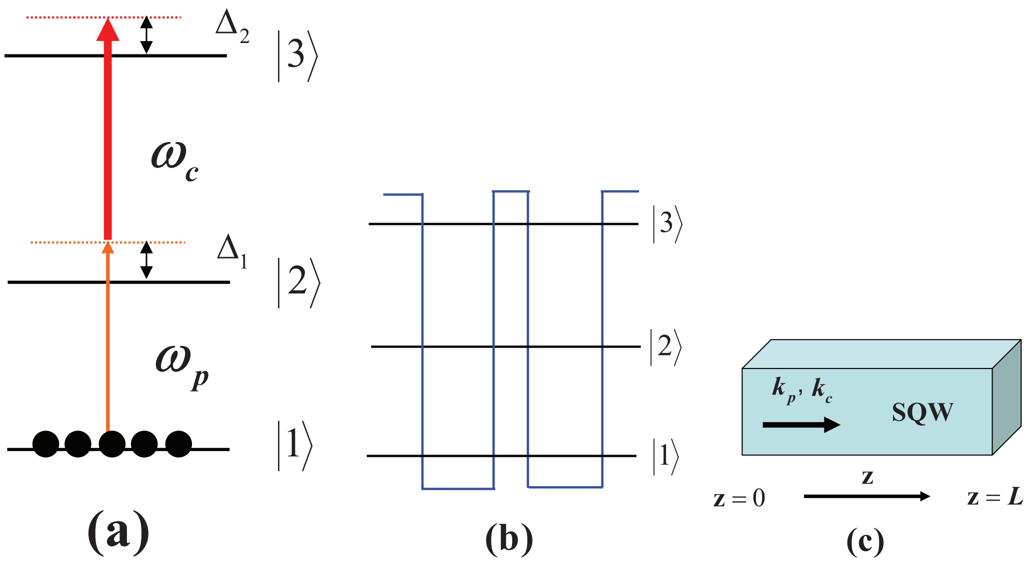

Fig. 1: (a) Schematic of the energy level arrangement for the quantum wells under consideration here. Subband levels are labeled as $|1\rangle$, $|2\rangle$, and $|3\rangle$, respectively. The subband transition $|1\rangle \leftrightarrow |2\rangle$ is driven by a weak probe field with central frequency ω_p and the subband transition $|2\rangle \leftrightarrow |3\rangle$ is coupled by a control field with central frequency ω_c. (b) Schematic of the three-level cascade electronic system synthesized in a semiconductor quantum well. (c) All the light propagates along the z-axis within our SQW sample.

can be controlled by an external bias voltage. The implementation of XPM phase shift in semiconductor-based devices is very attractive from the viewpoint of applications, such as the electroptical modulator.

In this paper, we show the formation of ultra-slow bright and dark solitons in a semiconductor double quantum well using intersubband transitions by applications of a pulsed probe field and a continuous-wave (cw) strong control laser field. With appropriate parameters choices, we also show the generation of a large XPM phase shift. As shown in fig. 1, we consider an quantum well structure with three energy levels that forms the well-known cascade configuration [24]. ω_{21} and ω_{32} presents the energy differences of the $|1\rangle \leftrightarrow |2\rangle$ and $|2\rangle \leftrightarrow |3\rangle$, respectively. As a rule, such SQW samples are grown by the molecular beam epitaxy (MBE) method. The sample consists of 30 periods, each with 4.8 nm $In_{0.47}Ga_{0.53}As$, 0.2 nm $Al_{0.48}In_{0.52}As$, and 4.8 nm $In_{0.47}Ga_{0.53}As$ coupled quantum wells, separated by modulation-doped 36 nm $Al_{0.48}In_{0.52}As$ barriers. The sample can be designed to have desired transition energies, *i.e.*, E_{12} in the range of 185 meV and E_{23} in the range of 124 meV. As shown in fig. 1(c), all the light propagates along the z-axis within our SQW sample, and we consider a transverse magnetic polarized probe incident at an angle of 45 degrees with respect to the growth axis so that all transition dipole moments include a factor $1/\sqrt{2}$, as intersubband transitions are polarized along the growth axis. The sheet electron density is about $4.7 \times 10^{11}\,\mathrm{cm}^{-2}$. By using the standard approach (this method has described quantitatively the results of several papers [13,15,16,18, 20,25,26]), under the rotating-wave and electro-dipole approximations the semiclassical Hamiltonian describing the electron-field interaction for the system under study in the Schrödinger picture, is given by

