Phase knob for switching steady-state behaviors from bistability to multistability via spontaneously generated coherence

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We conducted a theoretical investigation of the optical steady-state behavior in N four-level Y-type atoms driven coherently by a probe laser and a single elliptically polarized field (EPF) by means of a unidirectional ring cavity. It was found that the optical bistability can be observed for a wide regime of frequency detuning of the probe field, intensity of the EPF, and the atomic cooperation parameter. Interestingly, in principle the optical steady-state behavior can be switched from optical bistability to multistability or vice versa by adjusting the phase difference between two components of the polarized electric field of the EPF if the perfect spontaneously generated coherence of atoms is included. Our results illustrate the potential to utilize EPF for all-optical switching in atomic systems through the phase control, as well as provide guidance in the design for possible experimental implementations. © 2014 Optical Society of America

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1. INTRODUCTION

The spontaneous emission effect of excited atoms is a general process in light–matter interactions. It is responsible for many important physical phenomena that have been discovered in quantum optics [1]. Over the past few decades, many efforts have been put into finding an approach for the reduction and cancellation of spontaneous emissions of an excited atomic system. However, now it is well known that the closely lying levels of the excited atom can produce new types of coherence via interaction with the vacuum of the radiation field [2–10]. This coherence, called spontaneously generated coherence (SGC), can modify the transient and steady-state response of the medium. In these regards, we note that the effects of SGC on the optical steady-state behaviors in atomic systems have been widely analyzed, and such studies have investigated gain without population inversion [11,12], the evolution of the dark state [13], and the enhanced nonlinearity [14–16]. Especially, it should be noted that the behaviors of optical bistability (OB) and multistability (OM) in atomic systems also can be modified by the SGC effect [17–26], such as the threshold and the shape of the bistable hysteresis cycle.

On the other hand, the relative phase of coherent driving fields has been widely used for controlling several important processes in atomic, molecular, and solid-state systems [27–31], which is usually termed phase control. Recently, intensive interest has been given to the phase control of steady-state behaviors in the presence of the SGC effect. Menon and Agarwal [8] have shown that the SGC brings about quantitative changes in line profiles of absorption and dispersion, and this leads to the dependence of line shapes on the relative phase between the two applied fields when the two applied fields are both strong enough. Wu and Gao [11] have found that the inversionless gain will be related to the relative phase between the probe and the coherent driving fields when an incoherent process is used to pump a Λ atomic system. Furthermore, the phase effect on the optical steady-state behaviors including OB and OM have also been studied in atomic systems confined in an optical ring cavity [19–21]. For example, Gong and co-workers [19,20] showed that the phase fluctuation of the control field leads to OB even when both fields are in resonance with the three-level atoms, and the threshold intensity can be controlled by changing the bandwidth of the control field. Hu and Xu [21] have suggested a method for achieving phase control of the amplitude-fluctuation-induced OB with two-photon resonance in such three-level atoms.

In this paper, we analyze the optical steady-state behaviors via SGC in N four-level Y-type atoms driven coherently by a probe laser and a single elliptically polarized field (EPF) by means of a unidirectional ring cavity. Our results show that the intensity of the EPF driving field and the atomic cooperation parameters can efficiently modify the threshold and the shape of the bistable hysteresis cycle even in the absence of a perfect SGC effect. Moreover, adjustments of the frequency detuning of the probe laser can be used to switch on or off the OB phenomenon. More interestingly, our results also reveal that the conversion between OB and OM can be realized by adjusting the phase difference between the two components of the polarized electric field of the EPF in the presence of a perfect SGC effect. Different from the relative phase between the probe and control fields, the phase difference between the two electric components of the EPF can be changed easily by using the wave plates.
2. MODEL AND MOTION EQUATIONS

