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MISCELLANEOUS

Hyperbolic Metamaterials Based on Bragg Polariton Structures

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A new hyperbolic metamaterial based on a modified semiconductor Bragg mirror structure with embedded periodically arranged quantum wells is proposed. It is shown that exciton polaritons in this material feature hyperbolic dispersion in the vicinity of the second photonic band gap. Exciton–photon interaction brings about resonant nonlinearity leading to the emergence of nontrivial topological polaritonic states. The formation of spatially localized breather-type structures (oscillons) representing kink-shaped solutions of the effective Ginzburg–Landau–Higgs equation slightly oscillating along one spatial direction is predicted.

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1. INTRODUCTION

The propagation of electromagnetic radiation in artificial sophisticatedly structured inhomogeneous media known as metamaterials suggests the analogy between their unusual properties and the (curvilinear) geometry of spacetime used to describe physical phenomena in general relativity (see, e.g., [1]). From the viewpoint of basic research, this circumstance offers a promising way to search for analogs of fundamental effects in gravitation and cosmology that could be simulated in these media (see [2]). The idea of creating media where light propagates along designed trajectories, which is known as transformation optics [3], opens the way for the simulation of various gravitational effects like gravitational lensing [4], event horizon [2], etc. Media of this kind include hyperbolic metamaterials (HMMs), i.e., materials characterized by hyperbolic spatial dispersion [5]. The analogy between the propagation of electromagnetic waves described by the Helmholtz equation in HMMs and the effective Klein–Gordon equation describing the dynamics of a massive particle with a fictitious time coordinate can be employed to simulate the Minkowski spacetime using HMMs [6]. However, obtaining a more complete analogy with problems in gravitation, field theory, and cosmology also requires a strong Kerr-type nonlinearity in HMM structures. Meanwhile, the nonlinear response in "traditional" HMMs is fairly weak. In addition, HMMs containing metallic elements feature high ohmic losses, which lead to the strong damping of propagating electromagnetic fields.

Here, we propose a novel approach to the simulation of phenomena taking place in "curved spacetime" by means of resonant HMMs implemented as multilayer semiconductor exciton–polariton structures, which can be fabricated on the basis of modified Bragg mirror structures containing periodically arranged semiconductor quantum wells (QWs) [7, 8]. These structures are characterized by strong optical nonlinearity resulting from exciton–exciton interaction [9]. The possibility of controlling the dispersion properties of resonant HMM structures makes them suitable for studying the properties of the Higgs field (see [10]).

2. MODEL OF RESONANT HMM

Let us consider a modified semiconductor Bragg mirror shown schematically in Fig. 1a. The structure represents a lattice formed by alternating dielectric layers, with a QW being embedded at the center of each layer of one type. It should be noted that the Bragg resonance condition is not satisfied in the struc-

