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Goos–Hänchen shifts of partially coherent light beams from a cavity with a four-level Raman gain medium

Ziauddin [a,b,*], Ray-Kuang Lee [a], Sajid Qamar [b]

[a] Institute of Photonics and Technologies, National Tsing-Hua University, Hsinchu 300, Taiwan
[b] Department of Physics, COMSATS Institute of Information Technology, Islamabad, Pakistan

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ABSTRACT

We theoretically investigate spatial and angular Goos–Hänchen (GH) shifts (both negative and positive) in the reflected light for a partial coherent light incident on a cavity. A four-level Raman gain atomic medium is considered in a cavity. The effects of spatial coherence, beam width, and mode index of partial coherent light fields on spatial and angular GH shifts are studied. Our results reveal that a large magnitude of negative and positive GH shifts in the reflected light is achievable with the introduction of partial coherent light fields. Furthermore, the amplitude of spatial (negative and positive) GH shifts are sharply affected by the partial coherent light beam as compared to angular (negative and positive) GH shifts in the reflected light.

1. Introduction

The spatial Goos–Hänchen (GH) shift means the lateral displacement of a light beam from its expected geometrical optics path. The existence of this shift was first observed experimentally by Goos and Hänchen in 1947 [1] in the phenomenon of total internal reflection from the interface of two different media. Since then, a lot of attention has been emerged to study the spatial GH shift (positive) using different systems [2–5]. In addition to positive GH shifts, several media also give negative GH shifts, such as negative permittivity media [6], negative refractive media [7,8] and absorptive media [9]. Furthermore, due to the fundamental nature of the lateral shift, there are also interesting application to measure various quantities such as beam angle, refractive index, displacement, temperature, and film thickness [10]. The phenomenon of spatial GH shift can also be used for the characterization of the permeability and permittivity of materials [11] and in the development of near-field optical microscopy and lithography [12].

In addition to spatial GH shifts due to the total reflection of light beam, angular GH shift also occurs due to the partial reflection [13,14]. The angular GH shift is a small deviation from the law of reflection i.e., $\theta_{Inc} = \theta_{Ref}$ [14,15] and has been investigated experimentally in optical and microwave regimes [16,17]. Even though, it is common thinking that the spatial and angular GH shifts are two different phenomena and do not depend on each other, the dual nature of angular and spatial GH shifts in the reflected light has been studied [18].

To have a coherent control on the GH shifts in the reflected light (both for spatial and angular shifts), several schemes are proposed by using different atomic media in a cavity [19–21]. In these investigations, a coherent light beam is considered for the investigations on the negative and positive GH shifts in the reflected light without changing the structure of the medium, which shows that the coherence plays an important role to study the GH shift.

Now a debate raises whether the GH shift is influenced by the partial coherent light beam or not. Simon and Tamir were the first to consider a partially coherent light beam incident on a multilayer structure and reported that the spatial GH shift could not be affected by spatial coherence [22]. Similarly, a theory of GH shift has reported for partially coherent light beams, revealing that the spatial coherence has no effect on spatial GH shift [23]. Later, the influence of spatial coherence on spatial GH shift has been studied experimentally, and concluded that the spatial coherence has no effect on the GH shift [24,25]. However, in some other studies

* Corresponding author at: Department of Physics, COMSATS Institute of Information Technology, Islamabad, Pakistan.
E-mail addresses: ziauddin@comsats.edu.pk (Ziauddin), rklee@ee.nthu.edu.tw (R.-K. Lee), sajid_qamar@comsats.edu.pk (S. Qamar).

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[26,27], the spatial GH shift in the reflected light depends on the spatial coherence.

This challenge has been solved thoroughly by Wang and coworkers for the first time [28], then by Ziauddin and coworkers in considering a partial coherent light incident on cavity having three-level Raman gain medium [29]. In our previous article [29], we concentrated the effect of partial coherent light field on negative and positive GH shifts in the reflected light. Even though control of negative and positive GH shifts was demonstrated with the corresponding probe field detuning, it is a difficult task experimentally to realize such a proposal. Therefore, it is constructive to revisit the problem on spatial and angular GH shifts (negative and positive) via external control field for a partial coherent light field. In this work, we consider a partial coherent light field incident on a cavity embedded with a four-level Raman gain atomic medium. We report the control of negative and positive GH shifts in the reflected light via external control field Ω_2. The spatial and angular GH shifts in the reflected light are studied, for different spatial coherence, beam widths, and mode indexes of partial coherent light beams. The current study of the GH shift in the reflected light has an advantage over the previous investigation [19], because giant spatial GH shift in the reflected light is investigated. Besides, we also investigate that the angular GH shifts in the reflected light are less affected via spatial coherence, beam width and mode index of partial light beam as compared to spatial GH shift. In contrary to the coherent counterpart, a completely different scenario for spatial and angular GH shifts in the reflected light happens with partial coherent light fields.