Let us consider a four-level Y-type atomic system driven by a linear-polarized probe field and a single EPF, as shown in Fig. 1. The linear-polarized probe field $E_p$ (with angular frequency $\omega_p$) couples the electric dipole transitions between levels $|2\rangle$ and $|3\rangle$ (with transition frequency $\omega_{23}$). A single EPF $E_c$ (with angular frequency $\omega_c$) drives the electric dipole transitions between levels $|1\rangle$ and $|2\rangle$ (with transition frequency $\omega_{12}$) and between levels $|4\rangle$ and $|2\rangle$ (with transition frequency $\omega_{42}$). The utilization of a quarter-wave plate (QWP) can obtain the $\sigma$-polarization of the EPF $[32]$. An initial $\sigma$-polarized field $E_c$ with a rotating angle $\phi$ can be elliptically polarized after entering into the QWP. Thus, the EPF can be decomposed as $E_c = E_c^+ \sigma^+ + E_c^- \sigma^-$ with the unit basis vectors $\sigma^+$ and $\sigma^-$, where $E_c^+ = (E_c/\sqrt{2})(\cos \phi + \sin \phi)e^{i\phi}$ and $E_c^- = (E_c/\sqrt{2})(\cos \phi - \sin \phi)e^{-i\phi}$. The amplitudes and the phase difference between two components of the polarized electric field, which is denoted by the rotating angle $\phi$ for convenience, can be modulated by the QWP.

Inspecting the system, we find that states $|3\rangle$, $|2\rangle$, and $|4\rangle$ are in a typical three-state cascade-type scheme while $|3\rangle$, $|2\rangle$, and $|1\rangle$ compose another cascade-type scheme. Hence, the Y-type system is composed of two cascade-type configurations, and both of them share the ground state $|3\rangle$ and lower excited state $|2\rangle$. The medium is subject to an applied longitudinal magnetic field $B$ that removes the degeneracy of the states $|1\rangle$ and $|4\rangle$, whose Zeeman shift is determined by $\Delta_\gamma = \mu_B m_g y B/\hbar$, where $\mu_B$ is the Bohr magneton, $y_B$ is the gyromagnetic factor, and $m_g = \pm 1$ is the magnetic quantum number of the corresponding state.

By choosing $H_0 = \omega_p |2\rangle\langle 2| + (\omega_c + \omega_{42}) |1\rangle\langle 1| + (\omega_c + \omega_{12}) |4\rangle\langle 4|$ and taking level $|3\rangle$ as the energy origin, under the dipole and rotating-wave approximation, the interaction Hamiltonian of the present atomic system is given by $(h = 1)$:

$$H_{\text{int}} = (\Delta_\gamma + \Delta_p) |1\rangle\langle 1| + (\Delta_\gamma - \Delta_c) |4\rangle\langle 4| + \Delta_p |2\rangle\langle 2| - [\Omega_p |2\rangle\langle 3| + \Omega_c |1\rangle\langle 2| + \Omega_c |4\rangle\langle 2| + \text{H.c.}],$$

where $\Delta_p = \omega_{23} - \omega_p$ is the frequency detunings of the probe field. The symbol H.c. represents the Hermitian conjugate. $\Omega_c = -\mu_{12} E_c^+ / h = \Omega_c (\cos \phi + \sin \phi)e^{i\phi}$ and $\Omega_p = -\mu_{42} E_c^- / h = \Omega_p (\cos \phi - \sin \phi)e^{-i\phi}$ are the half Rabi frequencies of the two components of the EPF, where $\Omega_c = -\mu E_c^+/\sqrt{2}h$. $\mu = \mu_e/2\hbar$ is the half Rabi frequency of the probe field with $\mu_1 = \mu$ denoting the dipole matrix moment for the relevant optical transition from level $|i\rangle$ to level $|j\rangle$. Then, the density matrix equations of the system can be obtained under the standard approach [1]:

$$\dot{\rho}_{11} = -\gamma_1 \rho_{11} + i \Omega_p \rho_{21} - i (\Omega_c^+) \rho_{12} - \frac{\eta}{2} \sqrt{\gamma_1 \gamma_2} (\rho_{14} + \rho_{41}),$$

$$\dot{\rho}_{22} = -\gamma_3 \rho_{22} + \gamma_1 \rho_{11} + \gamma_2 \rho_{44} + i (\Omega_c^+) \rho_{12} - i (\Omega_c^+) \rho_{21} + i (\Omega_c^+) \rho_{24} + \eta \sqrt{\gamma_1 \gamma_2} (\rho_{14} + \rho_{41}),$$

$$\dot{\rho}_{44} = -\gamma_3 \rho_{44} + i (\Omega_c) \rho_{24} - i (\Omega_c) \rho_{42} - \frac{\eta}{2} \sqrt{\gamma_1 \gamma_2} (\rho_{14} + \rho_{41}),$$

