Cavity electromagnetically induced transparency via spontaneously generated coherence

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ABSTRACT
A four-level $N$-type atomic ensemble enclosed in a cavity is revisited to investigate the influence of spontaneous generated coherence (SGC) on transmission features of weak probe light field. A weak probe field is propagating through the cavity where each atom inside the cavity follows four-level $N$-type atom-field configuration of rubidium ($^{85}$Rb) atom. We use input–output theory and study the interaction of atomic ensemble and three cavity fields which are coupled to the same cavity mode. A SGC affects the transmission properties of weak probe light field due to which a transparency window (cavity EIT) appears. At resonance condition the transparency window increases with increasing the SGC in the system. We also studied the influence of the SGC on group delay and investigated magnitude enhancement of group delay for the maximum SGC in the system.

1. Introduction
When two fields resonantly excite two different transitions having a common state, then the phenomenon is known as electromagnetically induced transparency (EIT) (1–4). EIT play a significant role to control the light pulse propagation through an opaque medium and have interesting applications, these include such as, in light storage, retrieval (5–10) and quantum memories (11–14). A slow or sub-luminal light pulse propagation has been exploited using the technique of EIT (15). Similarly, rubidium atomic medium has been used and studied a slow light propagation through the medium by decreasing the absorption in EIT configuration (16). Earlier, in EIT configuration it has been reported that light pulses could be halted and frozen in an ensemble of sodium atoms (18) and Doppler medium (17), respectively. The optical information in an atomic medium could be stored which has a direct application of the slow light (5, 6, 18, 19). Besides a linear medium, EIT has been studied in strong optical non-linear media (20, 21). In the above investigations, EIT has exploited without considering a medium in a cavity. When such an EIT atomic medium is placed in an optical cavity one can get the enhanced interaction of the atoms with spatiotemporal field modes. The cavity EIT has a key role in enhancing optical depth and high-efficiency quantum memories (22–25). The sensitivity of atomic magnetometers (26) can be enhanced using the EIT induced reduction of cavity linewidth (27), and get lasing without inversion (28), optical switching (29, 30). The cavity EIT characteristics have been studied with a single (31, 32) or an ensemble of atoms (33–35). Recently, an experiment was performed in which ion coulomb crystals are enclosed in a cavity and observed EIT as well as all-optical EIT-based light switching for the first time (36). In this experiment (36), a change from maximum absorption to maximum transmission of a single photon field is reported. Later, an ensemble of atomic medium having four-level atomic configuration are considered in a cavity and studied the characteristics of transmission probe light field (37). A semi-classical input–output theory for the intracavity medium is used where three optical cavity fields couple to the cavity mode. In this work (37), the transmission characteristics of the probe field have been controlled using a switching field.

Further, the SGC refers to the quantum interference of spontaneous emission channels in a system and has been extensively studied earlier. A SGC has been studied in a $\Lambda$-type system, where spontaneous emissions from a single excited state to closely spaced lower levels can interfere (38). Similarly, the SGC also studied in a $V$-type system where spontaneous emissions from two closely spaced upper levels to a common ground state can interfere (39). Earlier, some attempts have been made to investigate the SGC in a system experimentally. The first
experiment has been performed for the observation of SGC in the sodium molecule in 1996 [40], but unfortunately this experiment was condemned in repetition [41]. Another experimental confirmation of SGC in charged GaAs quantum dot has been reported [42]. In atomic media, it is a difficult job to investigate SGC experimentally, but there are several schemes to simulate this process, these include for example, the influence of SGC on the EIT characteristics in a four-level Λ-type configuration [43].

In this article, we consider a four-level N-type atomic medium in a cavity as depicted in Figure 1(a). We also consider a SGC between two closely spaced levels of intracavity medium. As reported earlier that the SGC influenced the Kerr non-linearity and group index of the medium when light is propagating through it [44–46]. Based on these ideas [44–46], we enclose an ensemble of atomic medium in a cavity and investigate the cavity EIT via the SGC. We believe that the SGC will influence the cavity EIT features. In the best of our knowledge, no one has considered the cavity EIT via SGC.