2. Model

We consider a TE-polarized partial coherent light field incident on a cavity, which contains an atomic vapor cell. The incident partial coherent light beam makes an angle θ with z-direction. Inside the cavity, each atom follows a N-type atomic configuration and the atom-field system behaves as a Raman gain process. The cavity consists of three-layers, labeled as 1, 2 and 3, see Fig. 1(a). The layers 1 and 2 are the walls of the cavity with thickness d_1 and permittivity ϵ_1; while the layer 3 is the intracavity medium with thickness d_2 and permittivity ϵ_2. The permittivity ϵ_2 of the intracavity medium is directly related to the susceptibility of the medium via the relation $\epsilon_2 = 1 + \chi$.

We follow the same approach as reported previously [28,29] and use Marcer's mode expansion. The mth-order mode of electric fields in a partial coherent light field at $z=0$ can be written as

$$E_m^i(z, y) = \frac{1}{\sqrt{2\pi}} \int E_m(k_y - k_{y0}) e^{i(k_z z + k_y y)}\, dk_y, \tag{1}$$

where

$$E_m\left(k_y - k_{y0}\right) = \frac{1}{(2c\pi)^{1/4}} \times \frac{(-i)^m}{\sqrt{2^m m!}} \times e^{\frac{(k_y - k_{y0})^2}{4c}} \times H_m\left(\frac{\left(k_y - k_{y0}\right)}{\sqrt{2c}}\right), \tag{2}$$

is the angular spectrum that can be calculated using a Gaussian Schell model (GSM) beam. For the normalized eigen-function in GSM light beams, one has [30]

$$E_m(y) = (2c/\pi)^{1/4} \times \frac{1}{\sqrt{2^m m!}} H_m[y\sqrt{2c}]e^{-cy^2}. \tag{3}$$

One can calculate the corresponding angular spectrum $E_m(k_y)$ by taking Fourier transform of Eq. (3). Here, $E_m(k_y)$ is replaced by $E_m(k_y - k_{y0})$, and k_y is the y-component of the wave vector k, $k_{y0} = k \sin\theta$, and θ is the incident angle. In Eqs. (2)–(3), H_m is the Hermite polynomials and $c = [a^2 + 2ab]^{1/2}$, which can be calculated using the eigenvalues $\beta_m = A^2[\pi/(a + b + c)]^{1/2}[b/(a + b + c)]^m$ of GSM beams with $a = (4w_s^2)^{-1}$, $b = (2w_g^2)^{-1}$. Furthermore, w_g and w_s are the spectral coherence and beam width of partial coherent light, respectively. We have the expression for $c = (q^2 + 4)^{1/2}/(4qw_s^2 \sec^2\theta)$ with $w_s \to w_s \sec\theta$ and $w_g \to w_g \sec\theta$. To describe a partially coherent light field, the parameter $q = w_g/w_s$ measures the degree of the coherence in a GSM beam, which is also denoted as the spatial coherence.

For the reflected partial coherent light in mth-order mode, we have [28,29]

$$E_m^r(y) = \frac{1}{\sqrt{2\pi}} \int r\left(k_y\right) E_m\left(k_y - k_{y0}\right) e^{ik_y y}\, dk_y, \tag{4}$$

where $r(k_y)$ is the reflection coefficient of the proposed cavity, which can be calculated using the characteristic matrix [20] as

$$m_j\left(k_y, \omega_p, d_j\right) = \begin{pmatrix} \cos(k_j^z d_j) & i \sin(k_j^z d_j)/q_j \\ iq_j \sin(k_j^z d_j) & \cos(k_j^z d_j) \end{pmatrix},$$

where $k_j^z = k\sqrt{\epsilon_j - \sin^2\theta}$ is the wave number of three-layer structure, $k = \omega/c$ in vacuum, c is the speed of light, $q_j = k_j^z/k$, d_j is the thickness of jth layer. Our system consists of three layers, therefore, the total transfer matrix for the system can be written as,