$$\dot{\rho}_{12} = -\left(\frac{\gamma_1}{2} + \frac{\gamma_3}{2} + i \Delta_c\right) \rho_{12} - i (\Omega_c^+) \rho_{12} - \frac{\eta}{2} \sqrt{\gamma_1 \gamma_2} \rho_{12},$$

$$\dot{\rho}_{14} = -\left(\frac{\gamma_1}{2} + \frac{\gamma_3}{2} + 2i \Delta_c\right) \rho_{14} + i (\Omega_c^+) \rho_{14} - \frac{\eta}{2} \sqrt{\gamma_1 \gamma_2} \rho_{14},$$

$$\dot{\rho}_{23} = -\left(\frac{\gamma_3}{2} + i \Delta_p\right) \rho_{23} + i (\Omega_c^+) \rho_{23} + i (\Omega_c^+) \rho_{23} - \frac{\eta}{2} \sqrt{\gamma_1 \gamma_2} \rho_{23},$$

$$\dot{\rho}_{42} = -\left(\frac{\gamma_3}{2} + \frac{\gamma_3}{2} - i \Delta_c\right) \rho_{42} + i (\Omega_c^+) \rho_{42} - i (\Omega_c^+) \rho_{42} + i (\Omega_c^+) \rho_{42} - \frac{\eta}{2} \sqrt{\gamma_1 \gamma_2} \rho_{42},$$

$$\dot{\rho}_{43} = -\left(\frac{\gamma_3}{2} + i \Delta_p - \Delta_c\right) \rho_{43} + i (\Omega_c^+) \rho_{43} - i (\Omega_c^+) \rho_{43} - \frac{\eta}{2} \sqrt{\gamma_1 \gamma_2} \rho_{43},$$

$$\dot{\rho}_{41} = -\left(\frac{\gamma_3}{2} + \frac{\gamma_3}{2} - i \Delta_c\right) \rho_{41} + i (\Omega_c^+) \rho_{41} - i (\Omega_c^+) \rho_{41} - \frac{\eta}{2} \sqrt{\gamma_1 \gamma_2} \rho_{41}.$$
levels is very small, which is necessary for the existence of the SGC effect. The term \((\eta/2)\sqrt{\gamma_1\gamma_2}(\eta = \mu_{42}\mu_{12}/|\mu_{12}|)\) in the above equations denotes the SGC effect between the spontaneous transitions \(|4\rangle\leftrightarrow|2\rangle\) and \(|1\rangle\leftrightarrow|2\rangle\). If the two induced dipole moments \(\mu_{42}\) and \(\mu_{12}\) are orthogonal, there is no interference and \(\eta = 0\). But, if the two dipole moments \(\mu_{42}\) and \(\mu_{12}\) are parallel, the interference is maximum and \(\eta = 1\).

Now, we consider a medium of length \(L\) composed by the atomic sample described above, which is embedded in a unidirectional ring cavity (see [33] p. 71 and references therein), as shown in Fig. 2. For simplicity, we assume that both mirrors 3 and 4 have a 100% reflectivity, and the intensity reflection coefficients for mirrors 1 and 2 are \(R\) and \(T\) (with \(R + T = 1\)), respectively. The propagation of the pulsed probe field in such a medium is governed by Maxwell’s wave equation:

\[
\nabla^2 \bar{E} - \frac{1}{c^2} \frac{\partial^2 \bar{E}}{\partial t^2} = \bar{P} / \varepsilon_0,
\]

with

\[
\bar{P} = N[\mu_{23}\mu_{21}e^{i(\bar{\omega}_p - \omega_0)t}] + (\mu_{12}\mu_{21} + \mu_{42}\mu_{24})e^{i(\bar{\omega}_p - \omega_0)t} + \text{c.c.}],
\]

where only the probe field \(\bar{E}_p\) circulates inside an optical cavity, and then under the slowly varying envelope approximation, Maxwell’s equation can be reduced to the first-order equation. Thus, we can obtain the relevant propagating equation characterizing the probe field [1]:

\[
\frac{1}{c} \frac{\partial E_p}{\partial t} + \frac{\partial E_p}{\partial z} = \frac{i \omega_p}{2c\varepsilon_0} P(\omega_p),
\]

where the parameters \(c\) and \(\varepsilon_0\) represent the velocity of light in a vacuum and the vacuum dielectric constant, respectively. \(P(\omega_p) = N\mu_{23}\mu_{21}\) is the slowly oscillating term for the induced polarization in the transition \(|2\rangle\leftrightarrow|3\rangle\), with \(N\) the atom number density in the atomic sample. In the steady-state case, the time derivative in Eq. (13) \(\partial E_p/\partial t\) can be neglected, thus the amplitude of the probe field \(E_p\) can be described by

\[
\frac{\partial E_p}{\partial z} = \frac{\omega_p}{2c\varepsilon_0} P(\omega_p).
\]