2. Model

We consider an ensemble of four-level N-type atomic configuration of rubidium atoms (85Rb) in a cavity as depicted in Figure 1(a). Each atom in a cavity having energy-levels |a⟩, |b⟩, |c⟩ and |d⟩. An intense field \( E_1 \) drives the transition |a⟩ and |b⟩ with a Rabi frequency \( \Omega_1 \). Correspondingly, a weak signal field \( E_p \) and a strong field \( E_2 \) drive the transition |b⟩ → |c⟩ and |c⟩ → |d⟩ with Rabi frequency \( \Omega_p \) and \( \Omega_2 \), respectively. We also consider an optical cavity which consist of two mirrors \( M_1 \) and \( M_2 \) having mirror transmissions, \( T_H \) (high transmission) and \( T_L \) (low transmission). In rubidium atoms (85Rb), \( \gamma_1 \) and \( \gamma_2 \) are the spontaneous decay rates of the excited level |b⟩ (\( 5P_{1/2}, F′ = 2 \)) to the ground levels |a⟩ (\( 5S_{1/2}, F = 3 \)) and |c⟩ (\( 5S_{1/2}, F = 2 \)) whereas \( \gamma_3 \) is the decay rate from level |d⟩ (\( 5P_{3/2}, F′ = 2 \)) to |c⟩ (\( 5S_{1/2}, F = 2 \)). To construct the Hamiltonian of the system, we follow the same approach as has been considered in the experiment (36) where the ensemble of the medium is much smaller than the Rayleigh range of the cavity. We ignore the longitudinal deviation of the waists and phases of the light fields over the length of the ensemble. We consider that the transverse structure of the Gaussian mode is the same for all fields i.e. \( \psi_s(r) = e^{-r^2/\omega^2} (s = 1, p, 2) \). Then the interaction Hamiltonian of the system can be expressed as:

\[
H_{af} = -\hbar \left( \sum_i N \psi_1 g_1 \hat{a}_1^{\dagger} \sigma_{ab}^{(i)} + \psi_p g_p \hat{a}_p^{\dagger} \sigma_{cb}^{(i)} + \psi_2 g_2 \hat{a}_2^{\dagger} \sigma_{cd}^{(i)} + h.c. \right).
\]  (1)

The total Hamiltonian for the cavity system can be written as:
The set of coupled differential equations can be calculated using the relation \( \langle \hat{\sigma} \rangle = \frac{1}{N} \langle \hat{\sigma}, H \rangle \) (where \( \hat{\sigma} \) is the mean value of observable \( \sigma \)) as:

\[
\begin{align*}
\hat{\sigma}_{bc}^{(j)} &= -(\gamma_1 + \gamma_2 - i\Delta_p)\hat{\sigma}_{bc}^{(j)} + i\psi_1\hat{g}_1\hat{a}_1\hat{\sigma}_{ac}^{(j)} + i\psi_p\hat{g}_p\hat{a}_p \times (\mu_{bc} - \rho_{bb}) - i\psi_{2g}\hat{a}_2\hat{\sigma}_{bd}^{(j)}, \\
\hat{\sigma}_{ac}^{(j)} &= -i(\Delta_1 - \Delta_p) + \Gamma[\hat{\sigma}_{ac}^{(j)} + i\psi_1\hat{g}_1\hat{a}_1\hat{\sigma}_{bc}^{(j)} - i\psi_p\hat{g}_p\hat{a}_p\hat{\sigma}_{ac}^{(j)} - i\psi_{2g}\hat{a}_2\hat{\sigma}_{bd}^{(j)} + 2q\gamma_2\gamma_1\hat{\sigma}_{bb}^{(j)}, \\
\hat{\sigma}_{ad}^{(j)} &= -(\Delta_1 - \Delta_p + \Delta_2 + \gamma_2)\hat{\sigma}_{ad}^{(j)} + i\psi_1\hat{g}_1\hat{a}_1\hat{\sigma}_{bd}^{(j)} - i\psi_{2g}\hat{a}_2\hat{\sigma}_{ac}^{(j)}, \\
\hat{\sigma}_{bd}^{(j)} &= -(\Delta_2 - \Delta_p)\hat{\sigma}_{bd}^{(j)} + i\psi_1\hat{g}_1\hat{a}_1\hat{\sigma}_{ad}^{(j)} + i\psi_p\hat{g}_p\hat{a}_p\hat{\sigma}_{cd}^{(j)} - i\psi_{2g}\hat{a}_2\hat{\sigma}_{bc}^{(j)}.
\end{align*}
\]