$$Q(k_y, \omega_p) = m_1(k_y, \omega_p, d_1) m_2(k_y, \omega_p, d_2) m_1(k_y, \omega_p, d_1),$$

and finally the reflection coefficient can therefore be calculated as,

$$r(k_y, \omega_p) = \frac{q_0(Q_{22} - Q_{11}) - (q_0^2 Q_{12} - Q_{21})}{q_0(Q_{22} + Q_{11}) - (q_0^2 Q_{12} + Q_{21})}, \tag{5}$$

where Q_{ij} be the elements of total transfer matrix $Q(k_y, \omega_p)$ and $q_0 = \sqrt{\epsilon_0 - \sin^2\theta}$ and $r(k_y, \omega_p) = |r(k_y, \omega_p)|e^{i\phi_r}$, where ϕ_r is phase

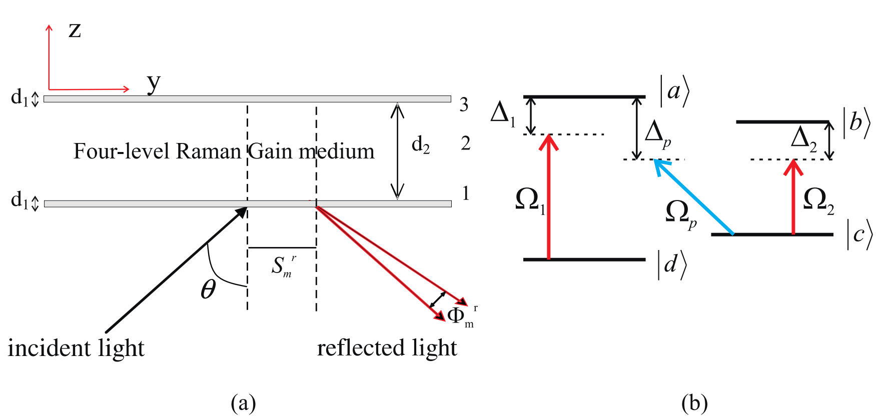

Fig. 1. (a) Schematics of the partial coherent light field incident on a cavity; (b) the energy-level configuration of a four-level Raman gain medium.

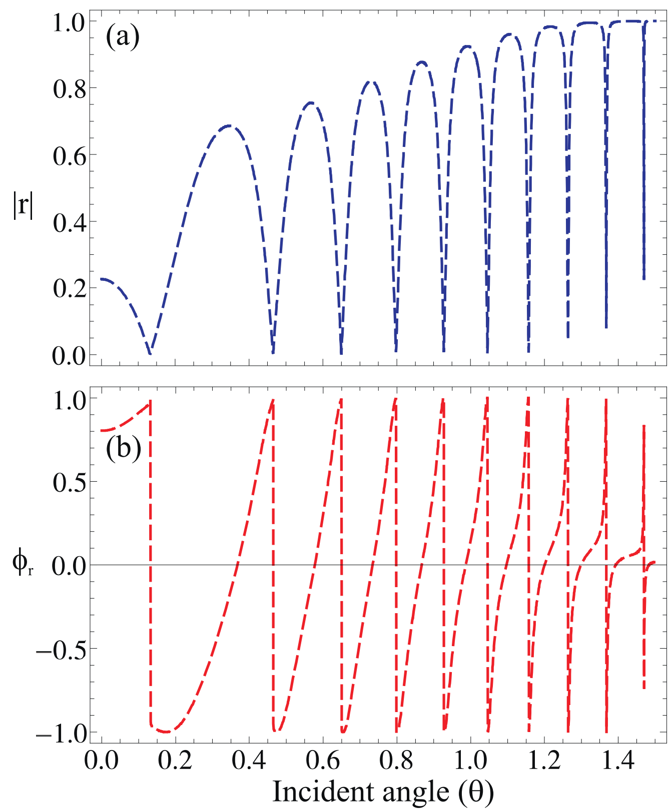

Fig. 2. (a) Reflection coefficient $|r|$ versus incident angle θ (b) Phase of the reflected light versus incident angle θ. The other parameters are $\Delta_1 = 50\gamma$, $\Delta_2 = 0$, $\Omega_1 = 4\gamma$, $\gamma_{da} = \gamma_{ca} = \gamma_{cb} = \gamma_{db} = 2\gamma$, $\Gamma_{dc} = 0.01\gamma$, $\Gamma_{bc} = \Gamma_{bd} = \Gamma_{ac} = \Gamma_{ad} = 2.01\gamma$, $\Gamma_{ab} = 4.01\gamma$, $d_1 = 0.2$ μm, $d_2 = 5$ μm, $\epsilon_1 = 2.22$, and $\Omega_2 = 4.9\gamma$.

of the reflected light.