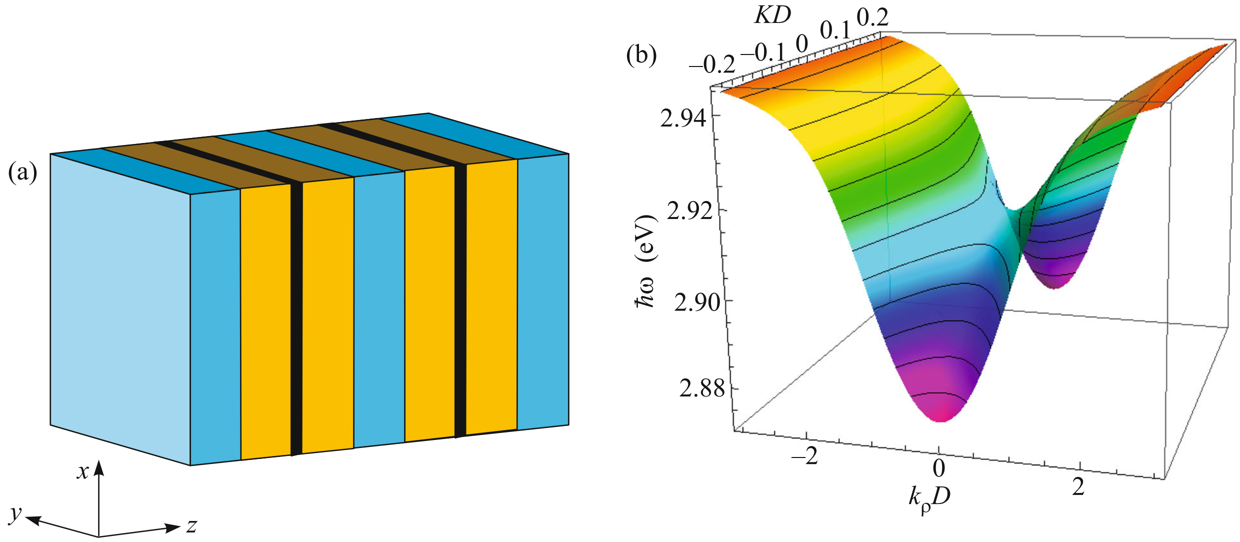

Fig. 1. (Color online) (a) Schematic layout of a spatially periodic resonant HMM structure. Narrow quantum wells at the center of each layer of one type are shown in black. (b) Dispersion surface of the lowest exciton–polariton branch in a $GaN/Al_{0.3}Ga_{0.7}N$ modified Bragg mirror structure with $In_{0.12}Ga_{0.88}N$ quantum wells located at the centers of the GaN layers. The parameters of the system used for the calculation are given in the main text.

ture under consideration; i.e., $n_1d_1 \neq n_2d_2$. This causes the opening of the second photonic band gap.

To find the eigenmodes of the structure, we use the transfer matrix method [11]. The dispersion equation for an infinite periodic structure is

$$\cos(KD) = \frac{1}{2}\mathrm{Tr}(\hat{T}), \qquad (1)$$

where K is the wave vector component in the direction perpendicular to the QW plane, D is the lattice period, and $\hat{T}$ is the transfer matrix for a single period of the structure. For the configuration under study, this transfer matrix can be expressed as

$$\hat{T} = \hat{T}_{d_1/2}\hat{T}_{\mathrm{QW}}\hat{T}_{d_1/2}\hat{T}_{d_2},$$

where $\hat{T}_{d_1/2}$, $\hat{T}_{\mathrm{QW}}$, and $\hat{T}_{d_2}$ are the transfer matrices for half of the first layer, the QW, and the second layer, respectively. Hereinafter, we consider the case of s-polarized light. Then, the transfer matrix for half of the first layer is written as

$$\hat{T}_{d_1/2} = \begin{pmatrix} \cos(k_{z1}d_1/2) & \dfrac{ik_{z1}}{k_0}\sin(k_{z1}d_1/2) \\ \dfrac{ik_{z1}}{k_0}\sin(k_{z1}d_1/2) & \cos(k_{z1}d_1/2) \end{pmatrix}, \qquad (2)$$

where $k_0 = \omega/c$, $k_{z1} = \sqrt{n_1^2k_0^2 - \mathbf{k}_\rho^2}$, $\mathbf{k}_\rho = (k_x, k_y)$ in the wave vector component in the QW plane, and n_1 is the refractive index of the first layer. The transfer matrix of the second layer can be written in a similar form by replacing $d_1/2$ with d_2 and k_{z1} with k_{z2} in Eq. (2). The transfer matrix of the QW is [11]

$$\hat{T}_{\mathrm{QW}} = \begin{pmatrix} 1 & 0 \\ 2\dfrac{k_z r}{k_0 t} & 1 \end{pmatrix},$$

where r and t are the reflection and transmission coefficients of the QW, respectively. For s polarization, these coefficients are expressed as

$$r = \frac{in_1k_0\Gamma_0/k_{z1}}{\omega_X - \omega - i(\Gamma + n_1k_0\Gamma_0/k_{z1})}, \qquad t = 1 + r,$$

where Γ_0 and Γ are the exciton radiative and nonradiative decay rates, respectively, and ω_X is the exciton resonance frequency.