Figure 2. The imaginary part of optical susceptibility \( \chi \) vs. probe field detuning \( \Delta_p \) for (a) \( q = 0 \), (b) \( q = 0.4 \), (c) \( q = 0.7 \) and (d) \( q = 0.99 \). The remaining parameters are \( \gamma = 1 \) MHz, \( \kappa = 2.2 \gamma \), \( g_p = 16 \gamma \), \( \gamma_1 = \gamma_2 = \gamma = 3 \), \( \Omega_{bc}^2 = \gamma \), \( \Omega_2 = \gamma \) and \( \Gamma = 0.003 \gamma \).
channels. For example, if \( q = 0 \) then the two dipole moments must be perpendicular to each other and there will be no quantum interference between the two spontaneous emission channels. In contrary, if \( q = 1 \) then the two dipole moments must be parallel and there will be maximum quantum interference between the two channels. Further, the angle \( \theta \) play a key role in the alignment between two dipole moments, where it can be adjusted with the help of \( E_1 \) and \( E_p \) fields, see Figure 1(c). The control and probe fields depends on the angle \( \theta \) and can be written as \( \Omega_1 = \Omega_1^q \sin \theta = \Omega_1^q \sqrt{1 - q^2} \). In Equation (4), \( \tau \) is the cavity round trip and \( \kappa_f \) (\( \kappa_f \)) is the decay rate corresponding to the mirror transmission whereas \( \psi_i(\tau) = e^{-\tau^2/\omega^2} \) and \( \Omega_f = g_i \hat{a}_i \).

We consider that all the intracavity atoms are initially prepared in level \( |c⟩ \). It is emphasized that the total volume of the ensemble is \( V \) and the transverse dimension of the ensemble is much larger than the cavity waist (47). Keeping the probe field in the first order while the other two fields are in all orders, then we can calculate the optical susceptibility of the intracavity medium by considering \( \Delta_1 = \Delta_2 = 0 \) as:

\[
\chi = \frac{g_p^2}{2} \int_V d\mathbf{r} \rho(\mathbf{r}) \frac{(i\Gamma + \Delta_p)\psi_p \times A - \Delta_p \psi_p^2 \Omega_p^2}{AB + \psi_p^2 \Omega_p^2 C - \psi_p^4 \Omega_p^4},
\]

(5)

where

\[
A = i\gamma_3 \Delta_p + \Delta_p^2 - \psi_p^2 \Omega_p^2,
\]

\[
B = (\Gamma - i\Delta_p)(\gamma_1 + \gamma_2 - i\Delta_p) + \psi_p^2 \Omega_p^2,
\]

\[
C = -\Gamma\gamma_3 + i(\Gamma + \gamma_1 + \gamma_2 + \gamma_3)\Delta_p + 2\Delta_p^2 + 2\psi_p^2 \Omega_p^2.
\]

(6)

where \( \rho(\mathbf{r}) = \frac{2N}{V_L} \) is the uniform density of the ensemble with \( L \) being the length of the intracavity medium and \( N \) is the effective number of atoms.