Note that if a set of reasonable parameters for the ring cavity can be found, the input field \(E_p^0\) and the transmitted field \(E_p^T\) will meet the conditions [33]

\[
E_p^T = \sqrt{T} E_p(L),
\]

\[
E_p(0) = \sqrt{T} E_p^T(L) + R E_p(L),
\]

where the term \(R E_p(L)\) in Eq. (16) represents a feedback mechanism that results from the reflection of the mirror \(M_2\) and is the main reason for the production of OB or OM. By normalizing the fields \((x = \mu_{23}\bar{E}_p^0/\hbar \sqrt{T}\) and \(y = \mu_{23}\bar{E}_p^0/\hbar \sqrt{T}\) and using the mean-field approximation [33], the input–output relationship equation can be described by

\[
y = x - i C \rho_{23},
\]

where \(C = LN\omega_{23}^2/2\hbar c\varepsilon_0 T\) is the cooperation parameter for atoms. We should note that the term \(i C \rho_{23}\) in Eq. (17) is very important for the OB or OM to occur. From Eq. (17), we find that the expression \(\rho_{23}\) is a complicated fractional form of two polynomials. Thus, it is difficult to get a straightforward relationship between the cavity input and output intensities. Therefore, we will solve the density matrix Eqs. (2)–(10) together with the input–output Eq. (17) in the following section.

3. NUMERICAL RESULTS AND PHYSICAL ANALYSIS

In this section, we present typical numerical results for the steady-state solution of the output field in terms of field intensities for different corresponding parameters. As mentioned in the above section, a realistic candidate for the proposed atomic system can be found in \(^{87}\)Rb atoms with the designated states chosen as follows: \([5S_{1/2}, F = 1, m_F = 0]\) as \(|3\rangle\), \([5P_{1/2}, F = 2, m_F = 0]\) as \(|2\rangle\), \([5D_{3/2}, F = 2, m_F = -1]\) as \(|4\rangle\), and \([5D_{3/2}, F = 2, m_F = 1]\) as \(|1\rangle\) [34]. To give a clear illustration, we selected \(\Delta_\gamma = 7 \text{ MHz} = 0.9\gamma_1\) and \(\gamma_1 = \gamma_2 = \gamma\), and \(\gamma_3 = 0.01\gamma\), and all the parameters used in the following numerical calculations are in the unit of \(\gamma\). For simplicity, we assume \(\Omega_\gamma^+ = \Omega_\gamma^- = \Omega_\gamma\). Subsequently, as shown in Figs. 3–7, we obtained a few numerical results for the steady behavior of the input–output field intensity with different values of the relevant parameters to illustrate that controllable OB and OM can be achieved in the present four-level Y-type atomic system by means of a unidirectional ring cavity.