The spatial GH shift for the mth-order mode (S_m^r) of the reflected field can be calculated using the normalized first moment of the reflected field, where the center of the incident field at $y = 0$, and then the shift can be written as [28,29]

$$S_m^r = \frac{\int y\left|E_m^r(y)\right|^2 dy}{\int\left|E_m^r(y)\right|^2 dy}. \tag{6}$$

The angular GH shift in the reflected light is proportional to the angular derivative of the amplitude reflectivity i.e., $|r|$ and can be expressed [31] as

$$\phi_r = \frac{1}{|r|}\frac{d|r|}{dk_y}. \tag{7}$$

The expression for the angular GH shift in the reflected light field for the mth-order mode can therefore be written [19] as

$$\Phi_m^r = \frac{\int k_y \phi_r \left|E_m\left(k_y - k_{y0}\right)\right|^2 dk_y}{\int\left|E_m\left(k_y - k_{y0}\right)\right|^2 dk_y}. \tag{8}$$

2.1. Atom-field interaction

We consider an atomic medium in a cavity where each atom follows four-level N-type atom-field configuration [32] as depicted in Fig. 1(b). The energy-levels of the proposed atomic system are $|a\rangle$, $|b\rangle$, $|c\rangle$ and $|d\rangle$. The energy-level $|d\rangle$ is coupled to the level $|a\rangle$ via a pump field say E_1 and $|c\rangle$ is coupled to $|b\rangle$ via a control field E_2. A partial coherent light field acting as probe pulse is incident from vacuum on a cavity and couples with the atomic transition $|c\rangle$ to $|a\rangle$. Under the dipole and rotating wave approximations the interaction picture Hamiltonian for the atomic system can be written as

$$V = -\frac{\hbar}{2}\left[\Omega_1 e^{-i\Delta_1 t}|a\rangle\langle b| + \Omega_2 e^{-i\Delta_2 t}|b\rangle\langle c| + \Omega_p e^{-i\Delta_p t}|a\rangle\langle c| + H.c.\right], \tag{9}$$

where Ω_1, Ω_2 and Ω_p are the pump, control and probe fields Rabi frequencies whereas Δ_1, Δ_2 and Δ_p are the detunings of pump, control and probe fields, respectively.

The nonlinear optical susceptibility $\chi = \beta\Omega_1^2 D$ of the proposed atomic system is calculated in a similar way as expressed in [32], where $\beta=(3N\lambda^3)/(32\pi^3)$ with N and λ being the atomic number density and wavelength of light whereas D can be calculated as

$$D = \frac{-i}{A}\left[\frac{2\Gamma_{ad}[\Gamma_{ab} - i(\Delta_p - \Delta_2)]}{(\gamma_{da} + \gamma_{ca})(\Gamma_{ad}^2 + \Delta_1^2)} + \frac{[\Gamma_{ab} - i(\Delta_p - \Delta_2)][\Gamma_{bd} - i(\Delta_p - \Delta_1 - \Delta_2)] - |\Omega_2|^2/4}{(\Gamma_{ad} + i\Delta_1)\left[\,[\Gamma_{dc} - i(\Delta_p - \Delta_1)][\Gamma_{bd} - i(\Delta_p - \Delta_1 - \Delta_2)] + |\Omega_2|^2/4\right]}\right], \tag{10}$$

with $A = (\Gamma_{ac} - i\Delta_p)[\Gamma_{ab} - i(\Delta_p - \Delta_2)] + |\Omega_2|^2/4$.

3. Results and discussion

Earlier, the spatial and angular GH shifts in the reflected light have been reported using a Gaussian beam incident on a cavity containing four-level Raman gain atomic medium [19]. The coherent control and influence of beam width of Gaussian beam are studied for both spatial and angular GH shifts in the reflected light. In the following, we proceed the problem to study the spatial and angular GH shifts in the reflected light [19,28,29] for partial coherent light beam incident on a cavity containing four-level Raman gain medium. We investigate the influence of spatial coherence, beam width and mode index of partial coherent light fields on both spatial and angular GH shifts in the reflected light. Initially, we study the reflection coefficient and phase of the reflected light. We plot the reflection coefficient $|r|$ and phase ϕ_r of the reflected light versus incident angle θ as depicted in Fig. 2. The dips in the reflection curve at different incident angles correspond to the resonance condition, see Fig. 2(a). Similarly, to know about the phase of the reflected light we plot ϕ_r versus incident angle as shown in Fig. 2(b). The plot shows that the phase change at those incident angles where dips occur in the reflected light. For different choices of spatial coherence q and beam width we can consider any incident angle θ where there is a dip and phase change of the reflected light. In the following, we consider incident angle $\theta = 0.12$ rad where there is a dip and investigate the spatial and angular GH shifts in the reflected light for partial coherent light fields.