In order to study the optical properties of transmitted field, we follow the same approach as used in Ref. (37). The input–output relation for the transmitted light can be written as:

\[
a_{tr}^t = \sqrt{2\kappa_L\tau} a_p,
\]

(7)

then the steady-state cavity of transmission can be written as:

\[
T = \left| \frac{a_{tr}^t}{a_p} \right|^2 = \left| \frac{\kappa - i\Delta_p - i\chi}{\kappa - i\Delta_p} \right|^2.
\]

(8)

where \( \kappa \) is the total decay rate of the cavity. Further, in the region of transparency window the dispersion changes rapidly, i.e. \( \psi(\omega_p) = \arg(T(\omega_p)) \) may induce the transmission group delay which can be expressed in the form as:

\[
\tau_g = \frac{d\psi(\omega_p)}{d\omega_p}.
\]

(9)

3. Results and discussion

In order to investigate the characteristics of transmitted probe light field incident on a linear optical cavity having four-level \( N \)-type rubidium atomic medium. We consider three optical fields, insert into the cavity and couple to the cavity mode. We investigate the influence of SGC using transverse structure of the fields. We study the effect of the SGC on absorption, transmission spectrum and group delay time of weak probe field. The fixed parameters we use in the investigation of the transmitted features of probe light field are \( \gamma = 1 \) MHz,
Figure 4. The plot of transmission $|T|^2$ vs. probe field detuning $\Delta_p$ for $q = 0.99$. The other parameters remain the same as those in Figure 2.

Figure 5. The plot of group delay $\tau_g$ vs. $q$ for $\Delta_p = 0$, $\Omega_1 = 0.5\gamma$ and $\Omega_2 = 0.5\gamma$. The other parameters remain the same as those in Figure 2.

Initially, we consider the intracavity medium and study the influence of SGC in the imaginary part of the optical susceptibility. We plot $\text{Im}[\chi]$ vs. probe field detuning $\delta_p$ using Equation (5) for different values of $q$ of SGC as shown in Figure 2. The plots show that the SGC affect the absorption of probe light propagating through the cavity having rubidium atomic medium. As mentioned earlier that for $q = 0$ there is no quantum interference between the two channels in the system, we investigate a high absorption at the resonance condition for $q = 0$, see Figure 2(a). We introduce the quantum interference in the system via increasing the value of $q$, then at resonance condition the absorption in the cavity system decreases gradually. To show the variation of absorption through cavity system for different values of $q$, we again plot the imaginary part of $\chi$ vs. probe field detuning $\theta_{\Delta \phi_p}$ for $q = 0.4, 0.7, 0.99$, see Figure 2(b)–(d). For maximum value of $q$ the absorption of weak probe field tends to zero as shown in Figure 2(d). This is due to the fact that there is a strong quantum interference effect of the spontaneous emission channels for the maximum value of $q$. This shows that the absorption of weak probe field through the intracavity medium decreases with increasing the value of $q$.

As reported earlier that all-cavity EIT characteristics have been controlled in an atomic medium enclosed in a cavity via switching field (37). We follow the same procedure as used earlier (37) and study the control of cavity EIT features via considering SGC in four-level atomic medium enclosed in a cavity. We use Equation (8) and study the influence of SGC of the output probe field through the cavity. We show a 3D plot of transmission of the probe field vs. probe field detuning $\Delta_p$ and SGC $q$, see Figure 3. To see a clear picture of transmitting probe light field, we show a 3D plot for smaller range of $q$ at resonance condition i.e. ranging from $q = 0$ to $q = 0.1$. The plot in Figure 3(a) clearly demonstrates that at resonance condition the transmission of the probe field increases gradually with increasing the value of $q$. It is also clearly
shown in Figure 3(a) that the variation of the transmitted probe light vs. \( q \) is very small. It is now more important to investigate the transmission spectrum of the probe light for the whole range of \( q \) i.e. from \( q = 0 \) to \( q = 0.99 \). Again we use Equation (8) and plot the transmission profile of the probe light field vs. probe field detuning \( \Delta_p \) and SGC \( q \). The plot shows that the transmission of probe field through the cavity increases gradually to its maximum value with increasing \( q \) at resonance condition, see Figure 3(b). The transmission spectrum for maximum values of \( q \) clearly demonstrates that cavity transparency occurs at resonance condition. This is due to the fact that quantum interference between two channels of atomic configuration of intracavity medium becomes maximum for maximum values of \( q \). The enhancement of transmission on resonance condition via increasing the SGC tends to cavity EIT in the scheme. Also the cavity EIT strongly depends on the SGC in the system.