$$H = \sum_{j=1}^{3} E_j|j\rangle\langle j| - \hbar(\Omega_c e^{-i\theta_c}|3\rangle\langle 2| + \Omega_p e^{-i\theta_p}|2\rangle\langle 1| + \text{h.c.}), \tag{1}$$

where the symbol h.c. means the Hermitian conjugate, $\theta_n = k_n \cdot r - \omega_n t$ corresponds to the positive-frequency part of the respective optical field, $\Omega_n (n = p, c)$ are one-half Rabi frequencies for the revant laser-driven intersubband transitions, and $E_j = \hbar\omega_j (j = 1\text{–}3)$ is the energy of the subband $|j\rangle$. For simplicity, in the following analysis we take $\omega_1 = 0$ for the ground-state level $|1\rangle$ as the energy origin. Turning to the interaction picture, with the assumption of $\hbar = 1$, the free and the interaction Hamiltonian can be respectively rewritten as follows:

$$H_0 = \omega_p|2\rangle\langle 2| + (\omega_p + \omega_c)|3\rangle\langle 3|, \tag{2}$$

$$H_I = -\Delta_1|2\rangle\langle 2| - \Delta_2|3\rangle\langle 3| - (\Omega_c e^{ik_c\cdot r}|3\rangle\langle 2| + \Omega_p e^{ik_p\cdot r}|2\rangle\langle 1| + \text{h.c.}), \tag{3}$$

where the intersubband transition detunings of the two optical fields are defined, respectively, by $\Delta_1 = \omega_p - E_2/\hbar$ and $\Delta_2 = \omega_p + \omega_c - E_3/\hbar$. Let us assume the electronic wave function of the form

$$|\psi\rangle = A_1|1\rangle + A_2 e^{ik_p\cdot r}|2\rangle + A_3 e^{i(k_p+k_c)\cdot r}|3\rangle, \tag{4}$$

together with $A_j (j = 1, 2, 3)$ being the time-dependent probability amplitudes of finding the electron in subbands $|j\rangle$. Making use of the Schrödinger equation in the interaction picture $i\partial|\psi\rangle/\partial t = H_I|\psi\rangle$ for the three level model, the equations of the motion for the probability amplitude of the electronic wave functions and the wave equation for the time-dependent probe field can be readily obtained as

$$\frac{\partial A_1}{\partial t} = i\Omega_p^* A_2, \tag{5}$$

$$\frac{\partial A_2}{\partial t} = i(\Delta_1 + i\gamma_2)A_2 + i\Omega_c^* A_3 + i\Omega_p A_1, \tag{6}$$

$$\frac{\partial A_3}{\partial t} = i(\Delta_2 + i\gamma_3)A_3 + i\Omega_c A_2, \tag{7}$$

$$\frac{\partial \Omega_p}{\partial z} + \frac{1}{c}\frac{\partial \Omega_p}{\partial t} = i\frac{2N\omega_p|\mu_{21}|^2}{c} A_2 A_1^*, \tag{8}$$

with N and μ_{12} being the concentration and the dipole mement between states $|1\rangle$ and $|2\rangle$, respectively. In writing eq. (8), we have assumed colinear propagation geometry and applied the slowly-varying-envelope approximation. γ_2 and γ_3 denote the total decay rates of the subbands $|2\rangle$ and $|3\rangle$, which are added phenomenologically [13,18] in the above coupled amplitude equations. In semiconductor quantum wells, the overall decay rate γ_i of the subband $|i\rangle$ comprises a population-decay contribution γ_{il} as well as a dephasing contribution γ_{id}, *i.e.*, $\gamma_i = \gamma_{il} + \gamma_{id}$. The former γ_{il} is due to longitudinal optical (LO) photon emission events at low temperature. The latter γ_{id} may originate not only from electron-electron scattering and electron-phonon scattering, but also from inhomogeneous broadening due to the scattering on interface roughness. The population decay rates can be calculated by solving the effective mass Schrödinger equation. For temperatures up to 10 K, and a carrier density smaller than $10^{12}\,\text{cm}^{-2}$, the dephasing decay rates γ_{ij}^{dph} can be estimated according to ref. [13]. For SQW our structure considered here, the total decay rates turn out to be $\gamma_2 = \gamma_3 = 5\,\text{meV}$. A more complete theoretical treatment taking into account these processes for the dephasing rates is though interesting but beyond the scope of this paper.