In the absence of the SGC effect \(\eta = 0\), we analyzed how the frequency detuning of probe field \(\Delta_p\) and the intensities of the EPF modify the steady-state behavior. In Fig. 3(a), we show the curve diagrams of the input–output field intensity with different frequency detuning \(\Delta_p\) of the probe field in the absence of the SGC effect (i.e., \(\eta = 0\)), in which the OB behavior can be obviously observed and can be manipulated by the frequency detuning. From Fig. 3(a), it is shown that the bistable threshold and the area of the bistable curve are controllable via changing the frequency detuning \(\Delta_p\). It was found that the OB disappears when the probe field is in resonance with the corresponding transition (\(\Delta_p = 0\)). In contrast, we can see that with increasing \(\Delta_p\) from \(\gamma\) to \(2\gamma\), the bistable threshold increases progressively and the area of the bistable curve becomes wider when all other parameters are kept.
fixed. The physics mechanism is rather clear and can be qualitatively explained as follows. By applying the EPF to driving the transitions $|1\rangle \leftrightarrow |2\rangle$ and $|4\rangle \leftrightarrow |2\rangle$, the absorption of the probe field applied to the transition $|2\rangle \leftrightarrow |3\rangle$ and the Kerr non-linearity of the atomic system can be modified, which leads to a change in the behavior of OB. Thus, it is illustrated here that the absorption-dispersion of the probe field can be modified in the present four-level Y-type atomic system. In Fig. 3(b), we display the effects of the intensity of the EPF $\Omega_p$ on the OB behavior without considering the SGC effect (i.e., $\eta = 0$). The data show that the bistable threshold and the area of the bistable curve decrease markedly as the amplitude of the EPF $\Omega_p$ increases, which can also be attributed to changes of the absorption-dispersion property resulting from the increases of $\Omega_p$. By applying an increasing intensity of the control field, the absorption of the probe field on the transition $|2\rangle \leftrightarrow |3\rangle$ can be reduced to an extreme extent. Thus, it is even easier for the absorption saturation of the ring cavity to take place.

The observation of the above steady-state behaviors can be interpreted with perturbation theory as follows [1]. First, we can obtain the expression of the first-order solution for $\rho_{23}$ in the steady-state case and the weak probe limit [185–40] (i.e., $\Omega_p \ll \Omega_c$, with the assumption of $\rho_{22}^{(0)} \approx 1$, $\rho_{11}^{(0)} = \rho_{22}^{(0)} = \rho_{44}^{(0)} = 0$). The first-order solution of $\rho_{23}$ is given by

$$\rho_{23}^{(1)} = \frac{i(\Delta_p - \Omega_p)^2 \Omega_p}{-4\eta^2 \gamma_1^2 - 4\eta(\cos(2\phi))\gamma_1 \Omega_2^2 + 4(abc + (b + c)\Omega_2^2 + (b - c)\sin(2\phi)\Omega_2^2)},$$

where $a = (\gamma_3/2) + i\Delta_p$, $b = (\gamma_2/2) + i(\Delta_p - \Delta_c)$, and $c = (\gamma_1/2) + i(\Delta_c + \Delta_p)$. From Eq. (18), one can also find the tedious and intricate expressions for the solutions, but without clear insight into the physics. In order to clearly show how the intensity of the EPF $\Omega_p$ and the frequency detuning $\Delta_p$ affect the OB behavior with neglecting the SGC effect (i.e., $\eta = 0$), according to Eq. (18), the density plot of the probe absorption $\text{Im}(\rho_{23})$ versus $\Omega_p$ and the frequency detuning $\Delta_p$ is shown in Fig. 4. Specifically, this figure displays the absorption of the probe field on the transition $|2\rangle \leftrightarrow |3\rangle$. As illustrated in Fig. 4, for the case of $\Delta_p = 0$, the absorption of the probe field can be completely suppressed and the atomic system becomes transparent to the weak tunable probe field by increasing the intensity of the control field $\Omega_c$, which can explain why the OB disappeared under $\Delta_p = 0$ at resonance.

In addition, with a certain frequency detuning of the probe field, Fig. 4 also demonstrates that the absorption of the probe field on the transition $|3\rangle \leftrightarrow |2\rangle$ and the Kerr nonlinearity can be modified when one increases the control field between the transitions $|2\rangle \leftrightarrow |1\rangle$ or $|2\rangle \leftrightarrow |4\rangle$. As we can see in Fig. 3(b), a smaller threshold intensity comes from the reduction in the effective saturation intensity due to the increasing intensity of the EPF.

It is worth noting that the cooperation parameter $C$ in Eq. (17) is linearly proportional to the atom number density $N$. One can easily find from Eq. (17) that the cooperation parameter will affect the input–output curves. Thus, we should consider the effect of the cooperation parameter $C$ on the OB behavior. Here, we present how the cooperation parameter $C$ affects the OB behavior for the case of $\eta = 0$. Figure 5 illustrates that the bistable threshold is reduced dramatically as $C$ decreases. From the term $C = L N \alpha \mu \nu_{23}^2 / 2 \hbar c N T$, it can be seen that $C$ changes with $N$ proportionally. Therefore, the decrease of the bistable threshold intensity originates from the probe absorption of the sample, which is enhanced with increasing $N$. Furthermore, we can optimize the optical switching process for controlling the OB behavior by choosing the electron sheet density of the corresponding system property [41–46]. We noticed that some works [47–49] reported the influence of the enhanced dipole coupling parameter $g \sqrt{N}$ (with $g$ being the atom-cavity coupling strength) on the OB behavior of multiwave mixing signals in coupled systems consisting of a specific ring cavity and a four-level atom assemble. Those results showed that the OB threshold decreases and the bistable region of the multiwave mixing signal becomes narrow as $g \sqrt{N}$ increases [47], which is different from the influence of the cooperation parameter $C$ on the OB behavior.