Thus, Eq. (1) determines the frequencies $\omega(K, k_\rho)$ of the exciton–polariton eigenmodes formed in the medium. The combination of the Bragg splitting of the photon dispersion branch and the Rabi splitting caused by the interaction with QW excitons leads to the formation of four exciton–polariton branches in this structure [8, 12]. From now on, we disregard the interaction between the polariton branches and consider only the lowest branch. The dispersion surface for the lowest branch in a resonant HMM structure is shown in Fig. 1b.

The calculations were carried out using the parameters of a $GaN/Al_{0.3}Ga_{0.7}N$ structure with narrow $In_{0.12}Ga_{0.88}N$ QWs. For this structure, the exciton binding energy is about 45 meV, the exciton Bohr radius is $a_b \approx 18$ nm, the width of the QW is $d_{\mathrm{QW}} =$

10 nm, and the Rabi frequency is $\Omega_P \approx 2\pi \times 7.1$ THz. The center of the second photonic band gap appears at $\hbar\omega_B = 3$ eV, and its width is $\hbar\omega_B = 0.05$ eV. The thicknesses of the layers are $d_1 = 64.8$ nm and $d_2 = 115.3$ nm and their refractive indices are $n_1 = n_{\mathrm{GaN}} = 2.55$ and $n_2 = n_{\mathrm{AlGaN}} = 2.15$. The full lattice period equals $D = d_1 + d_2 = 180.1$ nm. The radiative and nonradiative decay rates of excitons in InGaN are $\hbar\Gamma_0 = 2$ meV and $\hbar\Gamma = 0.1$ meV, respectively. The exciton energy is tailored to be at $\hbar\omega_X = 2.95$ eV.

Notably, the dispersion surface of the lowest polariton branch features a saddle point [13, 14]. Therefore, an interesting feature of this system is that the lowest branch polaritons are characterized by the effective mass tensor

$$\mathbf{m}^* = \begin{pmatrix} m_\parallel^* & 0 \\ 0 & m_\perp^* \end{pmatrix},$$

with the diagonal elements of opposite signs: $\mathrm{sgn}(m_\parallel^*) = -\mathrm{sgn}(m_\perp^*) = 1$. Indices $\parallel$ and $\perp$ characterize the components of the tensor parallel to the QW plane and to the structure growth axis, respectively. In the general case, the effective mass tensor depends on the wave vector; however, the tensor components for wave vectors lower than the reciprocal lattice period $1/D$ may be considered constant.

Let us derive analytical expressions for the components of the effective mass tensor of polaritons on the lowest branch near the saddle point. Taking into account that the Bragg condition is not satisfied in the structure under consideration, we introduce small parameters $\xi = n_1d_1/n_2d_2 - 1$ and $\delta = \omega/\omega_B - 1$, where $\omega_B = 2\pi c/(n_1d_1 + n_2d_2)$ is the center frequency of the second photonic band gap in the absence of QWs. Let us expand the right- and left-hand sides of Eq. (1) with respect to small parameters ξ and δ taking into account that $K, k_\rho \ll 1/D$. Then, we can find the components of the effective mass tensor of polaritons $m^* = \hbar\left(\partial^2\omega/\partial k^2\right)^{-1}$ at the center of the first Brillouin zone along the structure growth axis and in the QW plane:

$$m_\perp^* = \frac{-2\hbar\Omega_B}{(\omega_B - \Omega_B)\omega_B D^2}\frac{\pi^2(n_1 + n_2)^2}{n_1 n_2},$$

$$m_\parallel^* = \frac{2\hbar(\omega_B - \Omega_B)}{c^2}\frac{n_1^2 n_2^2}{(n_1^2 - n_1 n_2 + n_2^2)},$$

where $\Omega_B = \omega_B|(n_2 - n_1)(1 - \xi)/2(n_1 + n_2)|$ is the half-width of the second photonic band gap.