The Rabi frequencies \( (\Omega_1^0, \Omega_2) \) play an important role in the transparency, then it will be more constructive to study the influence of Rabi frequencies on cavity EIT. Initially, we consider the transmission of probe light field in the absence of control field i.e. \( \Omega_2 = 0 \), then the system becomes a simple three-level configuration. We plot the transmission of probe light vs. probe field detuning \( \Delta_p \) by considering \( \Omega_1^0 = 1\gamma \) and \( q = 0.99 \), see Figure 4(a). The spectrum of Figure 4(a) shows that a small narrow transparency window appears at resonance condition. To see the influence of driving field \( \Omega_1 \) on the transparency, we increase the strength of driving field from \( \Omega_1^0 = 1\gamma \) to \( \Omega_1^5 = 5\gamma \) and plot again the transmission vs. probe field detuning as shown in Figure 4(b). The peak and width of the transparency increase with increasing the strength of driving field \( \Omega_1 \). Next we introduce the fourth field i.e. \( \Omega_3 \), and the system then behaves as \( N \)-type four-level configuration. We keep all the other parameters unchanged and plot the transmission \( T \) vs. \( \Delta_p \) for two values of \( \Omega_2 \) i.e. \( \Omega_2 = 2\gamma \) and \( \Omega_2 = 5\gamma \) as depicted in Figure 4(c) and (d). The plots show that the control field \( \Omega_2 \) affect the transparency and the width of transparency window. The transparency and width increases with increasing the strength of control field \( \Omega_2 \). For further increase in Rabi frequencies \( \Omega_1 \) and \( \Omega_2 \) only the width of transparency window increases. Based on the above analysis it is concluded that the cavity EIT is strongly dependent on the Rabi frequencies \( \Omega_1 \) and \( \Omega_2 \).

The slow light propagation or group delay time has a key role in quantum information and all-optical communication system. In a different category of atomic and solid-state media slow light or group delay time has been reported using different schemes. It is now constructive to study the group delay time in the cavity EIT system via SGC. We use the relation (9) and show the enhancement of group delay with the variation of SGC \( q \). We plot the group delay \( \tau_g \) vs. \( q \) by considering the resonance condition i.e. \( \Delta_p = 0 \) ranging from 0 to 0.99, see Figure 5(a). The plot shows that the group delay increases gradually with increasing \( q \) and reaches to its maximum value for \( q = 0.99 \). We also show an enlarge view of group delay vs. \( q \) ranging from 0 to 0.3 as shown in Figure 5(b). We calculate the group delay for \( q = 0 \) is \( \tau_g = 3.48 \mu s \), while for maximum value of SGC i.e. \( q = 0.99 \), the group delay reaches to \( \tau_g \approx 809.5 \mu s \). This clearly indicates that the group delay increase 239.1 times by changing the value of \( q \) from 0 to 0.99 for this particular case. Finally, we notice that the Rabi frequencies have the same effect on the group delay as that of cavity EIT.

4. Conclusion

In conclusion, we considered a four-level \( N \)-type atomic medium in an optical cavity to investigate the cavity EIT features under the influence of SGC. We studied the influence of SGC on transmission properties of weak probe light field through the cavity. The increase of transmission of probe light field for maximum values of SGC is reported on resonance condition. The enhancement of transmission of probe light field in resonance condition tends to a transparency window (cavity EIT). We also investigated that the variation of transmission of probe light remained in the resonance condition with the variation of SGC in the system. Further, we studied the group delay and investigated a magnitude enhancement of \( \tau_g \) for maximum values of SGC. Our results correspond to very important highlights for future investigations of quantum information processing devices such as high-efficiency quantum memories (48). Quantum memory for light is a fast rising research topic in atomic spectroscopy, quantum optics and material science. Our scheme is another approach to control the light pulse propagation via SGC. If a device is developed based on SGC in a system, then it will be precious for quantum communication and optical quantum computation.

Disclosure statement

No potential conflict of interest was reported by the authors.

References