In order to describe clearly the interplay between the dispersion and nonlinear effects of the SQW system interacting with two optical fields (probe and control fields), we now first focus on the dispersion properties of the system. It requires perturbation of the system with respect to the first order of probe field Ω_p while keeping full orders of the control field Ω_c. In the following, we show effects that are due to higher-order Ω_p that are required for balancing the dispersion effect, thus we can show the formation of ultraslow solitons. Considering the situation that almost all electrons remain in the subband level $|1\rangle$ due to the fact that the laser-matter interaction is weak, hence we may assume that $A_1(t=0) = 1$, and the strong pump condition that the control laser is strong enough to make $\kappa = \Omega_p/\Omega_c$ be a small parameter (weak probe approximation). Then we take $A_j = \Sigma_n A_j^{(n)}$ with $A_j^{(n)} = O(\kappa^n)$ and assume the adiabatic condition $\hat{\omega}/\Omega_c = O(\kappa)$. Performing the time Fourier transformations for eqs. (6)–(8) [27–29], we can obtain the equations $(\hat{\omega} + \Delta_1 + i\gamma_2)\alpha_2 + \Omega_c^* A_3 = -\tilde{\Lambda}$, $(\hat{\omega} + \Delta_2 + i\gamma_3)\alpha_3 + \Omega_c\alpha_2 = 0$, and $\partial\tilde{\Lambda}/\partial z - i\tilde{\Lambda}\hat{\omega}/c = i\epsilon_{12}\alpha_2$, where $\epsilon_{12} = 2N\omega_p|\mu_{12}|^2/c$, and $\tilde{\Lambda}$, $\hat{\omega}$, and α are the Fourier-transform variable, the Fourier transforms of $A_j (j = 2, 3)$ and Ω_p, respectively. Those equations can be solved analytically, yielding $\tilde{\Lambda}(z, \hat{\omega}) = \tilde{\Lambda}(0, \hat{\omega})\exp(iK(\hat{\omega})z)$, where

$$\begin{aligned} K(\hat{\omega}) &= \frac{\hat{\omega}}{c} + \frac{\epsilon_{12}(\hat{\omega} + \Delta_2 + i\gamma_3)}{|\Omega_c|^2 - (\hat{\omega} + \Delta_1 + i\gamma_2)(\hat{\omega} + \Delta_2 + i\gamma_3)} \\ &\simeq K_0 + \frac{\hat{\omega}}{v_g} + K_2\hat{\omega}^2 + O(\hat{\omega}^3), \end{aligned} \tag{9}$$

with higher-order derivative terms have been neglected. The physical interpretation of eq. (9) is rather clear. $K_0 = \Phi + i\alpha$ describes the phase shift Φ per unit length and absorption coefficient α of the pulsed probe field, K_1 gives the group velocity $V_g = \text{Re}[1/K_1]$, and K_2 represents the group velocity dispersion that contributes to the probe pulse's shape change and additional loss of the pulsed probe field intensity. With the dispersion coefficients obtained, then we describe the nonlinear evolution of the probe field. We should emphasize that it is indeed possible to obtain a set of experimentally achievable parameters that will lead to the formation of ultraslow solitons, and solitons produced in this way generally travel with a group velocity given by $V_g = \text{Re}[1/K_1(0)]$.

By taking a trial function $\Omega_p(z,t) = \Omega_p(z,t)\exp(iK_0 z)$ and substitute it into the wave function (8), we can have the nonlinear wave equation of the slowly varying envelope $\Omega_p(z,t)$,

$$\frac{\partial \Omega_p}{\partial z} + \frac{1}{c}\frac{\partial \Omega_p}{\partial t} = iA_1^*\left[K(\hat{\omega}) - \frac{\hat{\omega}}{c}\right](\Omega_p A_0), \tag{10}$$

where the right-hand terms of eq. (10) can be obtained based on the following equations:

$$A_1^*\left[K(\hat{\omega}) - \frac{\hat{\omega}}{c}\right](\Omega_p A_1) = |A_1|^2\left[K(\hat{\omega}) - \frac{\hat{\omega}}{c}\right]\Omega_p + O(\kappa^4), \tag{11}$$

$$K(\hat{\omega})\Omega_p = \left[K_0 + \frac{\hat{\omega}}{v_g} + K_2\hat{\omega}^2\right]\Omega_p + O(\kappa^4), \tag{12}$$

$$|A_1|^2 \simeq 1 - |A_2|^2 + |A_3|^2, \tag{13}$$

with A_j, $(j = 2, 3, 4)$, given by

$$A_j = \frac{[(\Delta_2 + i\gamma_3)\delta_{j2} - \Omega_c^*\delta_{j1}]\Omega_p}{|\Omega_c|^2 - (\Delta_1 + i\gamma_2)(\Delta_2 + i\gamma_3)} + O(\kappa^2). \tag{14}$$