3. GROSS–PITAEVSKII EQUATION FOR BRAGG POLARITONS

The Hamiltonian of the system can be expressed via the direct-space boson-field operator $\hat{\Psi}^\dagger \equiv \hat{\Psi}^\dagger(\mathbf{r})$ as [11, 15]

$$\hat{H} = \int d^3\mathbf{r}\hat{\Psi}^\dagger\hat{H}_0\hat{\Psi} + \frac{g}{2}\int d^3\mathbf{r}\hat{\Psi}^\dagger\hat{\Psi}^\dagger\hat{\Psi}\hat{\Psi}. \quad (3)$$

In the effective-mass approximation, the kinetic energy operator for k_ρ, $K \ll \pi/D$ is expressed as

$$\hat{H}_0 = \hbar(\omega_B - \Omega_B - \Omega_P) + \frac{\hbar^2 K^2}{2m_\perp^*} + \frac{\hbar^2 k_\rho^2}{2m_\parallel^*}.$$

In the following calculations, the constant contribution $E_0 = \hbar(\omega_B - \Omega_B - \Omega_P)$ to the polariton energy will be omitted. The coefficient g describes nonlinear interaction between the excitons and can be estimated as $g \simeq 3E_b a_b^3 D/2d_{\mathrm{QW}}$ [8].

We assume that the ground state of the lowest polariton branch is macroscopically occupied. Then, in the mean-field approximation, the operator $\hat{\Psi}(\mathbf{r})$ can be replaced with its average value $\langle\hat{\Psi}(\mathbf{r})\rangle \equiv \Psi(\mathbf{r})$, which represents the polariton wavefunction [16]. As a result, we obtain the Gross–Pitaevskii master equation for the polariton wavefunction $\Psi(\mathbf{r})$:

$$i\hbar\frac{\partial\Psi}{\partial t} = \left[-\frac{\hbar^2}{2m_\parallel^*}\Delta_{xy} - \frac{\hbar^2}{2m_\perp^*}\frac{\partial^2}{\partial z^2} + g|\Psi|^2\right]\Psi. \quad (4)$$

To find the steady-state solution of Eq. (4), we introduce a new variable φ defined as $\Psi = \sqrt{\kappa^2\kappa_z}\varphi\exp(-iEt/\hbar)$ and dimensionless coordinates $X = x/\kappa$, $Y = y/\kappa$, and $Z = z/\kappa_z$, where the normalization parameters $\kappa = \hbar\sqrt{V/2m_\parallel^* g}$ and $\kappa_z = \hbar\sqrt{V/2|m_\perp^*|g}$ represent characteristic macroscopic scales of the polariton system. The parameter E is the energy of the system, which can be estimated as $E \simeq gn_\infty$, where n_∞ is the mean density of the exciton–polariton gas in the structure. Substituting Ψ into Eq. (4), we obtain the dimensionless steady-state equation

$$\partial_{ZZ}\varphi - (\partial_{XX} + \partial_{YY})\varphi - \eta\varphi + G|\varphi|^2\varphi = 0, \quad (5)$$

where $\eta = EV/g$ and $G = V/\kappa^2\kappa_z$. The dimensionless polariton wavefunction φ satisfies the normalization condition $\iiint_{L^2 L_X}\varphi^2 dXdYdZ = N$, where N is the number of polaritons in the structure and $L_X = L_Y = L$ and L_Z are the normalized characteristic sizes of the system.