As shown in Figs. 3–5, we have demonstrated the creation of the OB and studied the influences of parameters on the
bistable behavior in the present atomic system under consideration without including the SGC effect (i.e., \( \eta = 0 \)) and the effect of the phase difference between the two components of the polarized electric field (i.e., \( \phi = 0 \)). However, we should examine the influence of the phase difference \( \phi \) between the two components of the polarized electric field on the optical steady behavior, which is one of the most interesting characteristics of the present atomic system driven by the EPF. In Fig. 6, we present numerical results for the analyses of the SGC effect \( \eta \) and the phase difference between two components of the polarized electric field \( \phi \) on the steady-state of the output field amplitude \( |x| \) as a function of input field amplitude \( |y| \) for different cases. In contrast to the results of Fig. 3(a), Fig. 6(a) shows that OB output can still be achieved in the presence of the perfect SGC effect (i.e., \( \eta = 0.9 \)) even if the probe resonance condition is satisfied \( \Delta_p = 0 \). However, the OB phenomenon disappears again when the phase difference \( \phi \) is tuned from \( \phi = 0 \) to \( \phi = 2\pi/3 \). For the general case, Fig. 6(b) shows that the bistable threshold increases with an increasing SGC effect \( \eta \) when the phase difference \( \phi \) is equal to zero and the frequency detuning is changed from \( \Delta_p = 0 \) to \( \Delta_p = \gamma \), which demonstrates that the SGC effect can effectively modify the threshold intensity and the hysteresis loop of OB. For fully understanding the influence of the phase difference \( \phi \) on the OB behavior, we plotted Figs. 6(c) and 6(d).

As shown in Fig. 6(c), in absence of the SGC effect (i.e., \( \eta = 0 \)), one can find that the OB disappears when the frequency detuning \( \Delta_p \) and the phase difference \( \phi \) are both equal to zero, which is consistent with the results of Fig. 3(a) (solid line). In contrast, the bistable threshold increases progressively with increasing \( \phi \) from \( \pi/3 \) to \( 2\pi/3 \) when all other

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**Fig. 4.** Density plot of the probe absorption \( \text{Im}\rho_{22} \) versus the \( \Omega_c \) and the detuning \( \Delta_c \) with \( \Omega_c = \Omega_c^0 = 0.01\gamma \). Other values of the parameters were chosen as \( \gamma_1 = \gamma_2 = \gamma, \gamma_3 = 0.01\gamma, \Delta_c = 0.9\gamma, \eta = 0, \phi = 0, \) and \( \Omega_c = \gamma, \Delta_p = \gamma \).

**Fig. 5.** Curve diagrams of the input–output field intensity with different cooperation parameters \( C \). Other values of the parameters were chosen as \( \gamma_1 = \gamma_2 = \gamma, \gamma_3 = 0.01\gamma, \Delta_c = 0.9\gamma, \eta = 0, \phi = 0, \) and \( \Omega_c = \gamma, \Delta_p = \gamma \).