3.1. Influence of spatial coherence on spatial and angular GH shifts

As reported earlier that the spatial coherence q influenced both positive and negative GH shifts in the reflected light [28,29] and investigated that the magnitude of the GH shifts is high for small values of q. It is also studied that the magnitude of GH shift decreases with the increment of q. The physics behind this is, the light becomes more coherent when the values of q increase and for small values of q a large number of modes are needed to represent the field i.e., the light becomes incoherent. In the following, we study the effect of spatial coherence q on the negative and positive spatial GH shifts in the reflected light. For an incident angle $\theta = 0.12$ rad, we investigate the behavior of spatial GH shift in the reflected light using Eq. (6) for a partial coherent light beam.

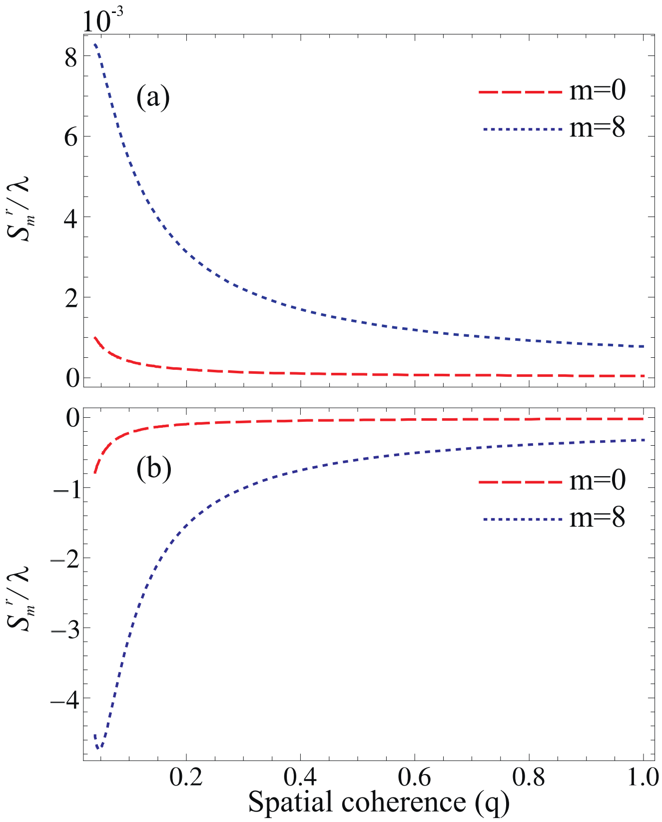

Fig. 3. (a) Spatial GH shift versus coherence q for normal dispersion ($\Omega_2 = 2.8\gamma$). (b) For anomalous dispersion ($\Omega_2 = 4.9\gamma$) by considering $w_s = 100\lambda$ and $\theta = 0.12$ rad. The other parameters remain the same as those in Fig. 2.