In the linear limiting case, equivalent to the case of low polariton density, where nonlinear effects caused

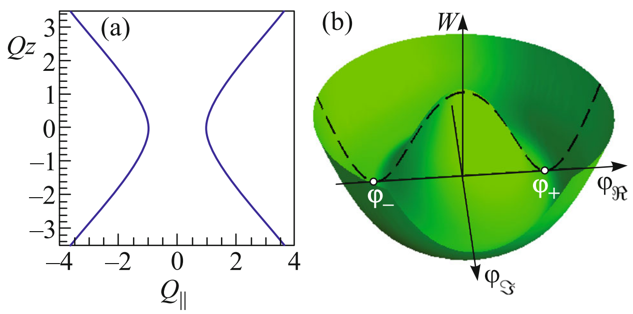

Fig. 2. (Color online) (a) Frequency contours for Bragg exciton polaritons in the linear regime. The values of $Q_{\|}$ and Q_Z are given in units of $\sqrt{\eta}$. The effective masses of polaritons in the structure are $m_{\|}^* \approx 5.6 \times 10^{-35}$ kg and $m_{\perp}^* \approx -0.8 \times 10^{-36}$ kg. (b) Schematic plot of the Higgs potential W as a function of the components $\varphi_{\Re}$ and $\varphi_{\Im}$ of the complex field φ.

by polariton–polariton interaction can be disregarded, the solution of Eq. (5) is a plane wave $\Psi \propto e^{i\mathbf{QR}}$. The components of the wave vector $\mathbf{Q}$ along the optical axis of the structure and in the QW plane obey the relation $Q_Z = \pm\sqrt{Q_{\|}^2 - \eta}$, where $Q_{\|}^2 = Q_X^2 + Q_Y^2$. The corresponding frequency contours are plotted in Fig. 2a.

4. POLARITON HIGGS FIELD

Nonlinear Eq. (5) with pseudotime coordinate Z is a Ginzburg–Landau–Higgs-type equation, which was discussed in relation to the issues of the evolution of the Universe [17, 18]. To analyze the properties of this equation, it is convenient to express the Higgs field φ as a complex scalar field $\varphi = \varphi_{\Re} + i\varphi_{\Im}$. The "Mexican hat" potential $W \equiv W(\varphi_{\Re}, \varphi_{\Im})$ is shown schematically in Fig. 2b. The false vacuum state corresponds to $\varphi = 0$, while the true vacuum states are described by $\varphi_{\pm} = \pm\sqrt{\tilde{n}_{\infty}} = \pm\sqrt{\eta/G}$ [10], where $\tilde{n}_{\infty}$ is the dimensionless density of the exciton–polariton gas, related to the dimensional density as $\tilde{n}_{\infty} = n_{\infty}\kappa^2\kappa_z$.

Let us consider the behavior of polaritons in the presence of weak perturbations $\delta\varphi_{\Re}$, $\delta\varphi_{\Im}$ of the ground state φ_0 of the system. In other words, we represent the solutions of Eq. (5) in the form $\varphi_{\Re} = \varphi_0 + \delta\varphi_{\Re}$ and $\varphi_{\Im} = \delta\varphi_{\Im}$ (where $\delta\varphi_{\Re}, \delta\varphi_{\Im} \ll \varphi_0$). Taking into account that the Lagrangian corresponding to Eq. (5) possesses global $U(1)$ symmetry, it can be assumed that the field $\delta\varphi_{\Re}$ has a characteristic analogous to the effective mass in (2 + 1)-dimensional space, while the field $\delta\varphi_{\Im}$ is massless and similar to the Nambu–Goldstone boson. Let us focus on the properties of the field $\varphi_{\Re}$. Equation (5) has a classical (steady-state) solution in the form of a kink or dark soliton $\varphi_0(X, Y) = \pm\sqrt{\tilde{n}_{\infty}}\tanh\left[\sqrt{\eta/2}(X - X_0 + Y - Y_0)\right]$, where the parameters X_0 and Y_0 define the position of the minimum of the envelope function. We rewrite the solution in a generalized form. In so doing, we perform a rotation of the reference frame by an angle $\pi/4$ about the Z axis; this is equivalent to the introduction of new Cartesian coordinates $X \to (X - Y)/\sqrt{2}$ and $Y \to (X + Y)/\sqrt{2}$. Upon such a rotation, the parameters X_0 and Y_0 are evidently transformed as $X_0 \to (X_0 - Y_0)/\sqrt{2}$ and $Y_0 \to (X_0 + Y_0)/\sqrt{2}$. Then, the steady-state soliton solution assumes the form

$$\varphi_0(X) = \pm\sqrt{\tilde{n}_{\infty}}\tanh\left[\sqrt{\frac{\eta}{2}}(X + X_0)\right]. \quad (6)$$