Equation (14) is readily obtained by solving eqs. (5), (6) under steady-state conditions, *i.e.*, $\partial A_{2,3}/\partial t = 0$ and $A_1^{(1)} = 1$. Here we have used the relations $\partial A_{2,3}/\partial t = O(\kappa A_{2,3}) = O(\kappa^2)$ and $A_1 = 1 + O(\kappa^2)$. Using the above results and discussion, it is then straight-forward to obtain the following nonlinear evolution equation, which

$$\alpha = \text{Im}\left[\frac{\epsilon_{12}(\Delta_2 + i\gamma_3)}{|\Omega_c|^2 - (\Delta_1 + i\gamma_2)(\Delta_2 + i\gamma_3)}\right], \tag{16}$$

$$W = \frac{\epsilon_{12}(\Delta_2 + i\gamma_3)(|\Omega_c|^2 + \Delta_2^2) + \gamma_3^2}{[|\Omega_c|^2 - (\Delta_1 + i\gamma_2)(\Delta_2 + i\gamma_3)]|[|\Omega_c|^2 - (\Delta_1 + i\gamma_2)(\Delta_2 + i\gamma_3)]|^2}. \tag{17}$$

$$\Omega_p = \Omega_{p0}\text{sech}(\eta/\tau)\exp[-i\xi W_r|\Omega_{p0}|^2/2], \tag{21}$$

$$\Omega_p = \Omega_{p0}\frac{4[\cosh(3\eta/\tau) + 3\exp(-8iK_{2r}\xi/\tau^2)\cosh(\eta/\tau)]\exp(-iK_{2r}\xi/\tau^2)}{\cosh(4\eta/\tau) + 4\cosh(2\eta/\tau) + 3\cos(8K_{2r}\xi/\tau^2)}, \tag{22}$$

is accurate up to the order $O(\kappa^3)$, for the slowly varying envelope $\Omega_p(z,t)$,

$$i\frac{\partial\Omega_p}{\partial\xi} - K_2\frac{\partial^2\Omega_p}{\partial\eta^2} = We^{-\alpha\xi}|\Omega_p|^2\Omega_p, \tag{15}$$

here we have assumed $\xi = z$, $\eta = t - z/v_g$. The velocity v_g and the dispersion coefficient K_2 are determined by eq. (9), the absorption coefficient $\alpha = \text{Im}(K_0)$ and the nonlinear coefficient W are explicitly given by

see eq. (16) *above*

see eq. (17) *above*

Now we briefly discuss the cross-phase modulation (XPM). Let us consider the following parameter condition: $\Delta_1 \simeq 0$, with other parameters unchanged and writing $K_0L = \Phi_{\text{XPM}} + i\alpha L$ (L is the length of the SQW system), it is straightforward to show that

$$\Phi_{\text{XPM}} \simeq \frac{|\Omega_c|^2\Delta_2\epsilon_{12}}{\gamma_2^2\Delta_2^2 + (|\Omega_c|^2 + \gamma_2\gamma_3)^2}, \quad \alpha \simeq \frac{|\Omega_c|^2\gamma_3\epsilon_{12}}{\gamma_2^2\Delta_2^2 + (|\Omega_c|^2 + \gamma_2\gamma_3)^2}. \tag{18}$$

These results in our structure are similar to those of the giant cross-phase modulation in cold-atom media [4], but, we only need one control laser field and do not need to introduce another control laser field. The ratio of $\Phi_{\text{XPM}}/\alpha L$, characterizing the ability achieving the cross-phase modulation phase shift without appreciated absorptions, has the form Δ_2/γ_3 and is independent of the coupling field intensity. Furthermore, since the intersubband energy level can be easily tuned by an external bias voltage, we may provide another possibility to realize an electrically controlled phase modulator at low light levels.