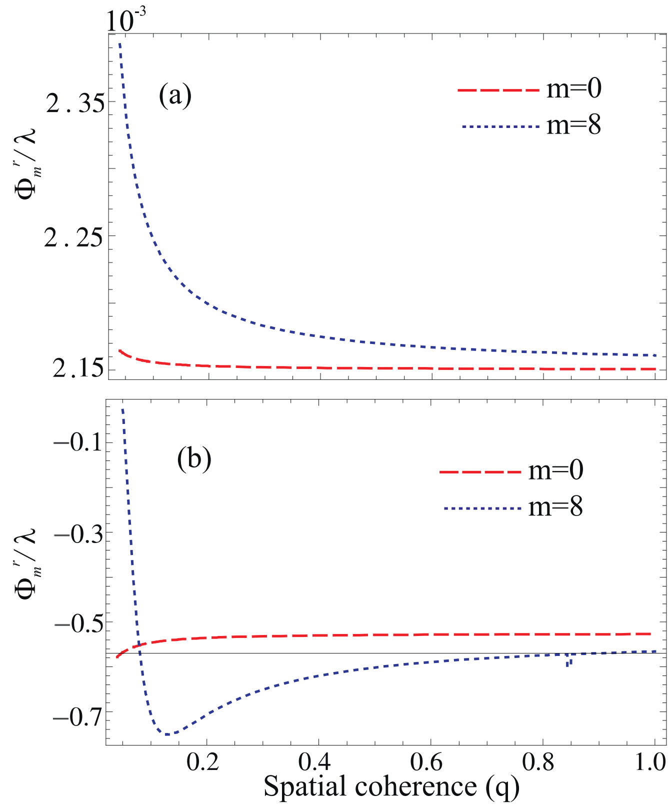

Fig. 4. (a) Angular GH shift versus spatial coherence q for normal dispersion. (b) For anomalous dispersion. The other parameters remain the same as those in Fig. 3.

For the normal dispersion i.e., by considering $\Omega_2 = 2\gamma$ of the intracavity medium, we plot the spatial GH shift in the reflected light versus coherence q for different modes ($m=0$ and $m=8$) of partial coherent light field, see Fig. 3(a). The plot shows that the magnitude of positive GH shifts in the reflected light is high for small values of q, and decreases exponentially when q increases. With further increase in the spatial coherence q the magnitude of the positive GH shift remains constant. For $m=0$, the magnitude of the GH shift increases 9 times for incoherent light ($q=0.09$) as compared to the magnitude at $q=1$.

Next, we change the control field from $\Omega_2 = 2.8\gamma$ to $\Omega_2 = 4.9\gamma$ of the intracavity medium and all the other parameters remain unchanged, anomalous dispersion appears for the intracavity medium. We plot again the spatial GH shifts in the reflected light versus spatial coherence q for two different modes *i.e.*, for $m=0$th- and $m=8$th-order modes. For anomalous dispersion of the intracavity medium we investigate negative GH shifts in the reflected light as depicted in Fig. 2(b). The behavior of the negative GH shifts in the reflected light remains the same as we reported in Fig. 3(b). We also investigate that for $m=0$, the magnitude of negative GH shift increases 13.3 times for incoherent light ($q=0.09$) as compared to the magnitude at $q=1$.

Further, we consider the positive and negative angular GH shifts in the reflected light and study the influence of spatial coherence q on it. We use Eq. (8) for different order modes and report the positive and negative angular GH shifts in the reflected light for normal and anomalous dispersion of the intracavity medium, respectively. For incident angle $\theta = 0.12$ rad, we plot the angular GH shifts in the reflected light for normal dispersion of the intracavity medium versus spatial coherence q for two different modes, see Fig. 4(a). We observe positive angular GH shift for the normal dispersion of the intracavity medium which has large amplitude at certain values of q and decreases to a constant amplitude with the increment of q. Then we change the normal dispersion to anomalous by changing the control field Ω_2 and plot the angular GH shift in the reflected light versus q for two modes. We achieve negative angular GH shift in the reflected light which increases to its maximum value first and then decreases to a constant value for further increasing the spatial coherence q. By comparing Figs. 3 and 4, it is observed that the behavior of spatial (positive and negative) GH shifts varies as same as that of angular (positive and negative) GH shifts in the reflected light.

3.2. Influence of beam width on spatial and angular GH shifts

It is reported earlier that the beam width plays an important role in the observation of GH shifts with a Gaussian beam [19], as well as a partial coherent light field [28,29]. For certain small values of beam width of Gaussian or partial coherent light fields, the amplitude of the GH shifts in the reflected light becomes maximum and decreases when the beam width increases [28,29]. Following this idea, here, we study the influence of beam width w_s of partial coherent light on the spatial and angular GH shifts in the reflected light. By considering the spatial coherence $q=0.09$ of the partial coherent light field while all the other parameters remain the same as those in Fig. 3. We plot the spatial GH shifts in the reflected light versus beam width w_s of partial coherent light field for normal and anomalous dispersion of the intracavity medium for $m=0$- and $m=8$th-order modes. For certain small values of beam width the positive and negative GH shifts become maximum and decrease for further increase in the beam width of partial coherent light beam, see Fig. 5. The effect of spatial coherence q suppresses by increasing the beam width of partial coherent light field. Next, we study the influence of beam width of partial