**Fig. 6.** (a) Curve diagrams of the input–output field intensity for different values of the SGC effect \( \eta \) and different phase differences \( \phi \) between two components of the polarized electric field of the EPF with \( \Delta_p = 0 \). (b) Curve diagrams of the input–output field intensity for different values of the SGC effect \( \eta \) with \( \Delta_p = \gamma \) and \( \phi = 0 \). (c) Curve diagrams of the input–output field intensity for different phase differences \( \phi \) with \( \Delta_p = 0 \) and \( \eta = 0 \). (d) Curve diagrams of the input–output field intensity for different phase differences \( \phi \) with \( \Delta_p = \gamma \) and \( \eta = 0.9 \). Other values of the parameters were chosen as \( \gamma_1 = \gamma_2 = \gamma, \gamma_3 = 0.01\gamma, \Delta_c = 0.9\gamma, C = 100\gamma, \) and \( \Omega_c = \gamma \).
parameters are kept fixed. Moreover, when the frequency detuning is changed from $\Delta_p = 0$ to $\Delta_p = \gamma$, and simultaneously the SGC effect is changed from $\eta = 0$ to $\eta = 0.9$, it was found from Fig. 6(d) that the phase difference $\phi$ plays an important role in the steady-state behavior. Compared with the curves ($\phi = 0$) in Figs. 6(a) and 6(b), Fig. 6(d) illustrates that the OB phenomenon can still be observed and the corresponding threshold increases slightly. When the relative phase is changed from 0 to $\pi/3$, the threshold of OB becomes more pronounced. In contrast to Fig. 6(c), it can be seen in Fig. 6(d) that the OM phenomenon appears when the relative phase $\phi$ is tuned to $2\pi/3$. In other words, in presence of the perfect SGC effect, we can easily realize a switch from OB to OM or vice versa just by adjusting the phase difference between the two components of the polarized electric field. These phenomena also demonstrate that the phase difference between the two components of the polarized electric field can effectively control the steady-state response in the present atomic system, which leads to the appearance of OM.

To obtain a better understanding of how the frequency detuning $\Delta_p$ and the phase difference $\phi$ between the two components of the polarized electric field of the EPF affect the steady-state behavior, we drew a density plot of the probe absorption $\text{Im}[\rho_{23}]$ versus the frequency detuning $\Delta_p$ and the phase difference $\phi$ as shown in Fig. 7. The reason for these interesting phenomena that were shown in Fig. 6 can be found from the changes of the absorption properties illustrated in Fig. 7. In the absence of the SGC effect ($\eta = 0$), the probe field can transmit through the medium without absorption at the probing resonance position ($\Delta_p = 0$) when $\phi = 0$. In other words, the input field $E_1^0$ is approximately proportional to the output field $E_3^0$, which results in the case without the appearance of the OB phenomenon. When the perfect SGC effect is included ($\eta = 0.9$), the large absorption of the medium at the probing resonance position results in the output of the OB phenomenon with $\phi = 0$. However, the probe absorption is shown to be negligible when the phase between the two components of the polarized electric field of the EPF $\phi$ is tuned to $2\pi/3$. Therefore, one can find from Fig. 6(a) that the OB phenomenon disappears again. However, if the frequency detuning $\Delta_p$ of the probe field is tuned to $\Delta_p = \gamma$, from Fig. 7, we can see that the absorption of the medium clearly increases, which leads to the appearance of the OB phenomenon as shown in Fig. 6(d). Especially, the corresponding absorption is dramatically enhanced under the condition of $\phi = 2\pi/3$, which causes the parameter $y$ in Eq. (17) to be a cubic polynomial of the variable $x$, and thus the OM phenomenon appears. Based on the absorption spectrum shown in Fig. 4, the results shown in Figs. 6(b) and 6(c) can also be understood clearly.

4. CONCLUSIONS

In conclusion, we have analyzed in detail the steady-state behaviors in a four-level Y-type atomic system driven coherently by a probe laser and a single EPF by means of a unidirectional ring cavity. It was clearly shown that a controllable OB can be achieved by adjusting different parameters in the present atomic system. Under the steady-state condition, the threshold intensity and related hysteresis cycle area for the OB can also be modified by modulating the frequency detuning of the probe field, the intensity of driving fields, and the atomic cooperation parameter, as well as the phase difference between the two components of the polarized electric field. To obtain the two components of the polarized electric field $\Omega_+^c$ and $\Omega_-^c$ of the control field, one can let the control beam pass through 1/2 and 1/4 wave plates in turn properly [32]. After passing through the 1/2 wave plates, the vertical polarized beam can be obtained. The rotating angle $\phi$ can be modulated by the 1/4 wave plate. Thus, one can obtain the EPF containing the two components (i.e., $\Omega_+^c$ and $\Omega_-^c$). More interestingly, we demonstrate the switch from OB to OM with the existence of a perfect SGC effect. With these results, we believe that phase controlled OB and OM will provide another feasible approach for applications in quantum information science.

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REFERENCES