If a reasonable and realistic set of parameters can be found so that $\exp(-\alpha L) \simeq 1$, *i.e.*, the losses of the probe pulse are small enough to be neglected, then the balance between the nonlinear self-phase modulation and the group velocity dispersion (described by the coefficient K_2) may keep a pulse with shape-invariant propagation, which yields $K_2 = K_{2r} + iK_{2i} \simeq K_{2r}$, and $W = W_r + iW_i \simeq W_r$. Thus eq. (15) can be reduced to the standard nonlinear Schrödinger equation governing the pulsed probe field evolution [3,4]

$$i\frac{\partial\Omega_p}{\partial\xi} - K_{2r}\frac{\partial^2\Omega_p}{\partial\eta^2} = W_r|\Omega_p|^2\Omega_p, \tag{19}$$

which admits of solutions describing bright ($K_{2r}W_r < 0$) and dark ($K_{2r}W_r > 0$) solitons, including the N-soliton ($N = 1, 2, 3, \ldots$) for dark and bright solitons. And whether the solutions to eq. (19) are the bright solitons or the dark solitons depends on the sign of the product $K_{2r} \cdot W_r$. The single soliton is called as the fundamental soliton, and the N-soliton ($N = 2, 3, \ldots$) is named as the higher-order soliton.

The fundamental dark soliton of eq. (19) with $K_{2r}W_r<0$ is

$$\Omega_p = \Omega_{p0}\tanh(\eta/\tau)\exp[-i\xi W_r|\Omega_{p0}|^2], \tag{20}$$

where amplitude Ω_{p0} and width τ are arbitrary constants subjected only to the constraint $|\Omega_{p0}\tau|^2 = -2K_{2r}/W_r$.

The fundamental bright soliton, and the bright 2-soliton (bright second-order soliton) of eq. (19) with $K_{2r}W_r > 0$ are given respectively by

see eq. (21) *above*

see eq. (22) *above*

where the amplitude Ω_{p0} and width τ are arbitrary constants subjected only to the constraint $|\Omega_{p0}\tau|^2 = 2K_{2r}/W_r$. It is worth noting that the bright 2-soliton solution in eq. (22) satisfies $\Omega_p(\xi = 0, \eta) = 2\Omega_{p0}\text{sech}(\eta/\tau)$.

Our scheme is different from the EIT in a SQW structure, in which solitons cannot be formed. Because slow group velocity propagation requires weak driving conditions, this leads to very narrow transparency windows. Thus the EIT operation with weak driving conditions requires single- and two-photon resonance excitations, *i.e.*, $\Delta_1 = \Delta_2 = 0$ in eq. (17). Deviations from these conditions will result in significant probe field attenuation and distortion. Besides, one can find the nonlinear coefficient W is almost purely imaginary under these EIT conditions. This is contradictory to the requirement of $W \simeq W_r$ in order to preserve the complete integrability of the standard nonlinear Schrödinger eq. (19). However, here we have found that by appropriate choosing the intensities and detunings

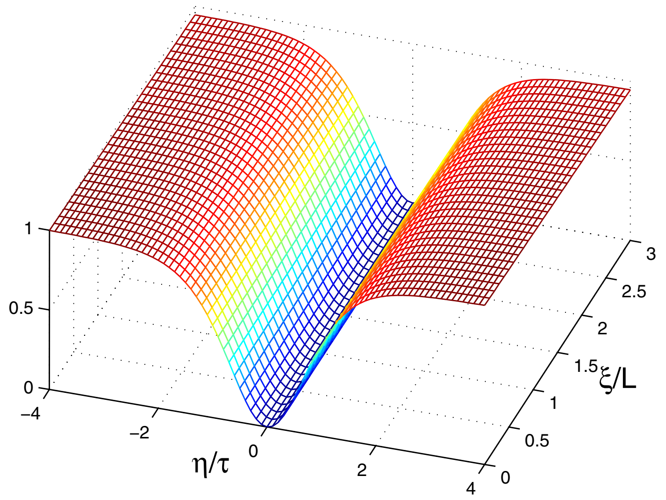

Fig. 2: Surface plot of the amplitude for the generated fundamental bright soliton $|\Omega_p/\Omega_{p0}|^2\exp(-2\alpha\xi)$ with $|\Omega_p|^2$ being the numerical solution to eq. (15) *vs.* dimensionless time η/τ and distance ξ/L under the boundary condition $\Omega_p(\xi=0,\eta)=\Omega_{p0}\tanh^2(\eta/\tau)$, where $L=1.0\,\mathrm{cm}$, and $\tau=1.0\times10^{-6}\,\mathrm{s}$ and other fitting parameters are explained in the main text.

of laser fields, we can achieve $\exp(-\alpha L)\simeq 1$ for L within a few centimeters, $K_2\simeq K_2r$, $W\simeq W_r$, and ultraslow group velocities for both bright and dark solitons studied in this paper with the typical population decay and dephasing decay rates of the transitions in SQW structures, for example, a system with total decay rates $\gamma_2=\gamma_3=5\,\mathrm{meV}$.