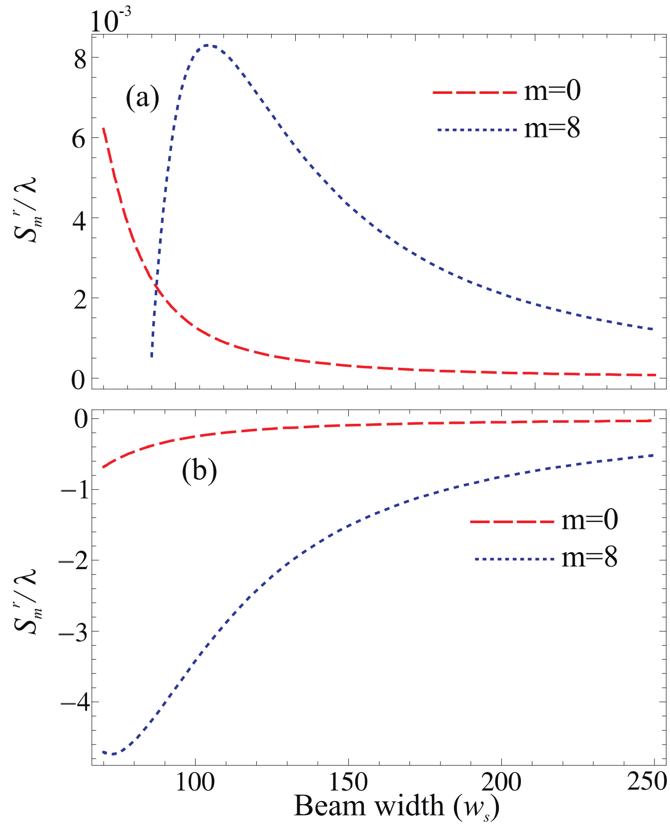

Fig. 5. (a) Spatial GH shift versus beam width w_s for normal dispersion when $q=0.09$. (b) For anomalous dispersion when $q=0.09$. The other parameters remain the same as those in Fig. 3.

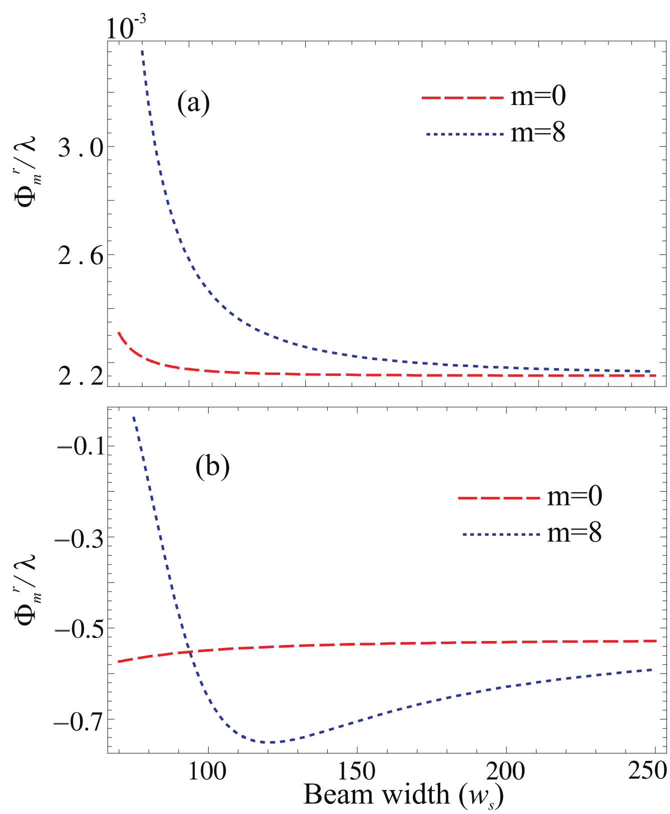

Fig. 6. (a) Angular GH shift versus beam width w_s for normal dispersion when $q=0.09$. (b) For anomalous dispersion when $q=0.09$. The other parameters remain the same as those in Fig. 3.