For $X \to \infty$, the soliton solution given by Eq. (6) approaches the vacuum states $\varphi_{\pm}$. For simplicity, we assume that the minimum of the kink is located at the center of the structure; i.e., we assume that, in the new coordinates, $X_0 = L/\sqrt{2}$ and $Y_0 = 0$. The normalization condition on φ yields the expression for the critical number of particles in the kink: $N \simeq L_Z(L^2\eta - 8\ln[\cosh[\tilde{L}]])/G$, where $\tilde{L} = L\sqrt{\eta}/2$ is a dimensionless parameter.

According to [10], in the limiting case $\tilde{n}_{\infty} \gg 1$, a soliton may be considered as a classical object. We consider a field φ that is weakly fluctuating along the Z direction; i.e., the field can be represented as a sum $\varphi = \varphi_0 + \varepsilon\delta\varphi$ ($\delta\varphi_{\Re} \equiv \varepsilon\delta\varphi$, $|\varphi_0| \gg \varepsilon|\delta\varphi|$), where the oscillon solution $\delta\varphi = \delta\varphi(X)\cos(\Omega(Z - Z_0))$ characterizes longitudinal oscillations with a spatial frequency Ω. Substituting this expression for φ with φ_0 given by Eq. (6) into Eq. (5) and linearizing the latter with respect to $\delta\varphi$, we obtain the first excited state of the system

$$\delta\phi(X)\sqrt{\tilde{n}_{\infty}}\tanh\left[\sqrt{\frac{\eta}{2}}(X + X_0)\right] \times \operatorname{sech}\left[\sqrt{\frac{\eta}{2}}(X + X_0)\right]$$

with $\Omega^2 = 3\eta/2$. As a result, we have a state called the "Higgs oscillon," where a classical kink φ_0 is accompanied by the low-amplitude oscillations of the Higgs field φ.

Figure 3a shows the excited kink φ^2 as a function of the dimensional spatial coordinates x and z for a fixed value of y. The darkened plane in Fig. 3a corresponds to the vacuum state $\varphi_{\pm}^2$. Taking into account that the solution is periodic along the direction Z (with a period of $2\pi/\Omega$), we consider the formation of an oscillon in three-dimensional volume $L_X \times L_Y \times L_Z$. The condition $\eta = 2\pi^2 j^2/3L_Z^2$ corresponds to a nor-