As an example, we now present numerical examples to demonstrate the existence of ultraslow dark solitons in the system studied through simulating eq. (15) under the boundary condition $\Omega_p(\xi=0,\eta)=\Omega_{p0}\tanh(\eta/\tau)$. Take $\epsilon_{12}=80\,\mathrm{cm}^{-1}\,\mathrm{meV}$, $\Omega_c=8\,\mathrm{meV}$, $\Delta_1=-10\,\mathrm{meV}$, $\Delta_2\simeq 0$, and $\gamma_2=\gamma_3=5\,\mathrm{meV}$, we have $V_g/c\simeq 2.7\times10^{-4}$, and $\alpha\simeq 0.00019\,\mathrm{cm}^{-1}$. With these parameters, the standard nonlinear Schrödinger equation (19) with $K_{2r}\cdot W_r<0$ is well characterized, and thus we have demonstrated the existence of dark solitons that travel with ultraslow group velocities in SQW structures. As shown in fig. 2, the numerical simulation of eq. (15) for the fundamental dark soliton shows an excellent agreement with eq. (20).

In our scheme, all the parameter sets also lead to negligible loss of the probe field for both the bright and dark solitons (including 2-soliton) described. Besides, we have used the one-dimensional model in calculation where the momentum dependency of subband energies has been ignored. However, there is no large discrepancy between the reduced one-dimensional calculation [13] and the full two-dimensional calculation [19,30–33]. In addition we have mentioned that our scheme and the strength of quantum interference in SQW depends on carrier density. Although the present study focuses only on the low temperatures up to 10 K, the results of additional broadening effects can be included by first rewriting the corresponding detunings, *i.e.*, $\Delta_1=\omega_p-E_2/\hbar\to\Delta_1-\Delta_{a1}$, $\Delta_2=\omega_p\pm\omega_c-E_3/\hbar\to\Delta_2-\Delta_{a2}$ with $\Delta_{a1}\sim k_{pz}$, $\Delta_{a2}\sim k_{pz}\pm k_{cz}$ being the additional broadening effects, which can be suppressed by a proper choice of the propagation directions of the lights. By considering ref. [30], we find that even for the moderate density, additional broadening effects are on the order of 1 meV. Combining the effect of the additional broadenings and the parameter values above mentioned, the absorption coefficient of the probe field is still on the order 10^{-4}. It is worth noting that some other many-body effects also contribute to the broadening effects except for the relaxation, for example, the depolarization effect, which renormalizes the free-carrier and carrier-field contributions. These contributions and their interplay have been investigated quite thoroughly by some authors in refs. [31,34,35]. Note that, due to the small carrier density considered here, these effects only give a small extent.

Just as done before by some authors [13,23], we have used the Maxwell-Schrödiner formalism with decay rates included in order to obtain the analytical results. It can readily be checked by numerical simulations that the results of such a treatment are essentially the same as those from the usual density matrix formalism under the condition of weak probe approximation.

In conclusion, using the coupled Schrödinger-Maxwell equations in a three-level system of electronic subbands, we have presented and analyzed a novel scheme to achieve ultraslow bright and dark optical solitons, and a large XPM phase shift can also be obtained with appropriate parameters. Such investigation of ultraslow optical solitons in the present work may lead to important applications such as high-fidelity optical delay lines, optical buffers in SQW structure. Besides, achieving a large XPM phase shift in a SQW structure may open up an avenue to explore possibilities for nonlinear optics and quantum information processing in solid system and may result in substantial impacts on technology of electrically controlled phase modulator.

Before ending, we note that there are some relevant works on optical solitons and optical breathers in semiconductor devices (for example [36,37]). The optical breathers discussed in refs. [36,37] are under the condition of self-induced-transparency (SIT) in multilevel quantum dots. Unlike SIT systems, where a single optical field with 2π or 0π area is needed to induce coherent pulse propagation, here we use two input optical fields, *i.e.* the control and probe beams, to provide the quantum interference channels. With the suppression of linear absorption and a large XPM phase shift induced by the control field, our proposed cascade quantum structure supports the shape-invariant transport of optical pulse not only on the nanoscales but also at low light powers.

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