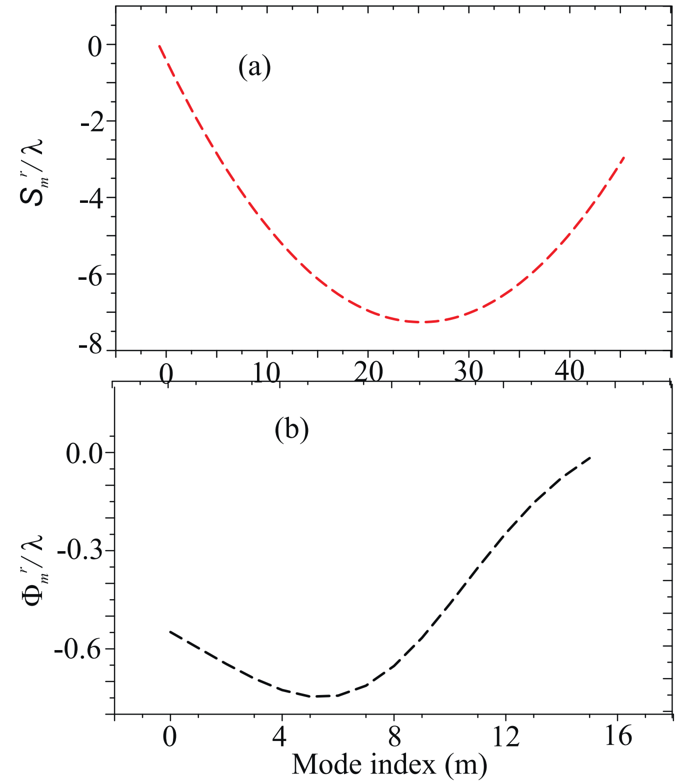

Fig. 7. (a) Spatial GH shift versus mode index m for anomalous dispersion. (b) Angular GH shift versus mode index m for anomalous dispersion by considering $q=0.09$ and $w_s = 100\lambda$. The other parameters remain the same as those in Fig. 3.

coherent light field on positive and negative angular GH shifts in the reflected light. We consider the normal and anomalous dispersion of the intracavity medium and plot the positive and negative angular GH shifts in the reflected light versus beam width w_s of partial coherent light field, respectively. In comparison of Figs. 5 and 6, it is found that the spatial coherence q and beam width w_s of partial coherent light field have the same effect on the positive and negative angular GH shifts in the reflected light.

We also study the influence of different modes on the negative spatial and angular shifts in the reflected light. We consider the anomalous dispersion of the intracavity medium and plot the spatial and angular GH shifts in the reflected light versus mode index m for an incident angle $\theta = 0.12$ rad, beam width $w_s = 100\lambda$ and spatial coherence $q=0.09$ of the partial coherent light fields. We investigate both negative spatial and angular GH shifts in the reflected light, see Fig. 7. The amplitude of spatial (negative) GH shift increases to its maximum value around $m=26$ and then decreases for further increasing in the mode index m of partial coherent light beam, see Fig. 7(a). Similarly for anomalous dispersion of the intracavity medium, we plot the angular GH shift in the reflected light versus mode index m of partial coherent light field. For different choices of mode index m we investigate negative angular GH shifts, which increases first to its maximum value around $m=6$ and then decreases for further increasing the mode index m as depicted in Fig. 7(b). From Fig. 7, it is clear that both spatial and angular GH shifts in the reflected light have the same behavior versus mode index m of partial coherent light field, i.e., the amplitude of the shifts increases to its maximum values and then tends to zero for large mode index m. This is due to the fact that the spatial and angular GH shifts i.e., $S_m^r = 0$ and $\Phi_m^r = 0$, respectively for any value of q when m is very large and since the effective width of partially coherent light tends to infinity [28].

4. Conclusion

In conclusion, by considering a four-level Raman gain medium inside the cavity, we study the spatial and angular GH shifts in the reflected light for a partial coherent light beam. In such a four-level Raman gain medium, the normal and anomalous dispersions are achieved via a single knob i.e., the control field Ω_2. The normal and anomalous dispersions of the intracavity medium then lead to positive and negative (spatial and angular) GH shifts, respectively. The present work has an advantage over our previous investigation [29], by considering the effect of partial coherent light field on angular GH shift. It is noted that the partial coherent light beam sharply influences the spatial GH shift as compared to the angular one. We investigated the effects of spatial coherence q, beam width w_s, and mode index m of partial coherent light on the spatial and angular GH shifts. For spatial and angular GH shifts, we find that the amplitudes of both positive and negative phase shifts in the reflected light increase first for a small degree of incoherence and a small beam width. However, the GH shifts decrease as the spatial incoherence and beam width of partial coherent light increase. It is also investigated that the magnitude of spatial (negative and positive) GH shifts increased more as compared to angular (negative and positive) GH shifts in the reflected light using partial coherent light field. Our results provide an interesting result for the applications in controlling and measuring GH shifts with partially coherent light fields.

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