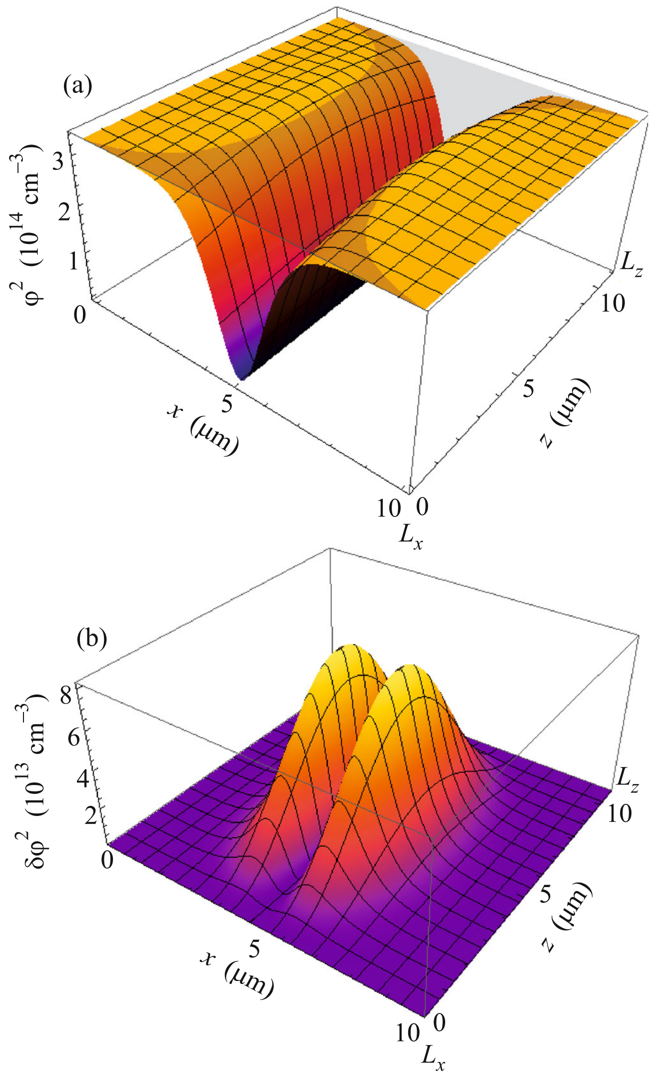

Fig. 3. (Color online) (a) Excited state φ^2 of the Higgs field and (b) excitation $\delta\varphi^2$ versus spatial coordinates x and z. The values of φ^2 and $\delta\varphi^2$ are shown in dimensional units: $\varphi^2 \to \varphi^2/\kappa^2\kappa_z$ and $\delta\varphi^2 \to \delta\varphi^2/\kappa^2\kappa_z$. The parameters are $X_0 = Y = Y_0 = L/2$, $Z_0 = \pi/2\Omega$, $\varepsilon = 0.2$, $L_z = L_y = 10$ μm, $L_z \approx 10.9$ μm, $g \approx 7.1$ meV μm^3, $n_\infty \approx 3.3 \times 10^{14}$ cm^{-3}, and $E_1 \approx 2.36$ meV. The darkened plane $\varphi^2 = n_\infty$ in panel (b) corresponds to the vacuum solutions.

malized energy of $\eta = E_jV/g$. Figure 3b shows the squared amplitude $\delta\varphi^2$ of a quantized Higgs oscillon in the ground state ($j = 1$).

The energy density J of the polariton Higgs field is given by

$$J = \frac{1}{2}\left[(\partial_Z\varphi)^2 + (\partial_X\varphi)^2 + (\partial_Y\varphi)^2 - \eta\varphi^2 + \frac{G}{2}\varphi^4\right].$$

Integrating J with respect to the spatial coordinates X and Y taking into account that $|\varphi_0| \gg \varepsilon|\delta\varphi|$, we obtain the following expression for the energy density distribution along the Z direction:

$$E_{0,Z} = \frac{\eta}{3G}\left(2 - 3\tilde{L}^2 + 8\ln\left[\cosh[\tilde{L}]\right] - 2\,\mathrm{sech}^2[\tilde{L}]\right). \quad (7)$$

The energy density of vacuum states $\varphi_\pm$ in the Z direction is $E_{\pm,Z} = -\tilde{L}^2\eta/G$. The analog of the "effective mass" of the kink can be introduced as $M \sim E_{0,Z} - E_{\pm,Z}$ taking into account Eq. (7) [17].

5. CONCLUSIONS

We have suggested a physical principle for modeling the properties of hyperbolic metamaterials based on a spatially periodic structure representing a modified Bragg mirror with quantum wells. We have shown that the Gross–Pitaevskii master equation for polaritons can be transformed into a nonlinear Ginzburg–Landau–Higgs-type equation, which describes the physically nontrivial properties of the field. We have predicted the formation of kink-shaped patterns in a system of weakly interacting polaritons. Low-amplitude oscillations (oscillons) appear in the polariton Higgs field owing to fluctuations. Resonant polaritonic hyperbolic metamaterials offer considerable promise for simulating fundamental processes of the evolution of the Universe.

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